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FINE TUNING AND EXCITATION OF THE 41st HARMONIC WITH THE TRIM QUADRUPOLES IN THE MAIN RING

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INTRODUCTION

Trim quadrupoles are being installed in the main ring in order to facilitate fine tuning of the ν-values at injection and to provide compensation for stopbands arising from quadrupole error fields, also at injection. Thus, the trim quadrupole system is required to provide:

a. fine tuning on ν_x (horizontal) and ν_y (vertical) independently,

b. excitation of the 41st harmonic with changeable amplitude and phase to cancel the half-integer stopband at $\nu = 20.5$. Of course, this is required independently in both the horizontal and vertical planes. Other quadrupole stopbands are likely to be of less importance, and their compensation will not be considered here.

When the quadrupoles are used in the "fine tuning" mode, the system is not to excite any harmonic in the neighborhood of $2\nu_x$ or $2\nu_y$, such as the 40th, 41st or 42nd.

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Similarly, when the quadrupoles are used in the "41st harmonic excitation" mode, the following harmonics are not to be excited: 0th, 40th, 42nd.

The functions above are provided by a proper gradient distribution among the quadrupoles. Superimposing the appropriate gradient distributions will allow fine tuning as well as 41st harmonic at the same time, amplitudes and phases being limited, of course, by the maximum quadrupole gradient.

We shall analyze in this paper the two gradient distributions for the fine tuning and 41st harmonic separately. The analysis is carried on treating the trim quadrupoles as perturbation elements and taking into consideration only the terms up to the second order. In this case, according to Courant and Snyder,[1] the ν-shift $\Delta\nu$ and the stopband width $\delta\nu$ are given by

$$\Delta\nu = \frac{\ell}{4\pi} \sum_{n=1}^{N} \beta_n K_n \tag{1a}$$

$$\delta\nu = \frac{\ell}{2\pi} \left| \sum_{n=1}^{N} \beta_n K_n e^{ip\phi_n} \right| \tag{1b}$$

where

N = number of quadrupoles

ℓ = length of each quadruple

p = the harmonic number

β_n, ϕ_n = the beta value and the phase advance at the center of the n-th quadrupole

K_n = $g_n/(B_o\rho)$, where g_n is the gradient of the n-th quadrupole and $B_o\rho$ is the magnetic rigidity of a proton. At the kinetic energy of 7 GeV it is

$$B_o\rho = 263 \text{ kG}\cdot\text{m}.$$

All the calculations are carried on for the kinetic energy of 7 GeV.

THE TRIM QUADRUPOLES

There are 36 trim quadrupoles each 30 cm long and a gradient range of ±4 kG/m, which in terms of K gives $\pm 0.015 \text{ m}^{-2}$. The quadrupoles are divided in two groups of 18. Those magnets of the first group are placed each in one of the short straight sections (6'11" long) after a focusing (F) main quadrupole magnet; the others of the second group are each after one of the defocusing (D) main quadrupole magnets. There are six trim quadrupoles per superperiod: Three after F-main quadrupoles and three after D-main quadrupoles.

Tables I and II list the trim magnets and their position according to two distributions which have been proposed with different criteria.

The beta values are estimated assuming that the quadrupoles are placed in the middle of the short straight sections.

Δ is the phase distance (normalized to 2π per turn) between two consecutive quadrupoles in normal cells. It is supposed to be the same in the horizontal and vertical plane:

$$\Delta = 0.030584 \text{ radians}.$$

The origin of the phase may be chosen arbitrarily in our calculation; we call the phase of the quadrupole #1 $\phi = 0$.

THE FINE TUNING

Fine tuning is achieved by controlling the eighteen F trim quadrupoles simultaneously and independently of the eighteen D trim quadrupoles that are also controlled simultaneously. By using identical current settings in each sector, the only harmonics present will be multiples of 6. To eliminate the 42nd harmonic it is necessary to modulate the gradient among the quadrupoles in the same superperiod, and in the same ways for all the superperiods.

Looking only at the contribution from the Sector A, the following conditions have to be satisfied

$$\sum_{n}^{1,3,5} K_n e^{i42\phi_n} \propto 1+a_3 e^{i42\phi_3} + a_5 e^{i42\phi_5} = 0 \text{ (horizontal)}$$

$$\sum_{n}^{2,4,6} K_n \ell^{i42\phi_n} \propto 1+a_4 e^{i42\phi_4} + a_6 e^{i42\phi_6} = 0 \text{ (vertical)}$$

where the index n refers to the number (#) of the quadrupole.

We are assuming here that if g is the gradient in the radial plane, -g is the gradient in the vertical plane. The above equations are solved for the a_i's. The results are shown in Table III. This table must be read in the following way.

For example, in the distribution B, if we apply 1 unit of current to the magnet #1 or #2 we have to provide 1.01 units and 0.83 units, respectively, to magnets #3 and #5, or to provide 1.22 units and 1.20 units, respectively, to the magnets #4 and #6. In this way there will be no 42nd harmonic in either mode vertical or horizontal. But if a relative error ε is introduced into the gradient modulation, a stopband at $\nu = 21$ would be enhanced. The width of this stopband is given in the third column of Table III.

Comparison between the two cases shows that the distribution B provides a wider range for tuning with a fairly small gradient modulation.

In absence of any error ($\varepsilon = 0$) and calling I_1 and I_2 the current units (4 kG/m $\equiv$ 127 units) supplied, respectively, to the quadrupoles #1 and #2 we have, summarizing,

in the horizontal plane:

$$\Delta\nu_x = (624\ I_1 + 96\ I_2) \times 10^{-5} \quad \text{(Distr. A)}$$

$$\Delta\nu_x = (453\ I_1 + 165\ I_2) \times 10^{-5} \quad \text{(Distr. B)}$$

in the vertical plane:

$$\Delta\nu_y = -(188\ I_1 + 319\ I_2) \times 10^{-5} \quad \text{(Distr. A)}$$

$$\Delta\nu_y = -(136\ I_1 + 549\ I_2) \times 10^{-5} \quad \text{(Distr. B)}$$

Besides, because of the amplitude modulation, I_1 and I_2 are limited in magnitude as follows:

$$I_1 < 66 \qquad I_2 < 120 \qquad \text{(Distr. A)}$$

$$I_1 < 125 \qquad I_2 < 104 \qquad \text{(Distr. B)}$$

EXCITATION OF THE 41st HARMONIC

Two quadrupoles, at opposite ends of a diameter of the ring, with the same gradient but opposite signs, have the effect of exciting the 41st harmonic but not the 0th, 40th and 42nd harmonics. This is the method we shall adopt; each quadrupole has a counterpart on the opposite side of the machine. In the discussion and enumeration that follow we will normally not make explicit reference to these counterparts.

In principle, only four quadrupoles are required for the complete cancellation of the stopbands at $\nu = 20.5$ in both planes: two to control the amplitude of the 41st harmonic and two to control the phase of the harmonic. Since we do not know a-priori the amplitude and the phase of the stopband region, it is required that the system provides a range wide enough of amplitudes, and that the phase of the harmonic can be changed over all the interval from 0 to 2π. In practice, because of the gradient limitation, this cannot be accomplished with only four quadrupoles.

Let us decide to take four groups, each of three quadrupoles that will be controlled in the same way, and each group independently of the others.

The four groups constitute the four "knobs" we need to provide amplitudes and phase of the 41st harmonic. The first requirement implied is, of course, that the three quadrupoles in the same group have to stay as close as possible in phase (apart from multiple of 2π). With this requirement we perform a first selection. The groups selected with the quadrupole numbers are listed in Table IV. All the quadrupoles in one group have the same β-value. As we can see from Table IV, some groups have the maximum value of β_x (quadrupoles with odd number). All the others have the minimum value (quadrupoles with even number).

From Eq. (1b) we have

$$\delta\nu = \frac{\ell}{2\pi} \sum_{n=1}^{4} \left[\beta_n K_n \left(\sum_{m=1}^{3} e^{ip\phi_{nm}} \right) \right]$$

$$= We^{i\psi}, \qquad (p = 41).$$

Let us call

$$\sum_{m=1}^{3} e^{ip\phi_{nm}} = A_n e^{ip\bar{\phi}_n}$$

so that

$$\delta\nu = \frac{\ell}{2\pi} \sum_{n=1}^{4} \beta_n A_n K_n e^{ip\bar{\phi}_n}.$$

Splitting this equation in two equations, one for the amplitude W and one for the phase ψ, we obtain

$$W^2 = 4 \sum_{i,j}^{1,4} a_{ij} K_i K_j \tag{2}$$

$$a_{ij} = \left(\frac{\ell}{2\pi}\right)^2 A_i A_j \beta_i \beta_j \cos p \, (\bar{\phi}_i - \bar{\phi}_j) \tag{2a}$$

and

$$\sum_{n=1}^{4} \beta_n A_n \left(\sin p\bar{\phi}_n - \text{tang}\ \psi \cdot \cos p\bar{\phi}_n \right) K_n = 0. \tag{3}$$

The A_n's and $\bar{\phi}_n$'s are given, for each group of quadrupoles selected, in Table IV.

The factor 4 in front of the summation in Eq. (2) is to take into account the contribution of the other 12 quadrupoles placed opposite to avoid any harmonic of 0th, 40th and 42nd order.

The Eqs. (2) and (3) are understood to apply twice: once for the horizontal plane and once for the vertical plane.

Eq. (2) is the equation of a 4-dimensional conic which is symmetric about the origin $K_1 = \ldots = K_4 = 0$. Eq. (3) is the equation of a 4-dimensional plane which crosses the origin $K_1 = \ldots = K_4 = 0$. There is one plane for each value of tang ψ as well as there is one conic (2) for each value of W^2. The intersection of the two conics W_x^2, W_y^2 with the two planes tang ψ_x, tang ψ_y provides values of K_1, K_2, K_3 and K_4 for which the 41st harmonic is excited in both planes with the above-mentioned amplitudes and phases.

Because the intersection is "real," Eq. (2) should represent a 4-dimensional closed ellipsoid. This can be verified diagonalizing the symmetric matrix with the a_{ij}'s, Eq. (2a), as elements. If $b_1, \ldots, b_4$ are the eigenvalues, Eq. (2) can be rewritten as

$$W^2 = 4 \sum_{i=1}^{4} b_i K_i'^2 .$$

This is the equation of an upright 4-dimensional ellipsoid if, and only if, the b_i's are all positive numbers. Besides the sizes of the ellipsoid are given by $1/\sqrt{b_i}$, $(i = 1, \ldots, 4)$; so that to excite the 41st harmonic with changeable phase and maximum W_{max} it must be

$$\sqrt{b_i} < \frac{1}{2} \frac{W_{max}}{K'_{i_{max}}} = \frac{W_{max}}{0.030} \, m^2$$

for any $i = 1,2,3,4$.

We listed in Table V all the possible combinations of 4 quadrupoles among all the quadrupoles of the first columns in Table IV, satisfying

$$0 < b_i < 1$$

for i = 1,2,3,4 and in both planes at the same time. Only combinations of two F-trim quadrupoles and two D-trim quadrupoles are taken into consideration, for symmetry reasons.

These combinations of 4 quadrupoles (or, 4 groups of one quadrupole each) would provide, at least, amplitudes up to 0.03 with changeable phases over all the interval $(0,2\pi)$. Since we expect that the stopband at $\nu = 20.25$ is fairly narrow, this should be enough in the case fine tuning is not performed.

In practice, the two modes of operation will overlap and two or three quadrupoles per group, in this case, are required.

The Table VI shows the three best 12 quadrupole combinations for both distributions. All can provide amplitudes fairly close to 0.12.

It might be useful to relate <u>linearly</u> the real part and the imaginary part of the 41st harmonic to the gradients of the four groups of quadrupoles. For this purpose let us introduce the following two vectors

$$\vec{W} \equiv (W_x \cos \psi_x,\ W_x \sin \psi_x,\ W_y \cos \psi_y,\ W_y \sin \psi_y)$$

$$I \equiv (I_1,\ I_2,\ I_3,\ I_4)$$

where I_i is the number of gradient units (4 kG/m $\equiv$ 127 units) supplied to the i-th group. It is easily verified that

$$\vec{W} = mA\vec{I} \times 10^{-5} \qquad (4)$$

where m (= 1,2,3) is the number of magnets employed in each group and A is a 4x4 matrix.

Setting $\phi = 0$ at the center of quadrupole #1 and ordering the I_i's in the same way the 4 groups are ordered in Table VI, we calculated the elements of the matrix A for all the six combinations in Table VI. The respective matrices are listed in the same order in Table VII.

The relation (4) might be useful in practice when for a given vector $\vec{W}$ it is required to calculate the vector $\vec{I}$.

Reference

1. E.D. Courant and H.S. Snyder, Ann. of Phys. 3, 1 (1958).

Table I: DISTRIBUTION A

Quad #	Position	β_x	β_y	ϕ
1	A 13 (F)	93m	28m	
2	A 21 (D)	28	93	7 Δ
3	A 26 (F)	93	28	12 Δ
4	A 35 (D)	28	93	19 Δ
5	A 42 (F)	93	28	24 Δ
6	A 47 (D)	28	93	29 Δ
7	B 13 (F)	93	28	$\pi/3$
8	B 21 (D)	28	93	$\pi/3 + 7\ \Delta$
9	B 26 (F)	93	28	$\pi/3 + 12\ \Delta$
10	B 35 (D)	28	93	$\pi/3 + 19\ \Delta$
11	B 42 (F)	93	28	$\pi/3 + 24\ \Delta$
12	B 47 (D)	28	93	$\pi/3 + 29\ \Delta$
⋮	⋮	⋮	⋮	⋮
31	F 13 (F)	93	28	$5\pi/3$
32	F 21 (D)	28	93	$5\pi/3 + 7\ \Delta$
33	F 26 (F)	93	28	$5\pi/3 + 12\ \Delta$
34	F 35 (D)	28	93	$5\pi/3 + 19\ \Delta$
35	F 42 (F)	93	28	$5\pi/3 + 24\ \Delta$
36	F 47 (D)	28	93	$5\pi/3 + 29\ \Delta$

Table II: DISTRIBUTION B

Quad #	Position	β_x	β_y	ϕ
1	A 13	93m	28m	
2	A 14	28	93	Δ
3	A 22	93	28	8Δ
4	A 35	28	93	19Δ
5	A 44	93	28	26Δ
6	A 45	28	93	27Δ
7	B 13	93	28	$\pi/3$
8	B 14	28	93	$\pi/3 + \Delta$
9	B 22	93	28	$\pi/3 + 8\Delta$
10	B 35	28	93	$\pi/3 + 19\Delta$
11	B 44	93	28	$\pi/3 + 26\Delta$
12	B 45	28	93	$\pi/3 + 27\Delta$
'	'	'	'	'
'	'	'	'	'
'	'	'	'	'
'	'	'	'	'
31	F 13	93	28	$5\pi/3$
32	F 14	28	93	$5\pi/3 + \Delta$
33	F 22	93	28	$5\pi/3 + 8\Delta$
34	F 35	28	93	$5\pi/3 + 19\Delta$
35	F 44	93	28	$5\pi/3 + 26\Delta$
36	F 45	28	93	$5\pi/3 + 27\Delta$

Table III: FINE TUNING

	Relative Mod. Coefficients	ν-shift	Full Stopband Width at p=42
Distribution A	$a_3 = 1.91$ $a_5 = 1.00$	± 0.53	$0.21\ \varepsilon$ (horizontal)
	$a_4 = -0.053$ $a_6 = 1.05$	± 0.51	$0.38\ \varepsilon$ (vertical)
Distribution B	$a_3 = 1.01$ $a_5 = 0.83$	± 0.74	$0.39\ \varepsilon$ (horizontal)
	$a_4 = 1.22$ $a_6 = 1.20$	± 0.74	$0.33\ \varepsilon$ (vertical)

Table IV: 41st HARMONIC QUADRUPOLES GROUPING

	Distribution A			Distribution B		
Group #	Quad #	A_n	$41\ \overline{\phi}_n$ (rad)	Quad #	A_n	$41\ \overline{\phi}_n$ (rad)
1	10 12 32	2.95	-2.49	3 13 29	2.80	-2.54
2	11 13 33	2.89	-2.41	10 20 36	2.80	-2.35
3	4 6 26	2.95	-1.44	4 14 30	2.80	-1.30
4	5 7 27	2.89	-1.36	7 17 33	2.91	-1.15
5	20 34 36	2.95	-0.34	1 11 27	2.91	-0.10
6	1 15 35	2.89	0.04	8 18 34	2.90	0.10
7	14 28 30	2.95	0.65	2 12 28	2.90	1.15
8	9 23 31	2.85	1.43	5 15 31	2.90	1.29
9	8 22 24	2.95	1.70	6 16 32	2.91	2.54
10	3 17 25	2.85	2.48	9 19 35	2.80	2.69
11	2 16 18	2.95	2.74	-		

Table V: BEST COMBINATIONS OF 4x1 QUADRUPOLES TO PROVIDE 41st HARMONIC

Distribution A				Distribution B			
1	2	8	9	1	4	5	10
1	2	9	10	2	3	6	7
1	4	5	14	2	3	6	9
1	8	9	10	2	5	6	9
2	3	9	14	2	5	8	9
2	5	10	11	4	5	8	9
3	4	10	11				

Table VI: BEST COMBINATIONS OF 4x3 QUADRUPOLES TO PROVIDE 41st HARMONIC

	1st Group			2nd Group			3rd Group			4th Group		
DIST. A	1	15	35	2	16	18	9	23	31	10	12	32
	2	16	18	5	7	27	10	12	32	11	13	33
	3	17	25	4	6	26	10	12	32	11	13	33
DIST. B	2	12	28	3	13	29	6	16	32	7	17	33
	2	12	28	3	13	29	6	16	32	9	19	35
	4	14	30	5	15	31	8	18	34	9	19	35

Table VII: THE A-MATRIX FOR THE EXCITATION OF THE 41st HARMONIC

Distribution A				Distribution B			
53.1	-12.8	7.25	-11.3	4.98	-43.6	-12.2	26.6
0.0	9.64	52.6	-11.3	15.2	-30.3	10.3	-46.0
-16.0	42.4	-2.18	37.5	-16.5	13.1	40.6	-7.99
0.0	-32.0	-15.8	37.6	-50.5	9.12	-34.3	13.8
-12.8	13.1	-11.3	-38.0	4.98	-43.6	-12.2	-48.0
9.64	-51.5	-11.3	-37.1	15.2	-30.3	10.3	22.6
42.4	-3.95	37.5	11.4	-16.5	13.1	40.6	14.5
-32.0	15.5	37.6	11.2	-50.5	9.12	-34.3	-6.82
-41.9	4.16	-11.3	-38.0	4.16	19.9	15.6	-48.0
32.6	-15.4	-11.3	-37.1	-15.4	49.2	3.28	22.6
12.6	-13.8	37.5	11.4	-13.8	-5.99	-52.0	14.5
-9.81	51.3	37.6	11.2	51.3	14.8	-10.9	-6.82