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**SOME ASPECTS OF THE MECHANISM OF A  
CHARGE PARTICLE PENETRATION THROUGH A MONOCRYSTAL**

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Penetration of charge particles through a monocrystal parallel to its axis is sharply different from one through a matter with nonregular positioning of atoms. If a motion of a positive charged particle is close to the direction of the crystal axis, a multiple Coulomb scattering of the particle vanishes, nuclear interaction is sufficiently suppressed, and ionization losses are lowered to a large degree. This phenomenon, called channeling, was best described theoretically by Lindhard.<sup>1</sup> For the axial case, the critical angle of a capture of a particle with a charge +1 can be written as follows:

$$\psi_c \approx 2 (Ze^2 / pvd)^{1/2},$$

where  $Z$  is the atomic number of the lattice atoms,  $p$  and  $v$  are the momentum and velocity of the particle, correspondingly, and  $d$  is the distance between consecutive atoms along the crystal axis.

The channeling is usually treated in the approximation where processes of an energy transmitting to lattice atoms are neglected. Taking these interactions into account shows that the excitation of the vibrations of the lattice atoms will cause the effect of the self-stabilization of a positively charged particle trajectory. During this process, the amplitude of the initial oscillations of the particle will degrade, and the particle trajectory will approach the line of the lower potential in the channel region. The effect results in cooling of the beam.

In semiclassical consideration, which can be applied in this case, we have decreasing of energy  $\epsilon$ , associated with transverse motion of the

particle, of the form

$$\epsilon(1) = \epsilon_0 e^{-1/\lambda}, \quad (1)$$

where

$$\lambda = \frac{(M/m_0) c^2}{\pi^2 \omega^2 \gamma d}. \quad (2)$$

Here  $\omega$  is a frequency of oscillation of the particle captured in the channeling process,  $M$  is a mass of lattice atom,  $m_0$  is the rest mass of a moving particle,  $\gamma = E_0/m_0$  is a relativistic factor,  $1$  is a length of a crystal. Expression (2) is written for the case of planar channeling. The averaged continuum potential in this case has a shape very close to a parabolic one, and the frequency of oscillation can be given by

$$\omega = (kZe/\gamma m_0)^{1/2},$$

where  $Ze$  is a charge of a particle, and  $k$  is a parameter defined by the potential of the lattice plane. In the case of silicon the frequency of oscillation of a 1000 GeV particle is  $\omega \approx 2 \times 10^{13} \text{ sec}^{-1}$ . For this case  $\lambda \approx 0.1$  cm, i. e. cooling process is fast enough. Parameter  $\lambda$  is not dependent on the incident energy of the particle.

As a result of this process a spatially homogeneous beam with a divergence of  $\psi_c$  converts itself to separate parallel jets. A size of a single jet will be defined mostly by the residual thermal vibrations of the lattice atoms. The amplitude of these thermal vibrations can be reduced by the cooling of the crystal up to  $10^{-2} d$ . An averaging of these perturbations by the path of the particle will reduce the rest of the deviations of the

particle to a large degree. Remember that, for example, 1000 GeV particles travel a distance of up to  $10^5 d$  during one oscillation. For more detailed estimations the real frequency spectrum of the oscillations should be taken into account. It is obvious that this averaging will reduce the size of a single jet and its angular divergence by a factor of not less than 10. So, the active area of the beam will be reduced by a factor of more than  $10^6$ , and the divergence of the beam will be reduced more than  $10^3$  times. For 1000 GeV particles, for example, the angular divergence of the cooled beam should be less than  $10^{-8}$  radians.

The second effect, which can be predicted, is connected with a relatively stable positioning of a trajectory of a positively charged particle in a lattice along the line of a minimum potential. What will happen with the trajectory of the channeled particles if we bend the crystal? Up to some critical value of the bending radius a particle trajectory will repeat the shape of a bent crystal. This unexpected phenomenon caused by the fact that the particle in this case gradually goes to high electric field of the atoms. It then starts to be bent in the direction of the crystal bending. A stable trajectory in this case goes away from the potential minimum, in the region where electrical fields are strong enough to create the necessary transverse acceleration. The critical radius of bending is given by the following expression:

$$R_c = \frac{pv}{e E_c} \quad (3)$$

Here  $E_c$  is the electrical intensity at the boundary of the region of stable trajectories. In the case of the planar channeling, this boundary is located at the distance of "a" from centers of atoms, where "a" is the Thomas-Fermi screening radius. In the planar continuum approximation the averaged potential of the atom plane is given by

$$V(\rho) = 2\pi n Z_1 Z_2 e^2 a \cdot f(\rho/a) ,$$

where  $\rho$  is the distance from the plane of atoms, and  $n$  is the density of atoms per unit area. For tungsten the Moliere approximation for  $f(\rho/a)$  gives an electrical field of  $0.7 \times 10^{13}$  volts/meter. The corresponding critical radius for 100 GeV particles is 1.5 cm. We can see that the bending ability of crystals is greater than one for the usual magnets by a factor of  $10^4$ .

It is interesting to point out that at this magnitude of a curvature the 400 GeV pion can already be detected by its synchrotron radiation.

So, crystals could be used for bending, focusing and cooling of high energy beams, at least for positively charged particles.

Theoretical treatment of the channeling of negatively charged particles is not good enough. We believe trajectories of axial channeled particles to be stable and of spiral shape around rows of atoms. It is natural to expect that the radius of such spirals at  $p_{\perp} \approx p_0 \psi_c$  should be of the order of the size of an atom. Calculations confirm this. The cooling process, as mentioned above for positively charged particles, will decrease the radius of

this spiral up to some value where consecutive collidings of a particle with atoms in a row will start to increase the transverse momentum of this particle. For example, thermal irregularity of atom positionings might cause this. These cooling and warming processes will compensate for each other at some equilibrium radius. If such a mechanism takes place, the use of a bent crystal will bend trajectories of particles, including negatively charged particles.

We appreciate many friends for their useful discussions.

Reference

1. J. Lindhard, Phys. Lett. 12, 126 (1964).