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BOOSTER R.F. CAVITY MODEL CALCULATIONS

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A model of the booster R. F. Cavity has been developed, considering it to consist of a series of discrete, lumped elements and step-wise continuous coaxial waveguide transmission lines. This model enables one to calculate many quantities of interest for the cavities - hub impedance, voltage step-up ratios, hub current, voltage and current distributions, stored electrical and magnetic energy, and so forth, as functions of frequency and gap capacitance. The model calculations agree well with measured quantities. This model may be used to predict performance for cavities of various dimensions and at different frequencies.

1. Model

A booster R. F. Cavity consists of an inner cylinder or drift tube of longitudinally varying radius connected to a uniform outer cylinder by means of ceramic vacuum seals and coupled to the beam tube by large capacitance gaps. The electrical configuration of a booster cavity, exclusive of tuners and power amplifiers is that of a non-uniform, coaxial waveguide containing lumped capacitors. The properties of such a cavity may be calculated by considering the cavity to be composed of discrete, uniform straight sections and describing each section in terms of a lumped or distributed waveguide transmission matrix. The transmission matrix for the entire cavity is then simply the product of the individual terms, and the electrical characteristics at any point may be found from the

propagation of an initial condition to the point in question.

1.1 Transmission matrix description of waveguides.

A waveguide is a two-port junction having a linear relation between the input voltage and current, V_1, I_1, and the output voltage and current V_2, I_2, which may be expressed as

$$V_2 = AV_1 + BI_1$$
$$I_2 = CV_1 + DI_1$$

or, in matrix form[1]

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} .$$

The ABCD matrix for a straight section of transmission line is given by

$$\begin{bmatrix} \cos(\beta\ell) & iZ_o \sin(\beta\ell) \\ i \sin(\beta\ell)/Z_o & \cos(\beta\ell) \end{bmatrix} \qquad (1)$$

where ℓ = length of transmission line

$\beta = 2\pi f/c = \omega/c$

$\omega = 2\pi f$ = angular frequency

c = speed of light

Z_o = characteristic impedance of the line.

For an air-filled coaxial line of inner radius a and outer radius b, the characteristic impedance is[2]

$$Z_o = 60 \ln (b/a) \text{ ohms.}$$

The transmission matrix for a lumped capacitor is simply

$$\begin{bmatrix} 1 & 0 \\ i\omega c & 1 \end{bmatrix}$$

where c = capacitance.

Finally, a transmission line of non-uniform cross-section may be simply divided into many short sections each approximated by a constant cross-section and each calculated by the matrix in Eq. 1.[3]

1.2 Booster cavity

One half of a booster cavity, from the gap in toward the hub, is parameterized with the following sequence:

(1) an effective lumped capacitor for the gap,

(2) a length of straight, coaxial transmission line for the distance from the gap to the ceramic seal,

(3) an effective lumped capacitance for the ceramic seal

(4) a series of very short, step-wise continuous, coaxial transmission lines representing the tapered drift tube, and

(5) a length of straight, coaxial line going into the hub.

The above five sets of quantities are obtained as follows:

(1) a variable input parameter. This may be either a measured capacitance, or conversely, used to determine the actual capacitance of the gap;

(2) a transmission matrix using the measured, or desired, radius;

(3) the capacitance for a thick, conical ring with a certain dielectric constant can be calculated. This is then considered to be an effective, lumped capacitor at the position of the ceramic seal along the drift tube;

(4) the radius of the tapered drift tube can be given by

$$R(x) = 1.391 + 0.0268\ x + 0.0034\ x^2$$

where x is the distance in inches from the ceramic seal and R(x) is the drift tube radius in inches. The series of step-wise continuous transmission matrices may then be formed as in Ref. 3;

(5) another transmission matrix using either measured or desired radii.

2. Cavity Calculations

The present booster cavities have been examined via this model. Figure 1 shows the voltage step-up ratio from the hub to the gap vs. frequency for several different gap capacitances. The impedance of a drift tube looking out from the hub vs. frequency for several different gap capacitances is shown in Figure 2. As an example of predicting the characteristics of different geometries, these same calculations were performed for a booster cavity with a drift tube radius of 1.4 times the present one. These results are shown in Figures 3 and 4.

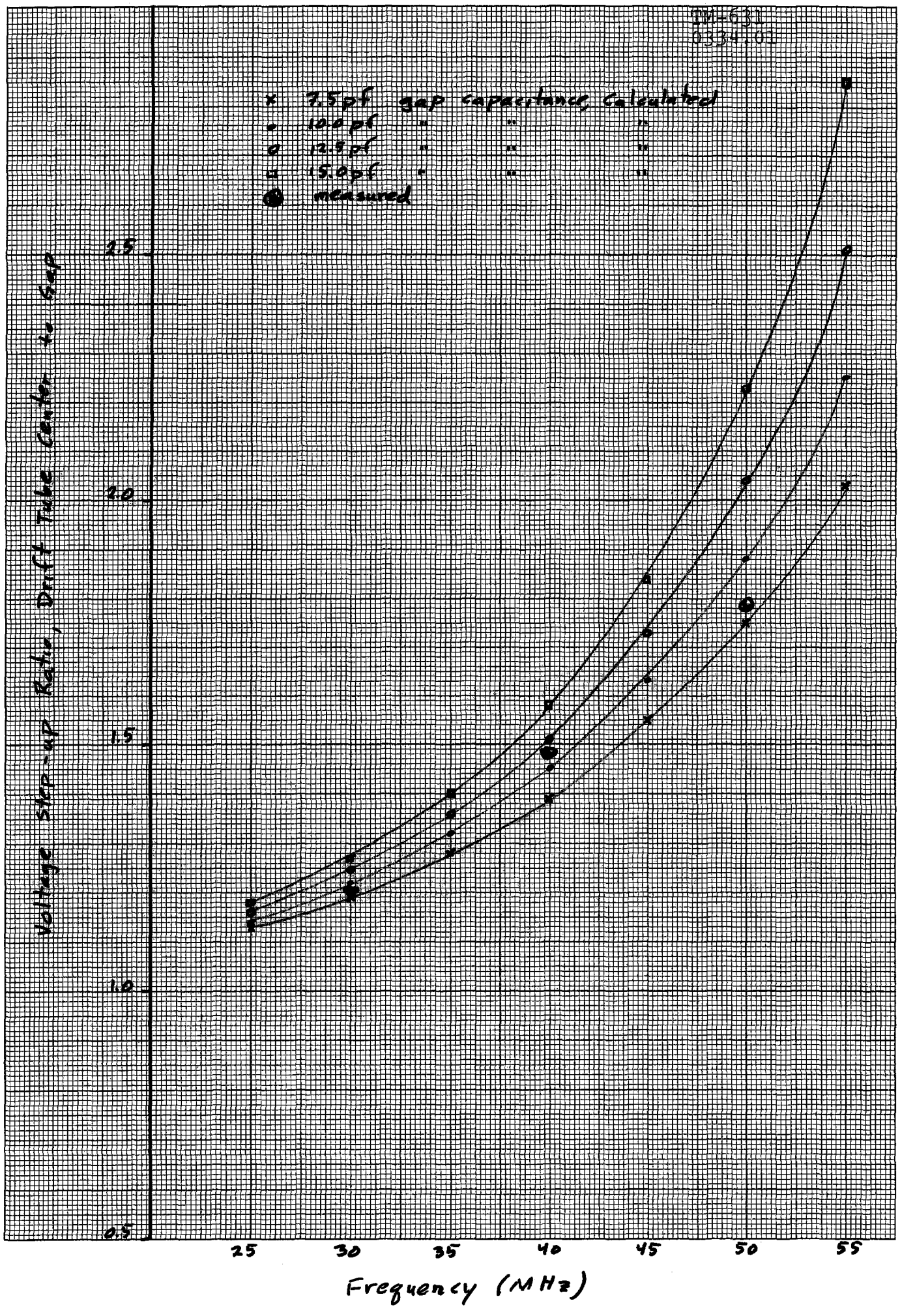
References

1. Collins, R.E.: "Foundations of Microwave Engineering", McGraw-Hill Book Company, New York, 1966

2. Ramo, S., and Whinnery, J.R.: "Fields and Waves in Modern Radio", John Wiley and Sons, Inc., New York, 1953

3. Protonotarios, E.N., and Wing, O.: "Analysis and Intrinsic Properties of the General Nonuniform Transmission Line", IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-15, No. 3, pp. 142-150, March 1967

Booster Cavity Step-up, Hub to gap vs f.



Booster cavity Z_{hub} vs f

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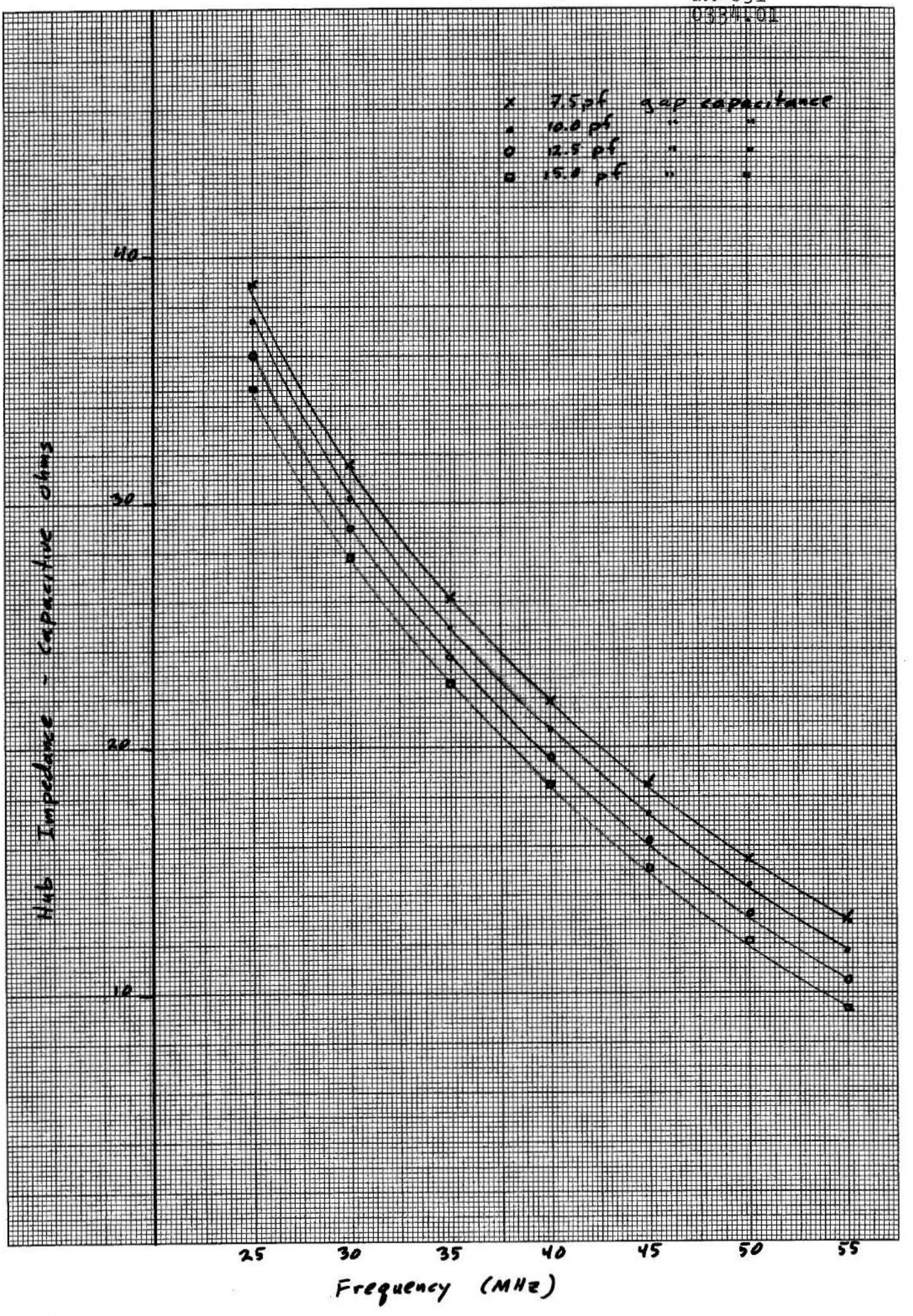
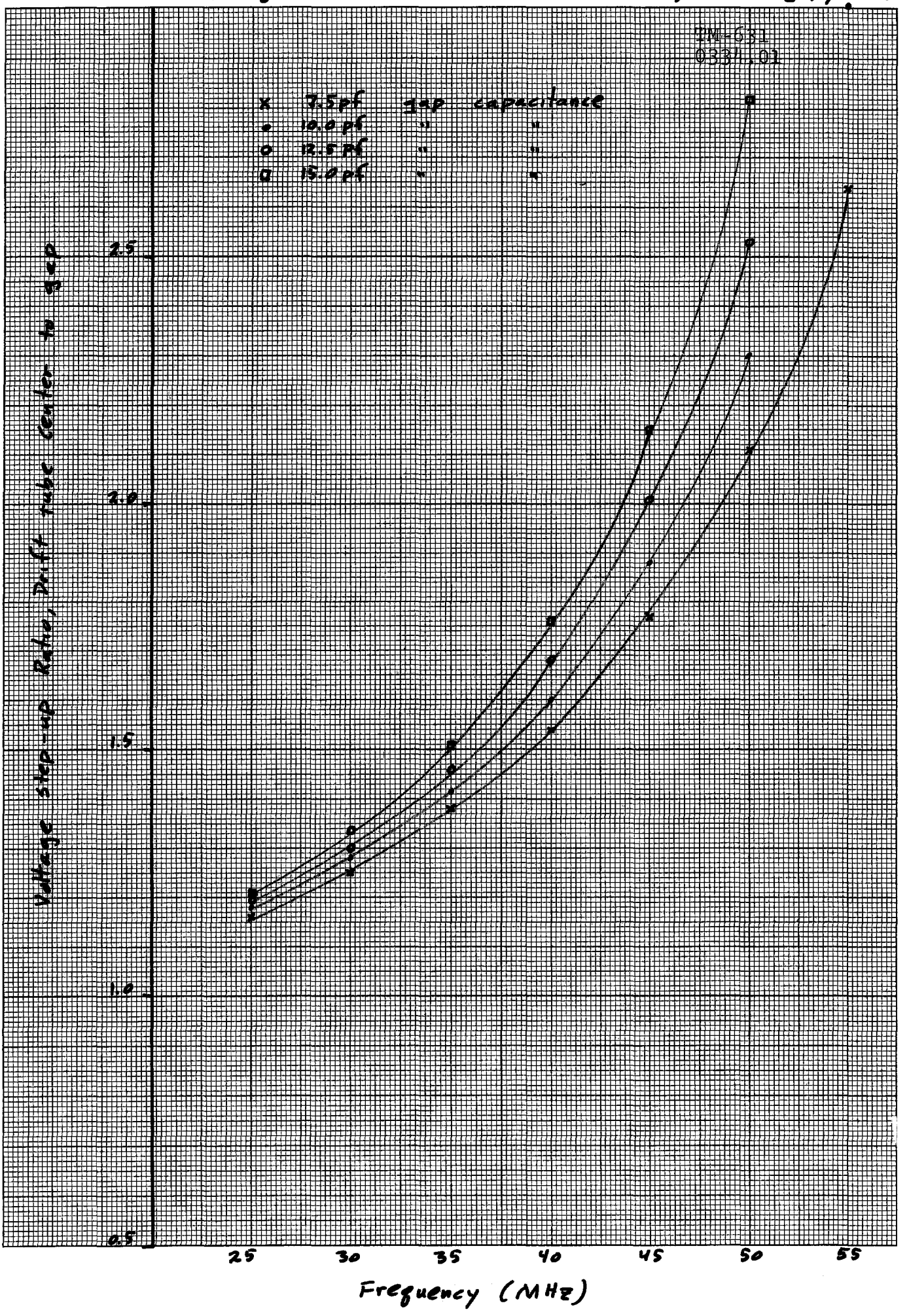
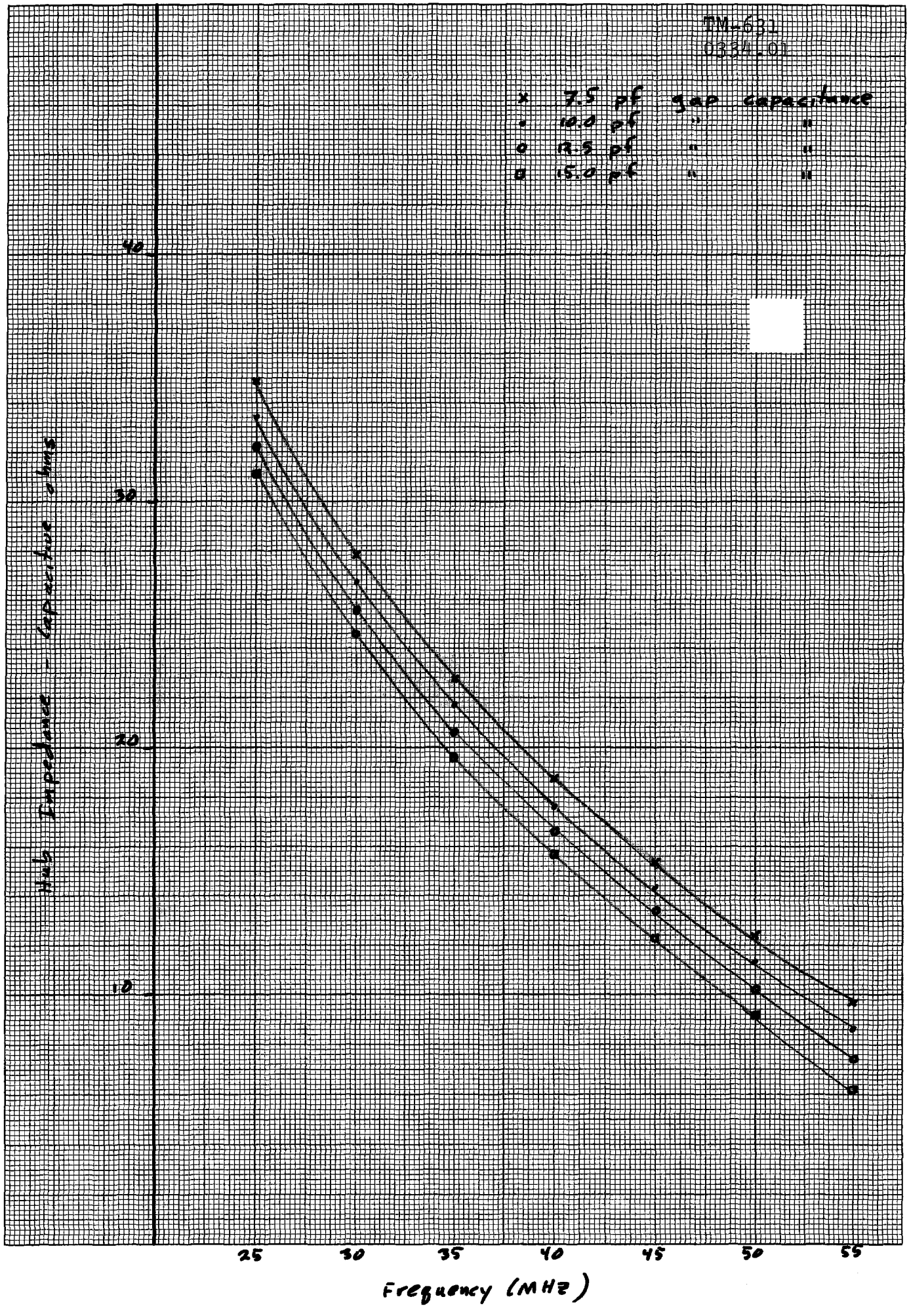


Enlarged aperature Cavity step-up, Hub to gap, vs freq



Enlarged aperature cavity Z_{hub} vs freq.