TM-268
2660.2

SPECULATIONS ON THE PRODUCTION OF QUARKS AND MONOPOLES

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I. Introduction

There have been extensive and unsuccessful searches made for both the quark and the Dirac monopole. The quark is sought because its existence would provide us with a simple conceptual picture of the structure of the elementary particles, while the search for the monopole is motivated by more aesthetic reasons (primarily because its existence would introduce a new symmetry into the Maxwell equations). More recently, the dyon (a particle carrying both magnetic charge and a fractional electric charge) has been proposed which would fill both of the above roles and adds a third type of basic, as yet unobserved, particle to the list.

One thing that all of these particles have in common is that we stand in total ignorance of the method by which they might be produced. Thus, the design of search experiments and the interpretation of a null result becomes extremely difficult. It is the purpose of this report to make some attempt to alleviate this problem by assuming a production mechanism and examining the consequences of this assumption on quark and monopole search experiments.

II. Presentation of the Model

The model which we shall use to make estimates of various properties of the production of new particles has already been presented by Hagedorn[1,3] and Hagedorn and Ranft,[2] and is based on a statistical thermodynamical approach to the strong interactions. In this model, one pictures the production process as taking place through the conversion of the kinetic energy into "thermal" energy--i.e., into heating up the hadronic matter which makes up both the projectile and the target. In the highly excited fireball which results from the collision, all possible particles and resonances can exist. If we assume that each fireball is, in turn, composed of other fireballs, and that what are normally called particles and resonances are also made up of the same sort of combination of particles and fireballs, (i.e. that the mass spectrum of the fireballs is identical to the mass spectrum of the particles), we find that two interesting results follow immediately from standard statistical mechanical arguments. These are:

1. at high energies, the density of particle states approaches $m^{-5/2}\ e^{m/T_o}$
2. there is a highest temperature, T_o, which the hadronic matter approaches asymptotically, and above which the matter will not go.

Since this second property will have important consequences for what follows, we note that there is a simple physical interpretation of this limiting temperature. When energy is added to a system of particles, it can either raise the kinetic energy (change the temperature) or create new particles. When the number of possible new particles to be created grows very rapidly (see point 1 above), one approaches a situation in which all the increase in energy goes into the creation of new particles, and there is no increase in kinetic energy. This corresponds to reaching the temperature T_o. From the distribution of large angle elastic p-p scattering, it is found that[1]

$$T_o = 160 \text{ MeV} \quad (1)$$

From this simple model, one can calculate the number of particles of mass m and momentum between p and p+dp in the excited hadronic matter. It turns out to be

$$\nu(p,m)d^3p = \frac{\Delta V}{(2\pi)^3}\, z\, \frac{d^3p}{\exp\left(\frac{\sqrt{p^2+m^2}}{T}\right)\pm 1} \quad (2)$$

where the plus (minus) sign goes with Bose (Fermi) statistics. The factor z=2j+1 takes account of the spin states allowed for a particle of spin j, but from this point on we will set z=1 for convenience. Also, we note that for heavy particles like the quark and the monopole, the factor ±1 in the above is insignificant, so that we will drop it as well.

The factor ΔV requires some further discussion. This is the volume over which thermodynamic equilibrium is established, so that the methods of statistical mechanics can be applied. It is, in principle, not known and must be determined experimentally, although it should not be wildly different from the volume of the hadrons themselves. To handle the problem of the volume correctly, one should look only at an infinitesimal volume element, where equilibrium at some temperature T can be assumed, and then sum over all such volume elements in the two colliding hadrons, taking into account the fact that each element may have different temperatures at different times. In fact, when detailed fits to particle production data are made in Ref. (2), this is what is done. For our purposes, however, such refinements are not necessary, since we seek only the general features of the production of the new types of particles. Hence we shall assume that ΔV is a large volume which has been raised to a temperature T_o in the collision, and that the magnitude of ΔV is the same as that derived in Ref. (3) from considerations of multiplicity in high energy collisions, namely

$$\Delta V = 0.4\ V_o \tag{3}$$

where $V_o = \frac{4\pi}{3} m_\pi^{-3}$ is the "natural" volume over which one might expect thermodynamic equilibrium to be established.

In the production of quarks, with their fractional charge, and monopoles, with their magnetic charge, a conservation law, hitherto not invoked, must be put into the model. This is the statement that such particles must be produced in particle-antiparticle pairs, so that electrical and magnetic charge can be conserved. It is shown in Ref (3) that the number of particles of momentum p, and antiparticles with momentum p_2 in an initial fireball is just

$$\nu(p_1 p,_2)\ d^3p,\ d^3p_2 = \frac{(\Delta V)^2}{(2\pi)^6}\ z^2\ e^{-\frac{\sqrt{p_1^2+m^2}}{T}}\ e^{-\frac{\sqrt{p^2_1+m^2}}{T}}\ d^3p, d^3p_2$$

$$= \nu(p_1,m)\quad \nu(p_2,m) \tag{4}$$

(This conclusion seems obvious, but a rather subtle proof is needed to establish it). This will be our basic working result when we go on to calculate the production of the new particle pairs.

A word is necessary at this point about the possibility that the quarks and monopoles might themselves have excited states, and therefore that processes might occur in which an excited quark or monopole is produced, and the final observed particle is the result of a chain of decays. We shall not allow this possibility in our calculations for a variety of reasons, which can be grouped roughly as physical, practical, and aesthetical.

Physically, the existence of excited states implies some sort of internal structure of the object in question. The excited states correspond to rearrangements of this internal structure. If the components of the structure themselves have structure (as in the case in nuclei, where the nucleon also has excited states) it usually takes much more energy to excite the components than to simply rearrange them. Thus, the possibility of excited states of quarks and monopoles affecting our results is probably greatly reduced by energy considerations.

Practically, Hagedorn[3] has shown that if we include the possibility of the quarks having a finite number of discrete excited states, the result in Eq. (4) is simply multiplied by a factor or order unity. Thus, even including such effects does not, change any of the conclusions which we draw.

Finally, there is the aesthetic point that if we wish to find quarks or monopoles because they would provide us with a simple picture of the structure of elementary particles, it certainly is unappealing to suppose that they themselves have structure, because one quickly is involved in an infinite regression. While it may turn out to be that this is what nature has in store for us, there does not seem to be any point in taking this option until we are forced to. Certainly such a step would be out of place in the type of preliminary calculations which are presented here.

III. Quark Momentum Spect

One of the standard means of searching for quarks is to look at a target through a spectrometer, setting the spectrometer in such a way that a normally charged particle with momentum up to that of the beam incident on the target will not get through. In this way, simple energy conservation insures that nothing will come down the spectrometer except particles with less than the normal charge. For example, if we had an incident beam of 200 GeV/c, and tuned the spectrometer to 210 GeV/c, quarks of charge 1/3 and momentum 70 GeV/c would come through. One can then logically ask whether this is a reasonable way to conduct the search in the light of our model for production, and whether there is any way of optimizing the chances of finding a quark through a judicious choice of experimental parameters.

The result in Eq. (4) can be used to predict the momentum distribution of quarks in the statistical model. Since we observe only one quark, we can integrate over the momentum of the other. We find

$$\int \nu(p_2, m)\, d^3p_2 = \frac{\Delta V}{2\pi} m^2 T \quad K_2\left(\frac{m}{T_o}\right) = \Delta V \left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T_o} \qquad (5)$$

where the second equality holds provided that m/T>>1, which is a good approximation for quarks.

Thus the probability of finding one quark with momentum p_1 is given by

$$\nu(p_1)\ d^3p_1 = \frac{(\Delta V)^2}{(2\pi)^3}\left(\frac{mT}{2\pi}\right)^{3/2} p_1^2\ e^{-\frac{\sqrt{p_1^2+m^2}}{T_o}}\ e^{\frac{-m}{T_o}}\ \Delta p\Delta\Omega \tag{6}$$

where $\Delta p\Delta\Omega$ is the momentum and solid angle bite of the apparatus in the c.m. system.

We shall consider two questions concerning this distribution: (1) where is maximum of the distribution and how wide is the spread for a given quark mass, and (2) for typical settings of a spectrometer, how many quarks per collision will be seen?

In considering the first of these, we note that the distribution will have a maximum when

$$p_1(\max) = 2\ T_o\ \left(1 + \ 1 + \left(\frac{m}{T_o}\right)^2\right) \tag{7}$$

If we consider only the normalized distribution

$$R = \frac{\nu(p_1)}{\nu\left(p_1(\max)\right)} \tag{8}$$

we see that it is independent of ΔV, and is therefore a slightly more reliable quantity than the distribution itself. In Fig. (1) we show the normalized distribution for various choices of the quark mass. We see that, as could have been expected, the distributions are fairly smooth, and exhibit no sharp peaks which might create difficulties in a quark search which is limited to a small momentum bite. In the figure we have defined

$$\xi = p_1/T_o \qquad (9)$$

for convenience, and scaled all energies and momenta in terms of the basic temperature.

Having established that there should be no sharp enhancements or suppressions of quark production in this model, we can now turn to the more interesting question of how many quarks one would expect to see coming down the spectrometer for typical spectrometer settings. It should be noted that the momenta p_1 and p_2 in Eq. (4) are defined in the rest frame of the fireball, which is, of course, the CM system. Thus, if we set the spectrometer for a lab momentum p_L, this would correspond to

$$p_1 = \frac{\gamma}{2p_L}\left[\left(\frac{p_L}{\gamma}\right)^2 - m^2\right] \qquad (10)$$

Defining the quark multiplicity M as

$$M = \frac{\nu(p_1)\ d^3p_1}{\Delta p \Delta \Omega} \qquad (11)$$

we can easily calculate the number of quarks per momentum and solid angle interval for various p_L and quark masses. The results of these calculations are shown in Table (1). For reference, Eq. (6) can easily be seen to hold for antiprotons as well as quarks (antiprotons must be created in conjunction with protons to conserve baryon number). Thus, the results for a mass of 1 GeV should be read as giving the multiplicity of antiprotons of the momentum inquestion, and the higher mass results interpreted accordingly.

We see that the number of quarks falls off exceedingly rapidly with the mass, and also that one expects more quarks at the lower lab momenta. In addition, in this model, we see that once we are past the threshold for the production of a quark-antiquark pair, nothing whatsoever is gained by increasing the momentum of the incident beam, since doing so requires an increase in p_L (in order that the spectrometer be set above the beam momentum), and hence produces a big drop in the cross section. This rather paradoxical result is a direct consequence of the existence of a maximum temperature. The energy which is added to the system by increasing the beam momentum does not go into making more quark pairs, but into making the large numbers of high mass normal particles whose threshold we passed in raising the momentum in the first place. Thus, it may be advantageous at times in making quark searches of the type described here to actually lower the momentum of the incident beam, working with the increased cross section, rather than using the highest available energies to get more cm energy.

IV. Production of Free Monopoles

In principle, everything which we have derived so far is valid for the production of magnetic monopoles as well as quarks. They, like quarks, must be produced as a particle-antiparticle pair, so that at first glance one might think that one could estimate their production cross section from Eq. (4). There is an effect, however, which makes the production of free monopoles somewhat more difficult.

In order for a particle to be seen in a search experiment, it must not only be produced, but it must be produced in such a way that it can get out of the interaction region. If the interactions between the two constituents of the pair is very strong and attractive, as would be the interaction between a Dirac pole and the anti-pole, then, even if the particles are copiously produced, it may well be that very few will ever get out of the interaction region and actually be seen in an experiment. This effect for monopoles has been discussed previously.[4,5] If we write the potential between a pole and an anti-pole as

$$V = g^2/r \tag{12}$$

then it is well known[6] that the coupling constant g is given by

$$g = \frac{n}{e} = 4\pi n\ (137) \tag{13}$$

where n is an integer. In the case where n=1 and the distance between the pole and anti-pole is 1 fermi, the potential energy becomes[4]

$$V \approx 20\ \text{GeV} \tag{14}$$

This is the minimum value of V for monopoles. Raising n or decreasing r can only raise it. Thus in order to be seen experimentally, the monopole pair must be created with at least 20 GeV of relative kinetic energy in order to overcome their mutual attraction. If we wished to find the total number of monopoles created per collision, we would integrate Eq. (4) not over all values of p_1 and p_2, but over those values of the momenta where the relative kinetic energy is greater than the value of V.

It should be noted that there is a certain ambiguity in the model as to the spatial location of the particles which are produced. However, we are really only interested in those particles which emerge from the interaction region ΔV, and the dimensions of this region are roughly one fermi. Thus the choice of the length in our evaluation of V is not entirely unreasonable.

If we denote the number of free monopoles created in a collision by N_f, then

$$N_f = \int_{K.E.>V} d^3p_1 \, d^3p_2 \quad \nu(p_1, p_2) \tag{15}$$

where KE denotes the relative kinetic energy between the monopoles. It is relatively simple to show that if the pair have momenta in the cm system of p_1 and p_2 respectively, and the angle between these momenta is denoted by θ, then

$$KE = \frac{E_1E_2 - p_1p_2 \cos\theta - m^2}{m} \tag{16}$$

so that, making use of the symmetry of the problem,

$$N_f = 8\pi^2 \int_{KE>V} p_1^2 p_2^2 \quad \nu(p_1, p_2) \, dp_1 \, dp_2 \, d(\cos\theta)$$

$$= \frac{(\Delta V)^2}{8\pi^4} \int_{KE>V} p_1^2 p_2^2 \quad e^{-\frac{(E_1+E_2)}{T}} \quad dp_1 \, dp_2 \, d(\cos\theta) \tag{17}$$

Before actually performing the integral (which, because of the messy kinematics, we will do numerically), a few observations might be in order. First, we note that for a given cm energy W, there will be a maximum V which can be overcome. From Eq. (16), we see that this occurs when

$$E_1 = E_2 = W/2$$
$$\cos\theta = -1 \tag{18}$$

at which point

$$V = W^2/4m \tag{18}$$

Thus for a 20 GeV cm energy system (which is what would obtain when a 200 GeV/c proton strikes a hydrogen target), there is a maximum value of m of about 5 GeV. Above this value of the mass, simple kinematics will not allow free monopole pairs to be made. Of course, there will be considerable suppression of the cross section for masses near 5GeV. At 30 GeV cm energy, this absolute maximum mass is about 11 GeV. Thus, in the event of a negative result, one must take this final state interaction effect into account in setting lower limits on the monopole mass.

Secondly, we note that although energy conservation is not built into the statistical model normally, we can take it into account (at least partially) by using the condition that

$$E_1 + E_2 < W \tag{19}$$

to put the upper limit on the integrals in Eq. (17).

If we now go ahead and do the integral numerically, we get the results shown in Table 2. We show the number of free monopoles produced and the total number (i.e., the result of Eq. (17) with no final state interaction) for various values of of the monopole mass and cm energy.

We see that the reduction in the number of monopoles is considerable, ranging from a factor of 10^{10} at low masses to 10^{20} at higher ones. We also note that there is no appreciable variation in this factor with W over NAL energies, so that once we are at a sufficiently high energy so that the thermodynamic model can be applied, and so that we are past the threshold for monopole production, there is nothing to be gained by going to high energies, particularly if one must sacrifice flux to do so.

Finally, we note that this reduction due to the pole-antipole attraction is, for the light masses, very close to the reduction predicted on the basis of a non-relativistic model,[4] even though we are dealing here with highly relativistic particles.

Discussion

This report has dealt with the production of quarks and monopoles in terms of a statistical-thermodynamic model of particle production. This model is interesting in that it allows us to calculate many qualities which we wish to know about the production processes for heavy particles, and it has enjoyed considerable success in dealing with the production of particles at a present accelerator energies.

However, it must be noted that the model which we are using has several defects, particularly for quark and dyon production. If one imagines elementary particles as being composed of either of these objects, then one can think of the production of the quark proceeding via a "stripping" reaction. This is an essentially peripheral process, and is not included in our model. If such processes were important, the mechanism for production by statistical processes would only set a lower limit on the production cross sections. One would presumably add the quarks produced in peripheral processes to the results quoted in this paper.

Second, the smallness of the quark and monopole production cross sections and the magnitude of the suppression due to final state interactions is directly related to the existence of a highest temperature, and to the fact that this temperature is rather small, being on the order of a pion mass. One could imagine modifying the model to allow for a much hotter central core in the fireball, in a region whose dimensions are set by the quark mass, and one could imagine the quarks and monopoles being produced primarily in this region. In this case the exponential suppression of the higher masses would be much less, since T_o would be higher, and one would see many more quarks and monopoles produced once theshold had been passed.

FIGURE AND TABLE CAPTIONS

Fig. 1 The distribution of quarks with momentum according to Eq (8). The curves are normalized to unity at their maximum value.

Table 1 The multiplicity of quarks as a function of quark mass and the laboratory momentum of the quark. The units are Gev/c for Δp and steradians for $\Delta\Omega$, both of which refer to c.m. and not lab quantities.

Table 2 The values of N_f calculated from Eq (17) for various monopole masses, and two different choices of the c.m. energy W. On the right hand column, we show the total number of monopoles which would be produced if we could neglect the final state interaction between the pole and the antipole. In all of these calculations, we have taken V = 20GeV, thereby overestimating the production rate.

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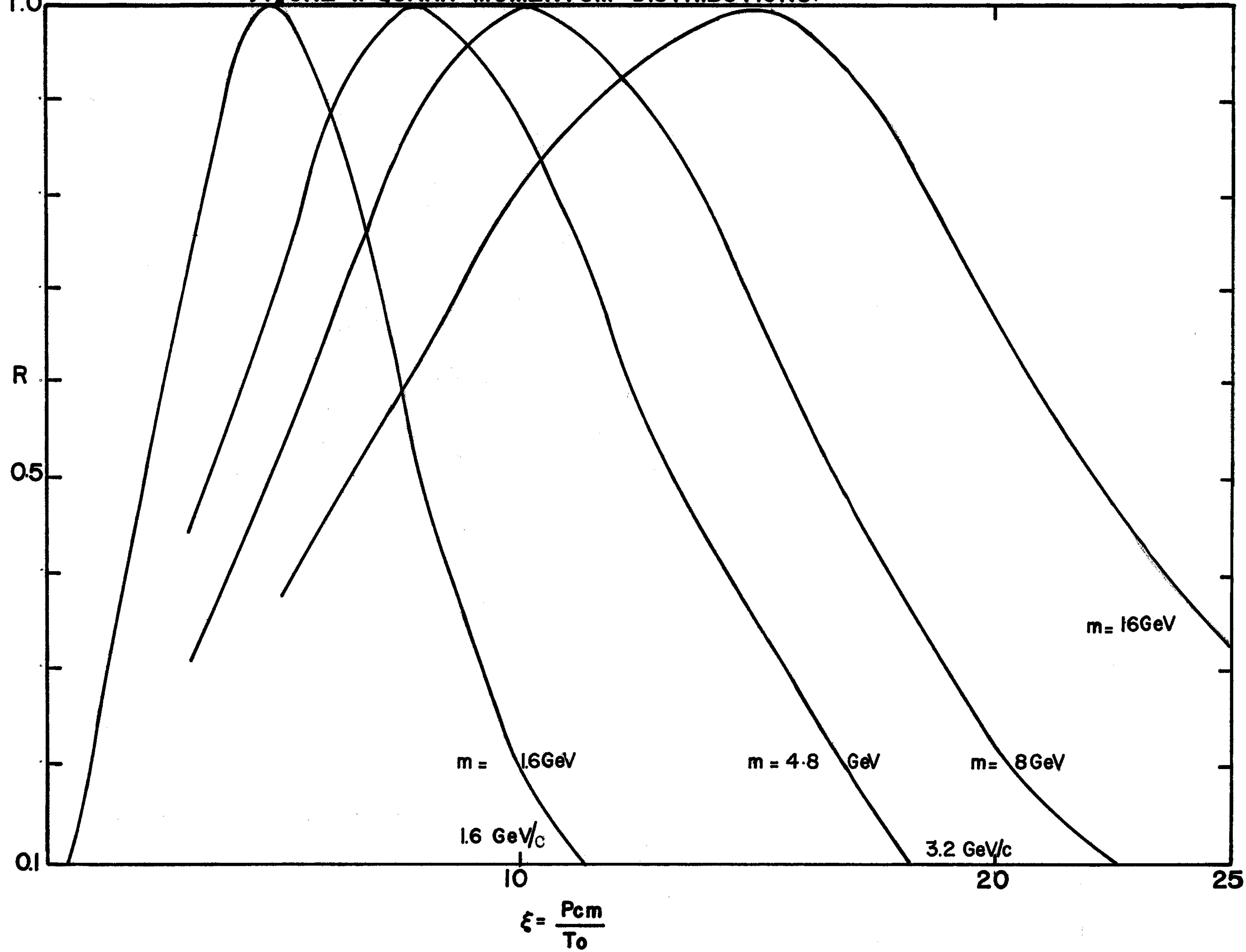
FIGURE 1. QUARK MOMENTUM DISTRIBUTIONS.



TM-268
2660.2

TABLE I.

$m \backslash P_{lab}$	30	40	50	60	70	80
1	5×10^{-11}	1×10^{-12}	4×10^{-14}	2×10^{-15}	1×10^{-16}	1×10^{-17}
2	2×10^{-17}	1×10^{-19}	1×10^{-21}	1×10^{-23}	3×10^{-25}	7×10^{-27}
3	7×10^{-23}	1×10^{-26}	3×10^{-28}	1×10^{-30}	1×10^{-32}	1×10^{-34}
4	6×10^{-28}	3×10^{-31}	4×10^{-34}	8×10^{-37}	3×10^{-39}	2×10^{-41}
5	8×10^{-33}	2×10^{-36}	1×10^{-40}	1×10^{-42}	3×10^{-45}	8×10^{-48}
6	2×10^{-37}	2×10^{-41}	7×10^{-45}	4×10^{-48}	5×10^{-51}	8×10^{-54}
7	5×10^{-42}	4×10^{-46}	6×10^{-50}	2×10^{-53}	1×10^{-56}	2×10^{-59}
8	1×10^{-46}	7×10^{-51}	8×10^{-55}	2×10^{-58}	7×10^{-62}	5×10^{-65}
9	4×10^{-51}	2×10^{-55}	1×10^{-59}	2×10^{-63}	5×10^{-67}	2×10^{-70}
10	1×10^{-55}	5×10^{-60}	3×10^{-64}	3×10^{-68}	5×10^{-72}	2×10^{-75}

TABLE II.

m(GeV)	N_f(W=30)	N_f(W=20)	N_{total}
1	3×10^{-14}	2×10^{-14}	2×10^{-4}
3	2×10^{-29}	3×10^{-29}	9×10^{-14}
5	9×10^{-42}	8×10^{-42}	6×10^{-24}
7	8×10^{-54}	0	2×10^{-35}
9	4×10^{-65}	0	6×10^{-45}