



NON HOMOGENEOUS MUON SHIELDING FLUX PERTURBATIONS  
DUE TO BENDING MAGNETS

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The effects of secondary beam magnets on muon fluxes outside a shield mainly designed for undeflected particles are estimated. The most important effect is the deflection of muons both away from the magnet and away from the beam direction. In this case the muons do not undergo compensating deflections from other parts of the magnet and they encounter the least amount of shielding material. This occurs when muons strike the return leg of a magnet.

Crude estimates of muon intensities may be obtained by representing the relevant part of a magnet as a single uniform field. The required input is then the dimensions of this region, field strength and a differential muon momentum spectrum at the magnet position. A typical layout is shown in Figure 1. If the magnet is not along a radius vector from the target ( $\beta \neq \alpha$ ), then along with muons striking the front, a significant number will enter the side of the magnet. These two contributions (front and side entrance) are treated separately.

If the distribution at the end of the decay space is called  $N(p, \alpha, \phi)$  where  $p \equiv$  momentum,  $\alpha, \phi \equiv$  polar, azimuthal angle of the muon in a frame with the beam as axis and the target as origin, then the distribution at the magnet position is given by  $N'(p', \alpha, \phi) \approx N(p - \Delta p, \alpha, \phi)$  where  $\Delta p$  is some average momentum loss in the hadron shield. This neglects multiple scattering in the hadron shield and slightly underestimates the momentum loss by very high and low momentum muons.  $N'(p', \alpha, \phi)$  could also be obtained from more detailed calculations which include these effects.

Front Entrance.

For the case of muons entering the front of the magnet they will be assumed to be normally incident (neglecting the difference between  $\alpha$  and  $\beta$ , see Figure 1) and uniformly distributed over the area of the return leg of the magnet. If the magnetic field is assumed to be perpendicular to the muon trajectory everywhere in this region, the (nearly constant) radius of curvature,  $R$ , is related to the momentum by

$$p = 0.03BR \quad (1)$$

$p$  is in GeV/c;  $B$ , the magnetic induction is in kG and  $R$  is in meters. The distribution for muons leaving the magnet is obtained using the relation (see Figure 2)

$$r = kp'(1 - \cos\theta) \quad (2)$$

where  $r \equiv$  distance from the edge of the region ( $r = 0$  : no deflection) and  $k = 1/0.03B$ . For typical parameters only muons leaving the side of the magnet need be considered. The desired distribution is

$$G(p', \theta, \phi) = N'kp' \sin\alpha / D_M \quad (3)$$

where  $D_M \equiv$  distance of the magnet from the target along the axis.  $G$  is the number of muons per GeV/c and steradian leaving the magnet.  $G$  is zero for polar angles  $\theta \geq (2r_f/kp')^{1/2}$  (where  $r_f$  is the width of the magnet region) and for azimuthal angles outside limits determined by the height of the magnet. The number of muons per unit area taken perpendicular to the flux at a distance  $y$  from the magnet measured along the axis is

$$I(p'', y) = N'kp' \sin\alpha / D_M (y^2 + r_s^2) \quad (4)$$

$r_s \equiv$  shield thickness;  $p'' = p' - \Delta p'$ , where  $\Delta p'$  is an average momentum loss in the muon shield and in the magnet. To get the total muon flux, eq.(4) is integrated between the limits

$$\begin{aligned} p'_{\min} &= (dp/dx)_{\text{soil}} y' / \{1 - k\theta (dp/dx)_{\text{Fe}}\} \\ p'_{\max} &= 2r_f y^2 / kr_s^2 \end{aligned} \quad (5)$$

$y' \equiv$  thickness of shield traversed,  $p'_{\min}$  = minimum momentum to traverse the shield assuming a constant momentum loss in the shield and the magnet. The integral was calculated for the layout of Area 2 shown in Figure 3<sup>(1)</sup>. A and B are locations of large bending magnets in the front end gallery.

The muon spectrum was taken from the program POWERM<sup>(2)</sup> using a pion source spectrum calculated using the Hagedorn-Ranft model for 200 GeV protons on a Be target, by means of the program SPUKJ<sup>(3)</sup>. The momentum loss was assumed to be  $2(\text{MeV}/c)/\text{gm}\cdot\text{cm}^{-2}$ . The width of the return leg,  $r_f$ , was taken as 0.15m. The result is plotted in Figure 4. The fluxes reach above the tolerance level ( $10^{-13}\mu\cdot\text{cm}^{-2}$ ) only in a relatively narrow region. By neglecting multiple scattering in the hadron shield and by representing the magnet region as a uniform field, these calculations probably overestimate the muon fluxes. On the other hand, there are many more magnets present than the two prominent ones considered and there is a large uncertainty in the assumptions for the calculations of the incident muon fluxes. Furthermore, a brief calculation shows that pre-target beam steering (through angles of about 7 mr) could increase the fluxes by an order of magnitude. These estimates refer to the plane perpendicular to the magnetic field and containing the beam. If the height of the magnet is assumed to be 0.5m, the corresponding heights at the shield would be about 4.0m and 2.5m respectively for the contributions from magnets A and B, at the shield boundary.

#### Side Entrance.

Muons striking the side of the magnet all enter at approximately the same angle  $\gamma$ . They will also leave at the

same angle  $\theta = \beta + \gamma = 2\beta - \alpha$  (Figure 5) i.e., there is no dispersion by the magnet. If they are not stopped by the magnet a narrow hot spot would appear. The number of muons per unit area and per unit momentum interval there being (neglecting edge effects)

$$D(p'', y) = N' \sin \alpha / \sin \theta (y^2 + r_s^2). \quad (6)$$

For the layout of Figure 1 only the magnet at A need be considered. For this case the angle  $\theta$  is about 16.5 mr. This means that these muons travel the entire length of the shield and no extra precaution is needed. When the magnet is placed at a larger angle to the beam direction, the bending angle is also larger but less muons will be reflected since the more energetic muons will cross the return leg (for  $p > 0.17/\theta^2$ , with  $p$  in GeV/c and  $\theta$  in radians) and more will escape through the back of the magnet (with less deflection and an additional dispersion).

#### Conclusions.

The case of front entrance predicts muon fluxes outside the shield which barely exceed tolerance levels. If pre-target beam steering is employed, tolerance levels may be exceeded by one or more orders of magnitude. At the large depths where the disturbances occur, about 1m of soil would be needed to reduce the muon flux by an order of magnitude<sup>(4)</sup>. It is therefore recommended that a 2m clearance around the presently adopted shield be provided at large depths. This

will enable eventual alterations to the shield guided by actual measurements.

#### ACKNOWLEDGEMENT

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#### REFERENCES

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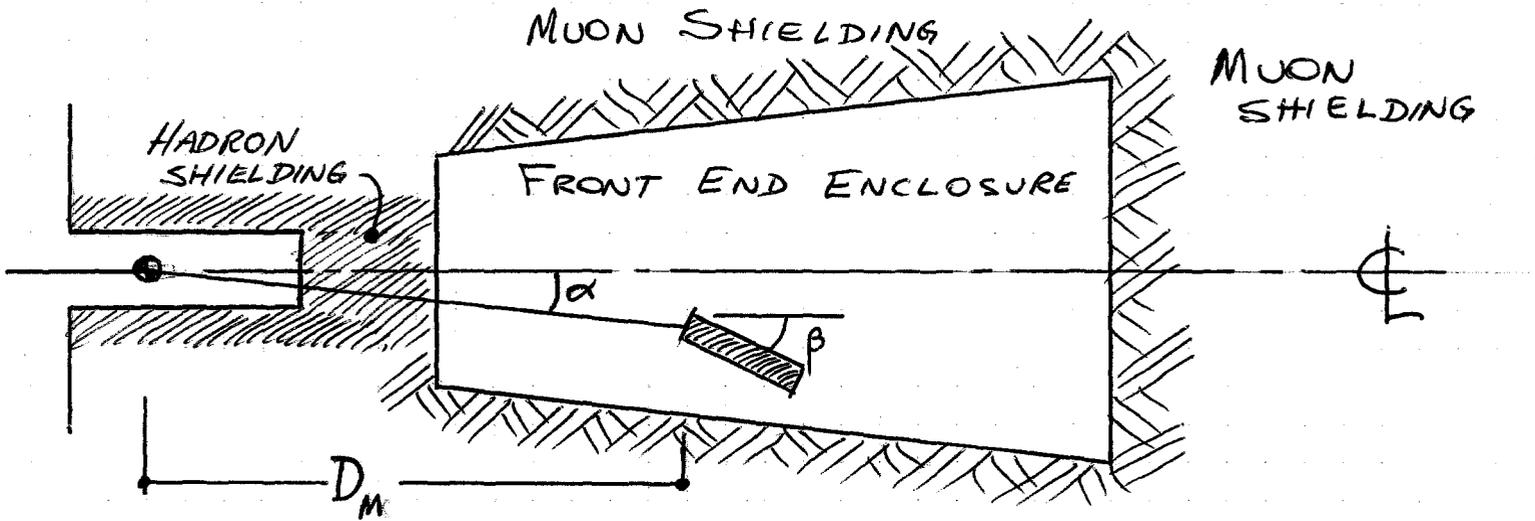


FIGURE 1.

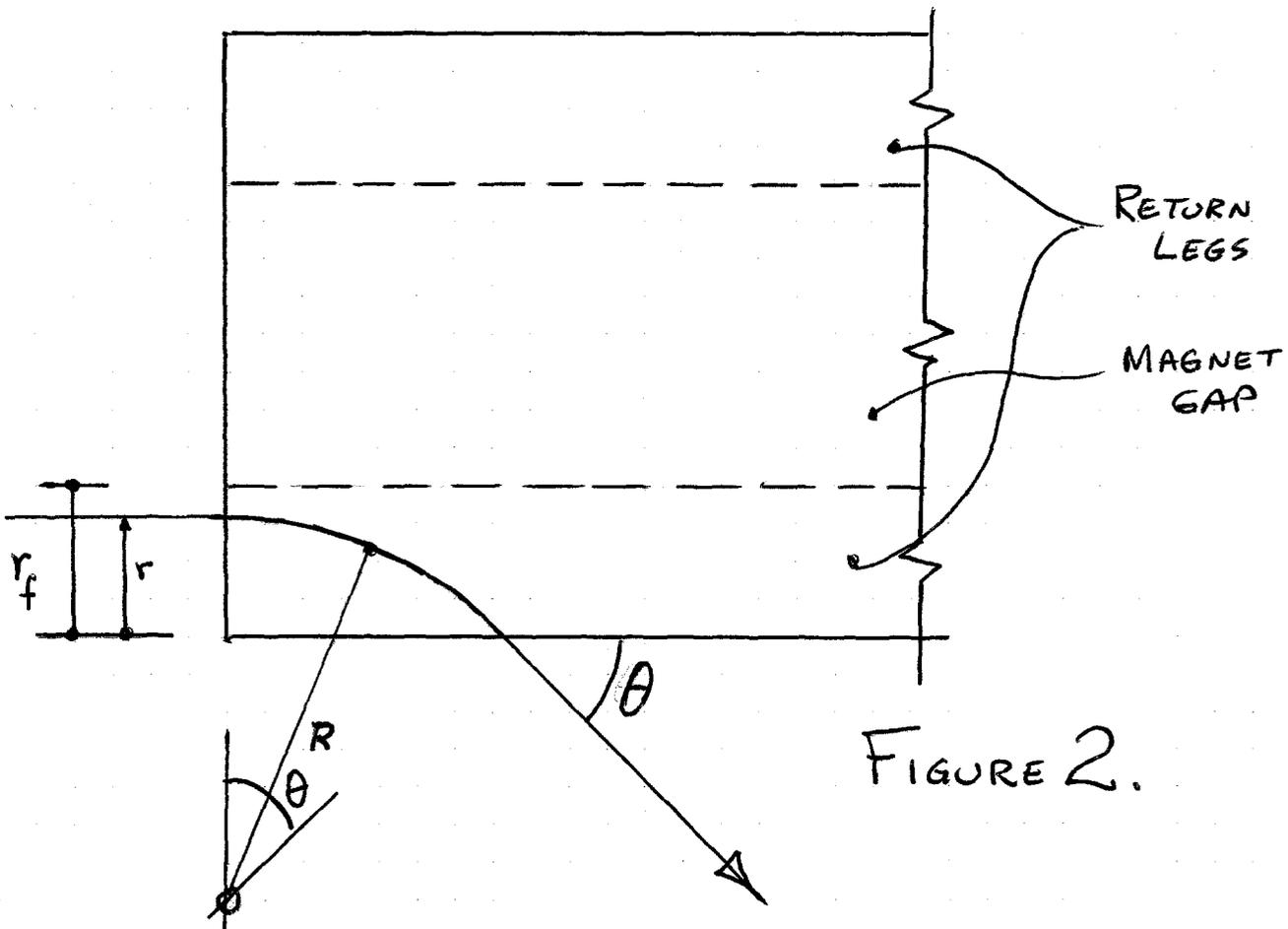


FIGURE 2.

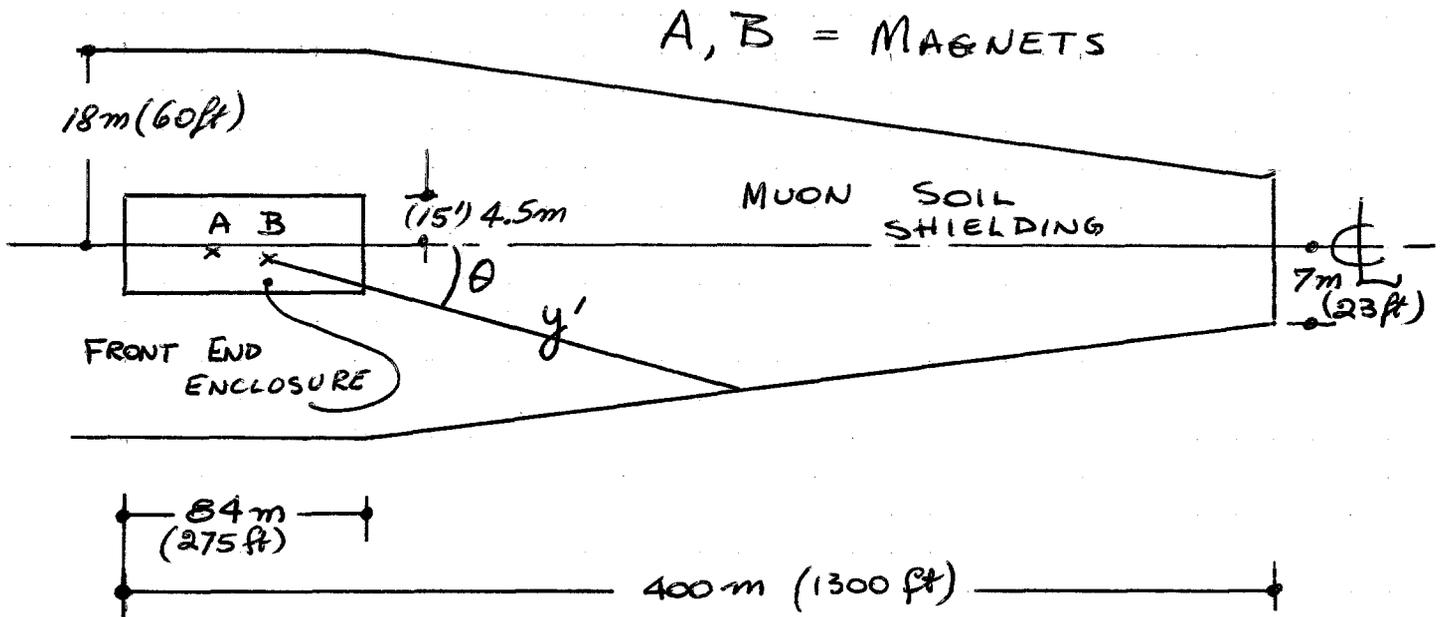


FIGURE 3.

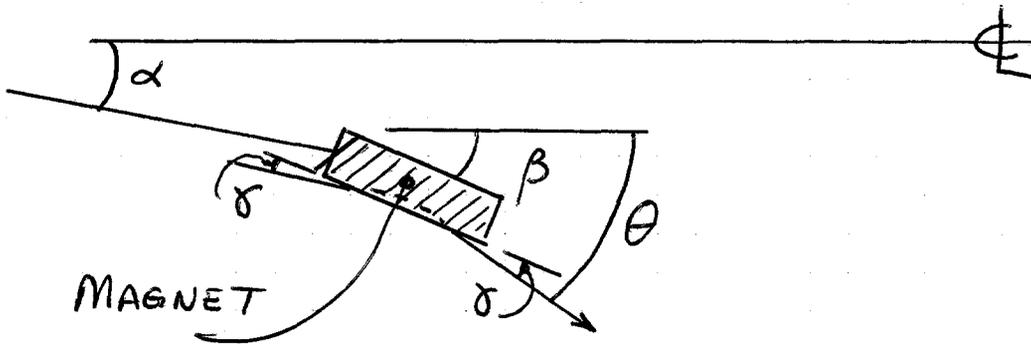


FIGURE 5.

