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QUADRUPOLE LONG-SPILL NEUTRINO BEAMS

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INTRODUCTION

A horn focusing system requires a short spill beam: The horns can be pulsed for only short time periods, because high current densities necessitate operation of the horn walls at extreme temperatures. Present best estimates indicate that conventional horns could not withstand current pulses longer than 10 msec. This is a major disadvantage in that it severely restricts the range of experimental apparatus that can be used with a horn neutrino beam.

In particular, the use of counter and spark chamber arrays is better suited to long spill operation. And as a rule, counter experiments do not have the severe shielding requirements of bubble chamber experiments, while they offer the possibility of doing some experiments with better statistics than is possible with bubble chambers. The purpose of this article is to demonstrate a simple quadrupole focusing system for neutrino energies above 60 GeV. Also, some considerations will be given to the possible development of a low-energy long-spill beam.

Focusing Devices for a Long-Spill Neutrino Beam

There are four types of focusing devices for a slow spill neutrino beam: (1) plasma focusing, (2) cryogenic horns, (3) low current horns, and (4) quadrupole focusing.

Plasma lens systems have had difficulties with stability, and their development has not been complete. No proposals have been made to extend the use of plasma lenses to NAL.

A wide-band long-spill neutrino beam could be achieved with cryogenic focusing devices. (An example of such a device is the Palmer Fish).[1] Efficiencies comparable to the pulsed horn system might be realized. However, the development of a cryogenic horn would require a long lead time, and cost estimates of the system are high (about $1 million, according to Palmer).

The extension of the conventional horn technique to slow-spill operation might be accomplished by making the horn very long (e.g., Fresnel lens),[2] and thereby allowing the use of low current densities. Such a system has been proposed for a very high energy slow-spill neutrino beam. The technique achieves results very near perfect focusing for energies in excess of 50 GeV. It involves, however, the expense of development of a somewhat new device.

A few quadrupole systems have been proposed in the past.[3] The most actively studied system involves the use of a long quadrupole channel, filling the decay tunnel with quadrupoles, to achieve a high flux of neutrinos within a narrow band of energies. Adiabatic systems have also been devised to study the transmission of neutrinos over a broad band of energies. In the future, we hope to extend our own calculations to include these sophisticated quadrupole focusing devices.

In this paper, the results of a simple quadrupole doublet system[4] will be presented. The results are encouraging in the very high energy region.

The Design of the Quadrupole Beam

The beam configuration of specific interest is that of a quadrupole doublet as the focusing element located near the target at the start of the decay tunnel. The target is put as near to the front face of the first quad as is practicable to achieve maximum acceptances. A high density collimator encloses the target to eliminate low energy pions from the beam and to protect the first quad from irradiation.

The particle production spectrum for the calculations is taken from the CKP model. A study of the various models[5] available indicates a large range of uncertainty in predicted particle yields. The CKP model was chosen as a first pass approximation because of its ease of application. Later calculations will study the range of variation predicted by the various models.

A useful way to study the effect of the focusing element is to consider the phase space distribution of parent particles that contribute to neutrinos in an energy range $E_{\nu 1}$ to $E_{\nu 2}$. Thus, if an optimal beam for neutrino energies from 50 GeV to 200 GeV is desired, only parents that contribute to these energies are considered. The limits on parent particle energies can be found analytically and are given by:

$$E_K^{min} = \frac{1}{1-\left(\frac{m_\mu}{m_K}\right)^2} E_{\nu 1}$$

$$E_K^{max} = \text{primary beam energy.}$$

Figure 1a gives the phase space distribution for perfect focusing, while 1b gives an example of the effects of a quadrupole doublet. The boundaries of the distribution in 1b are meant only for illustration; in general, they depend on the parameters of the quadrupoles and the features of the production spectrum. The quadrupole in 1b is chosen to do point to parallel focusing for momenta equal to P_o, where

$$P_o = \frac{1}{2}\left(E_K^{min} + E_K^{max}\right)$$

Because of the non-uniformity of the particle production spectrum, a higher absolute flux for this energy interval would result if P_o were biased toward E_K^{min}. However, if the highest energies in the interval are of most interest in the experiment, P_o should be chosen nearer to E_K^{max}.

The choice of parameters for the quadrupole is based on point to parallel focusing for momenta P_o: This gives the field gradients necessary for fixed quadrupole lengths. The separation between quadrupoles is made minimum in order to maximize the acceptance of the system. The pole tip fields are chosen at 15 kG, since these are the maximum fields that are easily attainable.[6] The aperture of the quadrupoles is then found from the field gradients. Thus, for any given combination of quadrupole lengths, there exists a set of field gradients to give point to parallel focusing for momenta of P_o. These gradients, combined with the 15 kG pole tip field, then give the maximum aperture of the system.

The quadrupole lengths are chosen in the ratio of 2:1 for front quadrupole length to rear quadrupole length. In general, this recipe gives field gradients in the two quads that are not too different. As the length of the doublet is increased, the field gradients can be decreased and the aperture of the quadrupole can be increased allowing for a larger geometric acceptance: The angular acceptance of a doublet is proportional to the square root of the radius of the aperture. However, because of the exponential fall-off of the particle production spectrum with transverse momentum, a point of diminishing returns is reached beyond which an increase in aperture will yield little increase in flux for the energy region of interest.

The results of a calculation for a thirteen-meter long quadrupole system is given in Figs. 2a and 2b. The calculation was done separately in the FD and DF planes: No attempt was made to study the effects of a random passage of particles through the quads. The front and rear quadrupoles were eight and four meters long, respectively; the field gradients were 1.9 and 1.3 kG/cm, respectively; and the aperture radii 8 and 11.5 cm, respectively. The first quad was one cm from the target, and a one-meter drift space was used between quadrupoles. The beam parameters used in the calculation are given in Table I. The results show that the quad doublet system competes very well with the horn system for energies in excess of 70 GeV. (The horn curves are taken from a design optimized for low energy operation). The low energy cut-off is due to the finite aperture of the doublet; it could be reduced by varying the lengths and aperture radii of the quadrupole.

The calculations shown in Fig. 2 were made with a modified version of the program NUFLUX. This program traces particle trajectories through a focusing system: The particle yield spectrum is divided into bins of equal momentum and angle; a ray from each bin is traced through the system; allowed to decay at various points on its path; and the flux at the detector is weighted according to the yield spectrum. Subroutines were added to the program to track parent particle trajectories through a quadrupole system. The calculation is based on quadrupole transfer matrices given by:

$$\begin{pmatrix} \cos\phi & \frac{1}{\omega}\sin\phi \\ -\omega\sin\phi & \cos\phi \end{pmatrix} \qquad (2a)$$

in the focusing plane and

$$\begin{pmatrix} \cosh\phi & \frac{1}{\omega}\sinh\phi \\ \omega\sinh\phi & \cosh\phi \end{pmatrix} \qquad (2b)$$

in the defocusing plane. Here ϕ is given by

$$\phi = \sqrt{\frac{B'}{B\rho}}\,\ell$$

where

ℓ = length of quadrupole
B' = field gradient
$B\rho$ = particle rigidity

A future modification of the program will track trajectories through any part of the quadrupole.

The beam was designed with the intention of determining what could be achieved in the horn beam if quadrupoles were interchanged with the horn. As such, no attempt has been made to optimize the beam parameters for the quadrupole beam. Of special note is that the high energy beam might benefit from a shorter decay tunnel due to the effects of the short kaon lifetime. For example, for a 100 GeV beam of K^+'s, a shield length of 173m, and a detector radius of 1m, one finds that a tunnel length of 520 meters maximizes the neutrino flux above 48 GeV.[7]

On the basis of the results in Fig. 2, we conclude that a quadrupole focusing system looks promising for a high energy neutrino beam. It has the advantage of offering a neutrino flux at high energies comparable to the horn system. Also, the background due to interactions of low energy neutrinos will be reduced because of the low-energy cut-off of the quadrupoles. Comparable results also hold true for 400 GeV operation.

Further revisions and improvements are being planned for the computer program used in these calculations. The decision to develop a quadrupole neutrino beam awaits the results of the final calculations.

Extension of a Low Energy Beam

A low energy quadrupole focusing system must be quite different in character than the high energy beam because the low energy pions are produced at larger angles. This requires that the geometric acceptance of the system be larger to focus more of the useful parent particles. But also, this larger spread in angles will require more than one doublet to get useful results. The improvement in parent particle focusing for a two doublet system is illustrated in Fig. 1c. The two doublet system offers the possibility of better approximating the perfect focusing phase space distribution in any given neutrino energy interval. This is extremely important if the angular distribution is very large, as seen by comparing Fig. 1b with 1c.

The calculations for the low-energy slow-spill beam are currently in progress. The results are not yet available.

REFERENCES

[1] R. B. Palmer, 1969 Summer Study Vol. 1, p. 383.

[2] D. H. Frisch, Y. W. Kang; 1969 Summer Study Vol. 1, p. 389.

[3] L. C. Teng, 1969 Summer Study Vol. 1, p. 297; S. L. Meyer and T. E. Toohig, 1969 Summer Study Vol. 1, p. 305; T. E. Toohig, UCRL 16830, p. 409 (1966); C. Matsubara, UCRL 16830, p. 395 (1966); N. C. Dutta, UCRL 16830, p. 378 (1966); V. Z. Peterson and D. Keefe, UCRL 16830, p. 364 (1966); V. Z. Peterson, UCRl 16830, p. 324 (1966); D. Keefe, UCRL 16830, p. 311 (1966).

[4] A. Roberts, NAL Internal Report FN-124 (1968).

[5] F. A. Nezrick, R. J. Stefanski, Y. W. Kang, D. Carey (NAL-Internal Report TM-196).

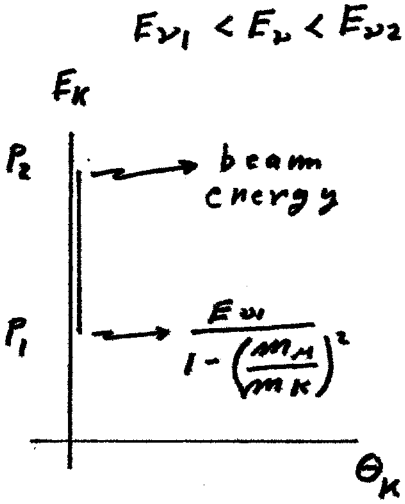
[6] See, e.g., Klaus G. Steffen, High Energy Beam Optics (Interscience Publishers, New York 1965), page 70.

[7] Obtained from a calculation by M. L. Stevenson; see 1968 Summer Study, Vol. 1, page 265.

TABLE I

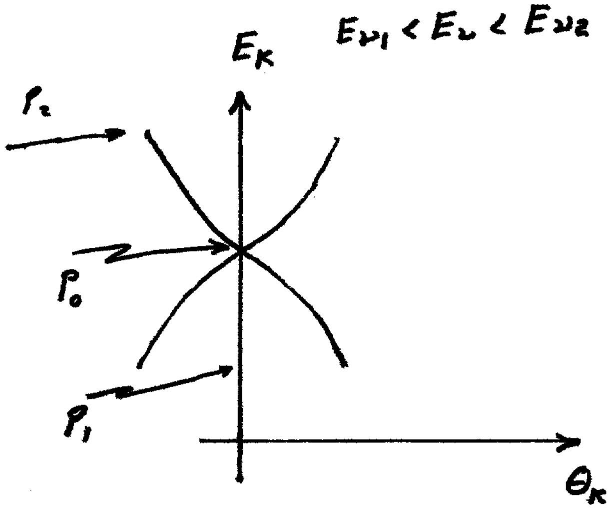
BEAM PARAMETERS

Target	2mm dia. .03 interaction lengths long
Decay Tunnel	600 m long .75 m radius
Shield	150 m long
Recess	22.5 m long
Detector	1 m radius
First Quad	8 m long 16 cm ϕ aperture 1.9 kG/cm gradient
Second Quad	4 m long 23 cm ϕ aperture 1.3 kG/cm gradient



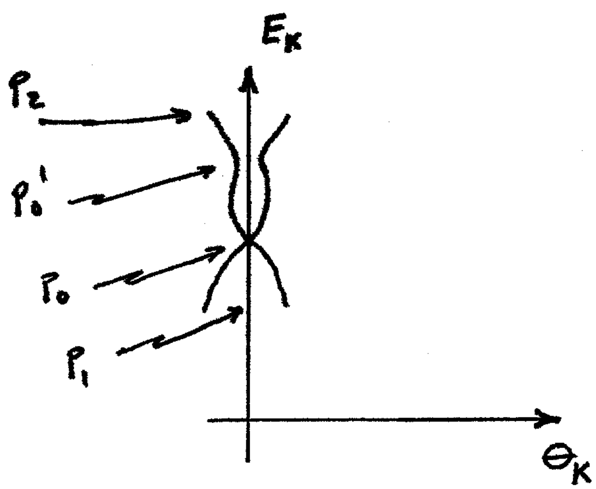
Perfect focusing: parent particles that contribute neutrinos of energy $E_{\nu 1} < E_\nu < E_{\nu 2}$ have momenta P_1 to P_2 and lie at $\theta_K = 0$.

Figure 1a.



One doublet focusing system: point to parallel focusing chosen for momentum P_o.

Figure 1b.



Two doublet focusing system.

Figure 1c.

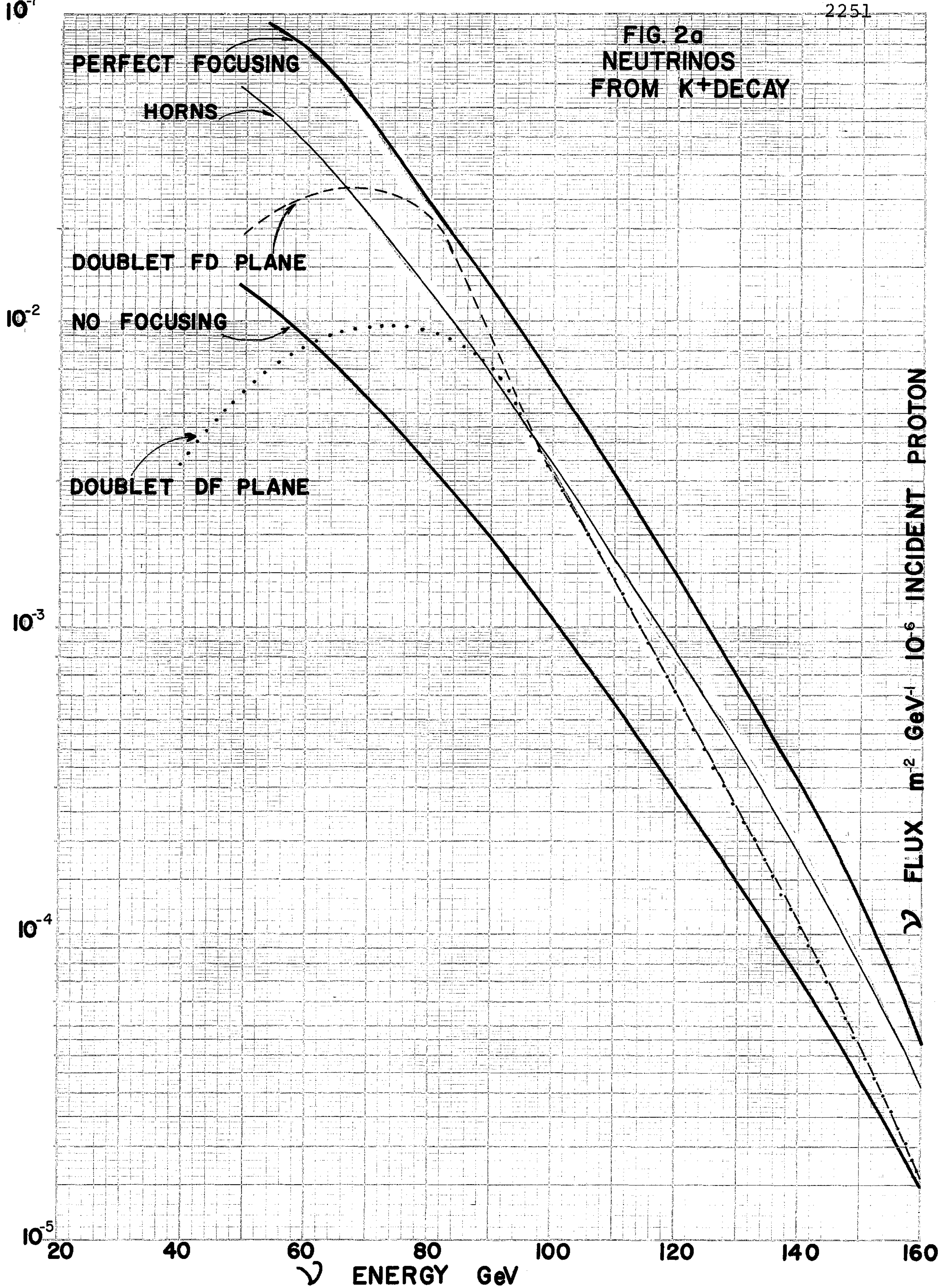


FIG. 2a
NEUTRINOS
FROM K+DECAY
PERFECT FOCUSING
HORNS
DOUBLET FD PLANE
NO FOCUSING
DOUBLET DF PLANE
10-1
10-2
10-3
10-4
10-5
20
40
60
80
100
120
140
160
ν ENERGY GeV
ν FLUX m-2 GeV-1 10-6 INCIDENT PROTON

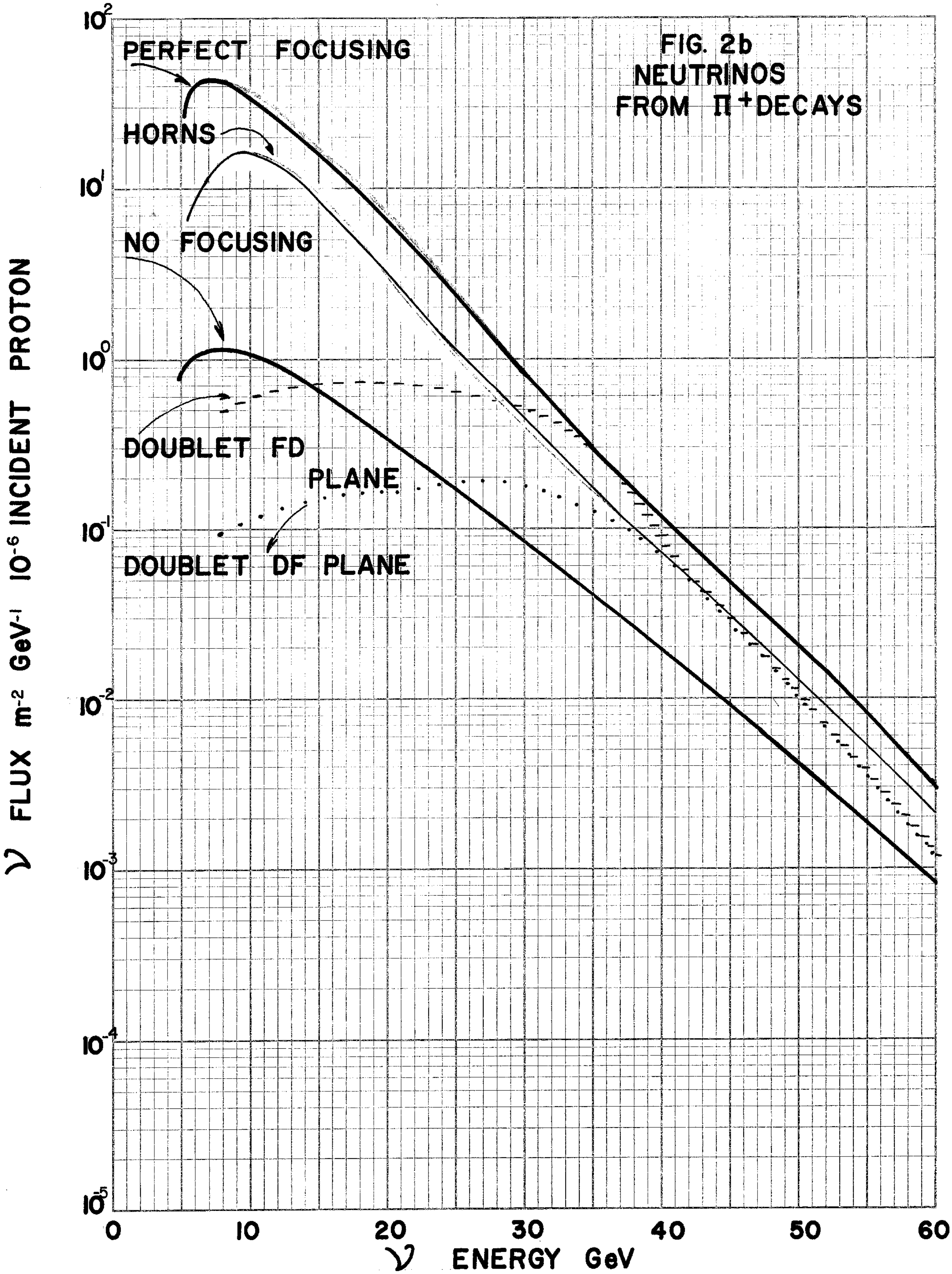


FIG. 2b
NEUTRINOS
FROM Π+DECAYS
PERFECT FOCUSING
HORNS
NO FOCUSING
DOUBLET FD
PLANE
DOUBLET DF PLANE
ν FLUX m-2 GeV-1 10-6 INCIDENT PROTON
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
0
10
20
30
40
50
60
ν ENERGY GeV