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AN INTERACTIVE RAY TRACING PROGRAM

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A quasi-ray-tracing program named TURTLE (trace unlimited rays through lumped elements) has been written and is available on the NCN commercial time sharing service. Its usefulness is in determining the higher order aberrations in a beam line given a first order solution. The format of the input data is similar to that of TRANSPORT and most of the TRANSPORT elements suitable for ray tracing programs are available in TURTLE. Below are explained the theory behind the program and the procedure for its use.

I. Theory of Ray Tracing Through Lumped Elements

A true ray tracing program computes the trajectory of a particle through a magnetic field directly. It therefore does not distinguish between different order effects on a ray, but would represent all orders to the accuracy of the numerical integration of the equations of motion. However, if one is interested in tracing a large number of rays through a complicated system, this procedure can prove both time consuming and unnecessary.

A considerable simplification can be achieved by treating each element as lumped and evaluating its effect by a transformation which gives the exit ray directly from the entrance ray. Such a transformation would correspond roughly to the use of the first and second order transform matrices calculated by the program TRANSPORT. However, unlike the case with TRANSPORT, there are several good reasons for considering the results of TURTLE to be good to all orders for typical NAL beams.

1) Transfer matrix elements for quadrupoles and sextupoles are evaluated directly for each ray from the actual momentum of the ray, and are, therefore, exact to all orders in chromatic effects. Chromatic effects are evaluated only to second order in bending magnets, but since the chromatic effects (as contrasted with first order dispersive effects) of bending magnets are small, the higher order terms may be considered to be negligible.

2) Due to the small phase space of the beams, the third and higher geometric effects of a given element will also be small. Higher order aberrations will occur primarily as successive influences of lower order teams, an effect not included in TRANSPORT.

3) The high energies of the NAL beams necessarily imply large lever arms for aberrations, thus enhancing the production of higher order aberrations due to cumulative effect of lower order ones.

4) As the energy of beams increases, typically the number rather than the size of magnets is increased. Therefore, the lumped element approximation represents a finer structure relative to the entire beam line and we come closer to the numerical integration of a true ray tracing program.

In the respect that it does not distinguish among effects of different orders, TURTLE resembles a true ray tracing program. It therefore does not produce transfer matrices or beam envelopes. However, by giving up some of the versatility of TRANSPORT or other programs, much is gained in efficiency. A typical second order TRANSPORT run for the NAL high energy high resolution beam takes about eight minutes on the IBM 360/75 at Argonne. To run a given ray through the same beam line with TURTLE, evaluating the effect of each element as we progress, takes but a few seconds. This makes this program ideal for such things as sextupole filling. The reasons for such an increase in speed will be elucidated below.

II. Use of the Program TURTLE

Data for use of the program is input in two data sets-one for the beam line and another for the set of rays one is interested in tracing. Each of these data sets may either be stored as a data file, or input from the teletype. This way one can have a fixed beam line and run different sets of rays through it, depending on what effect one wished to investigate. Conversely one can run the same set of rays through different beam lines.

The program is currently on the NCN commercial time sharing service and is activated by the command LRUN TURTLE. It then prints the message

INPUT UNITS FOR BEAM LINE AND RAYS!

One then enters two integers on the same line and presses carriage return. Unit 9 is the teletype and 1 through 8 are for accessing stored files. A typical choice would be 1,9 where the beam line is in a stored file and the initial rays are to be entered via the teletype. If both data sets are stored, files a typical choice might be 1,2.

The beam line is read first. If the unit specified is 1 through 8, the teletype will print

DEFINE FILE(S)
N =

where N is the unit number you specified. You then type the name of the disk file containing the beam line and it is read directly from the disk. If the unit specified is 9, the program will type out the message

INPUT BEAM LINE
!

Following the exclamation, one types in the data for the first element, followed by a carriage return. Another exclamation will appear and one types in all remaining elements in a similar manner.

Each element is specified by four parameters separated by commas. There is a limit of 200 elements. Their format is integer, floating point, floating point, integer. The first quantity is the type code of the element, and is the same as that used in TRANSPORT. In the explanation of each element below these quantities will be referred to as I1, FP2, FP3, I4. At the end of any element which actually occupies physical space, I4 is a print command. If it is equal to 1, the coordinates of the ray are printed out.

Type Code 3 - Drift Space

For a drift space FP2 is the length, FP3 is irrelevant and I4 is a print control. The transform for a drift space can be found in the report SLAC-75, as can all transformations not given here.

Type Code 4 - Bending Magnets

Zero gradient rectangular bending magnets are assumed by the program. The results will agree with those of TRANSPORT provided that in using TRANSPORT one employs the pole face rotations (type code 2.0) to simulate a rectangular magnet. This simplifying assumption about the bending magnets is the primary reason for a gain in speed over a second order TRANSPORT run. The first and second order transfer matrix elements for a zero gradient rectangular magnet are given in Appendix A.

Chromatic effects are included in the transfer matrix by setting

$$R_{ij} = R^{o}_{ij} + T_{ij6} \frac{\Delta p}{p}$$

where R^{o}_{ij} are the on-momentum first order matrices. If second order geometric effects are desired, one inserts a type 17 element at the beginning of the beam line. Then the ray is also affected by T_{ijk} elements where j and k are both spatial indices.

The length of the magnet is FP2, its field is FP3, and I4 is a print control. A positive field is a bend to the right.

<u>Type Code 5 - Quadrupole</u>

The quadrupole transfer matrix is as given in SLAC-75. Here we evaluate the matrix elements for the actual momentum of the particle, thus eliminating the need for higher order chromatic matrix elements. The length of a quadrupole is FP2, its gradient FP3, and I4 is a print control. Positive gradient means a horizontally focussing quad.

<u>Type Code 6 - Slit</u>

In this case I4 indicates the orientation of the slit - 1 for a horizontal slit and 3 for a vertical slit. FP2 is the half width. When a ray hits a slit, a message is printed out along with the distance along the beam line at which the slit occurs. The program then proceeds to the next ray.

<u>Type Code 15 - Units</u>

Normal units for this program are meters for lengths, centimeters for x and y coordinates, milliradians for angles, kilogauss for field, GeV/c for momentum, and percent for $\Delta p/p$. If one wishes to change units I4 indicates which unit one wishes to change. FP2 gives the size of the new unit, in terms of the old. Below is a table of codes for unit changes

I4	Quantity	Standard Unit
1	Transverse dimensions	cm
2	Transverse angles	mr
6	Momentum spread	%
8	Longitudinal length	m
9	Magnetic fields	kg
11	Momentum	GeV/c

Type Code 16 - Multipole Aberrations for Quadrupoles

Multipole aberrations up to and including dodecapole may be represented. A 2n-pole is represented by setting I4 equal to n. FP2 is equal to B/a^{n-1} where B is the pole tip field of a given multipole component and a is the aperture. FP3 is a phase angle, equal to the angle the field must be rotated from a midplane symmetric configuration. With positive z pointing along the beam line, positive x to the left, and positive y up the rotation is in a positive sense around the z axis.

The aberration is considered to be lumped at the center of the quad. A ray is traced half way through a quad, affected by the multipole, then traced the remainder of the way through.

Any 16 element applies to all quadrupoles after it. Since the field of the multipole is given, quads with the same relative multipole strength but different excitations require different 16 elements to precede them.

Type Code 17 - Second Order Geometric Aberrations

In the normal mode, only the chromatic effects are calculated to orders higher than first. The inclusion of a

17 type element initiates the calculation of the effect of higher order geometric aberrations. These aberrations occur only for sextupoles and bending magnets. The multipole aberrations specified by the 16 type code are not affected.

Type Code 18 - Sextupole

Unless a 17 type code precedes this element, it has the same effect as a drift space. The sextupole transfer matrix elements are given in SLAC-75, save that in TURTLE they are calculated using the actual particle momentum. FP2 is the length, FP3 is B_o/a^2, and I4 is a print control.

Type Code 20 - Coordinate Rotation

Coordinate rotation is in a positive sense about the z axis. FP2 is the angle of rotation in degrees, and I4 is a print control.

Type 73 - End of Data

In keeping with TRAN2, use a type code higher than 21 signals the end of the beam line. We now proceed to ray tracing.

Tracing Rays Through the Beam Line

Having set up our beam line, we now proceed to trace rays through it. The first line of data is the number of rays to be run. If this is unknown, as in the case one wishes to input rays from the teletype and explore properties of the system, this number can be set as high as one wishes and execution then terminated with a control P. There is no limit to the number of rays one can look at since the rays are treated one at a time. The second line of data is the central momentum of the beam. All subsequent lines are the coordinates of the initial rays. Each line contains five floating point quantities, separated by commas, which are x, x', y, y', and $\Delta p/p$ for the given ray.

If the rays are to be read from a saved file, the teletype will ask for a file name. It will then take each ray, number it, and type out its coordinates at the initial point and at all locations where a print control is given. If the rays are input from the teletype, it will ask for the number of rays and the central momentum. It then writes

RAY NO 1!

at which time you print in the initial coordinates of the ray you want traced. It then traces the ray through the system, printing out its coordinates as requested. When done, it proceeds to the next ray and treats it in a similar fashion.

Changing the Data File(s)

It is often convenient to store the data for the beam line in a file. To correct errors or make changes in the data, one can use the NCN text editor. This is made much easier by use of the command ALN to add line numbers to the data set. One can then use the LOCATE command to determine the line numbers one is interested in. (For example, to find all the lines containing sixteen elements, one would type in L /16/ *.) After appropriate editing one types in DLN to delete the line numbers and obtains a usable data file.

SUMMARY OF ELEMENTS

Element	Type Code T1	FP2	FP3	I4
Drift Space	3	Length	---	Print Control
Bending Magnet	4	Length	Field	Print Control
Quadrupole	5	Length	Gradient	Print Control
Slit	6	Half Width	---	Orientation of Slit
Units Change	15	Size of New Unit	---	Indicates Unit
Multipoles for Quads	16	B_n/a^{n-1}	---	Multipole No.
Second Order	17	---	---	---
Sextupole	18	Length	B_o/a^2	Print Control
Rotation	20	Angle in Degrees	---	Print Control

APPENDIX A

First and second order transfer matrix elements for a rectangular, zero gradient bending magnet.

t = length of trajectory in magnet

$\rho = 1/h = P/eB$ = radius of curvature of trajectory

$R_{11} = 1$

$R_{12} = \rho \sin ht$

$R_{16} = \rho\,(1 - \cos ht)$

$T_{112} = 2 \tan \frac{1}{2}ht$

$T_{116} = 2 \tan^2 \frac{1}{2}ht$

$T_{122} = \frac{1}{2}\,\rho\,(1 - \cos ht)$

$T_{126} = 2\,\rho\,(1 - \cos ht\,) \tan \frac{1}{2}ht$

$T_{133} = h^2 t \tan \frac{1}{2}ht\,(1 - \frac{1}{2}ht \tan \frac{1}{2}ht) \sec^2 \frac{1}{2}ht$

$T_{134} = -ht \sec^2 \frac{1}{2}ht\,(1 - ht \tan \frac{1}{2}ht)$

$T_{144} = -\frac{1}{2}\rho\,(1 - \cos ht) - \frac{1}{2}ht^2 \sec^2 \frac{1}{2}ht$

$T_{166} = -\,2\rho \cos ht \tan^2 \frac{1}{2}ht$

$R_{21} = 0$

$R_{22} = 1$

$R_{26} = 2 \tan \frac{1}{2}ht$

$T_{222} = -\tan \frac{1}{2}ht$

$T_{226} = -\,2 \tan^2 \frac{1}{2}ht$

$T_{266} = -\,2 \tan \frac{1}{2}ht \sec^2 \frac{1}{2}ht$

$T_{233} = -\,2\,h^2 \tan^3 \frac{1}{2}ht + 2h^3 t \tan^4 \frac{1}{2}ht - \frac{1}{2}h^4 t^2 \tan^5 \frac{1}{2}ht$

$T_{234} = 2\,h \tan^2 \frac{1}{2}ht - 3h^2 t \tan^3 \frac{1}{2}ht + h^3 t^2 \tan^4 \frac{1}{2}ht$

$T_{244} = -\tan \frac{1}{2}ht + ht \tan^2 \frac{1}{2}ht - \frac{1}{2}h^2 t^2 \tan^3 \frac{1}{2}ht$

$R_{33} = 1 - ht \tan \frac{1}{2}ht$

$R_{34} = t$

$T_{313} = -2h \tan^2 \frac{1}{2}ht + h^2 t \tan^3 \frac{1}{2}ht$

$T_{314} = \sin ht - ht \tan \frac{1}{2}ht$

$T_{323} = -ht\,(1 + \cos ht \tan^2 \frac{1}{2}ht) - 2\,(1 - \cos ht) \tan \frac{1}{2}ht$

$T_{324} = \rho\,(1 - \cos ht) - t\,(1 - \cos ht) \tan \frac{1}{2}ht$

$T_{336} = (1 - \cos ht) \left[1 - \tan \frac{1}{2}ht\,(1 - ht \tan \frac{1}{2}ht)\right]$

$T_{346} = -\rho \sin ht + t \left[1 - \tan \frac{1}{2}ht\,(1 - \cos ht)\right]$

$R_{43} = -2h \tan \frac{1}{2}ht + h^2 t \tan^2 \frac{1}{2}ht$

$R_{44} = 1 - ht \tan \frac{1}{2}ht$

$T_{414} = -h \tan \frac{1}{2}ht \sin ht$

$T_{424} = ht \sec^2 \frac{1}{2}ht$

$T_{436} = 4h \tan \frac{1}{2}ht - h^2 t \tan^2 \frac{1}{2}ht - h^2 t \sec^4 \frac{1}{2}ht\,(1 - \cos ht)$

$T_{446} = 2 \tan^2 \frac{1}{2}ht + 2ht \tan \frac{1}{2}ht \sec^2 \frac{1}{2}ht$