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SECOND-ORDER OPTIMIZATION OF BEAM LINE DESIGNS WITH TRANSPORT

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I. Introduction

The aberrations present in a beam line typically represent the departure from the ideal first-order design. Such aberrations may be of second and higher order. Their effect becomes more bothersome as the phase-space volume (including momentum band pass) accepted by a beam line is increased.

We restrict ourselves here to strictly second-order terms. Such effects may be represented by a second-order transfer matrix T.[1] The net contribution of the aberrations to the beam spot size may be determined by including the effect of the second-order T matrix in the calculation of the beam matrix.[2]

By the inclusion of sextupoles or effective sextupole components in bending magnets, one may influence the second-order terms without affecting the first-order design. Typically one wishes to minimize the effect of the most damaging second-order terms without creating others more damaging. A selection of correcting elements and their placement may be done by examining the second-order coupling terms calculated by Brown.[1,3] Once one decides which aberrations are to be minimized, and selects

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an appropriate set of correcting elements, the problem reduces to numerical calculation.

This calculation may now be done automatically with a revised version of the program TRANSPORT.[4,5] One indicates the aberrations to be minimized and the parameters to be adjusted, and the program will now find the optimal solution. In Section II below we discuss the parameters which may be varied, and in Section III we describe the constraints that may be applied. Second-order fitting differs in certain ways from first-order problems. Such differences, plus the necessary restrictions to be observed, are mentioned whenever appropriate.

II. Parameters Which May Be Varied

Second-order investigations are usually begun only after a satisfactory first-order design is formed. Therefore, a problem to be solved typically is purely first or second order. At present, TRANSPORT cannot solve problems involving both first- and second-order fitting. One must proceed one order at a time. As usual,[4] one indicates a second-order run by including a 17. card in the deck. This restricts the parameters which may be varied to the ones mentioned below.

The desirability of being able to solve mixed order fitting problems has been pointed out by Brown.[6] The effect on second-order terms of varying first-order parameters is complicated and has yet to be derived. It is planned for a future version of TRANSPORT to include this capability.

The parameters which may be varied, with the type codes used by TRANSPORT to indicate them are as follows:

a) $\varepsilon(1)$ - The normalized second derivative of the central field of a bending magnet, indicated by a 16. 1. card.

b) $1/R1$ - The curvature of the entrance face of a bending magnet, indicated by a 16. 12. card.

c) $1/R2$ - The curvature of the exit face of a bending magnet, indicated by a 16. 13. card.

d) The pole tip field of a sextupole, indicated on an 18. card.

e) The length of a drift space, indicated on a 3. card.

For further clarification of the meaning of any of these parameters, the reader is advised to consult the TRANSPORT manual.[4]

The first four parameters have only second-order effects. The length of a drift space may have a first-order effect, so it should be varied with caution. The purpose of including drift length variation was to allow adjustment of sextupole positions. This question will be discussed more completely in Section III.

Varied parameters are indicated, as usual, by a vary code. For example, one might independently vary any of the first three listed parameters by writing the type code as 16.01. Direct or inverse coupling between varied parameters is accomplished by using vary codes two through nine, as with first order TRANSPORT.[4]

The parameters given on 16. cards apply to all subsequent magnets, until another similar 16. card is encountered. When such a parameter is varied the variation is done uniformly on all magnets to which the card applies. If one wishes, after a certain point, that a parameter should no longer be varied, one introduces another 16. card having no vary code. If one wishes to vary a given parameter independently for two different magnets, one must insert a 16. card for each magnet. An example of three bending magnets, the second derivative of the central field of the outer two being varied independently, is given below.

16.01	1.0	0.0	;
4.	--	--	;
3.	--		;
16.	1.	0.0	;
4.	--	--	;
3.	--		;
16.01	1.	0.0	;
4.	--	--	;

Entrance and exit pole face curvatures of a bending magnet are indicated similarly and may therefore be coupled.

Any second-order fitting problem will be linear in any of the first four listed parameters. So, if these are the only parameters being varied, the choice of initial values is unimportant, and will not affect the fitting. If a drift space is being varied, the problem becomes nonlinear. In this case one should choose reasonable initial values for both drift lengths and sextupole strengths to achieve satisfactory fitting.

III. Constraints Which May be Imposed

Either second-order transfer matrix (T matrix) or beam matrix (σ matrix) constraints (or both) may be imposed in a second-order run. A description of each plus some restrictions and other capabilities follows.

A constraint on a T matrix element is indicated as follows. There is no limit on the number of constraints that may be imposed.

1) Type code 10.

2) The negative of first or dependent index i of the matrix element T_{ijk}.

3) A combination of the second two or independent indices j and k.

4) The desired value of the matrix element.

5) Desired accuracy of the fit.

If one wished, for example, to constrain the value of the T_{122} matrix element to be 0.5 then one would insert a card which looked like:

10. -1. 22. 0.5 0.01 ;

The matrix being constrained is actually the second-order part of the R_2 matrix. Therefore, when activating second-order fitting, one must not include any element which causes an update of the R_2 matrix. Such elements are:

1. An rms addition to the beam.
6. An update of either R_1 or R_2
7. A centroid shift
8. A misalignment
21. A stray field

In some cases, one may wish to set the focal plane at a certain angle. One can, of course, compute the required value of the T_{126} matrix element and fit to that value. More simply, one can use the 16. 15. element to rotate to the desired focal plane angle, and then constrain T_{126} to be equal to zero.

A beam matrix constraint indicates that the net contribution of the second-order aberrations to the beam size in a given coordinate is to be minimized. The constraint is written as:

1) Type code 10.
2) An index indicating the coordinate in which the minimization is to be done.
3) A repeat of item 2.
4) The number 0.0.
5) Desired accuracy of the fit.

If, for example, one wished to minimize the net contributions of second-order aberrations to the horizontal divergence, one would insert the following card:

10. 2. 2. 0.0 0.01 ;

The quantity being minimized is the second-order part in the second of equations (23) of reference 2. This quantity is treated as the chi-squared of the problem, so the only meaningful desired value for the fit is zero. The procedure used for fitting is essentially that developed by Thiessen.[7]

The quantity minimized is the net increase due to second-order terms in the second moment of the beam about the origin. It is computed using the R_2 matrix. Therefore, once again, one must not include any element which updates the R_2 matrix. Centroid shifts must not be inserted when doing second-order fitting, even immediately following the beam card.

The second-order image of the initial beam centroid at some later point in the beam is not necessarily the beam centroid at the later point. The displacement of the centroid image is given by the first of equations (23) of reference 2. The parameters printed by TRANSPORT are the centroid image position and the beam matrix about the new centroid. One must therefore look at both of these to observe the effects of the fitting procedure. It may even happen that an improvement in one parameter will be accompanied by a slight deterioration in the other. Sometimes, also, fitting with TRANSPORT will show little difference in the printed beam parameters. However, in many cases, an investigation of the beam profile with the program TURTLE[8] will in fact show a significant improvement.

The beam profile at any point is a function of the initial beam parameters. One may therefore impose weights on the effect of the various aberrations by the choice of parameters on the beam card. One might, for example, adjust the strength of the correction of the chromatic aberrations by the choice of the $\Delta p/p$ parameter. In particular, one should not try to eliminate chromatic aberration by using a beam constraint with a monochromatic beam. Correlations (the 12. card) may also be included in the initial beam.

Sometimes not all aberrations are equally troublesome. One may have a spectrometer which measures certain coordinates of the initial phase space, such as momentum or scattering angle. Aberrations involving only those coordinates may simply require a calibration of the spectrometer, while others will cause a deterioration in its performance. What is desired is a beam constraint ignoring certain terms, but not necessarily a set of terms that can be eliminated by adjusting the initial phase space. In such cases, one wishes to constrain each of the relevant second-order matrix terms T_{ijk}, weighing each according to its importance. One does this by including a constraint card for each such matrix element, setting the tolerance equal to the inverse of the phase space factor which the matrix element multiplies. For a matrix element T_{ijk} acting on an uncorrelated initial phase space, this factor would be $1/(x_{oj}x_{ok})$, where x_{oj} and x_{ok} are the initial beam half widths. Because any fitting problem, where both varied parameters and constraints are strictly second-order, is linear, the absolute size of the tolerances is unimportant. Thus, even if certain tolerances seem large, the above procedure will produce the desired fit.

If one is correcting with sextupoles, there may be practical limitations on field strength. One may wish to weigh such limitations against the minimization of aberrations. The sextupoles will have minimum field if located where they are most effective. Such locations are where the coupling terms to the aberrations are largest.[1] If several sextupoles are used to adjust many aberrations, the precise positions may not be obvious. If one could vary the position of a sextupole while constraining both its field and certain aberrations, one might achieve the optimum fit. The position of a sextupole may be varied by straddling it with drift spaces whose lengths are inversely coupled.

A constraint on sextupole strengths is indicated by a 10. card with the first index set equal to eighteen. Such a card pertains to all sextupoles in the beam which follow. The desired value should be set to zero. The tolerance indicates the maximum desired pole tip field and like any other tolerance may be exceeded on the optimal solution. One will need to experiment to determine the best value for the tolerance in a given case.

If, for example, one wished to use a value of 5 kilogauss as a maximum for the sextupole pole tip field, one would insert the following card:

10. 18. 0.0 0.0 5.0 ;

If the only parameters being varied are among the first four listed in Section II, the problem will be linear. The program will then make a single iteration to find the solution. In TRANSPORT the value of chi-squared printed is that which existed before the corrections were made. Therefore the solution will be the best possible even though the chi-squared may look ghastly.

If drift spaces are varied, the problem will cease to be linear and the program proceeds as in the first-order case. Iterations are made until the best fit is found.

References

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