

NEUTRINO BEAMS AND SHIELDING

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Among the questions which the neutrino group must consider are the following:

1. Should NAL concentrate initially on a "wide-band system" or a "narrow-band system" ?
2. Should one concentrate on a low-energy neutrino beam or a high-energy neutrino facility?
3. Does the facility require a full muon shield capable of ranging out muons of the maximum energy or will a combination of magnetized iron, earth, Fe shield and magnetic field suffice?
4. Should the facility be primarily below ground or above?
5. Where should the facility be located? This matter must be settled at a fairly early date.

About the only advantage we can see at this time for the narrow-band system (excluding considerations pertaining only to a high-energy beam) is that it simplifies the shielding problem. The focusing problem also appears to be more straightforward for the NBS. However, as was

pointed out by D. H. Perkins,¹ the momentum resolution of the NBS for neutrinos will not in general be good enough to provide any advantage in analyzing events in the bubble chamber.

The difference in neutrino fluxes between the two systems is at least an order of magnitude (in favor of the wide-band system). As indicated in G. Snow's report,² the wide-band system appears to be reasonably well matched to the 25-ft bubble chamber in the sense of events/picture but not comfortably so. This matching would not obtain for the narrow-band system.

As Block³ has emphasized, much interest is attached to the low-energy events (below 2-3 GeV) in the comparison of neutrino and anti-neutrino interactions. Also, it is important to measure $d\sigma/dq^2$ over a range of neutrino energies to verify that it is, in fact, independent of energy above 3 or 4 GeV (test of locality).

It would thus appear that an important early program of the neutrino facility would utilize a beam emphasizing the lowest energies and a wide-band system including the energies between, say, 3 and 15 GeV. (This presupposes the availability of a large hydrogen/deuterium bubble chamber). Our first recommendation, therefore, is that such a facility be planned.

We recognize, however, that there will undoubtedly be great pressure to pursue the "glamorous" route which means a search for the intermediate vector bosons, W^\pm , and for other exotic interactions. This

requires a beam of high-energy neutrinos, and is the clearest justification for the 200-GeV machine (if not necessarily the most convincing on the grounds of physics). We must therefore plan to have available a facility which will emphasize neutrinos of the highest energy possible and realize that this may be the first activity of the neutrino facility. At this stage, we can only consider various alternatives to provide input for the decision makers.

We have not considered in detail the various focusing devices required for the facility. For the purposes of our calculations, we have assumed an "ideal" pencil beam incident from the target. We proceed on the assumption that one will always wish to separate ν from $\bar{\nu}$ by separating the positive and negative pions (kaons) before substantial decay has occurred. We have not at this time considered the various possible devices for making this separation in detail, but we propose that some sort of achromatic device such as indicated in Fig. 1 be used for this purpose.

As drawn, a pencil beam enters from the target. Some preliminary focusing is called for to minimize the divergence of the beam from the spread of production angles. This system provides a means for getting rid of the bulk of the neutrons which would otherwise plow into the muon shield with full energy and generate muons by pion decay in the shield. By moving the beam stop past the axis, a mechanism is provided for cutting out the high-energy part of the pion spectrum. This cutting out

of the highest energy pions has certain advantages as we shall discuss later.

The beam stop in Fig. 1 should be made of magnetized iron to deflect out those (high energy) muons from pion decay in the region before the charge separation. Since the detector is always of the order of at least 750 meters away, 5 meters of magnetized iron beam stop provides ample deflection for even the highest-energy muons. It may be possible to combine this magnetized iron with the lower deflection magnet in the figure.

" Full Muon Shield "

The major problem of the wide-band system is that of the shielding. To get a rough estimate of this problem, let us follow Perkins in making a very conservative estimate of the iron shielding required to range out all the muons up to the full 200-GeV energy of the beam. Neglecting the radiative energy losses which have too large a fluctuation for the purpose of shielding the bubble chamber, we estimate a shield of about 150 meters of Fe would adequately do the job. If we argue that the full extent of the bubble chamber must be shielded and that the cross section of the shield should be uniform to minimize the fabrication costs, we end up with a shield which is $150 \text{ m} \times 8 \text{ m} \times 8 \text{ m}$ which is 83,000 (American) tons of Fe. At a cost of 7 cents/lb, this entails a cost of \$11.6 million.

There are some immediate comments which one should make at this point. This shield is conservative--but not very. In principle, the shield need not be uniform in cross section and certainly a circular cross section may be used rather than the (larger) square cross section. The actual shape of the shield must be calculated with attention paid to the in-scattering of muons which are produced outside the shield or which hit the shield at a glancing angle. This calculation is, of course, dependent on the composition of the material outside the iron shield (which, of course, involves the question of whether or not the beam is above or below ground). A major factor, however, may turn out to be the fabrication and handling cost, and this may determine the dollar figure to a greater extent than the physics calculation. It may thus not be possible to reduce appreciably this cost estimate.

In any case, we feel that we must reduce this cost as much as possible. When one thinks of scaling this full iron shield to 400-GeV operation, this straightforward scheme looks even worse. We have, accordingly, spent some time in making preliminary considerations of alternative schemes to reduce, or largely eliminate, this expense.

An additional shielding problem is the interaction of neutrinos in the last few meters of the shield which produce muons. These muons, however, are predominantly of a few GeV energy and below and may be bent away from the detector by a sweeping magnet. It has been suggested that a "flux-grabber" attached to the bubble-chamber magnet could

provide this effect. We shall not discuss this problem further in this report.

There are several ways to reduce the shielding cost. As initially mentioned, there are many combinations of magnetized iron, iron shield, deflection magnets and earth shielding which are possible. It is expected a priori that, if these alternative schemes require a larger distance between the detector and the end of the drift space used to produce the neutrinos from pion (kaon) decay, the loss of solid angle occasions the loss of neutrino flux. As we shall see, this conclusion requires some qualification and the superiority of an iron shield is not equally great for all ranges of neutrino energy. It is clear that one may compensate so long as one gains significant flux by increasing the drift space.

We have calculated various combinations of drift-space lengths and shield lengths (here taken to mean the total distance between the end of the drift space and the detector; in general, this space may contain shield, sweeping field, and lever arm) to estimate how these affect the neutrino flux at different energies. We have assumed an ideal focusing system for these calculations (pencil beam of parent pions) and used the production spectra obtained from the formulae of Trilling. Results of these calculations are summarized in Figs. 2-6.

At this point we should mention certain difficulties with these calculations aside from the question of whether to use the spectrum predictions of Trilling, Cocconi-Koester-Perkins (CKP) or Hagedorn-Ranft.

In principle, when a computer becomes more accessible than the one at Aspen, one may repeat the calculations for all of these production spectra and compare the results. However, all of these prescriptions assume production from a nucleon (hydrogen) target. Of course, production will be on a complex nucleus.

We expect that the effects of Fermi momentum and secondary interactions will change all the results, especially those pertaining to the production of the lowest-energy neutrinos. Note that the flux of lowest-energy neutrinos is small for all the beams considered. This is probably unduly pessimistic. One should, in principle, perform nuclear cascade calculations of the type done by Riddell to optimize the target for the production of low-energy neutrinos. An obvious consideration is to make the target several nuclear interaction lengths long so as to have the pions produced from the primary interaction themselves interact to produce lower-energy pions. Neutrons produced in the primary collisions would also serve to provide pions through secondary interactions. For the purpose of performing the ν , $\bar{\nu}$ comparison at low neutrino energies, this mechanism would also go some way to improving the ratio of $\bar{\nu}/\nu$ production which is expected to be roughly 1/2 according to Trilling. (Note that our calculations have assumed 100% transport efficiency between target and the start of the drift space ignoring losses due to finite apertures and decay. The efficiency will be lower for the lower-energy pions. The production of lower-energy pions, on the

other hand, will be enhanced over our estimates by the nuclear cascade effects of complex target production. It may happen that our calculation may not be too far off in a relative sense over the spectrum of pion energies if these neglected effects tend to compensate.)

Earth Shield

The simplest cheap alternative to ranging out the highest-energy muons with an iron shield is to range them out with an earth shield. The ratio between the two absorbers is dependent on the approximate energy loss in Fe of 1.95 GeV/m and in earth of 0.45 GeV/m. Thus, the earth shield will have to be approximately four times as long. We have calculated this for the basic situation, and the results are summarized in Figs. 4 and 6 and in Table I.

As one expects a priori, the larger the shield length, the more one loses flux. However, this loss is dependent on the energy range of neutrinos in which one is interested since that determines the possible drift-space length. The tables include data for "compromise" drift-space lengths. We have had in mind the enhancement of the neutrino flux in the energy range between 3 and 15 GeV and arbitrarily chose the drift-space length corresponding to midway between the $E > 3$ GeV and $E > 5$ GeV maxima. "Optimization" of the drift space varies according as the choice of energy interval. The curves in the figures should be used if an "optimum" drift space is desired for an energy range different from that of the "compromise."

The net loss of flux for the earth shield compared to the iron shield appears to be of the order of a factor of ten below 10-GeV neutrino energy but only 25% above 10 GeV. In fact, above 20 GeV, the flux favors the particular earth shield configuration used. The neutrino fluxes from kaons are shown on Figs. 4 and 6 and do not change the conclusion: a full earth shield produces little if any loss in the flux of high-energy neutrinos compared to yields from a configuration using a full iron shield. (The fluxes of neutrinos from kaons in fact appear to be higher for the earth-shield configuration than for the iron-shield case. Unfortunately, the kaon flux in Fig. 4 was calculated using $1/10 \times$ CKP and that in Fig. 6 using $1/10 \times$ Trilling. Figure 6(a) shows the flux due to kaons using $1/10 \times$ CKP.)

The advantages of the earth shield for high-energy neutrino beams (above 10 GeV) are the following. It would certainly appear advisable to target this neutrino beam from the proton beam while it is yet below grade (as we state later, we recommend that this be downstream of the SA station). This saves the magnet system required to deflect the proton beam up. The beam line from the target through the end of the pion drift space requires excavation, of course, but the shield region need not be excavated at all. The tunnel to the shield and the tunnel leading to the detector after the shield would naturally be aligned but the shield itself could be unexcavated (at least if the high-energy experiment were run first).

While the loss of low-energy neutrino flux is undeniable, it is likewise true that the backgrounds also go down with the decreased solid angle. The muon background scales in approximately the same proportion. The increased lever arm will facilitate the use of magnetized iron slabs sunk into the earth shield to remove the (low energy) muons produced in the shield itself. The neutron background in the chamber itself has been investigated. The source of these background neutrons is the interaction of neutrinos in the magnet coils and in the stainless steel shell of the bubble chamber. These neutrons could provide an annoying source of background, especially to the polarization studies suggested in Block's report. The major handle against these background neutrons producing proton recoils is the coplanarity of the two-body neutrino interaction in the chamber. Not only will the number of neutrino-induced neutrons go down by a factor proportional to the decrease in the (good) neutrino flux, but the angular definition of the beam neutrinos improves as the lever arm from the drift-space region increases.

High-Energy "Clipping": Reduced Energy

A mode of operation which reduces the extent of the shield and has merit if it is not too costly in terms of neutrino flux is to remove the highest-energy pions from the beam produced by primary protons of the full energy. It would appear from our calculations that one loses mostly the shielding problem and gives up very little in the flux at the lower

energies considered important for a major part of the experimental neutrino program. This mode of operation also has the advantage that it scales immediately to machine operation at 400 GeV. Our calculations for this mode of operation are summarized in Figs. 7-10 and Table II. We have considered usage of both iron and earth shields (except that 100 meters is unduly pessimistic and was chosen only because of the constraints of the computer calculation). We do, of course, lose the very highest-energy neutrinos but the flux of neutrinos above 20 GeV is still substantial. The flux due to kaons has not yet been calculated. Note that the flux above 5 GeV neutrinos for an earth shield and pion cutoff of 100 GeV is quite comparable to that from the 150-m Fe shield case at 200 GeV with no cutoff.

We have also considered the possibility of running the accelerator at less than full energy. This has a twofold advantage: first, the increased repetition rate actually increases the effective number of pions at low energy; second, the fact that the maximum energy of the beam is reduced makes the shielding problem more tractable. We have calculated within our stated approximations the effect of this mode of operation on the neutrino flux in the vicinity of 3 and 5 GeV and above 10 GeV and 20 GeV. We note that it is possible to obtain low-energy neutrino fluxes which are quite comparable, and in some cases superior, to those obtainable from the basic beam (full muon shield of 150 meters of Fe) although we lose heavily at the high-energy end of the spectrum. The results of our

calculations, including the cases where a pion cutoff momentum is imposed, are summarized in Figs. 11-15 and 15(a) and in Tables III and V. Up to an energy of 10 GeV for the neutrinos, the most pessimistic conclusion is that one needs no more than 50 meters of Fe shielding at the outside.

To get a feeling for the space requirements when the machine goes over to 400-GeV operation, we have calculated the basic beams at 400 GeV for both the full iron shield and the full earth shield cases. The results are summarized in Figs. 16 and 17 and in Table IV. What is most relevant here is the scale of dimensions required for both the drift space and the shield length. The scale must be taken into consideration in deciding the placement of the neutrino facility.

Conclusions

Within the context of our approximations and assumptions, we conclude that it is not necessary to provide a full Fe shield 150 meters long. We further conclude that a wide-band system is the facility of choice for matching the event rate to the large bubble chamber. However, we must define wide band in such a sense that the running of the neutrino facility would involve the sequential running of the various energy ranges, utilizing different beam setups for each range. It appears from our work that this could probably be accomplished with two, or at most three, different beam configurations.

This mode of operation has at least three advantages over the "basic" system utilizing a full shield of 150 meters of Fe:

1. The major

while retaining the advantage of ranging out the highest-energy muons in the beam. At most, this mode of operation requires a shield of 1/3 the length of that of the basic system. The other beam to be utilized requires 600 meters of earth (at most). Earth has the virtue of being cheap and, as pointed out earlier, judicious scheduling might permit the use of unexcavated space as the shield.

2. As shown in Table VI, the use of the full-earth shield beam for the high-energy neutrinos (above 10 GeV) and one or more of the four beams listed (which each require only 50 meter iron shields and which are quite compatible with one another) for the low-energy end (up to about 10 GeV) may actually result in a more efficient use of machine time for the purpose of doing neutrino physics. If one has only a definite total amount of accelerator time which is to be devoted to neutrino physics, it is possible to obtain more total neutrinos in the range 1 GeV and above by this division of running than may be gotten with the expensive 150 meter Fe shield beam alone. The total neutrino flux may be allocated to the various neutrino energy bands in different ways depending on the fraction of the total running time allocated to each beam.

One can utilize this increased efficiency ~~when~~ one has a better idea of how much running time in each neutrino energy range is required.

This mode of "division of running" provides maximum flexibility for optimizing the programming of the neutrino facility. This flexibility is not available for the fixed spectrum yield of the basic beam. Note also that the yields are specified in Table VI for our "compromise" estimates. These are certainly not optimum for this purpose and can surely be improved.

3. The third advantage is the division of labor which occurs in the taking of bubble-chamber pictures. Instead of having all experiments in all pictures, the division of running results in the bubble-chamber pictures being divided between "high-energy neutrino film" and "low-energy neutrino film." This strikes us as an advantage in the scanning and measuring of the film but the guy who has to wait for his film may not think so!

Recommendations

1. We propose to repeat all the calculations performed with realistic focusing devices and dimensions pinned down in more detail. The various production spectra must be used and their results compared.

2. We propose that an intensive design study be made of the feasibility of a magnetic deflection system utilizing magnetized iron shielding and air magnets to obviate the need for ranging out the muons. This, however, is a problem of great complexity due to the need to reject muons by an enormous factor. As Jovanovic,⁴ Perkins¹ and others have pointed out, very detailed analysis and calculations are required to

account for the various exotic and devious routes a muon can take to get into the detector. Our personal bias is that such a study will succeed but it is impossible to know for sure since Monte-Carlo studies of the various scattering processes are necessary for each geometry considered. The advantage of such a study succeeding, however, in our opinion justifies the time and effort involved.

3. In any case, our recommendation is that one no longer think only in terms of constructing a full muon range shield of the order of 100-150 m in length. The idea of sequential running appears to us to be more attractive as discussed earlier. We propose that studies be continued to ascertain the most efficient program for running the accelerator for neutrino physics.

4. We recommend that an investigation be made of how fast the bubble chamber may be made to cycle. It is highly desirable that the bubble chamber be able to expand at two or four times the usual rate if the accelerator cycles faster as a result of running at lower primary energy.

5. We recommend that provision be made to install the neutrino facility underground so as to facilitate the use of earth shielding. This is not, however, a necessary requirement for our suggested mode of operation but the idea of using unexcavated space for shielding has appeal.

Recommendation Concerning the Placement of the Primary Neutrino Facility

We recommend that the basic neutrino beam facility be placed in proximity to beam switching station SA. To be specific, we believe that it should be taken off the primary proton beam somewhat downstream of SA. The reasons for this recommendation are as follows:

1. Beam station SA will undoubtedly be the first construction item. The importance of the neutrino facility is such that delay in its implementation is certainly to be avoided.

2. Placement close to SA permits the longest possible neutrino beam. This provides a degree of flexibility which is highly desirable whatever the initial choice of neutrino beam. It is particularly important to have this flexibility in view of the eventual upgrading of the machine energy. It is clear that the beam can always be shortened by transporting the proton beam further (assuming, naturally, the fixed positioning of the 25-ft bubble chamber) but the maximum length is always delimited by the space available. This available space is maximal for station SA.

3. Placement at this position permits geographic isolation of the bubble chamber from the rest of the detector area. Assuming the decoupling of strong-interaction physics from the chamber, this appears to be desirable. Should a decision be made at a later time to use the bubble chamber for this purpose, a beam originating from station SB could be brought to the chamber. We regard it as an advantage that the chamber placement would ab initio discourage this.

REFERENCES

- ¹D. H. Perkins, CERN/ECFA 67/16, Vol. II, 1967.
²G. A. Snow, NAL Summer Study Report B. 1-68-59, 1968.
³M. Block, NAL Summer Study Report B. 1-68-42, 1968.
⁴D. D. Jovanovic, "The Neutrino-Muon Facility at the 200 BeV Accelerator," Lawrence Radiation Laboratory UCRL-16830, Vol. III, 1966.

Table I. 200-GeV Primary Proton Energy: All Pions Accepted.

Shield Length (meters)	"Compromise" Drift Space (meters)	Number of neutrinos/10 ² interacting protons in energy range (E _ν in GeV)				
		1-3	3-5	5-10	>10	>20
50	700	0.3	3.5	3.2	2.3	0.50
100	750	0.1	1.8	2.7	2.5	0.54
150*	750	0.0	1.0	2.7	2.3	0.54
250	1000	0.0	0.0	1.6	2.4	0.69
600 ⁺	1500	0.0	0.0	0.3	1.7	0.79

* corresponds to full muon shield of Fe
⁺ corresponds to full muon shield of earth

Table II. 200-GeV Primary Proton Energy: Pions Cut Off Above p_{max}.

Pion Cutoff Momentum p _{max} (GeV)	Shield Length (meters)	"Compromise" Drift Space (meters)	Number of neutrinos/10 ² interacting protons in neutrino energy interval				
			1-3	3-5	5-10	>10	>20
100	100	700	0.0	2.0	3.0	2.2	0.45
100	300	1000	0.0	0.2	3.3	2.2	0.66
50	50	650	0.6	3.5	3.1	1.4	0.39
50	200	600	0.0	1.1	2.2	1.2	0.31

Table III. 100-GeV Primary Proton Energy*: Pions Cut Off Above p_{\max} .

Pion Cutoff Momentum p_{\max} (GeV)	Shield Length (meters)	"Compromise" Drift Space (meters)	Number of neutrinos/ 10^2 interacting protons in neutrino energy interval				
			1-3	3-5	5-10	>10	>20
no cutoff	50	550	0.4	3.0	2.3	0.88	0.03
no cutoff	100	600	0.1	1.5	2.1	0.9	0.035
no cutoff	300	1000		0.1	0.9	0.9	0.053
50	50	650	0.6	2.0	2.4	0.9	<0.01
50	200	700		0.6	1.5	0.9	<0.01

*N. B. The reduced machine energy permits an increase in repetition rate of about a factor of two. The flux/sec is, therefore, about twice that indicated in the table.

Table IV. 400-GeV Primary Proton Energy.

Pion Cutoff Momentum p_{\max} (GeV)	Shield Length (meters)	"Compromise" Drift Space (meters)	Number of neutrinos/ 10^2 interacting protons in neutrino energy interval				
			1-3	3-5	5-10	>10	>20
no cutoff	300	2400		0.0	1.1	3.7	2.5
no cutoff	1200	3400		0.0	0.2	1.8	1.7

Table V. 50-GeV Primary Proton Energy*:
All Pions Accepted: Shield = 50 meters⁺.

"Compromise" Drift Space (meters)	Number of neutrinos/10 ² interacting protons in neutrino energy range (E _ν in GeV)					
	<u>1-3</u>	<u>3-5</u>	<u>5-10</u>	<u>>10</u>	<u>>15</u>	<u>All</u>
400	1.01	2.55	1.34	0.06	<0.01	4.96
400 Contribution of kaons (1/10 CKP formulae)	<0.001	<0.001	0.002	0.038	<0.001	0.040
200	2.16	2.45	0.90	0.03	<0.01	5.54
200 Contribution of kaons (1/10 CKP Formulae)	<0.001	<0.001	0.051	0.083	0.020	0.134

*N. B. The reduced machine energy permits an increase in repetition rate of perhaps a factor of four. The flux/sec is, therefore, about 4× that indicated in the table.

⁺This corresponds to a full muon shield of Fe.

Table VI. Comparison of "Effective" Number of Neutrinos/ 10^2 Protons for Various Beams with the Repetition Rate Factor Included ($2\times$ for 100-GeV Protons; $4\times$ for 50-GeV Protons).

	Compromise Drift Space	E_ν in GeV					Effective Total Neutrinos >1 GeV	Total Length for Detector
		1-3	3-5	5-10	>10	>20		
Basic Beam								
150 m Fe Shield	750							
200-GeV Protons	meters	<0.01	1.0	2.7	2.3	0.54	6.0	900 m
Full Earth Shield								
600 m								
200-GeV Protons	1500	<0.01	<0.01	0.3	1.7	0.79	2.0	2100 m
These Utilize 50 meter Fe shields of less.	200 GeV $P_{\pi\max} = 50$	650	0.6	3.5	3.1	1.4	0.39	8.6
	100 GeV $P_{\pi\max} = 50$	650	1.2	4.0	4.8	1.8	<0.01	11.8
	50 GeV	400	4.04	10.20	5.36	0.24	<0.01	19.84
	50 GeV	200	8.64	9.80	3.60	0.12	<0.01	22.16

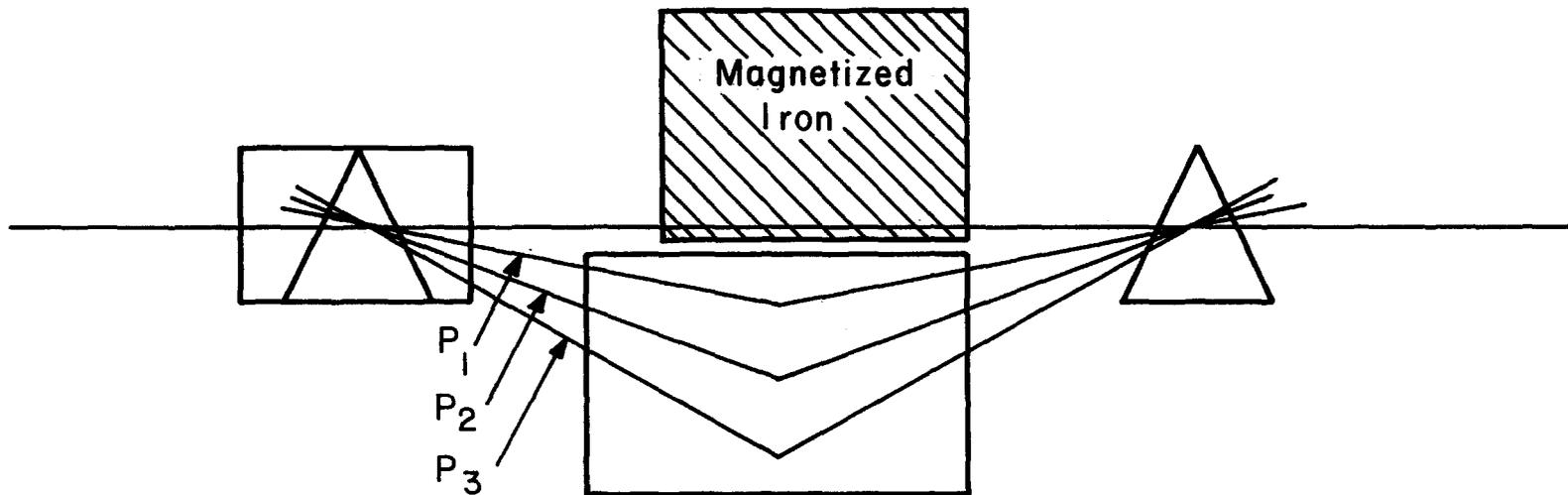


Fig. 1. Ideal achromatic device for separating positive and negative pions (kaons) and effecting a cutoff of maximum pion (kaon) momentum.

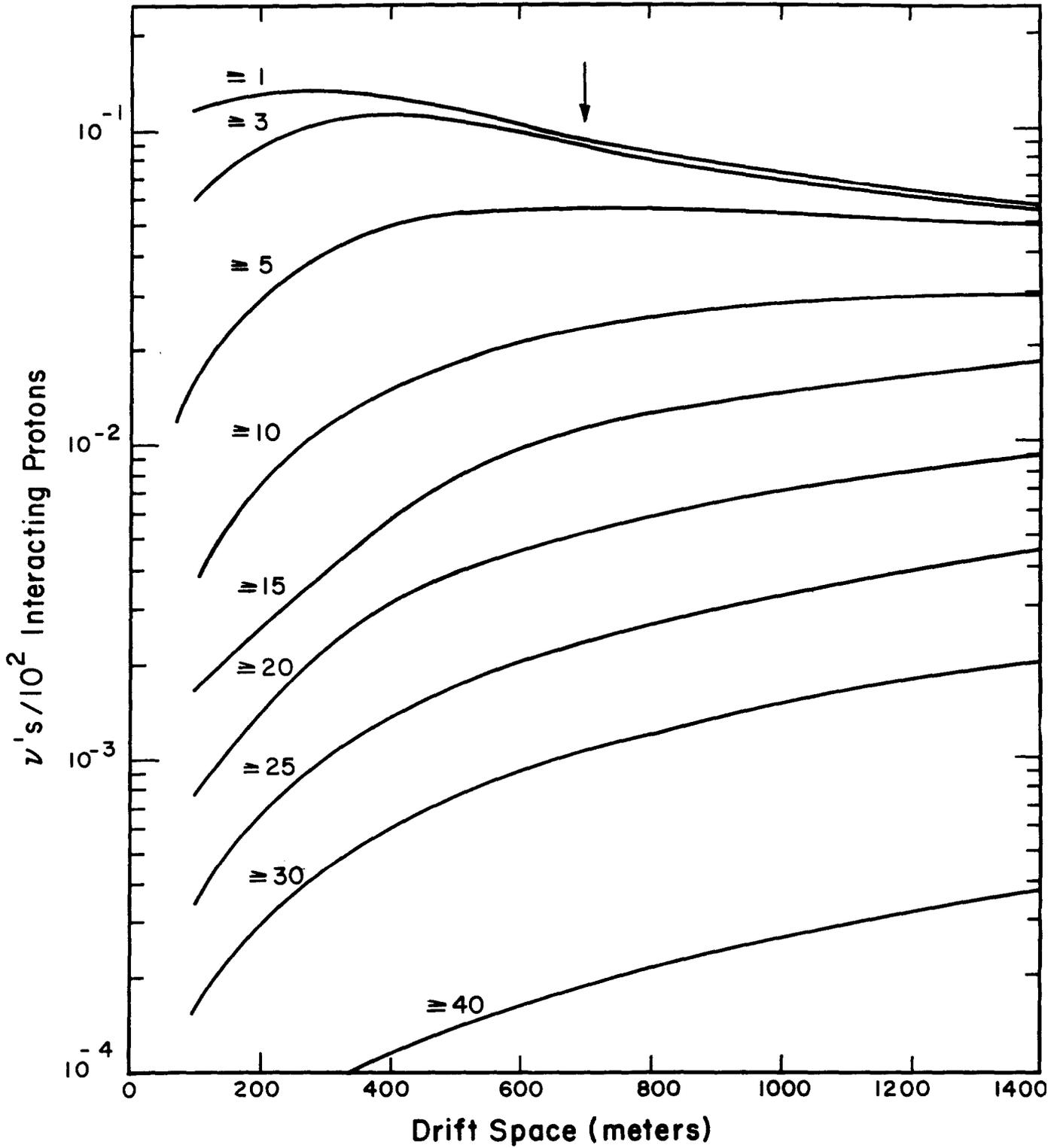


Fig. 2. Neutrino spectrum calculated for pencil beam and pions from Trilling. The chamber radius is 1.8 meters. Curves represent integral neutrino flux (for energies above E_ν) vs. drift space length. 200 GeV proton energy: shield = 50 meters.

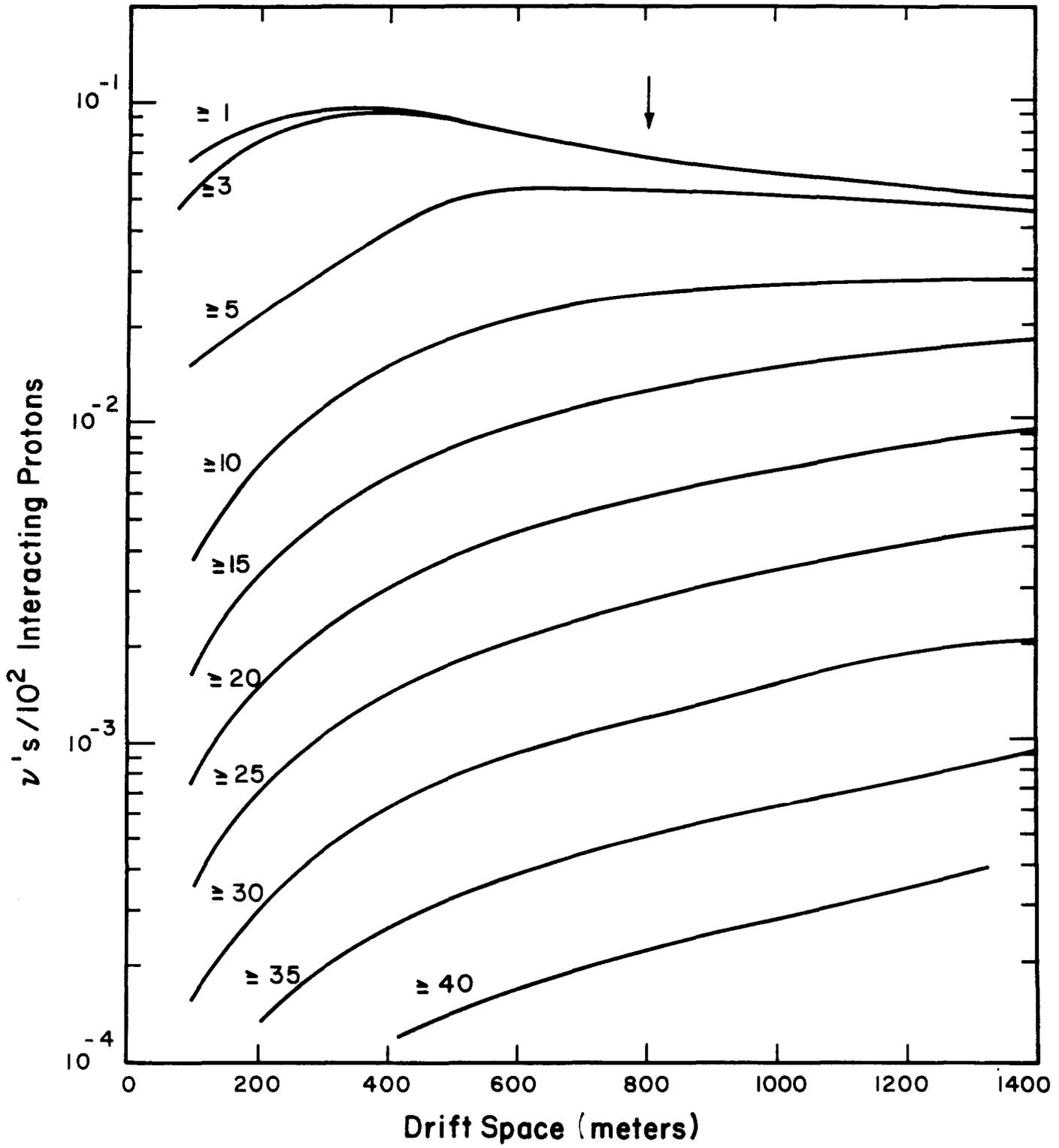


Fig. 3. Same as Fig. 2, but shield = 100 meters.

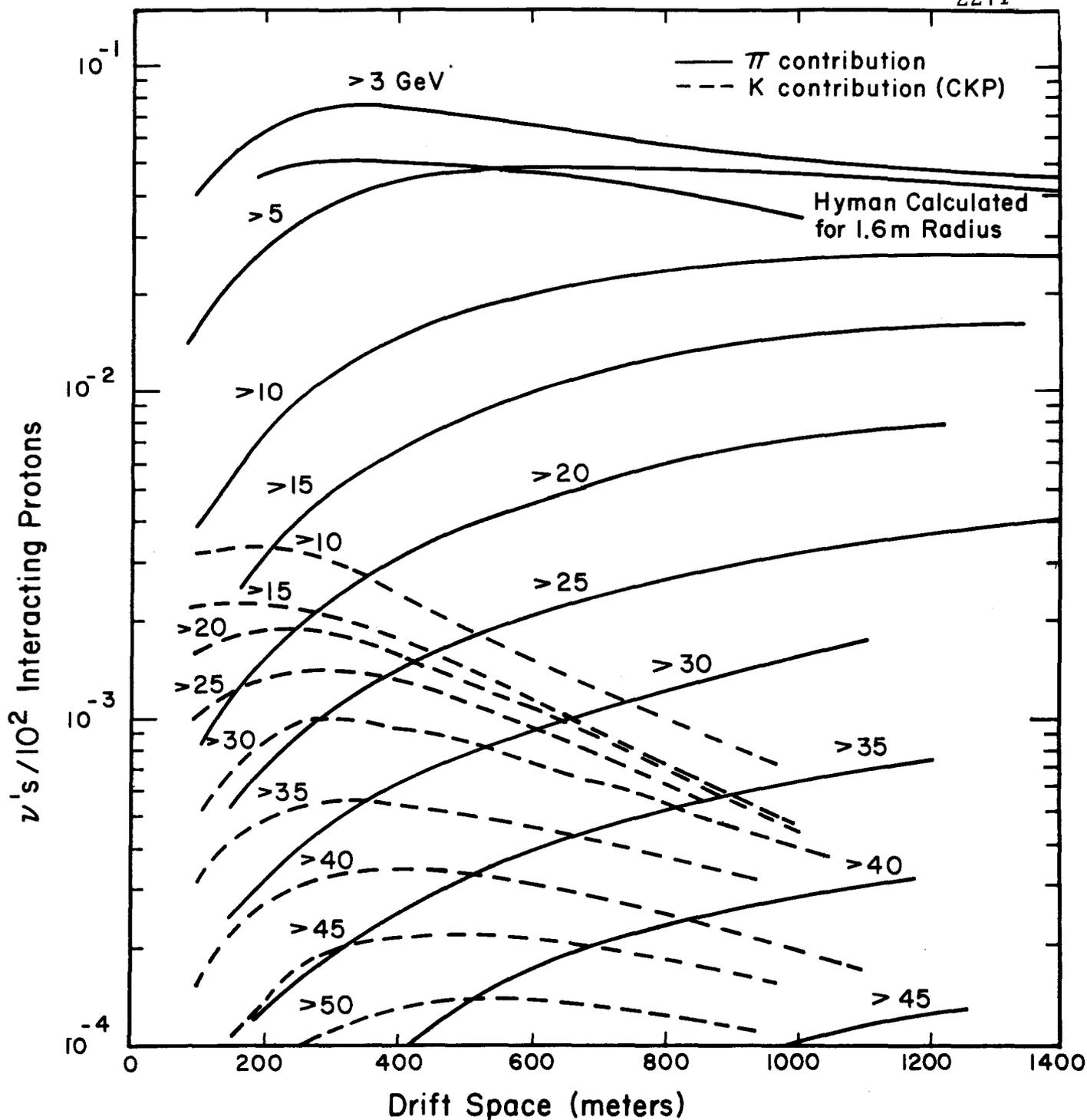


Fig. 4. Same as Fig. 2, but shield = 150 meters. Kaon contribution calculated from $1/10 \times \text{CKP}$ production formula. This plot corresponds to a (conservative) full muon shield of Fe at 200 GeV primary energy. The neutrino flux from L. Hyman's report may be compared with this except that Hyman used a chamber radius of 1.6 meters.

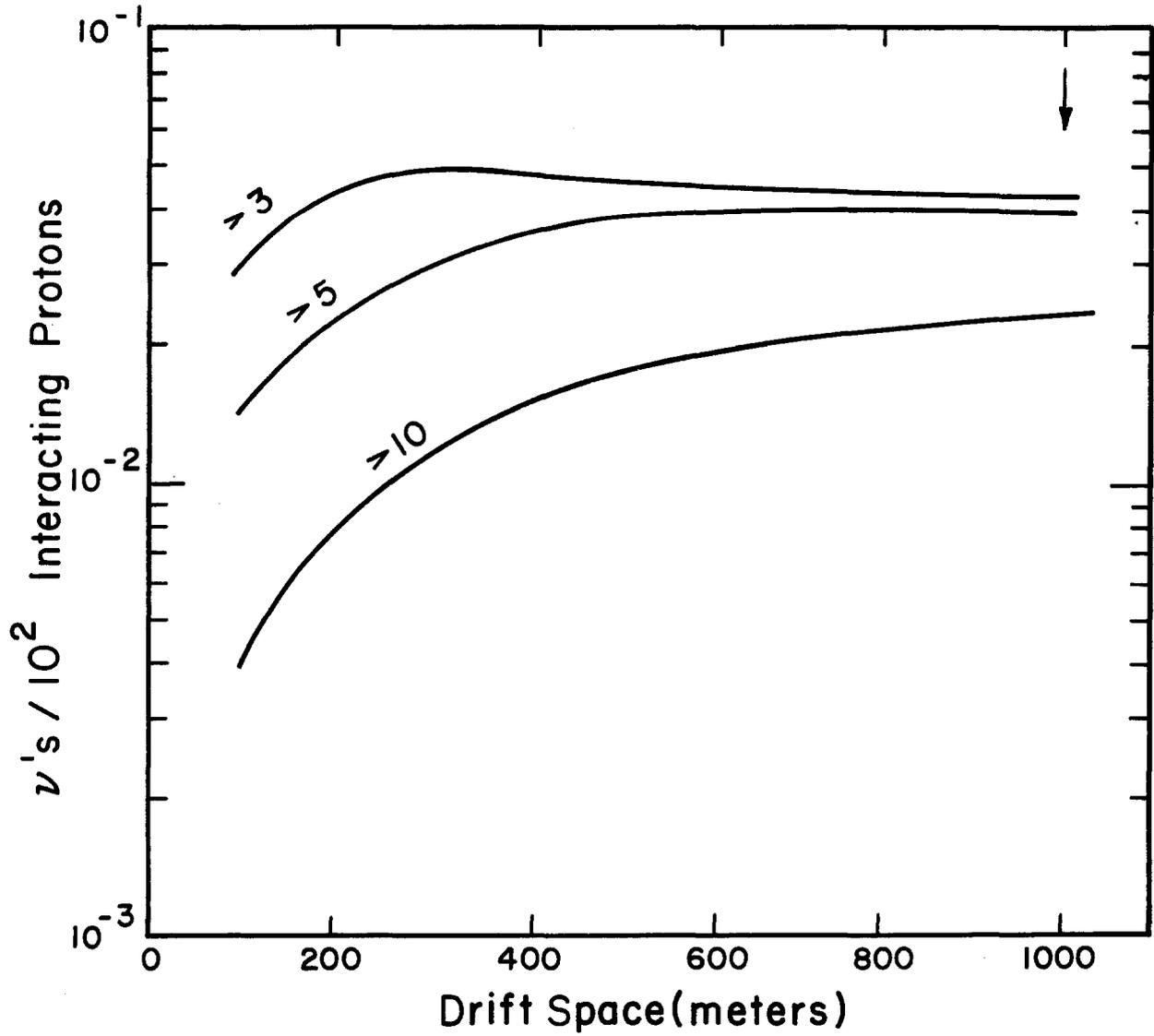


Fig. 5. Same as Fig. 2, but shield = 250 meters.

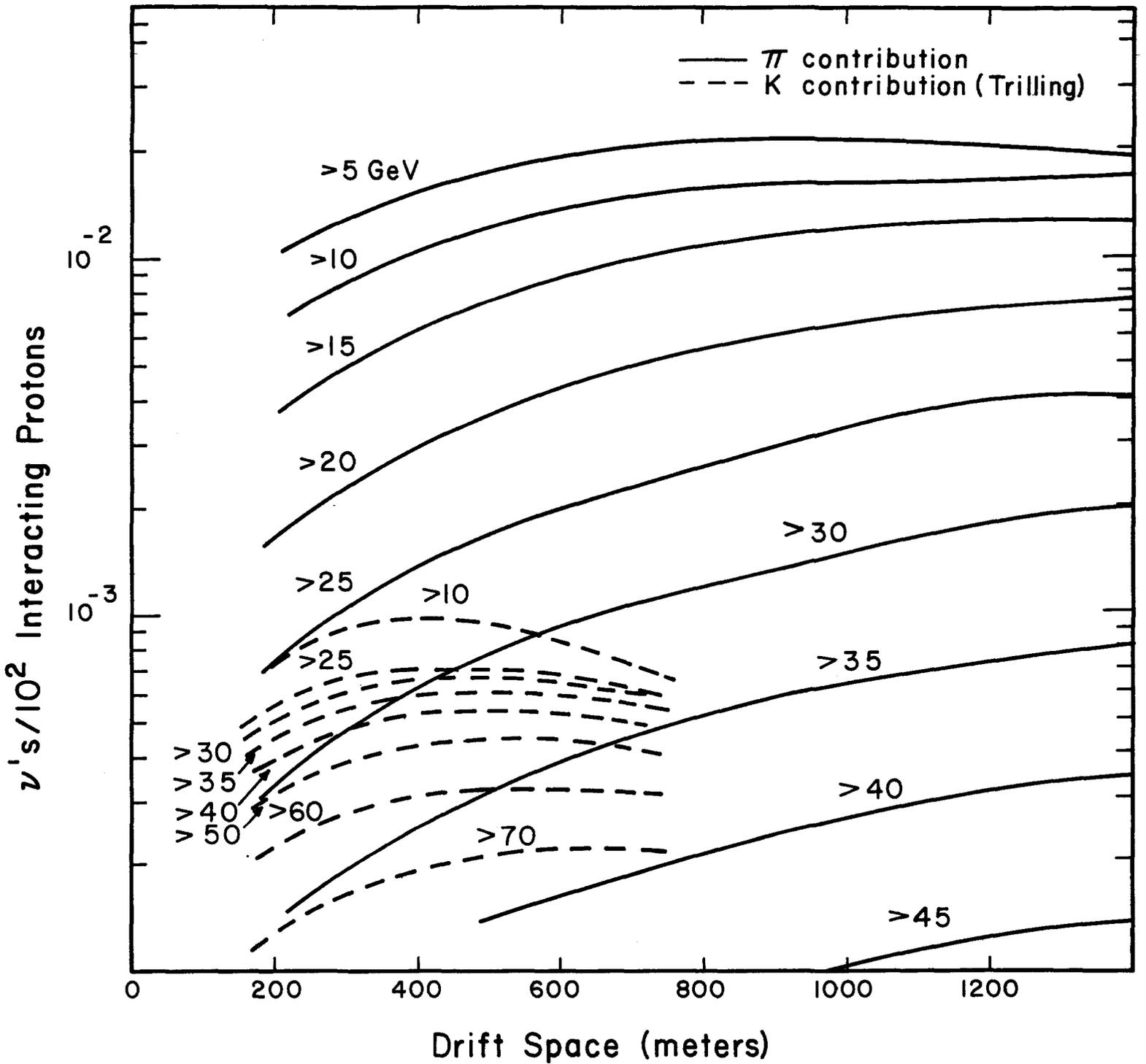


Fig. 6. Same as Fig. 2, but shield = 600 meters. Kaon contribution calculated from 1/10 Trilling formula. This plot corresponds to a full muon shield of earth at 200 GeV primary energy.

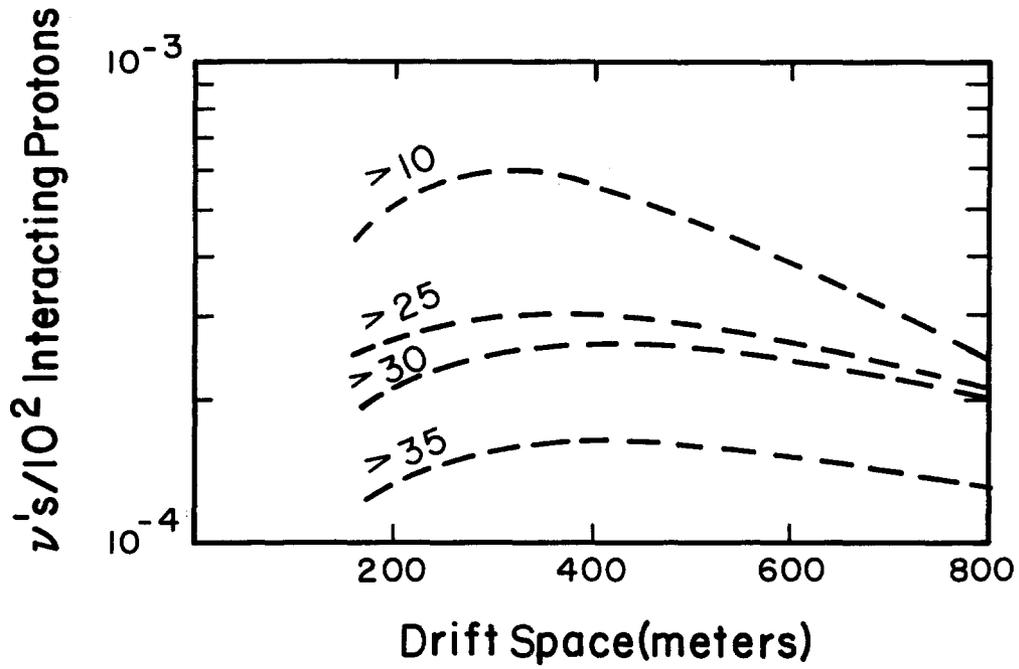


Fig. 6A. Same as Fig. 6, but kaon contribution calculated from 1/10 CKP formula. Full earth shield case.

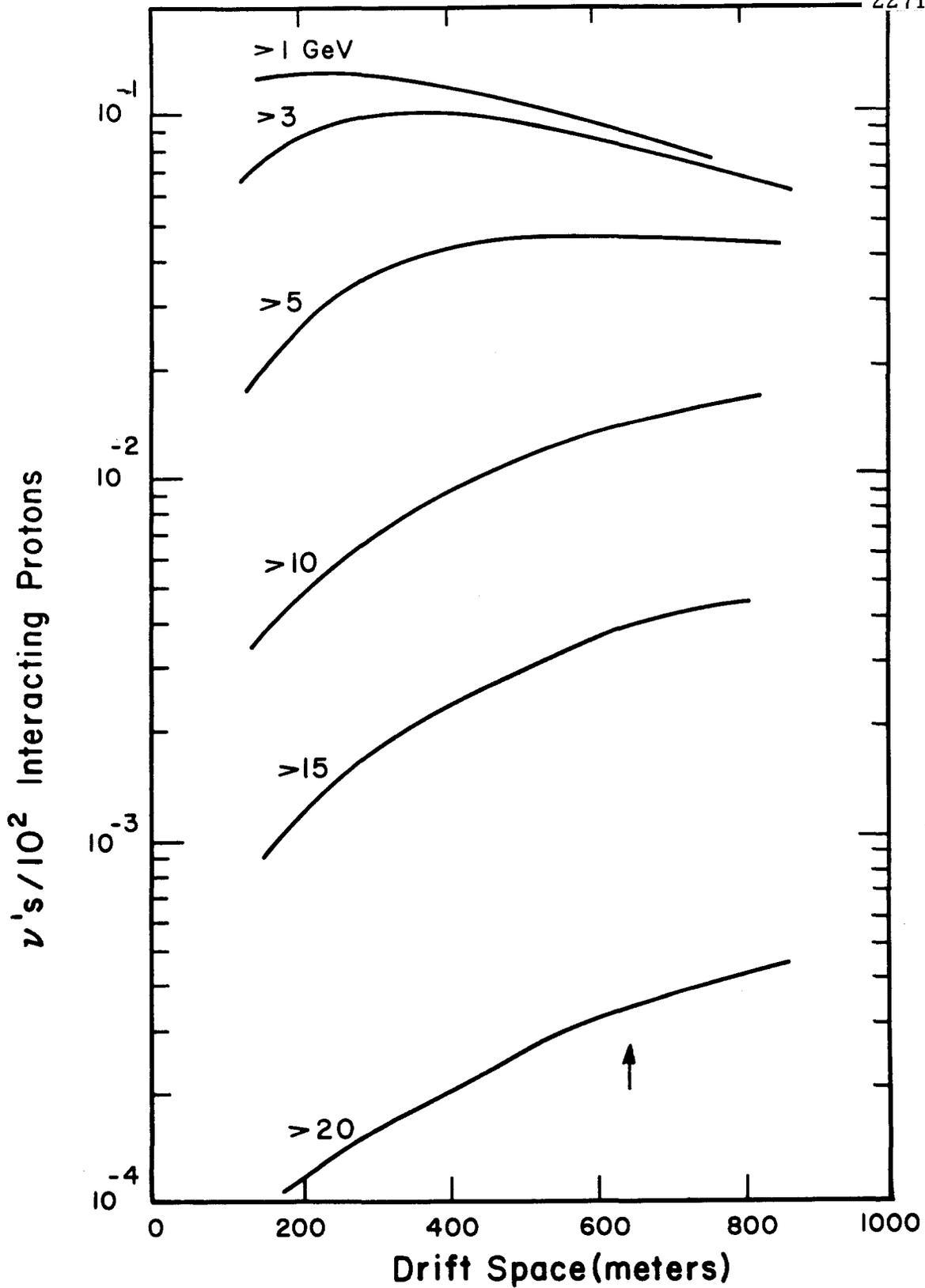


Fig. 7. Same as Fig. 2, with pions cut off above 50 GeV/c. This corresponds to a full muon shield of Fe for this case.

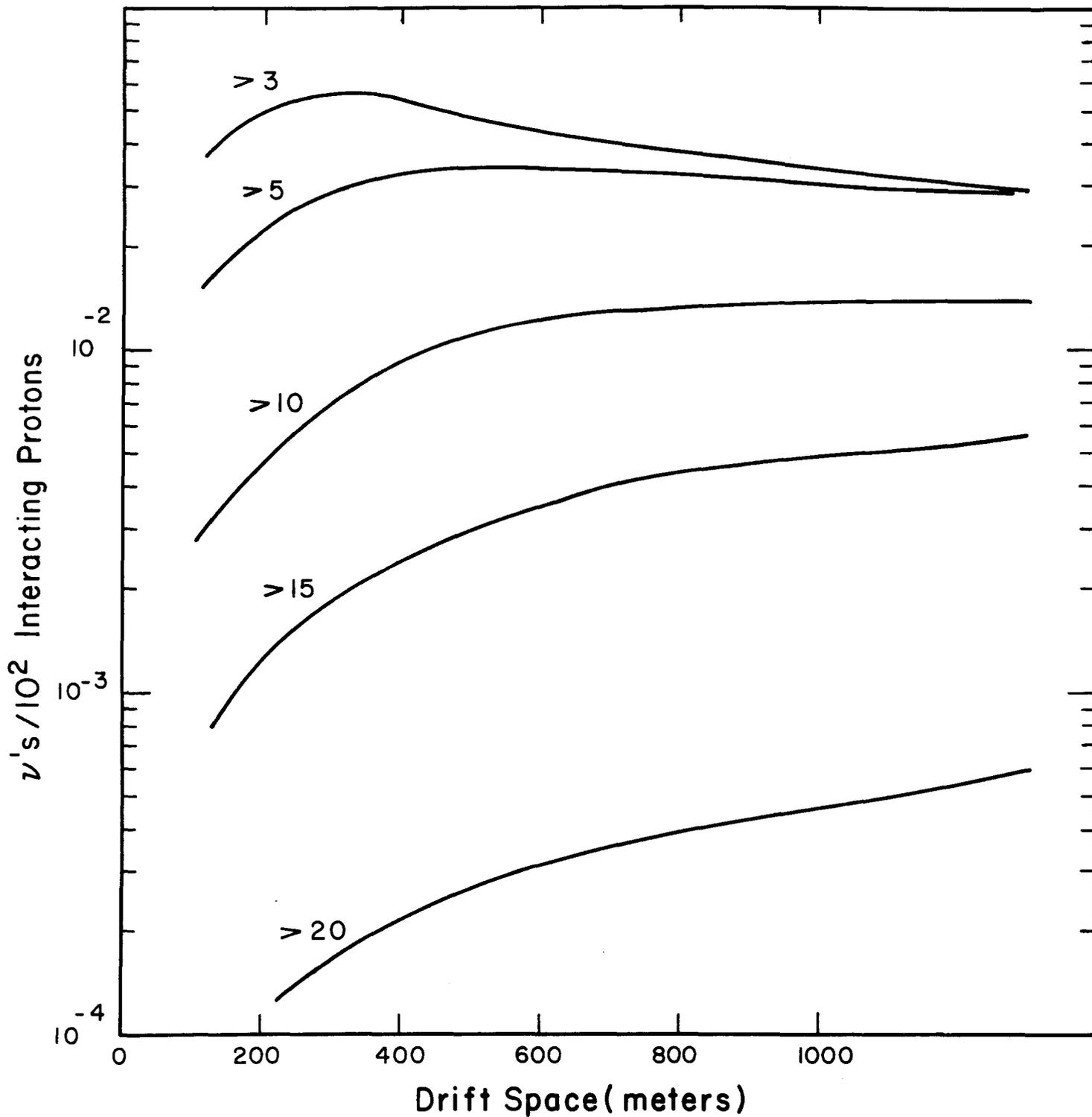


Fig. 8. Same as Fig. 2, with pions cut off above 50 GeV/c; shield = 200 meters. This corresponds to a full muon shield of earth for this case.

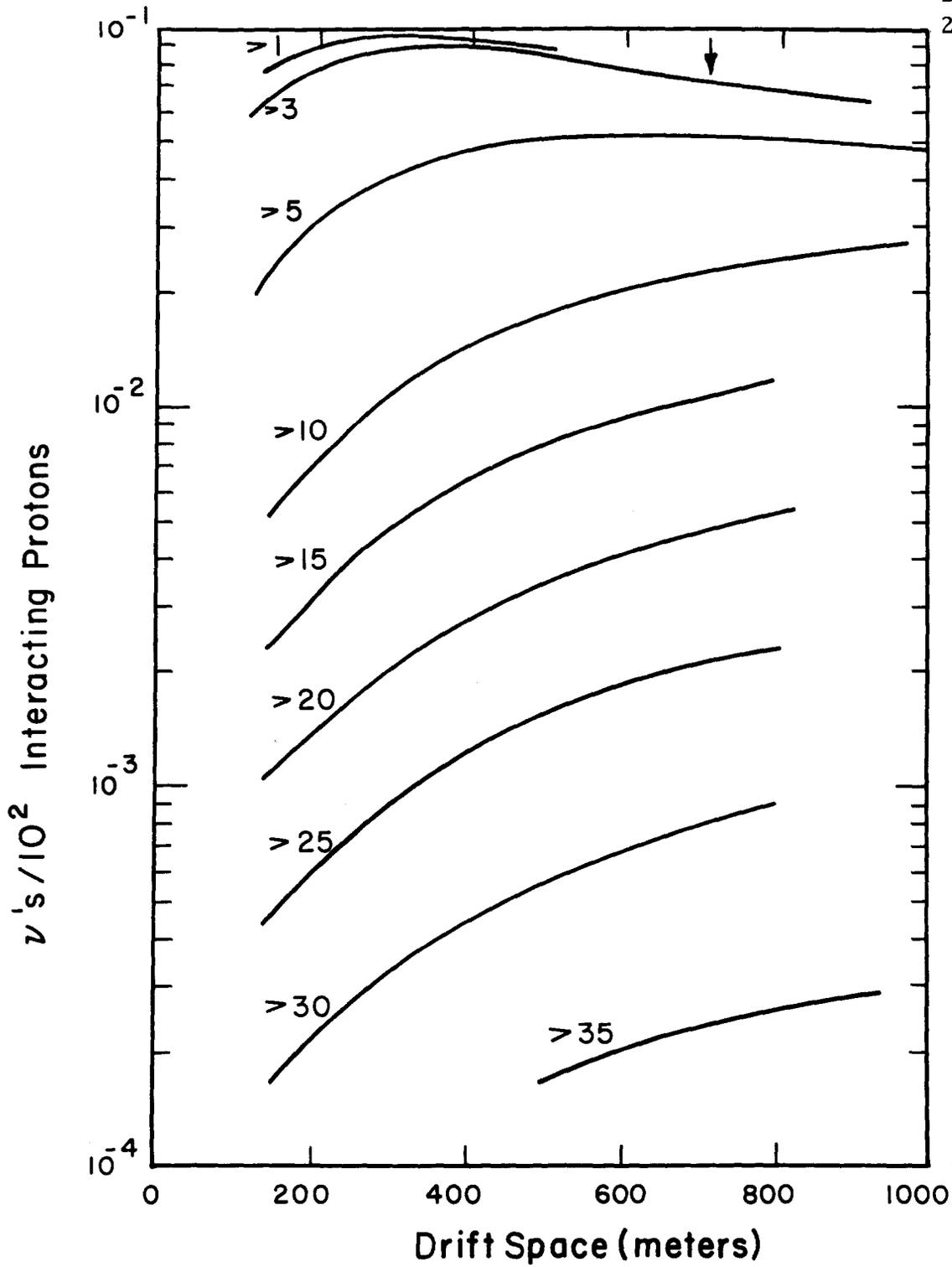


Fig. 9. Same as Fig. 3, with pions cut off above 100 GeV/c. This corresponds to more than is needed for a full muon shield of Fe.

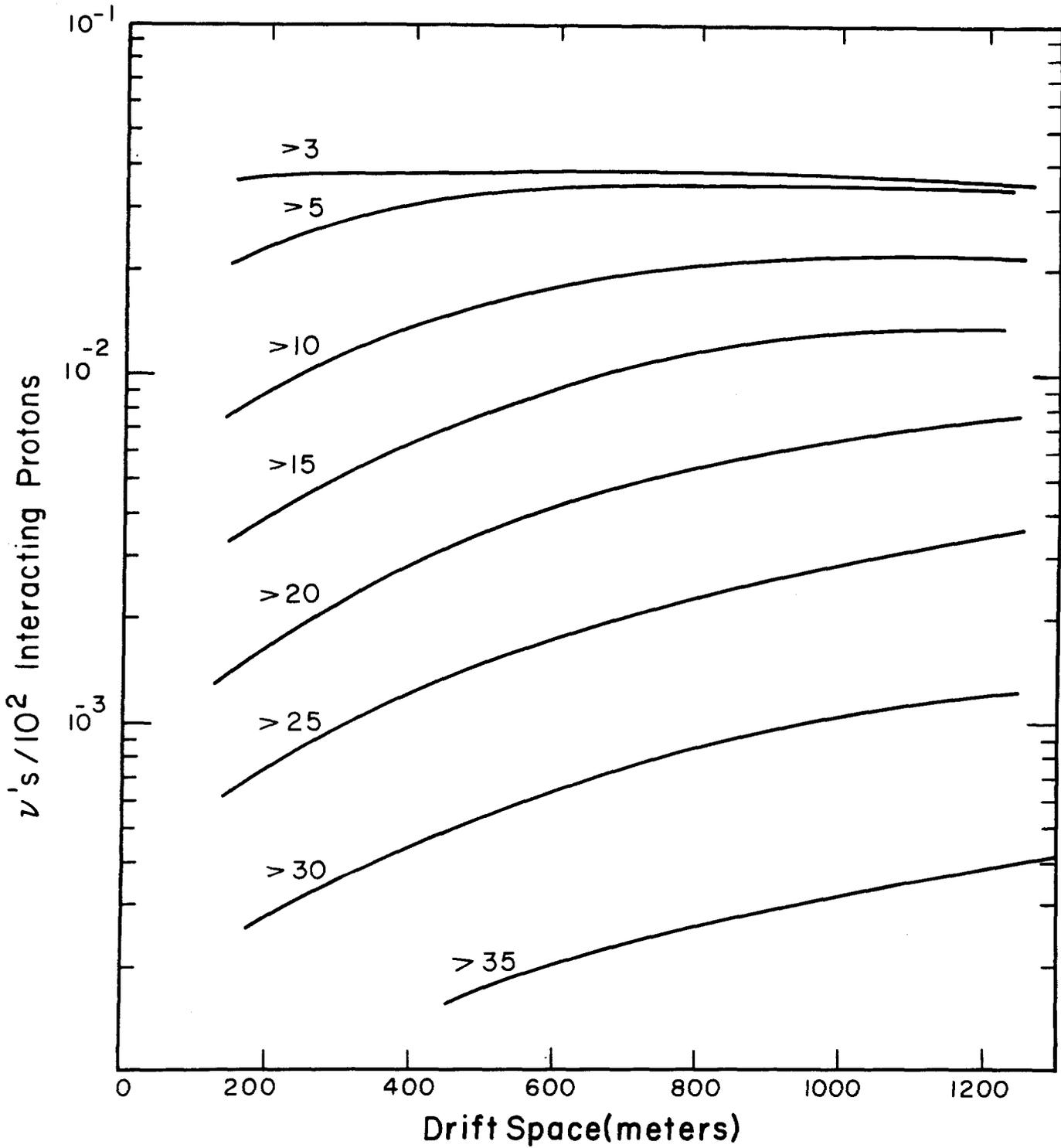


Fig. 10. Same as Fig. 9, but shield = 300 meters. This corresponds to a full moon shield of earth.

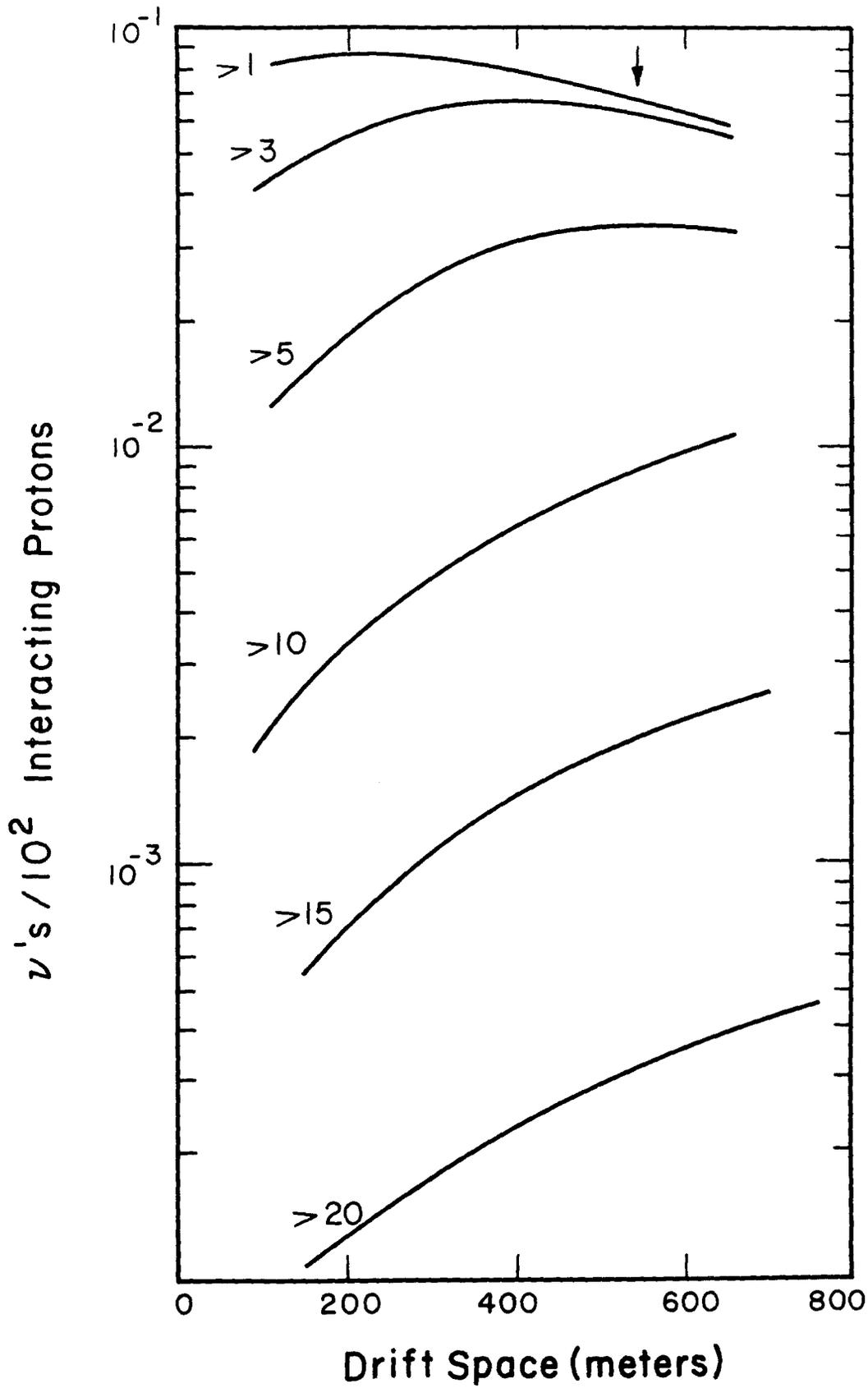


Fig. 11. Same as Fig. 2 but 100-GeV proton energy.

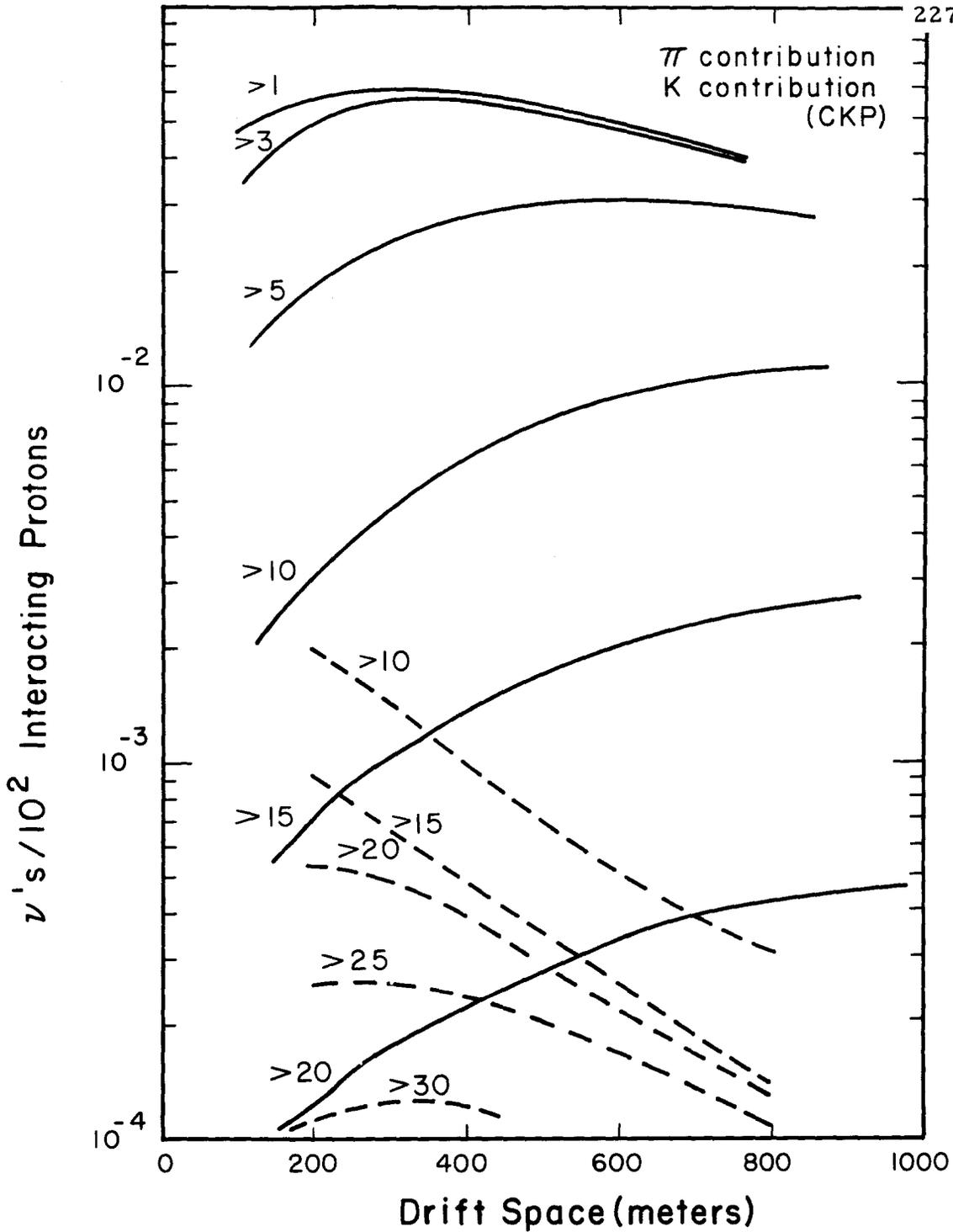


Fig. 12. Same as Fig. 3, but 100 GeV proton energy. This corresponds to more than is needed for a full muon shield of Fe. The contribution to the flux of kaons is plotted using 1/10 CKP.

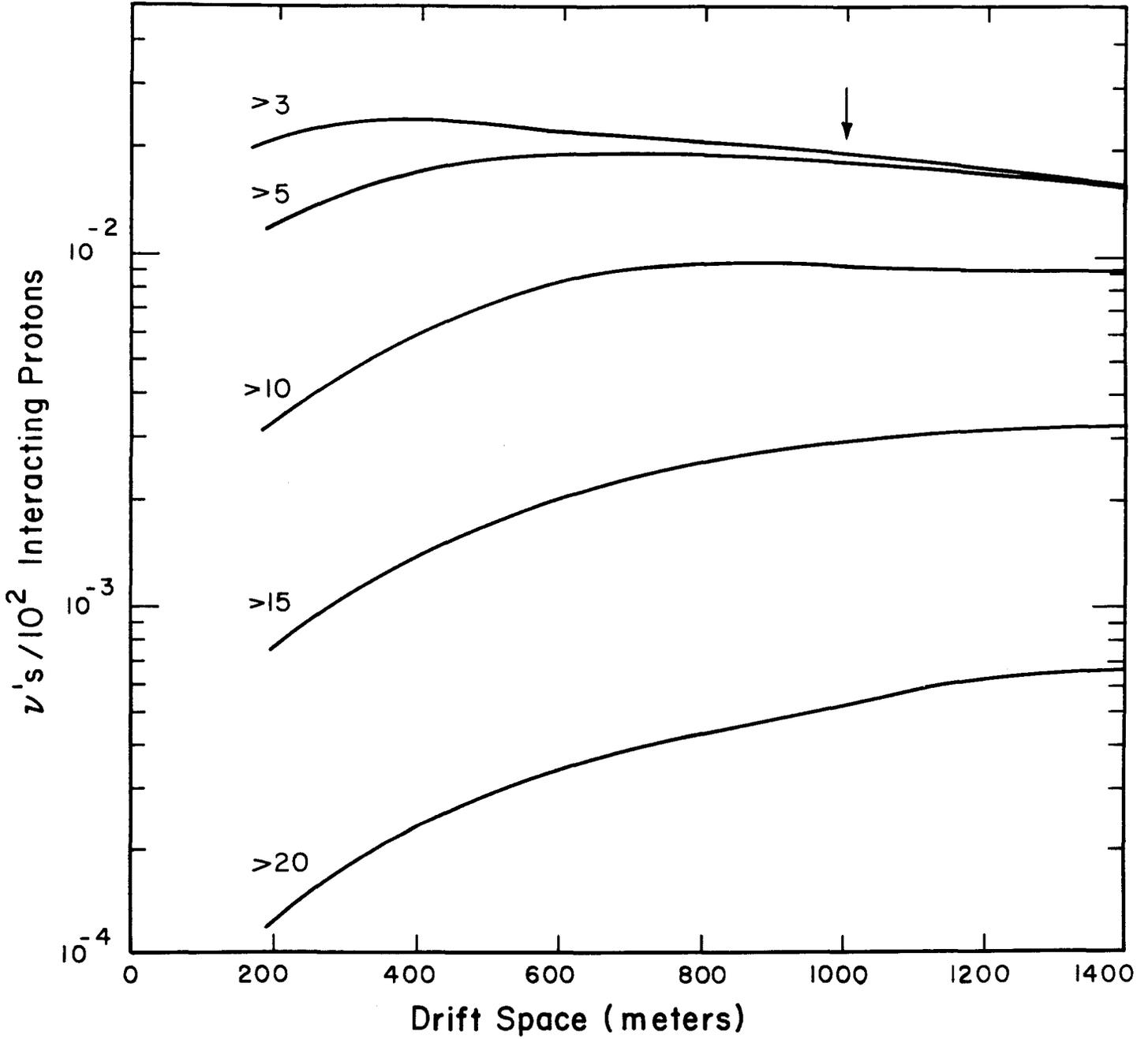


Fig. 13. Same as Fig. 4, but 100 GeV proton energy. This corresponds to a full muon shield of earth for this case.

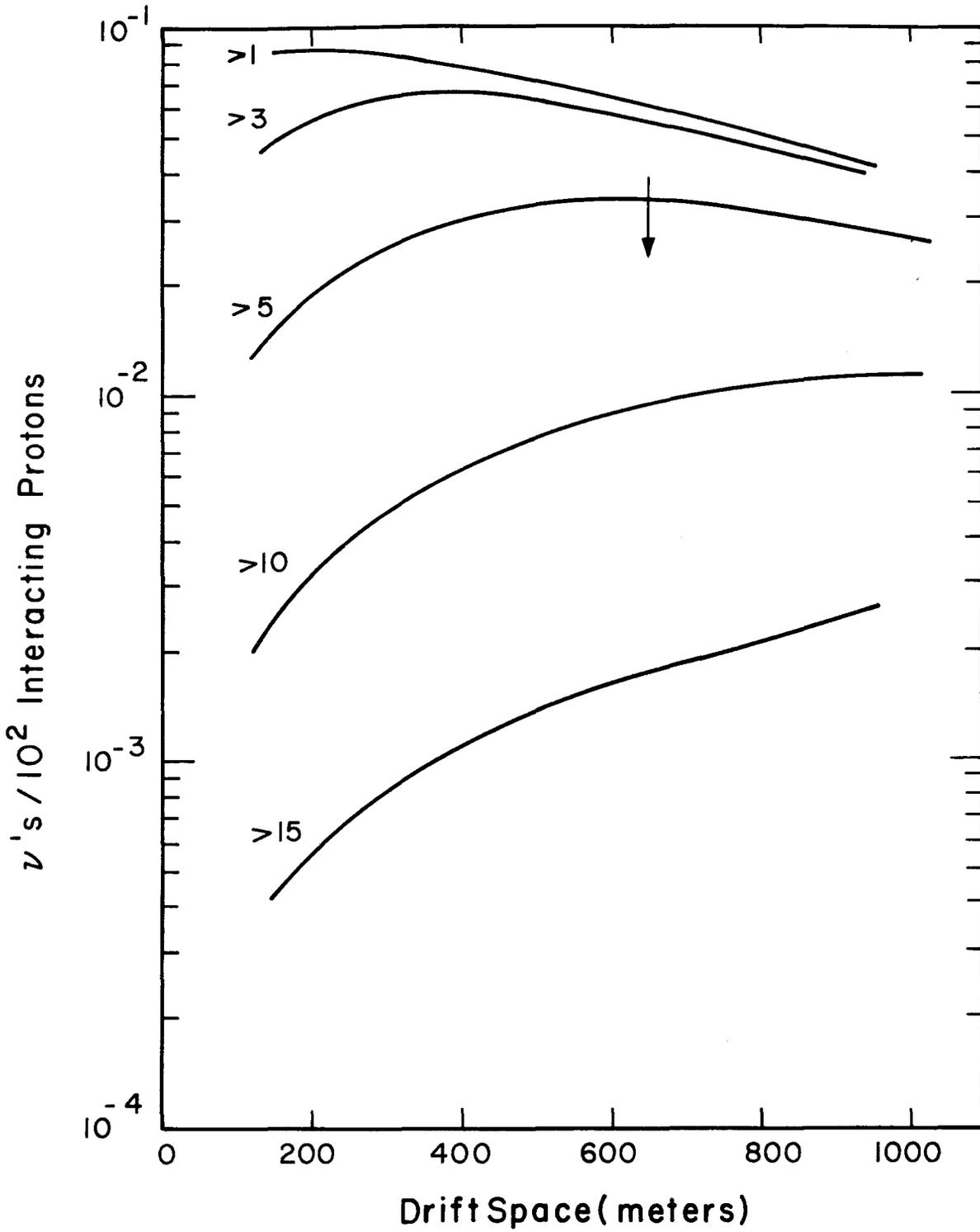


Fig. 14. Same as Fig. 7, but 100 GeV proton energy. This corresponds to more than is needed for a full muon shield of Fe.

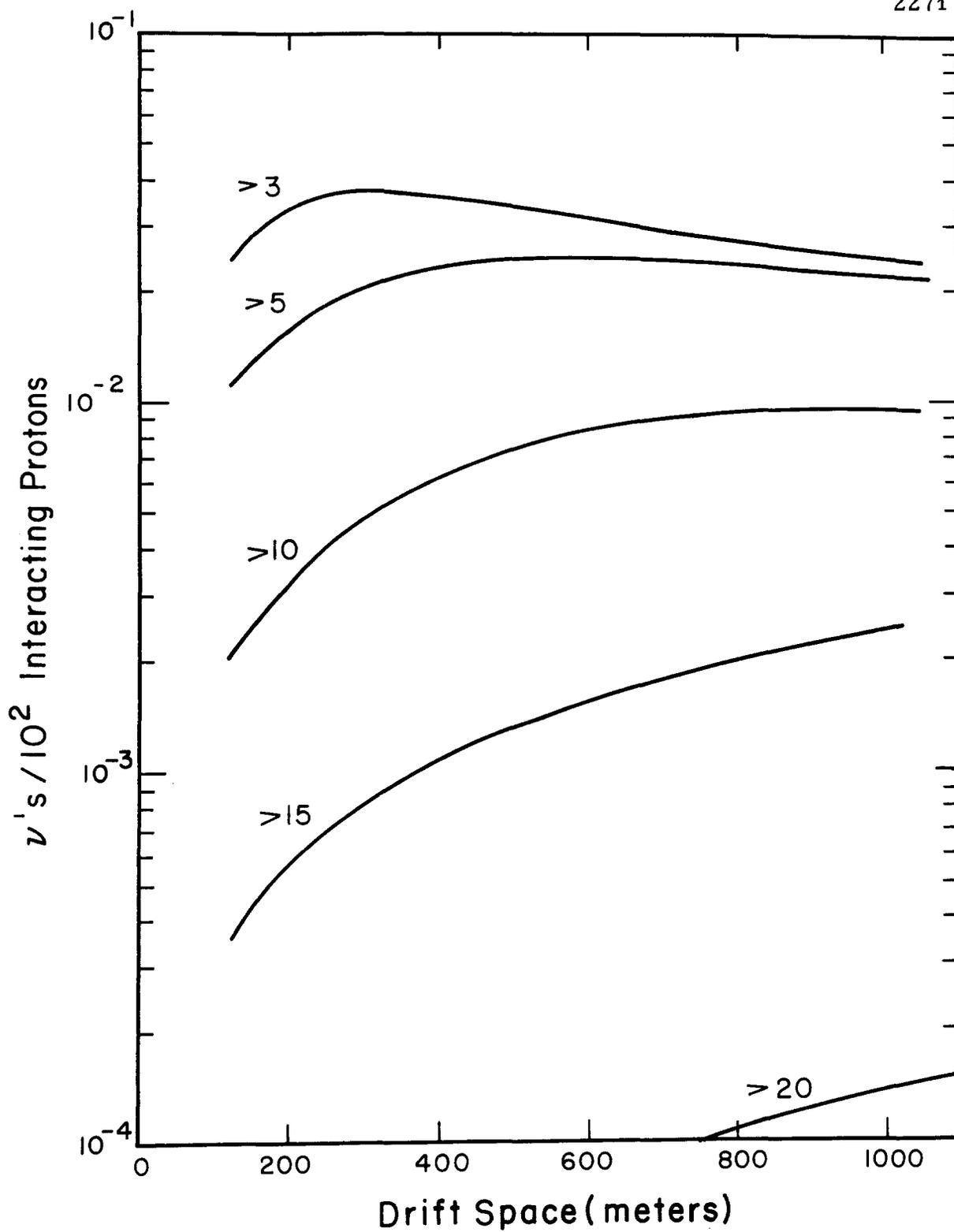


Fig. 15. Same as Fig. 8, but 100 GeV proton energy. This corresponds to more than a full muon shield of earth.

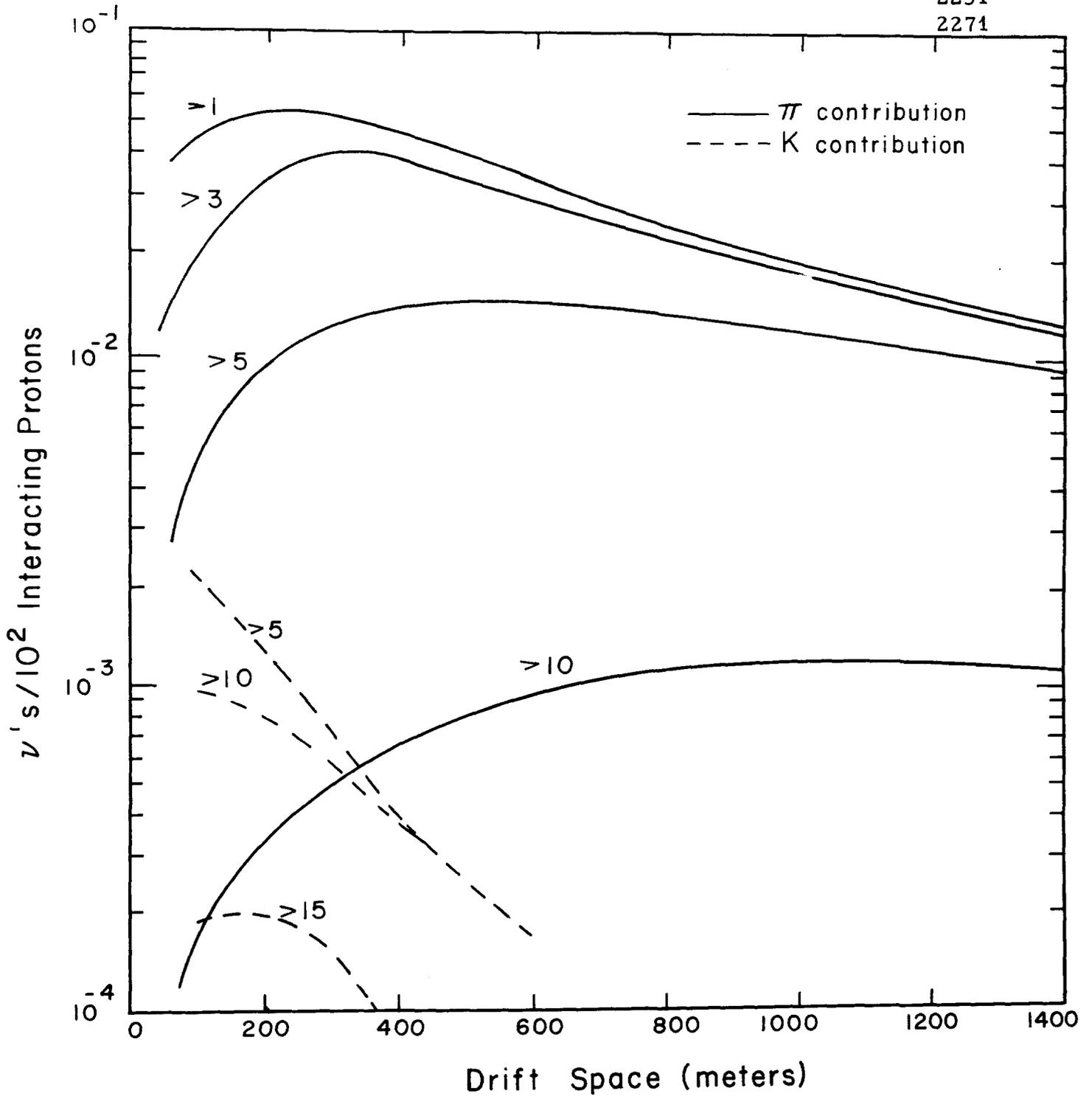


Fig. 15A. Same as Fig. 2, but 50 GeV proton energy. This corresponds to a full muon range shield of Fe which is more than needed. The contribution of kaons is plotted calculated from 1/10 CKP.

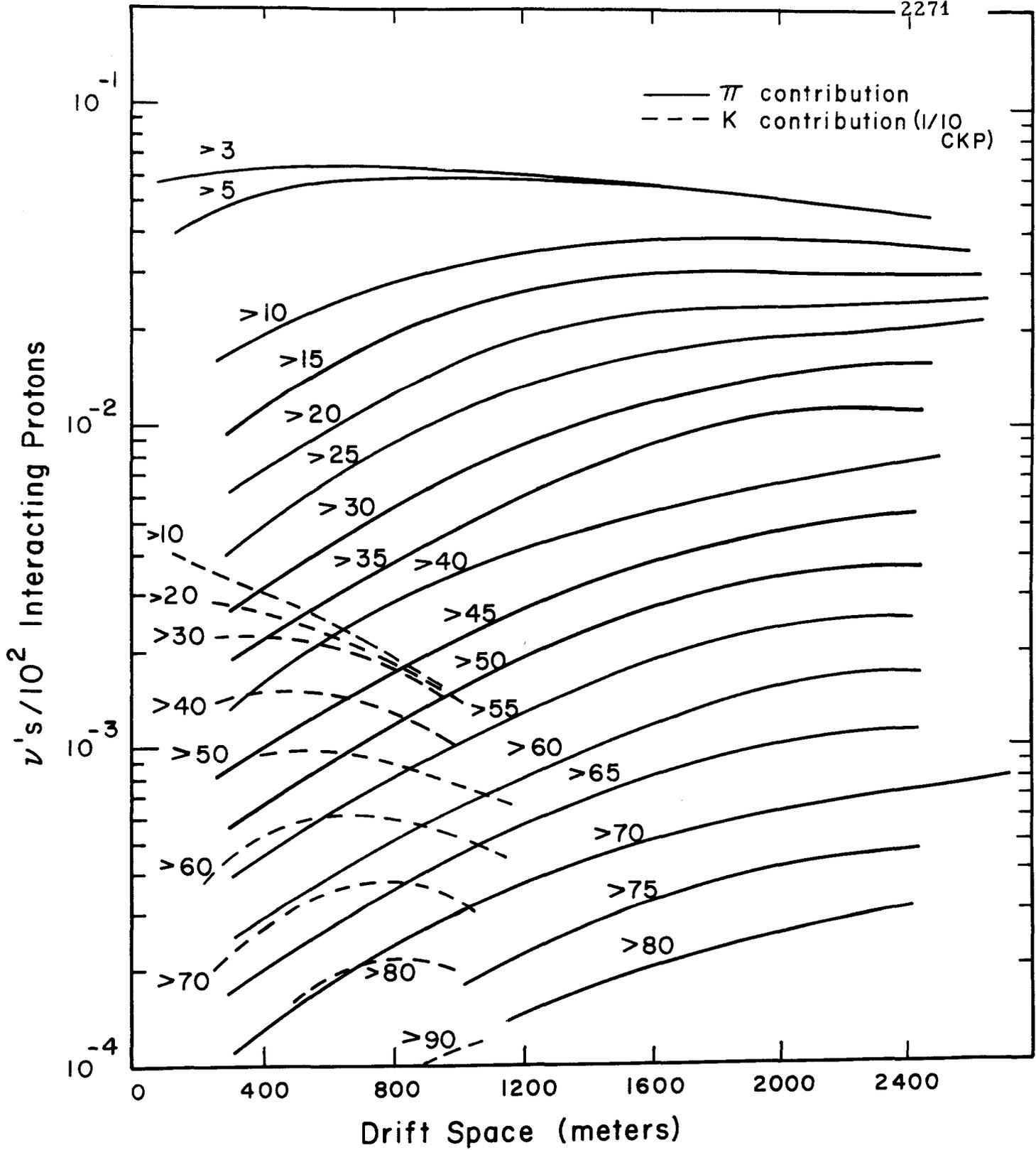


Fig. 16. Same as Fig. 2, except 400 GeV proton energy: shield = 300 meters. This corresponds to a full muon range shield of Fe. The contribution of kaons is plotted calculated from 1/10 CKP.

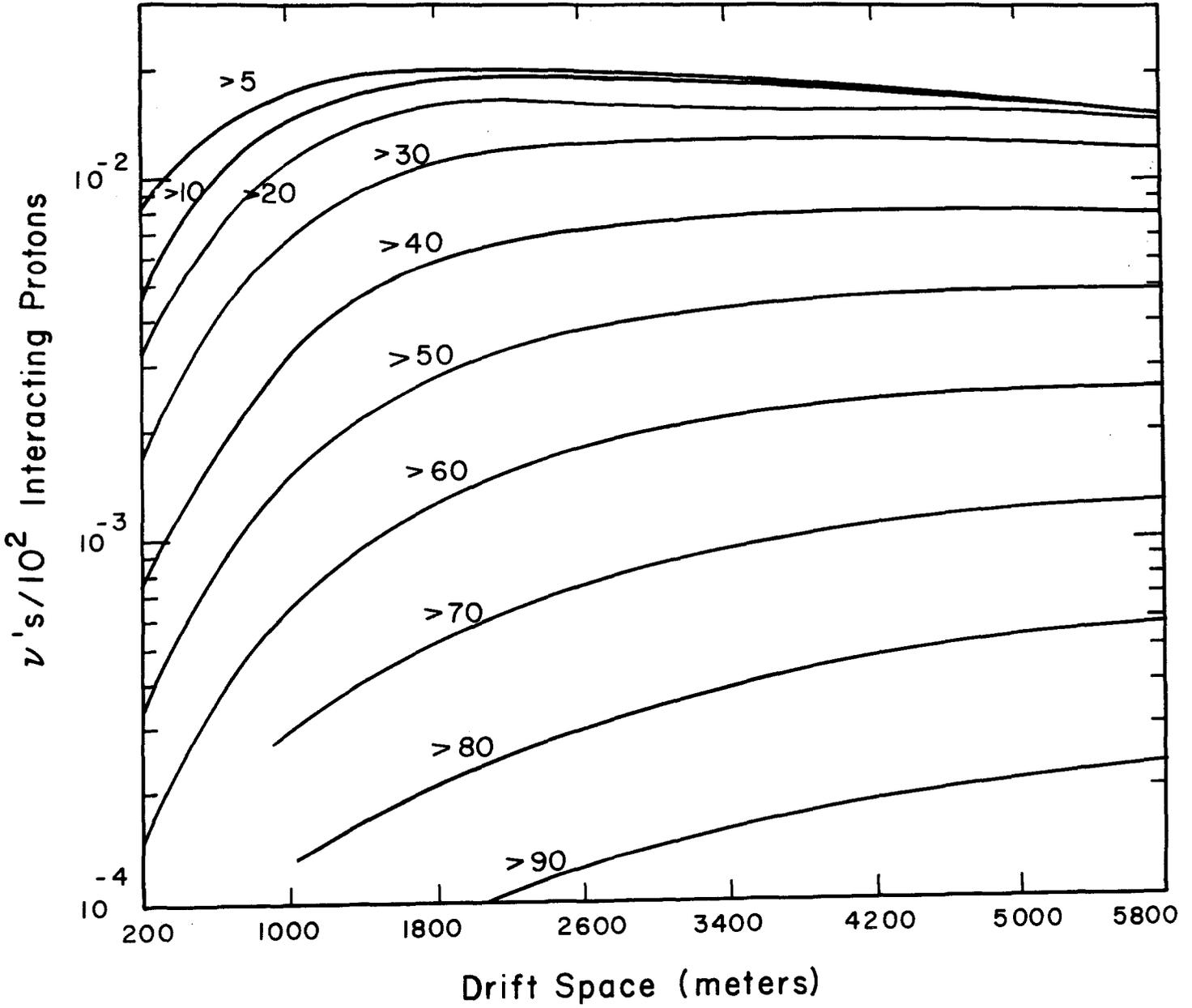


Fig. 17. Same as Fig. 16, except shield = 1200 meters. This corresponds to a full muon range shield of earth.