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MODIFIED PHASE ACCEPTANCE IN LINACS

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Purpose

To extend the calculation presented by Gluckstern[1] to obtain the phase width of the acceptance area of the synchronous energy.

Calculation

Gluckstern[1] gives as the relation for the modified phase acceptance boundary:

$$y^2(x) = f(x) + \frac{5\epsilon}{2}\int_x^2 y(u)\,du \tag{1}$$

where

$$f(x) = (2 - x)^2 (1 + x). \tag{2}$$

His approximate analytical solutions do not cover the range $x < -1$.

To obtain these differentiate Eq. (1):

$$\left(2\frac{dy}{dx} + \frac{5}{2}\epsilon\right) y \cong f'(x). \tag{3}$$

Near $y = 0$ and $x = -1$ one expects that $2dy/dx$ is quite large compared with $5\epsilon/2$. Hence, in this range

$$2y\frac{dy}{dx} \cong f'(x) \tag{4}$$

or

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$$y^2 = f(x) + \text{constant}. \tag{5}$$

The constant may be calculated using Gluckstern's solution for $x \to -1$. Thus,

$$y^2(x) = y^2(-1) + f(x). \tag{6}$$

At the intercept $y = 0$ one has

$$(2-x)^2(1 + x) = -y^2(-1) = -6\ 3\epsilon. \tag{7}$$

To first order in ϵ the left-hand intercept is

$$x = -1 - \frac{2\sqrt{3}}{3}\epsilon. \tag{8}$$

Gluckstern[1] gives $x = 2$ for the right-hand intercept. Hence the phase width at the synchronous energy is

$$\Delta\phi = \left[2 - \left(-1 - \frac{2\sqrt{3}}{3}\epsilon\right)\right] \cdot \left|\phi_s\right|. \tag{9}$$

or

$$\Delta\phi = \left(3 + \frac{2\sqrt{3}}{3}\epsilon\right) \cdot \left|\phi_s\right|. \tag{10}$$

Results

Gluckstern[1] gives

$$\epsilon \cong 0.83\sqrt{\frac{\Delta\beta}{\beta_s^i\left|\phi_s\right|}}, \tag{11}$$

where $\Delta\beta$ is the change in β per cell near injection. Using a PARMILA run, one finds

$$\begin{aligned} \Delta\beta &= 0.0015 \\ \beta_s^i &= 0.0413 \\ \phi_s &= -32°. \end{aligned} \tag{12}$$

Thus,

$$\epsilon = 0.21 \tag{13}$$

and

$$\Delta\phi = 103.76°. \tag{14}$$

Correction to Account for Proper Initial Bucket Width

Gluckstern[1] approximates the initial bucket width as $\Delta\phi = 3|\phi_s|$. To remove this approximation, one needs to employ the theory of Symon and Sessler.[2] For $\phi_s = -32°$ (Symon and Sessler use $90° - 32° = 58°$) the initial bucket width is

$$\Delta\phi = 122° - 24.93° = 97.07° \tag{15}$$

which is to be compared with the approximate value used by Gluckstern.[1]

$$\Delta\phi = 3 \times 32° = 96°. \tag{16}$$

Hence, a correction of $97.07 - 96° = 1.07°$ should be added to Gluckstern's estimate of the bucket width. Thus, for $\phi_s = -32°$

$$\Delta\phi = 103.76 + 1.07 = 104.83°. \tag{17}$$

REFERENCES

[1] R. L. Gluckstern, Phase Acceptance in Linacs, Proceedings 6th International Conference on High Energy Accelerators, Cambridge (1967), p. 153; see also, Linear Accelerators, P. M. Lapostolle and A. L. Septier, Editors; North Holland Publishing Company, Amsterdam (1970), p. 797.

[2] K. R. Symon and A. M. Sessler, Methods of Radio Frequency Acceleration in Fixed Field Accelerators with Applications to High Current and Intersecting Beam Accelerators, MURA-106 (1956). For numerical evaluations, see I. Gumowski, CPS RF-Bucket, Width, Height, and Area, CERN MPS/Int. RF 67-1.