national accelerator laboratory

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TRANSMISSION LINE MODES AROUND THE MAIN RING

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FORMALISM

Each magnet (dipole or quadrupole) or power supply is represented by a 4-terminal network. The steady-state behavior of such a network for a given frequency component (ω) is most easily studied using 3x3 matrices. The voltage-current vector is written as

$$\begin{pmatrix} V \\ I \\ 1 \end{pmatrix} e^{i\omega t} \rightarrow A e^{i\omega t}, \qquad A \equiv \begin{pmatrix} V \\ I \end{pmatrix} \tag{1}$$

where V and I are, in general, complex. A general propagation-source matrix is written as

$$\begin{pmatrix} a & b & e \\ c & d & f \\ 0 & 0 & 1 \end{pmatrix} \rightarrow (M|D) \qquad \begin{aligned} M &\equiv \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \text{propagation} \\ D &\equiv \begin{pmatrix} e \\ f \end{pmatrix} = \text{source} \end{aligned} \tag{2}$$

where we have adopted the notation introduced in FN-232. In this notation

$$(M|D)A \equiv MA+D$$

$$(M_2|D_2)(M_1|D_1) \equiv (M_2M_1|M_2D_1+D_2).$$

The 2x2 propagation matrix M is unimodular and, in general, complex. For a magnet the source vector D = 0 and for a power supply $D = \begin{pmatrix} \Delta V \\ \Delta I \end{pmatrix}$ where ΔV and ΔI are the voltage and the current (including phases) produced by the supply.

The propagation matrix all the way around the ring (say, N element) is, therefore,

$$(M_N|D_N)(M_{N-1}|D_{N-1}) \cdots\cdots (M_3|D_3)(M_2|D_2)(M_1|D_1) \equiv (M|D) \qquad (3)$$

where

$$M \equiv M_N M_{N-1} \cdots\cdots M_2 M_1$$

$$D \equiv D_N + M_N D_{N-1} + M_N M_{N-1} D_{N-2} + \cdots + M_N M_{N-1} \cdots M_2 D_1 .$$

The voltage-current vector at the entrance to element 1 is, then, given by

$$A_o = (M|D)A_o = MA_o + D$$

or

$$A_o = (1-M)^{-1}D = \frac{M^{-1}-1}{\mathrm{Tr}M-2}\, D. \qquad (4)$$

For a dissipative ring

$$\mathrm{Tr}M \equiv \Omega(\omega) + i\Gamma(\omega) = \text{complex}$$

and

$$A_o = \frac{(M^{-1}-1)D}{(\Omega-2)+i\Gamma} \qquad (5)$$

which has the standard form of a Breit-Wigner resonance. Resonances occur when $\Omega = 2$ and $|\Gamma|$ gives the width (and strength) of the resonance. The voltage-current vector at the entrance to element n+1 is, then, given by

$$A_n = (M_n|D_n)(M_{n-1}|D_{n-1}) \cdots\cdots (M_2|D_2)(M_1|D_1)A_o . \qquad (6)$$

The formulation is entirely analogous to that for the off-momentum (dispersed) closed-orbit given in FN-232, except here all quantities are, in general, complex. The resonance modes here are equivalent to integral resonances for orbits.

The analogy of the formulation can be carried further if the ring is electrically divided into N identical sectors each having the propagation-source matrix $(\overline{M}|\overline{D})$. All around the ring we, then, have

$$(M|D) = (\overline{M}|\overline{D})^N = \left(\overline{M}^N \,|\, (1-\overline{M}^N)(1-\overline{M})^{-1}\overline{D}\right) \qquad (7)$$

and from Eq. (4)

$$A_o = (1-M)^{-1} D = (1-\overline{M}^N)^{-1}(1-\overline{M}^N)(1-\overline{M})^{-1}\,\overline{D}$$

$$= (1-\overline{M})^{-1}\,\overline{D} = \frac{(\overline{M}^{-1}-1)\overline{D}}{(\overline{\Omega}-2)+i\overline{\Gamma}} \qquad (8)$$

where $\mathrm{Tr}\overline{M} \equiv \overline{\Omega}+i\overline{\Gamma}$. As in the case of particle orbit the number of (intrinsic) resonances is greatly reduced by sectorizing the ring. With identical sectors symmetry requires the ring to behave like a much smaller ring one sector in circumference and the resonance condition is applied to each sector instead of to the entire ring.

To proceed we shall parametrize the 2x2 unimodular matrix $\overline{M}$ in the usual manner

$$\overline{M} = \cos\mu + J\sin\mu. \qquad (9)$$

Then

$$M = \overline{M}^N = \cos N\mu + J\sin N\mu. \qquad (10)$$

So defined μ is the "propagation constant" (phase advance per sector) and J is the "impedance matrix." When $\alpha = 0$, $J = \begin{pmatrix} 0 & \beta \\ -\frac{1}{\beta} & 0 \end{pmatrix}$,

β is the "characteristic impedance." Eq. (8), then gives

$$A_o = \frac{1-\overline{M}^{-1}}{2-\mathrm{Tr}\overline{M}}\,\overline{D}$$

$$= \frac{1}{2\sin\frac{\mu}{2}}\left(\sin\frac{\mu}{2} + J\cos\frac{\mu}{2}\right)\overline{D} \qquad (11)$$

and

$$A_n \equiv \overline{M}^n A_o = \frac{1}{2\sin\frac{\mu}{2}}\left[-\sin\left(n-\frac{1}{2}\right)\mu + J\cos\left(n-\frac{1}{2}\right)\mu\right]\overline{D}. \qquad (12)$$

The effects of the power-supply source vectors can be linearly superposed. It is, therefore, instructive to look at the effect of one source vector D. In this case $(M|D) = (\overline{M}^N|D)$ and Eq. (4) gives

$$A_o = \frac{1-\overline{M}^{-N}}{2-\mathrm{Tr}(\overline{M}^N)}\,D$$

$$= \frac{1}{2\sin\frac{N}{2}\mu}\left(\sin\frac{N}{2}\mu + J\cos\frac{N}{2}\mu\right)D \qquad (13)$$

and

$$A_n \equiv \overline{M}^n A_o$$

$$= \frac{1}{2\sin\frac{N}{2}\mu}\left[\sin\left(\frac{N}{2}-n\right)\mu + J\cos\left(\frac{N}{2}-n\right)\mu\right]D. \qquad (14)$$

In general, $\mu \equiv a-ib$ is complex and resonances occur at $\mathrm{Re}\left(\sin\frac{N}{2}\mu\right) = 0$ or

$$\frac{N}{2}a = k\pi \qquad k = \text{integer} \qquad (15)$$

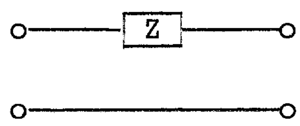
with the width

$$|\Gamma| = 2\left|\mathrm{Im}\left(\sin\frac{N}{2}\mu\right)\right| = 2\left|\sinh\frac{N}{2}b\right|. \tag{16}$$

PROPAGATION MATRICES

For the 2x2 propagation matrix the basic elements are

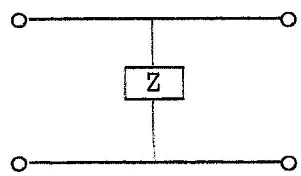
a. "Drift Space"



For this

$$M = \begin{pmatrix} 1 & -Z \\ 0 & 1 \end{pmatrix} \tag{17}$$

b. "Thin Lens"

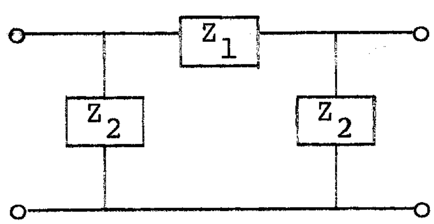


For this

$$M = \begin{pmatrix} 1 & 0 \\ -\frac{1}{Z} & 1 \end{pmatrix} \tag{18}$$

Both the magnet and the power supply can be represented by either the

a. Symmetric π



for which

$$M = \begin{pmatrix} 1 & 0 \\ -\frac{1}{Z_2} & 1 \end{pmatrix} \begin{pmatrix} 1 & -Z_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{Z_2} & 1 \end{pmatrix}$$

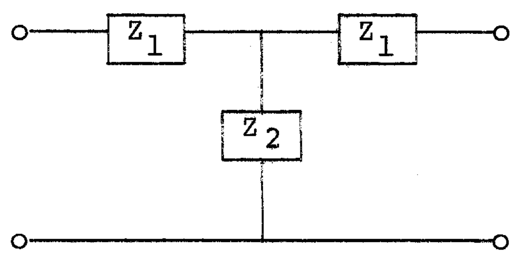
$$= \begin{pmatrix} 1+\frac{Z_1}{Z_2} & -Z_1 \\ -\frac{1}{Z_2}\left(2+\frac{Z_1}{Z_2}\right) & 1+\frac{Z_1}{Z_2} \end{pmatrix} \equiv \begin{pmatrix} \cos\mu & \beta \sin\mu \\ -\frac{1}{\beta}\sin\mu & \cos\mu \end{pmatrix} \qquad (19)$$

where

$$\begin{cases} \cos\mu = 1+\frac{Z_1}{Z_2} \\ \beta = iZ_2 \left[\frac{Z_1}{Z_2} \Big/ \left(2+\frac{Z_1}{Z_2}\right)\right]^{\frac{1}{2}} \end{cases} \qquad (20)$$

or the

b. Symmetric T



for which

$$M = \begin{pmatrix} 1 & -Z_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{Z_2} & 1 \end{pmatrix} \begin{pmatrix} 1 & -Z_1 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+\frac{Z_1}{Z_2} & -Z_1\left(2+\frac{Z_1}{Z_2}\right) \\ -\frac{1}{Z_2} & 1+\frac{Z_1}{Z_2} \end{pmatrix} \equiv \begin{pmatrix} \cos\mu & \beta\sin\mu \\ \frac{1}{\beta}\sin\mu & \cos\mu \end{pmatrix} \tag{21}$$

where

$$\begin{aligned} \cos\mu &= 1+\frac{Z_1}{Z_2} \\ \beta &= iZ_2\left[\frac{Z_1}{Z_2}\left(2+\frac{Z_1}{Z_2}\right)\right]^{\frac{1}{2}} \end{aligned} \tag{22}$$

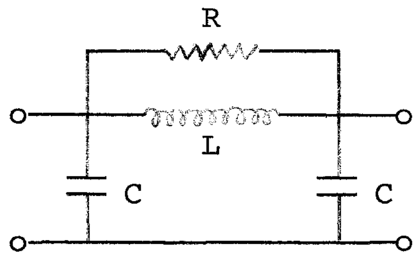
In both cases if $\mu \equiv a-ib$ we have

$$\begin{cases} \cos a = \left|1+\frac{Z_1}{2Z_2}\right| - \left|\frac{Z_1}{2Z_2}\right| \\ \cosh b = \left|1+\frac{Z_1}{2Z_2}\right| + \left|\frac{Z_1}{2Z_2}\right| \end{cases} \tag{23}$$

where $|\ |$ denotes the magnitude of a complex number.

CRUDE APPROXIMATION FOR MAIN RING

The frequency dependencies of Z_1 and Z_2 for a magnet are rather complicated but for a given frequency a main ring bending magnet can be approximated by the network



where 2C is the capacitance-to-ground of the coil, L is the inductance of the coil (resistance of the coil is neglected), and R represents the dissipation by eddy currents in the magnet laminations and the vacuum chamber plus the 10Ω damping resistor (see TM-325 by S. C. Snowdon). We shall assume that all magnets (B1 and B2) are identical with average circuit-parameter values and all power supplies have 2x2 propagation matrices equal to unit (short-circuit). Thus, each magnet is a sector and we have $N = 774$. This crude approximation will not properly describe the structure-dependent features such as the resonances. But it should be a fairly good approximation for structure-insensitive characteristics away from resonances such as the propagation constant μ and the characteristic impedance β, provided the average values of the circuit parameters are chosen properly.

We investigate only the effect of one power supply producing a peak-to-peak voltage jump of $2V_o$. In this case Eq. (14) gives

$$\begin{cases} V_n = \dfrac{V_o}{\sin\frac{N}{2}\mu} \sin\left(\frac{N}{2}-n\right)\mu \\[2ex] I_n = -\dfrac{V_o}{\beta\sin\frac{N}{2}\mu} \cos\left(\frac{N}{2}-n\right)\mu \end{cases} \qquad (24)$$

with μ and β given by Eqs. (20) and (23) and

$$Z_1 = \frac{i\omega L}{1+i\frac{\omega L}{R}} \qquad Z_2 = \frac{1}{i\omega C}. \tag{25}$$

For main-ring bending magnets the crude approximate numbers at 720 Hz ($\omega = 4.52\text{x}10^3\ \text{sec}^{-1}$) are

$$L = 6\text{x}10^{-3}\ \text{H} \qquad C = 3\text{x}10^{-8}\text{F} \qquad \frac{N}{2} = 387$$

$$R = 8\Omega\ (10\Omega \text{ and } 40\Omega \text{ in parallel}).$$

These values give

$$\mu = a-ib = .0365 - .0273\, i$$

and

$$\beta = (168\Omega)\ e^{-0.642\, i}.$$

For $\frac{N}{2} - n \gtrsim 100$ (within about 300 magnets on either side of the power supply) we have

$$\sin\left(\frac{N}{2}-n\right)\mu \cong \frac{1}{2i}\ e^{i\left(\frac{N}{2}-n\right)\mu}$$

$$\cos\left(\frac{N}{2}-n\right)\mu \cong \frac{1}{2}\ e^{i\left(\frac{N}{2}-n\right)\mu}$$

and we also have

$$\sin\frac{N}{2}\mu \cong \frac{1}{2i}\ e^{i\frac{N}{2}\mu}.$$

Eq. (24) then gives

$$\left\{\begin{aligned} V_n e^{i\omega t} &= V_o e^{i(\omega t-\mu n)} = V_o e^{bn}\ e^{i(\omega t-an)} \\ &= V_o e^{-0.0273n}\ e^{i(4524t-0.0365n)} \\ I_n e^{i\omega t} &= -i\ \frac{V_o}{\beta}\ e^{i(\omega t-\mu n)} = -i\ \frac{V_o}{\beta}\ e^{bn}\ e^{i(\omega t-an)} \\ &= \frac{V_o}{(168\Omega)e^{0.928i}}\ e^{-0.0273n}\ e^{i(4524t-0.0365n)}. \end{aligned}\right. \tag{26}$$

We see that both V and I are attenuated waves propagating away from the supply with the phase velocity

$$\frac{\omega}{a} = 1.24\text{x}10^5 \text{ magnets/sec} = 160 \text{ turns/sec}$$

and the attenuation rate

$$\frac{1}{e}\text{-folding in } \frac{1}{0.0273} = 36.6 \text{ magnets,}$$

and that the characteristic impedance of the ring is $(168\Omega)e^{0.928\,i}$.

To study in detail the resonance characteristics of the main ring, first one has to obtain the "circuit parameters" individually for different types of magnets and power supplies and for all frequencies. One can then compute the voltage-current vectors around the ring as given by Eqs. (5) and (6) using a computer.