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SEXTUPOLE AND OCTUPOLE FIELDS IN THE MAIN RING QUADRUPOLES

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Field Measurements

We have been analyzing data about measurements of the field gradient on the horizontal plane of 7-foot quadrupoles.[1]

The measurements have been made in the following way.[2] For every magnet, the excitation current I was set at the following values (in ampères):

100   1000   2000   3000   4000   5000   6000   6500   (1)

At each value of the current, first the gradient g_{in} at $x = 0$ was measured, then the relative variation of the gradient at $x = +2"$, $+1"$, $-1"$ and $-2"$ was detected and finally the gradient g_{fin} at $x = 0$ was measured again.

The following is the table of the measurements data relative to the magnet 7060 at location A1061 on Dec. 1, 1972:

7060   A1061

0.0283	0.0054	0.0043	0.0036	0.0000	-0.0140	-0.0392	-0.0560
0.0330	0.0060	0.0044	0.0037	0.0033	0.0021	0.0003	-0.0008
0.0285	0.0043	0.0027	0.0022	0.0019	0.0018	0.0019	0.0017
0.0336	0.0061	0.0044	0.0037	0.0034	0.0027	0.0013	0.0002
0.0335	0.0053	0.0041	0.0032	0.0001	-0.0128	-0.0365	-0.0523

The *nth* column of this table corresponds to the *nth* of the current values listed in (1).

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Every row corresponds to a horizontal position, respectively, from the first to the last row

+2" +1" 0" -1" -2"

These points are taken on the "reference horizontal plane" which is supposed to be coincident to the symmetry plane of the magnet. Also, the point x = 0" is supposed to be the horizontal centre of quadrupole. In the following, we shall disregard the errors introduced in setting the "reference horizontal plane" to make the measurements, as well as the location of the probe on this plane.

Let us denote the elements of the data table by

$$A_{ij}, \quad i = 1, \quad \ldots, 5 \quad \text{and} \quad j=1, \quad \ldots, 8$$

Also, let us call g_j the actual gradient at x = 0" and the current I_j. Then, the gradient G_{ij} at the location x_i ($i \neq 3$) and at the current I_j is

$$G_{ij} = g_j \, (1 + A_{ij}) \tag{2}$$

The numbers A_{3j} are merely a check on the stability of the measuring system

$$A_{3j} = \frac{g_{fin} - g_{in}}{g_{in}} \tag{3}$$

at the current I_j. This number, thus, can be regarded as the measurement relative error of the gradients G_{ij} in the same magnet.

Analysis of the Data

Since we were mostly interested in the knowledge of the field in the proximity of the magnet axis, we excluded from our analysis the data relative to the two extreme points x = ±2", as there the field undergoes very anomalous variation.

Besides, we assumed a vertical magnetic field on the "reference horizontal plane" of the type

$$B_y = gx + \frac{s}{2} x^2 + \frac{q}{6} x^3$$

The gradient of the magnet is, then, differentiating

$$G = \frac{\partial B_y}{\partial x} = g + sx + \frac{q}{2} x^2 \tag{4}$$

where g is the gradient at x = 0, s is the sextupole strength (B") and q the octupole strength (B"').

For every magnet and every current we write Eq. (4) twice, requiring that it fit the measurement data at x = ±1". We have

$$\begin{aligned} G_{2j} - g_j &= s_j x_2 + \frac{1}{2} q_j x_2^{\,2} \\ G_{4j} - g_j &= s_j x_4 + \frac{1}{2} q_j x_4^{\,2} \end{aligned} \tag{5}$$

where the index j refers to the current and g_j is the actual gradient at x = 0.

Since

$$x_2 = -x_4 = \ell = 1",$$

solving the system (5) we obtain

$$s_j = \frac{G_{2j} - G_{4j}}{2\ell}$$

$$q_j = \frac{G_{2j} + G_{4j} - 2g_j}{\ell^2}$$

But from (2) we have, finally,

$$s_j/g_j = s_j' = \frac{A_{2j} - A_{4j}}{2\ell}$$

$$q_j/g_j = g_j' = \frac{A_{2j} + A_{4j}}{\ell^2} \qquad (6)$$

here s_j' and q_j' are, respectively, the sextupole and octupole strength normalized to the local gradient. Analyzing s_j' and q_j' instead of s_j and q_j is much more useful because we eliminate from our calculation the gradient which changes from magnet to magnet.

It is reasonable to take $\pm A_{3j}$ as the fractional error of A_{ij} $(i \neq 3)$. In conclusion, the errors to attribute to s_j' and q_j' are

$$\Delta s_j' = \pm 3A_{3j} \frac{|A_{2j}| + |A_{4j}|}{2\ell}$$

$$\Delta q_j' = \pm 3A_{3j} \frac{|A_{2j}| + |A_{4j}|}{\ell^2} \qquad (7)$$

Processing and Results of the Data

We have calculated (6) and (7) at all the 8 current values listed in (1), for 144 7-foot quadrupoles, which on December 1, 1972 were all located in the main ring tunnel. All the other magnets have been eliminated from our data processing for one of the following three reasons:

(i) No measurements were taken in that particular magnet.

(ii) Only partial measurements were made, but not enough to calculate (6) and (7).

(iii) The measurements were taken but at different set of current values.

Note that, we do not have information regarding measurements of the field in the 4-foot quadrupoles.

Once we calculated (6) and (7) for each of the 144 magnets, we made histograms of the field distribution. In particular, we calculated, for every current listed in (1), the averages $\bar{s}_j'$ and $\bar{q}_j'$ and the rms averages $<s_j'>$ and $<q_j'>$ of the distributions. These quantities are plotted in Figs. 1 and 2. The central line in the plot refers to the averages. The two external lines show the full rms deviation of the distribution around the average curve. We adopted, of course, the same procedure for the errors (7).

The rms average of the errors are marked by a cross (x) in Figs. 1 and 2

The most interesting features of the results of our analysis are the following:

(a) *Sextupole Field*. The rms of the distribution is always much larger than the average at any current. While the average is practically constant, the rms deviation at low energy is a factor 2 or 3 higher than the rms deviation at high energy. This indicates that the remanent field contributes to the sextupole moment distribution but, also it is likely that there is a combination of systematic and random construction errors. Indeed, observe that the rms of the measurement error is larger than the average only at low energy, whereas at higher energy it is always much smaller.

Taking the nominal value of the gradient of 141 G/in at 8 GeV, we found an rms sextupole moment of 0.94 G/in^2 (at 8 GeV) when the field distribution has been corrected (quadratically) for the measurement error. This is about 3 times the average sextupole of the remanent field in the dipoles[2], and about 55% of the rms of the 30° rotated sextupole in the same quadrupoles.[3]

(b) *Octupole Field*. Also, here the rms of the distribution is always much larger than the average at any current. But in this case it seems that only the remanent field contributes. The data at high energy agree with the design values.

At the energy of 8 GeV (excitation current of about 100 A) the rms measurement error is much larger than the average of the distribution but lower than the rms deviation of the distribution. At this energy, when the distribution has been corrected for the measurement error, we found an rms octupole moment of about 5.5 G/in^3, which is about 10 times larger than that at higher energy.

Nonlinear Resonances

Recently, third-order and fourth-order nonlinear resonances have been observed[4] in the 8-GeV beam injected in the main ring. Both resonances occur in the horizontal, as well as on the vertical plane.

We believe the fourth-order resonances can be justified only by the larger amount of the octupole field we discovered by analyzing the field measurement data as it was explained in the foregoing sections. Besides, we might use this information to calculate the number and strength of correcting octupole elements we need to cope with both vertical and horizontal fourth-order resonances.

The third order vertical resonance was justified assuming that the source of the vertical sextupole field is from the field distortion (kink) has been analyzed in another paper.[3] With this assumption, we already can calculate the number of air-core sextupoles to be rotated by 30° to kill the resonance.

It seems to us that the sextupole remanent field in the dipoles has too narrow a distribution to build up enough 61st harmonic to make the resonance noticeable. Thus, we believe that most of the contribution to this resonance is from the wider sextupole distribution in the quadrupoles we calculated in the foregoing sections. Also, our results seem in agreement with the experimental observation that the third-order vertical resonance is stronger than the horizontal one.

References

1. J. Schivell, TM-366 (April 1972).
2. C. Schmidt, private communication.
3. A. G. Ruggiero, TM-387 (August 1972).
4. D. Edwards, R. Stiening, private communication.

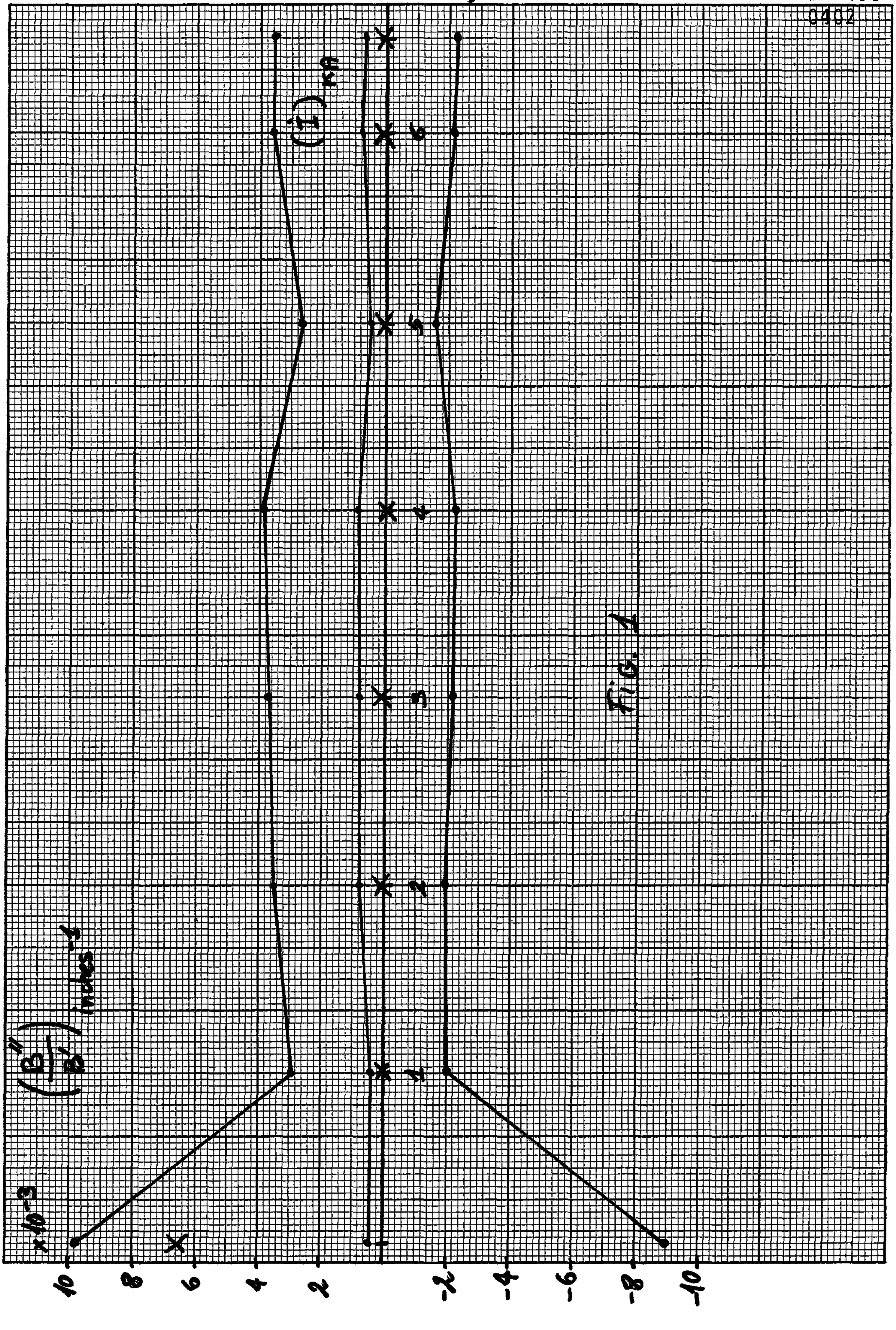

Fig. 1

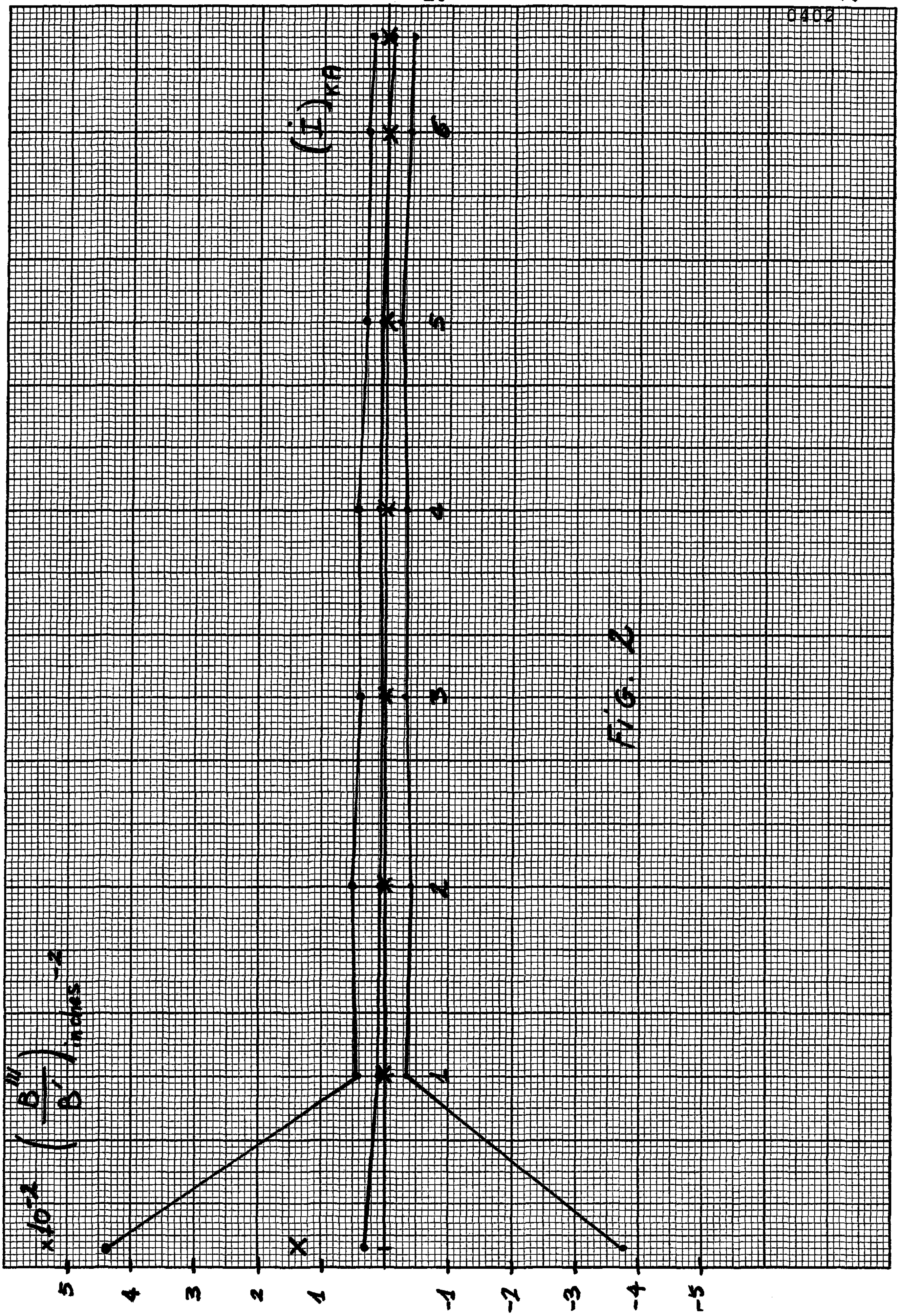

FIG. 2