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ESTIMATES OF COOLING AND BENDING PROCESSES FOR CHARGED PARTICLE PENETRATION THROUGH A MONOCRYSTAL

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In our previous paper[1] we presented the results of quantitative estimates of cooling and bending processes for charged particle penetrations through a monocrystal. We have been asked the details of these estimates, and we present them below.

A. Cooling a beam by a crystal

Let us consider a two-dimensional motion of a high energy particle in a plane of two rows of atoms in a crystal lattice (see Fig. 1). Let us designate:

ω - frequency of oscillation of a particle; $\omega = 1/T$, where T is a period of oscillation.

M - mass of a lattice atom.

d - distance between consecutive atoms in the lattice.

n - current number of a lattice atom along the Z-axis.

N - number of atoms along the Z-axis corresponding to one period of oscillation.

P_o - momentum of incoming particle.

E_o - energy of incoming particle.

v - velocity of incoming particle.

ψ - incoming angle.

$\Delta P_{\lambda/2}$ - momentum transfer per one half of an oscillation; the assumption is that momentum is tranferred to N/2 atoms during one half of the period of oscillation. We suppose also that the energy loss during one period of oscillation is much less than the initial transverse energy.

P_n - momentum transferred to n-th atom.

$\Delta E_{\lambda/2}$ - energy transferred to atoms during one half of the period of oscillation.

So,

$$\Delta P_{\lambda/2} = \sum_{n=1}^{N/2} P_n$$

Suppose that the motion of a charged particle in the transverse direction can be described classically, but with the mass increased by a relativistic factor γ[2]. Suppose also that the continuum planar approximation is valid, and the potential has a parabolic shape[3] (see Fig. 2). Then:

$$\Delta P_{\lambda/2} = \sum_{n=1}^{N/2} A \cdot \sin\left(\frac{2\pi\omega}{v} \cdot nd\right) ,$$

where A is an arbitrary constant, and

$$\Delta E_{\lambda/2} = \frac{A^2}{2M} \sum_{n=1}^{N/2} \sin^2\left(\frac{2\pi\omega}{v} \cdot nd\right) .$$

Let us go to an integral form:

$$\Delta P_{\lambda/2} = \frac{A}{d} \int_0^{v/2\omega} \sin\left(\frac{2\pi\omega}{v} \cdot z\right) dz ,$$

$$\Delta E_{\lambda/2} = \frac{A^2}{2Md} \int_0^{v/2\omega} \sin^2\left(\frac{2\pi\omega}{v} \cdot z\right) dz .$$

Then

$$\Delta P_{\lambda/2} = A \cdot \frac{v}{\pi\omega d} = 2P_0\psi ,$$

and

$$A = \frac{2\pi\omega P_0 \psi d}{v} .$$

Then

$$\Delta E_{\lambda/2} = \frac{\pi^2 \omega P_0^2 \psi^2 d}{2Mv} .$$

The number of periods per length ΔL along the Z-axis is $(\omega \cdot \Delta L)/v$, and the energy loss per length ΔL is equal to:

$$\Delta E = \frac{\pi^2 \omega^2 P_0^2 \psi^2 d}{Mv^2} \cdot \Delta L .$$

If we designate the energy associated with transverse motion as[1]

$$E_{tr} = E_0 \psi^2 ,$$

then

$$\frac{\Delta E}{\Delta L} = E_{tr} \cdot \frac{\pi^2 \omega^2 \gamma d}{(M/m_0) c^2} .$$ (Here m_0 is the rest mass of the particle)

This corresponds to an exponential loss of transverse energy:

$$E_{tr}(L) = E_0 \psi^2 \cdot e^{-\frac{L}{\Lambda}} ,$$

where

$$\Lambda = \frac{(M/m_0) c^2}{\pi^2 \omega^2 \gamma d} .$$

The frequency of oscillation is given by

$$\omega = (k/m_0\gamma)^{1/2}$$

where k is a constant defined by the potential. Let us remark that the cooling length Λ is independent of incoming particle energy. In the case of proton penetration in silicon, from Fig. 2,

$$E \simeq 4.4\cdot 10^{21}\cdot x \text{ volts/meter,}$$

where x is the distance from the median plane, and

$$k \simeq 4.4\cdot 10^{21}\cdot 1.6\cdot 10^{-19} \text{ volt}\cdot\text{coulomb/meter}^2.$$

So, for 1000 GeV protons

$$\omega \simeq \left(\frac{4.4\cdot 10^{21}\cdot 1.6\cdot 10^{-19}}{1.07\cdot 10^{3}\cdot 1.67\cdot 10^{-27}}\right)^{1/2} = 1.98\cdot 10^{13} \text{ sec}^{-1}.$$

Then,

$$\Lambda \simeq \frac{28\cdot(3\cdot 10^{8})^{2}}{3.14^{2}\cdot(1.98\cdot 10^{13})^{2}\cdot 1.07\cdot 10^{3}\cdot 5.43\cdot 10^{-10}} =$$

$$= 1.1\cdot 10^{-3} \text{ meters.}$$

B. Bending a beam by a crystal

In the case of a bent crystal, the equilibrium line around which the particles oscillate does not pass through the median plane. Instead of this, it is shifted in the region where the electrical field is strong enough to produce a necessary centripetal acceleration to the particle. So, the necessary electrical intensity should be equal to

$$E = \frac{m_0 \gamma v^2}{eR}$$

where R is the bending radius of the crystal, and e is the electron charge.

For the planar channeling of a single charged particle, the continuum approximation gives the following potential[2]:

$$V_{(\rho)} = 2\pi n Z e^2 a \cdot f(\rho/a) \ .$$

Here ρ is the distance from a plane of atoms, n is the density of atoms per unit area, Z is the atomic number of atoms of the crystal lattice, a is the Thomas-Fermi screening radius, and $f(\rho/a)$ is a function defined by the screening function of the atom. The corresponding electrical intensity is

$$E(\rho) = 2\pi n Z e^2 a \cdot \frac{d}{d\rho}\{f(\rho/a)\} \ .$$

For Lindhard's[2] screening function, this becomes

$$E(\rho) = 2\pi n Z e^2 \{\frac{\rho/a}{((\rho/a)^2 + C^2)^{\frac{1}{2}}} - 1\} \ .$$

Here $C^2 \simeq 3$. At $\rho = a$ (this is the critical distance for the barrier penetration)

$$E(\rho=a) = -\ 2\pi n Z e^2 \cdot 0.5\ .$$

For the tungsten $Z = 74$, $d = 3.16 \cdot 10^{-10}$ meters, and

$$E(\rho=a) = 0.34 \cdot 10^{13}\ \text{volt/meter}\ .$$

So, the bending radius for $P_0 = 100$ GeV/c

$$R \simeq \frac{107 \cdot 1.7 \cdot 10^{-27} \cdot 9 \cdot 10^{16}}{1.6 \cdot 10^{-19} \cdot 0.34 \cdot 10^{13}} = 3.0 \cdot 10^{-2}\ \text{meters.}$$

References

1. E. Tsyganov, Fermilab TM-682, Submitted to Physical Review Letters.
2. P. Lervig, J. Lindhard, and V. Nielsen, Nuclear Physics, A96(1967)481.
3. D. S. Gemmel, Reviews of Modern Physics, 46(1974)129.

Figure Captions

1. Diagram of the motion of a charged particle through a crystal.
2. Continuum potential energy for protons channeled in the (110) planes of Si. Figure taken from reference 3. Different solid lines correspond to different temperatures of the crystal. Dashed line represents the parabolic approximation.

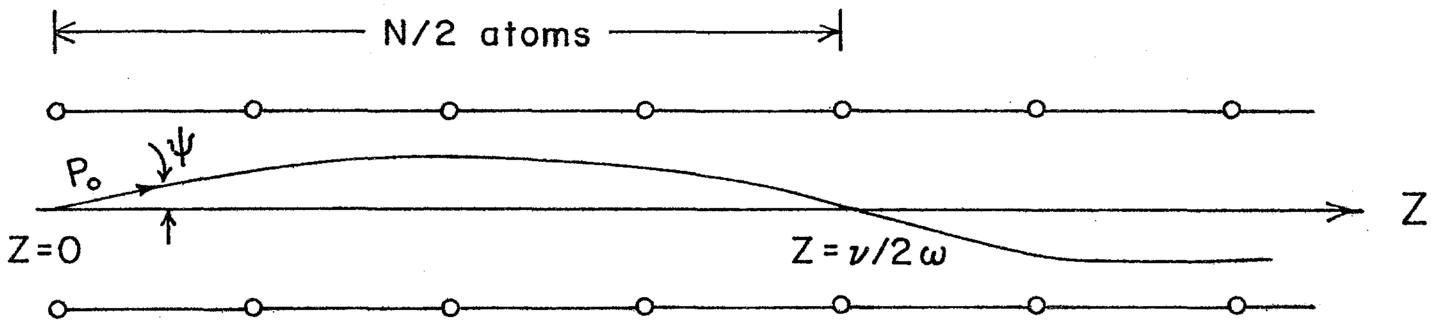

Figure 1.

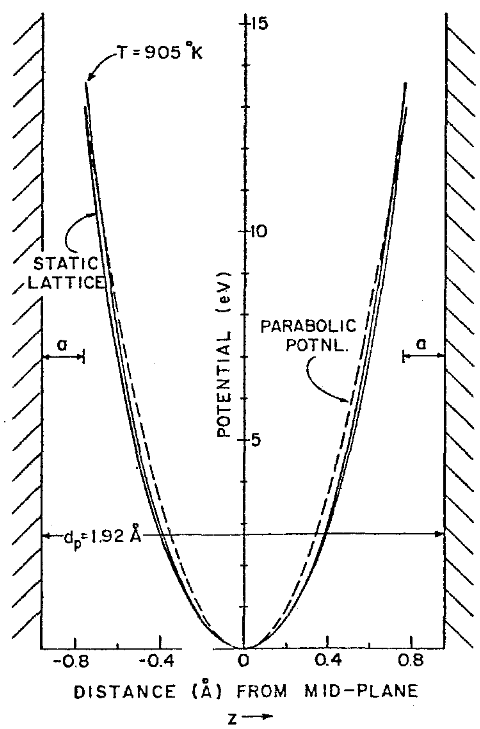

Figure 2.