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MAGNETIC FIELD IN THE 15-FT BUBBLE CHAMBER

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As there is no magnetic material in this chamber, it is possible, although somewhat lengthy, to compute the magnetic field by integrating, all over the coils, the contributions coming from each current's element. This method is used by a program written by H. Desportes at Argonne.

Here has been tried a different approach, which is to find a polynomial expansion for the field, the coefficients of which being computed by a fit. One uses as data a sample of points in the chamber where the field is known from the previous exact method. For that purpose 390 points are chosen at random inside the chamber. Results are the following:

Core used	Exact method	3052
	Polynomial method	174
Time (PDP10) per point	Exact method	266 msec
	Polynomial method	5 msec

The accuracy of the polynomial method with respect to the exact one is shown by the figures:

Maximum error on z-component	0.97% of the field at center of the chamber
Mean error on z-component	0.13%.

The preceding is valid for a 15-polynomial expansion. A better accuracy can be obtained by increasing the number of polynomials. As an example, for 16 polynomials one obtains 0.65% maximum error and 0.093% mean error.

The polynomial expansion used.

Inside the chamber the following two equations hold:

$$\text{curl } \vec{H} = 0$$

$$\text{div } \vec{H} = 0.$$

So one can write $\vec{H} = \vec{\nabla} F$ with $\Delta F = 0$.

One can make an expansion of F in terms of the harmonics polynomials $r^{\ell} e^{im\phi} P_{\ell}^{m}(u)$ with

$$\phi = \operatorname{Arg}(x + iy)$$

$$r^2 = x^2 + y^2 + z^2$$

$$u = \frac{z}{r}.$$

P_{ℓ}^{m} is a so called Legendre's associated function.

The cylindrical and up and down symmetries give the selection rules $m = 0$ and ℓ odd.

Taking advantage of various properties of the P_{ℓ}^{m}, one can write down for the field

$$\vec{H} = \Sigma\, a_{\ell} \vec{H}_{\ell}$$

$$\ell = 1, 3, 5, \ldots$$

with
$$\vec{H}_\ell = \left| \begin{array}{l} \ell r^{\ell-1} \frac{x}{r} I_\ell \\ \ell r^{\ell-1} \frac{y}{r} I_\ell \\ \ell r^{\ell-1} (u I_\ell + J_\ell) \end{array} \right.$$

where
$$J_\ell = \frac{P_{\ell-1} - uP_\ell}{1 - u^2} \quad I_\ell = \frac{P_\ell - uP_{\ell-1}}{1 - u^2} .$$

The polynomials I_ℓ and J_ℓ are computed by means of the recurrence formulas:

$$J_{\ell+2} = \frac{[(2\ell+1)(2\ell+3)u^2 - \ell(\ell+2)] J_\ell + (\ell+1)(2\ell+3)u I_\ell}{(\ell+1)(\ell+2)},$$

$$I_{\ell+2} = - \frac{(2\ell+1) u J_\ell + (\ell+1) I_\ell}{\ell+2},$$

and $\quad I_1 = 0 \quad J_1 = 1.$

Table I. Magnetic Field.

```
      SUBROUTINE BXYZ(H,X)
C       ---COMPUTES MAGNETIC FIELD
C       MAGNETIC FIELD IS SUPPOSED TO BE SYMETRICAL WITH RESPECT
C       TO THE CHAMBER SYSTEM.
      DIMENSION X(3),H(3)
      DIMENSION C(15)
      DATA C/3Ø.Ø714,-2.Ø8Ø96,-.6Ø8Ø78,.2Ø84715,-.Ø35Ø2178,
     ,Ø.3845957E-Ø2,-Ø.271151E-Ø3,Ø.5574568E-Ø4,-Ø.4479559E-Ø5
     5,-Ø.1316256E-Ø4,Ø.2457531E-Ø5,Ø.3167965E-Ø6,
     ,-Ø.4586444E-Ø7,Ø.2114467E-Ø9,Ø.429775E-Ø9/
      PIP=Ø.
      PJP=1.
      HI=Ø.
      HJ=C(1)
      DLP1=3.
      DLP2=5.
      ELP1=2.
      ELP2=3.
      R2=AMAX1(VDOT(X,X,3),1.E-26)
      R=1./SQRT(R2)
      R2=R2*1.E-Ø4
      U=X(3)*R
      U2=U*U
      DO 1 I=2,15
      PII=-(DLP1*U*PJP+ELP1*PIP)/ELP2
      PJP=((DLP1*DLP2*U2-ELP2*(ELP1-1.))*PJP+ELP1*DLP2*U*PI
     IP)/(ELP1*ELP2)
      PIP=PII*R2
      PJP=PJP*R2
      HI=HI+PIP*C(I)
      HJ=HJ+PJP*C(I)
      ELP1=ELP1+2.
      ELP2=ELP2+2.
      DLP1=DLP1+4.
      DLP2=DLP2+4.
    1 CONTINUE
      H(3)=HJ+U*HI
      HI=HI*R
      H(1)=X(1)*HI
      H(2)=X(2)*HI
      RETURN
      END
```