# A MUON BEAM AT NAL

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## Introduction

An example of the muon-neutrino beam for a 200-GeV accelerator has been discussed by Toohig.[1] The system requires a decay channel of approximately 1 km long, consisting of some 20 quadrupoles in a FODO configuration. It was estimated, under certain simplifying assumptions, that the system could produce a beam of muons with the intensity as high as $10^9$-$10^{10}$ per pulse in the energy range of 25 to 100 GeV ($\Delta p_\mu / p_\mu \sim 20\%$). This would seem to be more than adequate flux for most experiments.

The purpose of the present study is to investigate whether one can construct a simpler (cheaper) muon beam without too much loss in the intensity. A possible solution to this is to make a system just like Toohig's but shorter. Since the intensity of the beam is roughly proportional to the length of the system, if one takes Toohig's estimate literally, a system with 100-m flight path (using only few quadrupoles) can produce a beam of $10^8$-$10^9$ muons per pulse. This is still an adequate flux for many interesting experiments.

An important question now is to examine the validity of the assumptions which Toohig has used in his estimate. Another question is to

investigate if the other qualities of the beam, besides its intensity, are good enough for a practical use. Let me list the questions which I am going to study in some detail.

1. Pion-Production Spectrum. Toohig had used the Trilling formula in his estimate of the pion flux. Although the validity of this formula at 200 GeV was not established, there was no better formula available at that time. Recently a theoretical model proposed by Hagedorn and Ranft has shown remarkable success in predicting many features of particle productions at the energies up to 30 GeV. This model may well predict the characteristics of pion production at 200 GeV better than any empirical formula. Therefore, I decided to use this model for the present study. Even if the Hagedorn-Ranft model should fail at 200 GeV, a comparison of two results would give us a measure of sensitivity of our results to the uncertainty in the pion-production characteristics.

2. Acceptance Solid Angle of the Entrance (Front End) System. A triplet of quadrupole magnets with 12-in. aperture was used in Toohig's system. The half angle subtended by this system was 11.7 mrads. It was assumed that all the pions produced at angle less than 11.7 mrads were accepted by this entrance system. This means a rather large solid angle. In a practical application it might be difficult to obtain such large acceptance angle.

3. Probability of Muon Capture by the Decay Channel. In Toohig's estimate it was assumed that the muons from pion decay will all be

retained in the beam channel, provided that their momenta lie in the pass band of the channel. This may not be a safe assumption. Let us consider an example of a 50-GeV pion decaying into a 46-GeV muon. Under the present assumption this muon will be retained in the channel with 100 percent probability. The decay angle in this particular case is 0.5 mrads. This is non-negligible compared with the "typical" angular divergence of pions in the system of 1.25 mrads. It means that the phase space of the muons will be appreciably larger than that of the pions. There are a few problems of practical importance which were not considered by Toohig. Let me list these problems:

4. beam halo and shielding
5. tail in the muon momentum spectrum
6. beam purification (pion filter).

Of the six problems listed above 1, 2, and 6 are relatively simple and will be discussed in some detail later. The problems 3, 4, and 5 are quite complicated. The only way to answer these questions is to run a Monte-Carlo type program. We have such a program and have used it for the design of the muon beams at AGS.[2] We have used the same program, with few modifications, for the present study. Some results are already available and will be described briefly in the last section. They are, however, still preliminary and may well be far from the optimum. In the next three sections we shall present an estimate of the beam characteristics, on the basis of general considerations, taking practical limitations into account.

Entrance System

For the purpose of collecting a maximum flux of pions by the entrance system one would like to set the first quadrupoles as close to the target as one can. A serious limitation here is the radiation problem. Maschke has estimated that the (minimum) desirable distance between the target and the first magnet is 20 feet. Once this distance is fixed, the only way to increase the flux is to use a large-aperture quadrupole. However, there are a number of practical reasons which make the use of large quadrupole undesirable. For instance, it is difficult to capture those muons which originated in the large-aperture section into the following channel efficiently, unless one constructs the entire channel with large-aperture quadrupoles. Then the size (cross section) of the beam becomes large. If an appreciable amount of muons fail to be captured by the subsequent section of the channel, this may add a significant contribution to the beam halo. It seems better to use a standard-size aperture for the entire beam channel.

As a simplest example of the entrance system let us consider a quadrupole doublet. The full aperture of the quadrupole is chosen to be 4 in. A system which is capable of producing a parallel beam up to 100 GeV/c is given in Table I. Several variations of this system have been considered. None of them gives any better acceptance than this one. Let us use this system as the entrance system for the present study.

Table I.

| | | |
|---|---|---|
| Target to first quad | | 240 in. |
| Length of first and second quads | | 120 in. |
| Spacing between two quads | | 60 in. |
| Strength of first quad | | +7.3 kG/in. |
| second quad | | -4.4 kG/in. |
| Acceptance (Half) Angle | Horizontal | 2.3 mrads |
| | Vertical | 7.7 mrads |

Our next task is to calculate how many pions we can collect by this system. Four different configurations have been considered. In the first two cases the system is set at exactly zero degrees production angle to the primary proton. In the latter two the system is set at 2.5 mrads with respect to the primary proton.

| | |
|---|---|
| Case I : 0 degree | Simple collimator (rectangular hole) |
| Case II : 0 degree | A beam stopper is added. This masks $\lvert \theta_v \rvert < 1$ mrad (Fig. 1) |
| Case III : 2.5 mrads | Simple collimator. The primary beam will hit the collimator. |
| Case IV : 2.5 mrads | A beam stopper to mask $\theta$ prod. < 2 mrads is added (Fig. 2) |

In the Case I, all primary protons which did not interact at the target will go through the entrance system and may cause a serious radiation problem downstream of the channel. Although the beam stopper or any depature from zero-degree production will cost us some pion flux, there are simple solutions for getting rid of unwanted protons.

The pion-production spectrum predicted by the Hagedorn-Ranft model is shown in Fig. 3. Figures 4 and 5 are the angular distributions after integration over the azimuthal angles, i.e.

$$\frac{d^2 N_\pi}{dp \cdot d_\theta} = \frac{2_\pi \, d^2 N}{dp \cdot d_\Omega} \cdot \sin\theta \, .$$

The area under the curve represents the total pion flux for each momentum.

Let us now find out how much of these available fluxes can really be utilized by our system. For a given geometry of the system (i.e. Case I, II, III, or IV) and a given value of the production angle one can easily find out what fraction, $f(\theta_p)$, of the azimuthal angle will be accepted by the entrance system. Figure 6 is shown here as a typical example to illustrate the way I obtained the values of $f(\theta_p)$. The results are summarized in Fig. 7. The triangular points in Figs. 4 and 5 are the product

$$\frac{d^2 N_\pi}{dp \cdot d\Omega_p} \cdot f(\theta_p) = n(\theta_p) \, .$$

Only the Case I is shown for 100 GeV/c and the Case IV for 50 GeV/c. The integral

$$n(\theta_p) \, d\theta_p = N \, ,$$

will give us the total flux of pions accepted by the entrance system. The values of N thus obtained are tabulated in the third column of Table II.

Table II. Estimate of "Useful" Muon Flux at the End of Decay Channel.

| Momentum (GeV/c) | Entrance* System | Pion Yield (per GeV/c Interacting Proton) | Muon Flux (per $5 \times 10^{12}$ Proton $\Delta p_\mu / p_\mu \sim \pm 5\%$) | Toohig's Values for 200 m |
|---|---|---|---|---|
| 100 $\pi^+$ | 1 | $0.38 \times 10^{-3}$ | $5.3 \times 10^7$ | $(1.2 \times 10^8)$ |
| | 2 | 0.28 | 3.9 | |
| | 3 | 0.29 | 4.1 | |
| | 4 | 0.24 | 3.4 | |
| 100 $\pi^-$ | 1 | $0.74 \times 10^{-4}$ | 1.0 | |
| | 2 | 0.52 | 0.72 | |
| | 3 | 0.55 | 0.77 | |
| | 4 | 0.47 | 0.66 | |
| 50 $\pi^+$ | 1 | $0.96 \times 10^{-3}$ | $1.34 \times 10^8$ | |
| | 2 | 0.77 | 1.08 | $(4.4 \times 10^8)$ |
| | 3 | 0.89 | 1.24 | |
| | 4 | 0.82 | 1.15 | |
| 50 $\pi^-$ | 1 | 0.39 | 0.55 | |
| | 2 | 0.30 | 0.42 | |
| | 3 | 0.34 | 0.48 | |
| | 4 | 0.31 | 0.43 | |

*System # 1 0 mrads No beam stopper
# 2 0 mrads ±1-mrads beam stopper
# 3 2.5 mrads 0.5-mrads beam stopper (collimator well)
# 4 2.5 mrads 2-mrads beam stopper

**Targetting efficiency of ~ 1/3, decay path ~ 200 m, and no loss of "useful" muon in the beam channel are assumed.

Muon Flux

In the fourth column of Table II an estimate of the muon flux is given. Following assumptions have been made:

1. $5 \times 10^{12}$ protons hit a target and one third of them will interact.
2. A total decay path of 200 m.
3. Pions are momentum analyzed to $\Delta p/p \simeq \pm 5\%$.
4. All the muons with

$$0.9\ p_\pi < p_\mu < p_\pi$$

will be captured by the channel.[3]

The first assumption is completely arbitrary. One can scale up or down the flux depending on the intensity of the primary proton or the size of the target to be used. The second assumption is somewhat arbitrary too. One can make the system longer by adding more quadrupoles. The flux should increase linearly as the length increases. In order to minimize the problems of radiation, the beam halo etc., it is desirable to eliminate unwanted pions at an early stage. A rough momentum analysis for the pions seems to be necessary. The third assumption has been made for that reason. The fourth assumption is the one made by Toohig in his work and may not be quite valid as we discussed earlier. This remains to be a subject of further investigation.

A schematic drawing of the entire system under consideration is shown in Fig. 8.

Let us now compare our result to that of Toohig's. Since we have made a similar assumption for the decay channel, any discrepancy between two results must come from the difference in the pion-production spectrum or the entrance system. The last column of Table II gives Toohig's results for 200-m decay path. His values are 2-3 times larger than ours. The main source of the difference seems to come from the entrance system. Our system does not accept as large a solid angle as his.

Pion Filter

In order to achieve a reasonably good purity of the muon beam, one needs 20 mean free paths (mfp) of an absorber. The first 3 mfp is the transition region (i.e. more pions are produced than attenuated). The remaining 17 mfp give us an attenuation factor of $\simeq 3 \times 10^{-8}$. Since we start with $\pi/\mu \sim 30$ (for 100 GeV/c) this will give us a purity of $\sim 10^{-6}$. At this level the regeneration of pions by the muons starts competing with the attenuation. Therefore, one can hardly improve the purity by adding more absorber.

We have measured, at AGS, the attenuation mean free path of pions in carbon.[4] The result is shown in Fig. 9. Since there is no appreciable difference between 6-GeV data and 17-GeV data, let us assume that the same value on the mean free path can be applied up to 100 GeV. Then the thickness of the carbon absorber we need is 2100 gm/cm$^2$ ($\sim$ 13 m).

The energy loss of the muons in this material is ~ 4 GeV/c and the multiple scattering angle is $\theta_{proj}$ ~ 1.03 mrads (for 100 GeV/c). Neither the energy loss nor the multiple scattering seem to give us any problem to the quality of our beam.

Monte-Carlo Calculation[5]

Although our study of the beam using the Monte-Carlo program is in a preliminary stage, it is still interesting to see how close the result is to the (rough) estimate we have obtained from general considerations.

The program generates pions at the target with momentum and angular distributions as they are given by the Hagedorn-Ranft formula.[6] These pions are then traced through a given system of dipole and quadrupole magnets using standard matrix methods. For each pion generated, the distance at which it decays is chosen at random from an exponential distribution appropriate to the pion momentum. All pions striking a magnet are assumed to be lost at that point. The muon resulting from the decay is then traced to see if it reaches the end of the system.

Up to now we have investigated only the 100-BeV/c beam with this program. We are able to verify our estimates of the pion flux given in Table I. The useful muon flux obtained by the program, however, is lower by a factor of approximately two.

## REFERENCES

[1] T. E. Toohig, UCID-10180.

[2] The original version of this program was written by D. Birnbaum. Birnbaum, Tinlet, and Yamanouchi, IEEE Trans. Nucl. Sci., 12, [3], 895 (1965). Later the program has been extensively modified by J. Christensen and M. Kraemer. The author wishes to thank J. Christensen for providing him the latest version of the program.

[3] This does not imply that all the muons outside of this momentum band will be rejected by the system. There will be some low-energy tails. Those unwanted muons have to be eliminated by a momentum-analysis system at the end.

[4] L. Camilleri et al., Bull. Am. Phys. Soc., 13, 636 (1968).

[5] The author wishes to thank A. Entenberg and I-Hung Chiang for their assistance in the calculation described in this section.

[6] Hagedorn and Ranft, CERN/ECFA 67/16, Vol. I, 1966, p. 69.

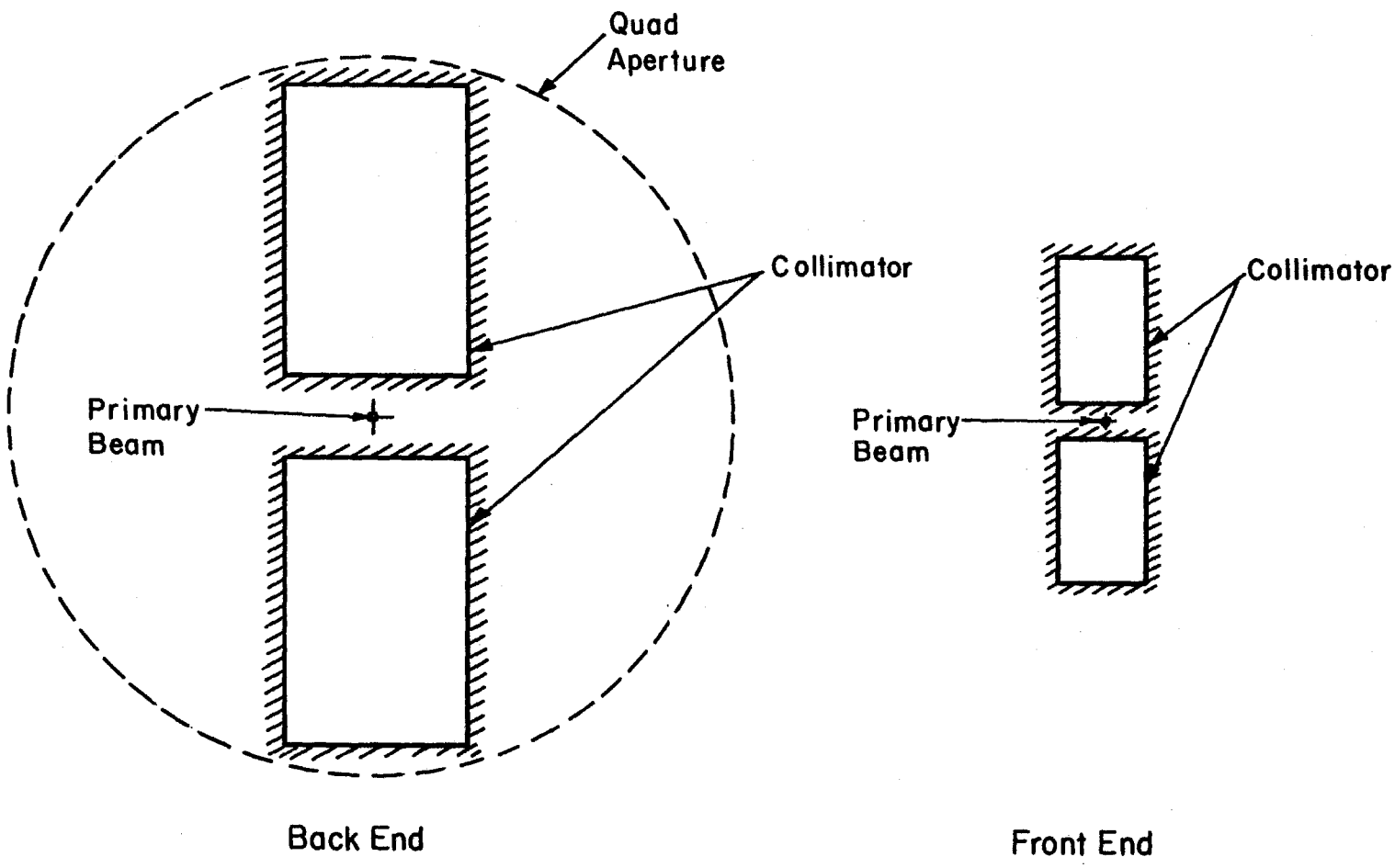


Fig. 1. Collimator for Case 2.

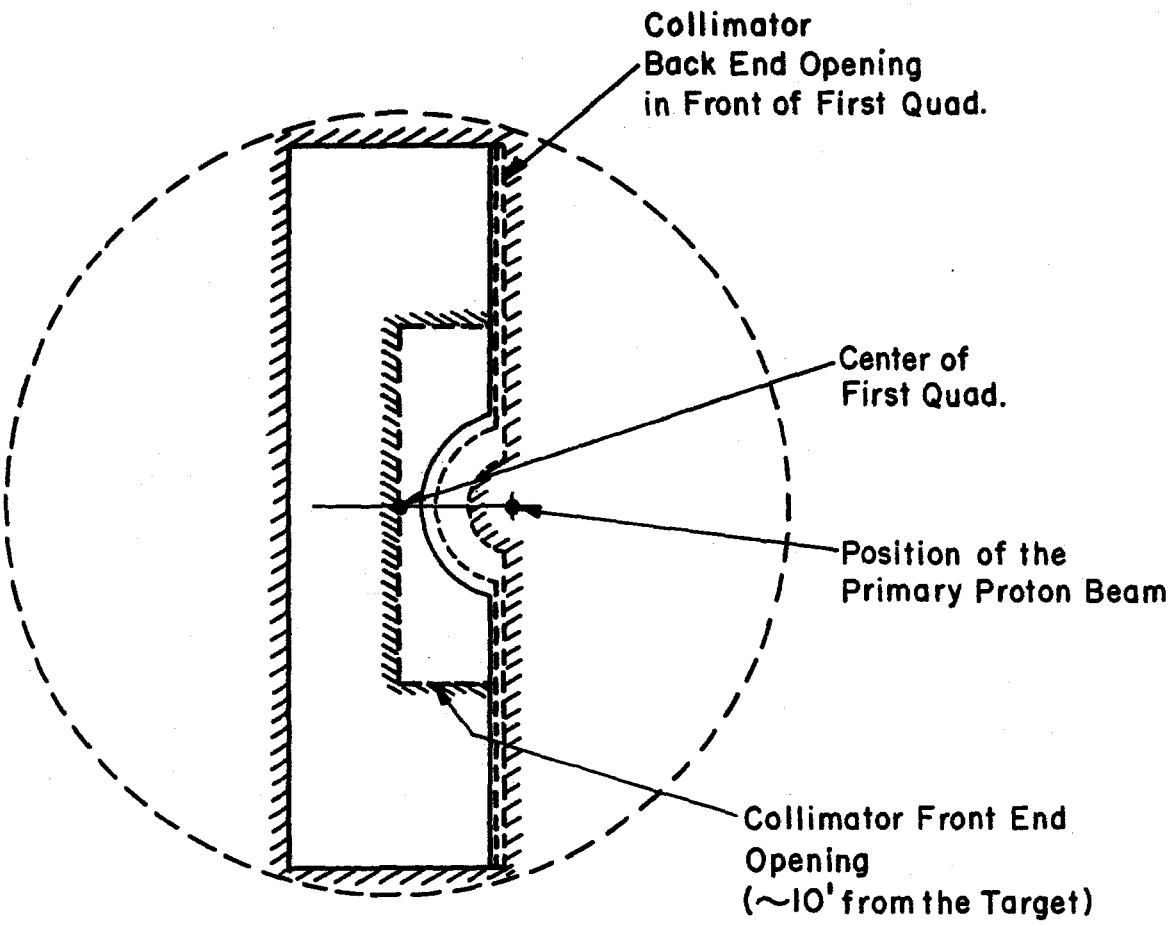


Fig. 2. Collimator for Case 4.

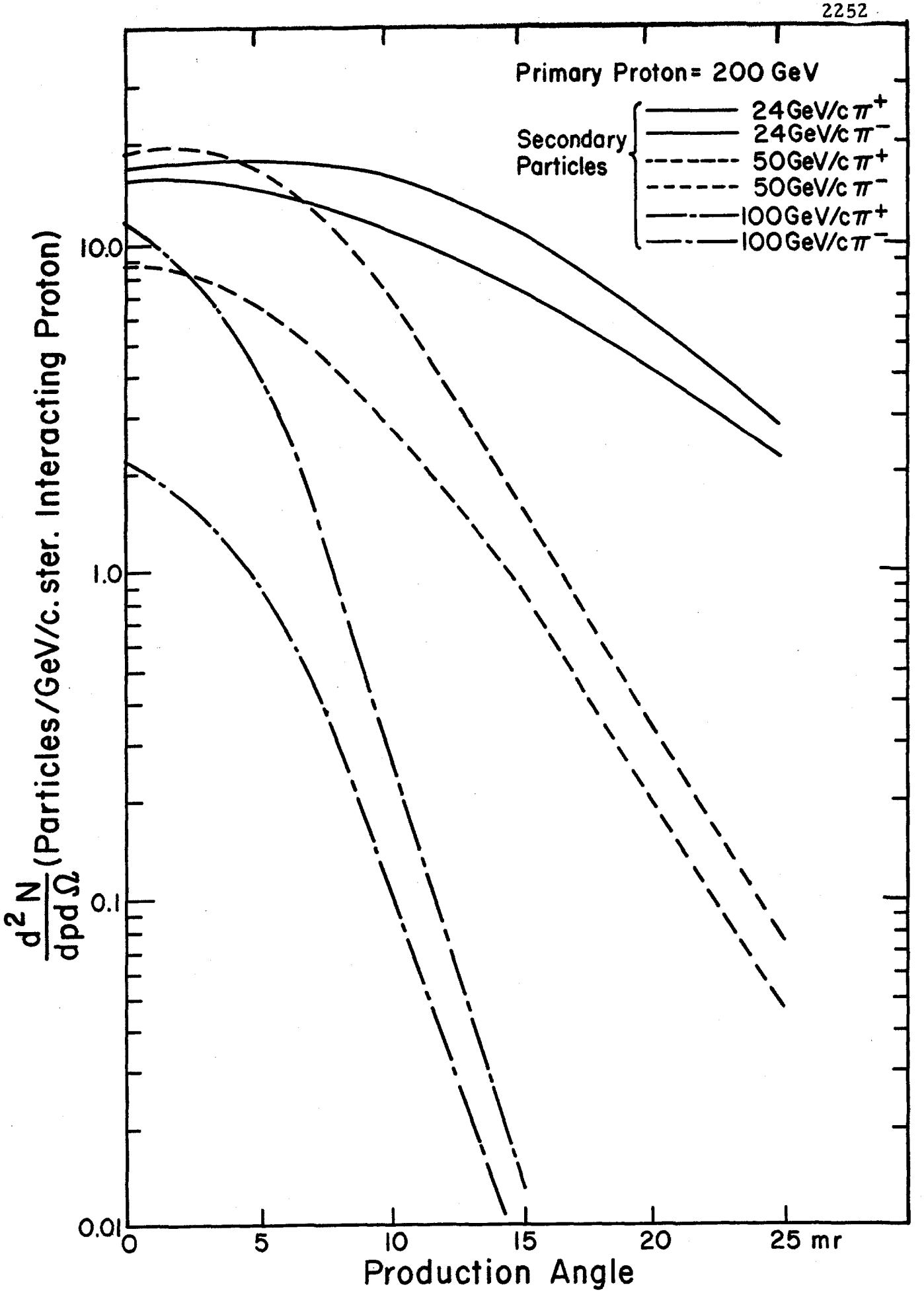


Fig. 3. Pion angular distribution (Hagedorn-Ranft).

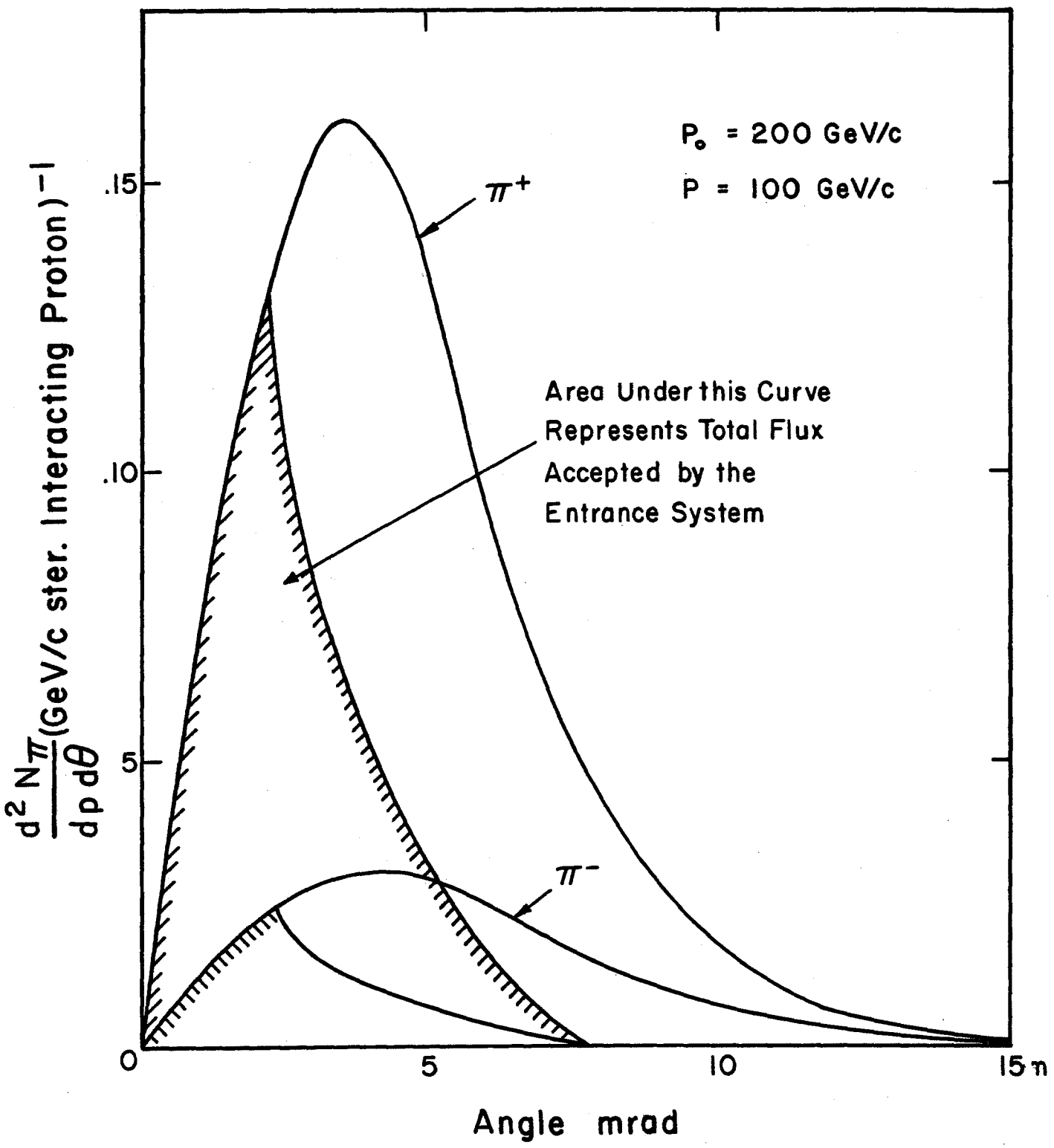


Fig. 4. Acceptance of 100 GeV/c pions, collimator as in Case 1.

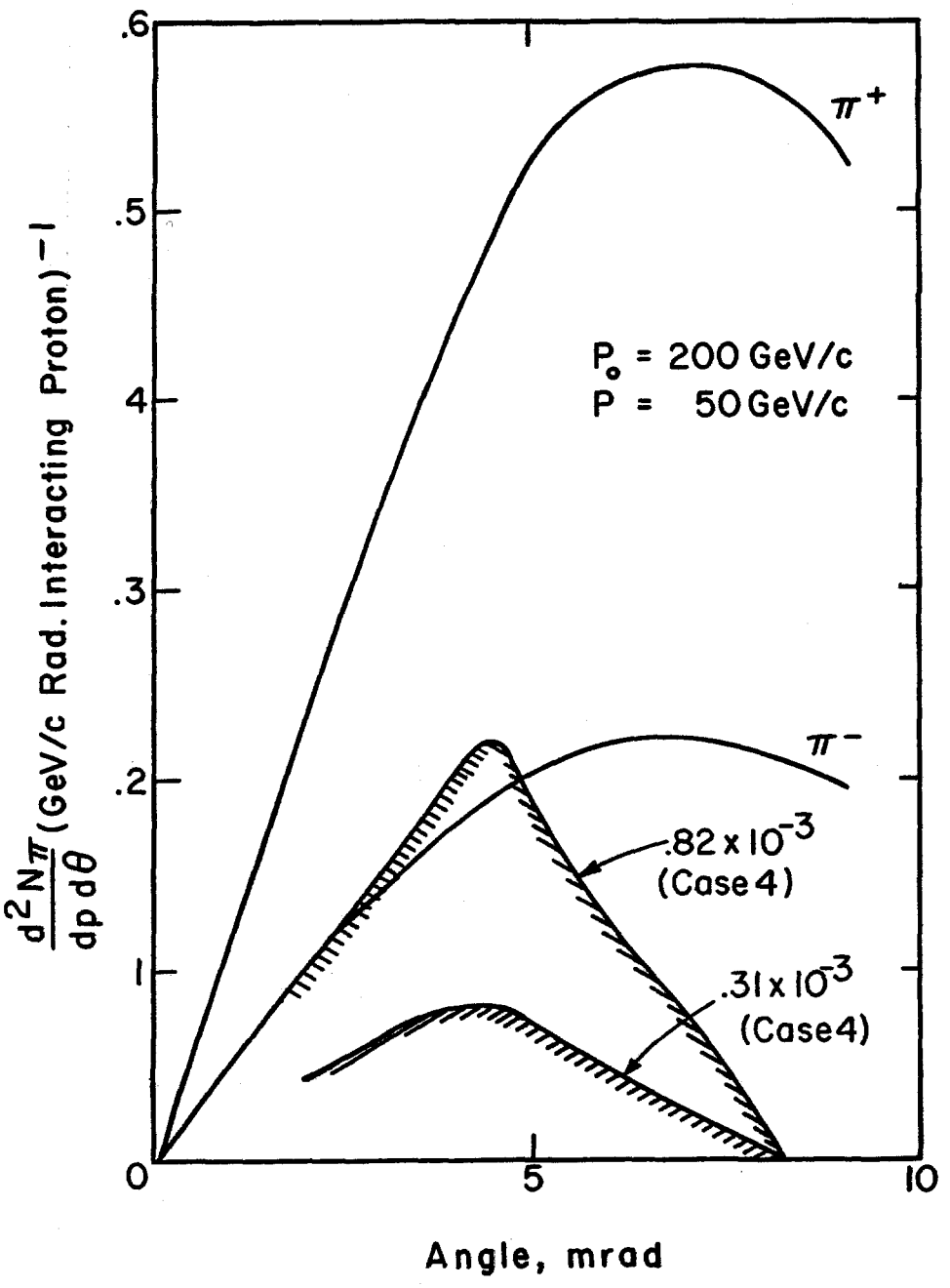


Fig. 5. Acceptance of 50 GeV/c pions, collimator as in Case 4.

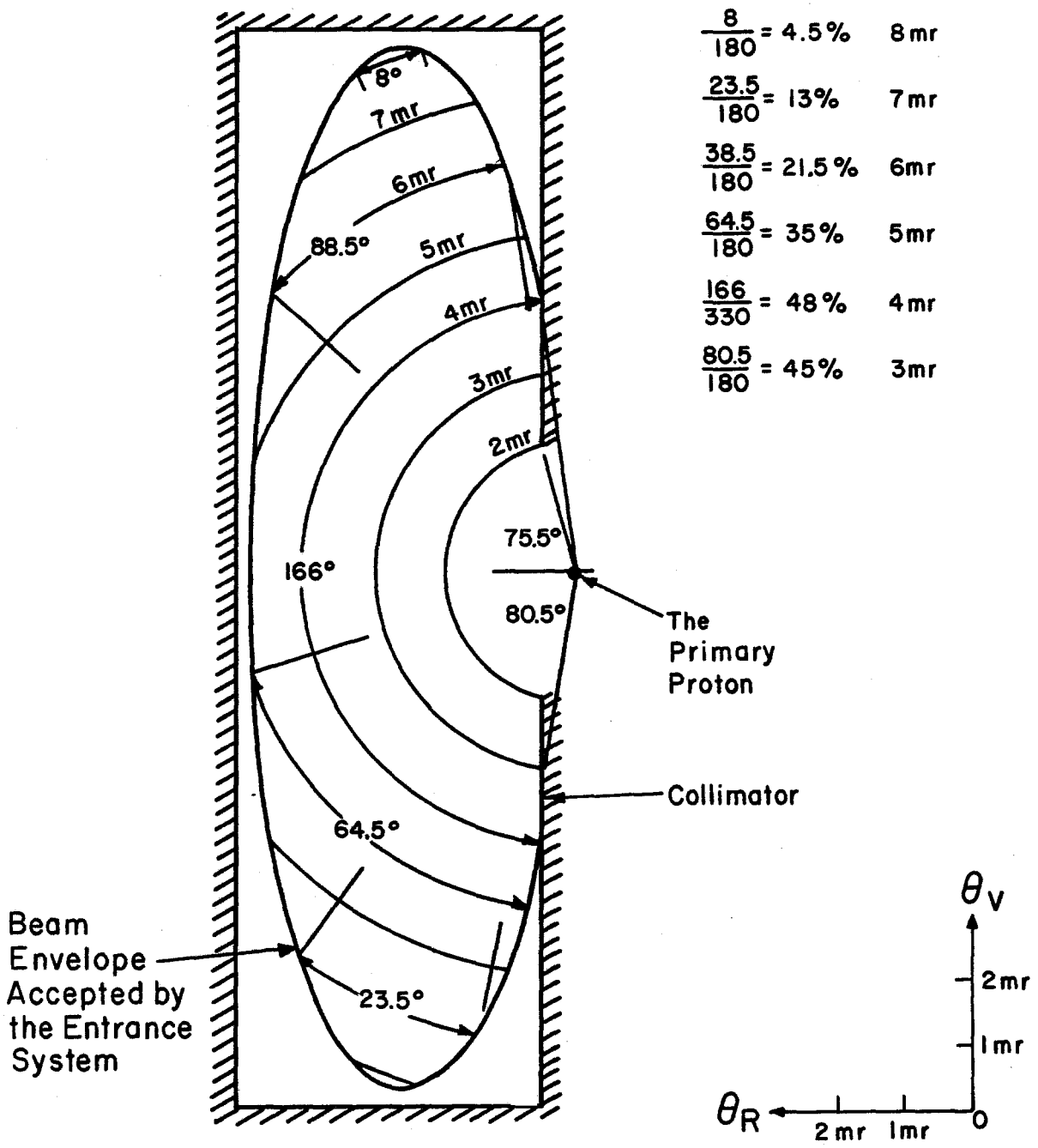


Fig. 6. Method for calculating beam acceptance.

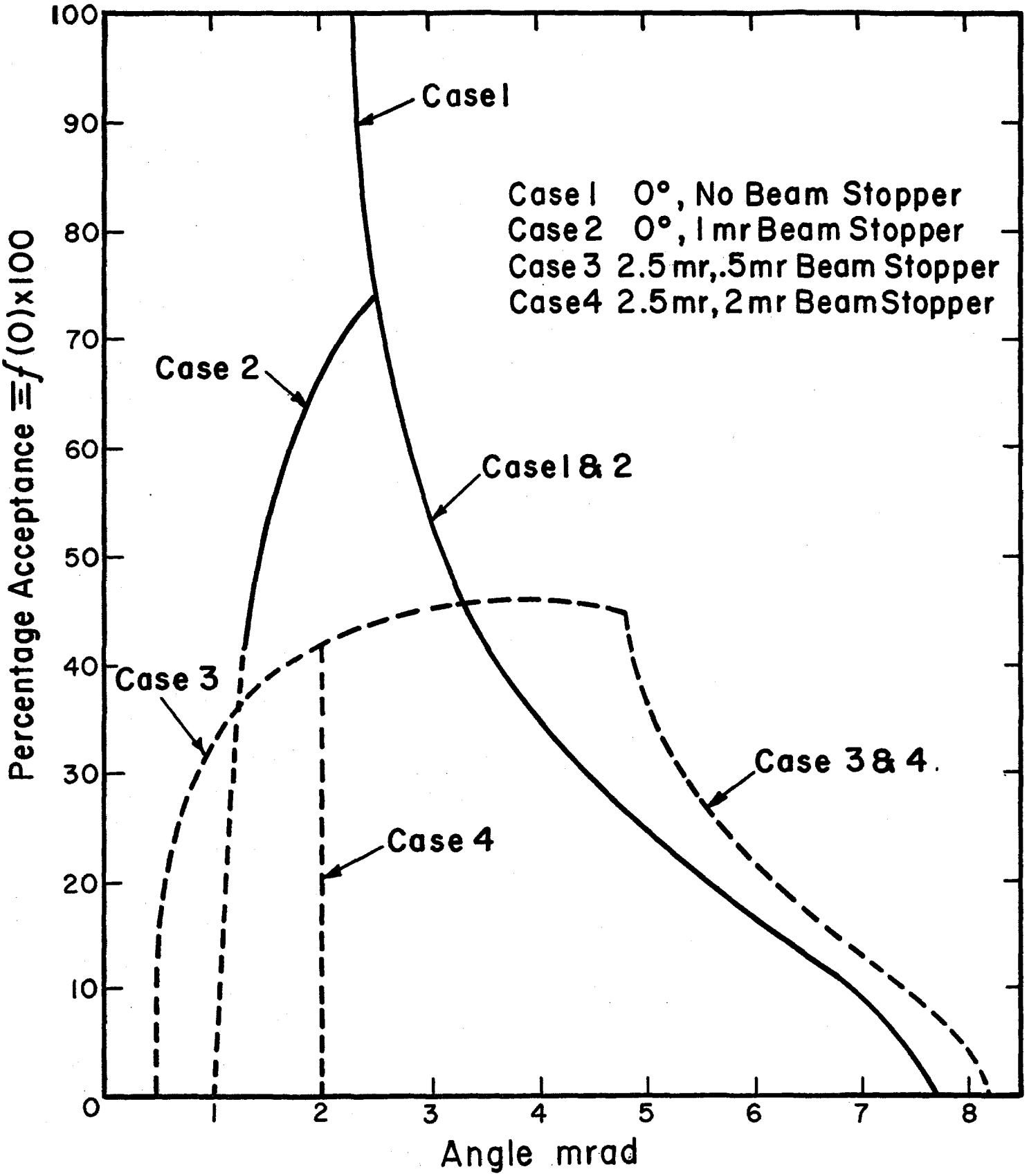


Fig. 7. Beam acceptance with various beam stoppers.

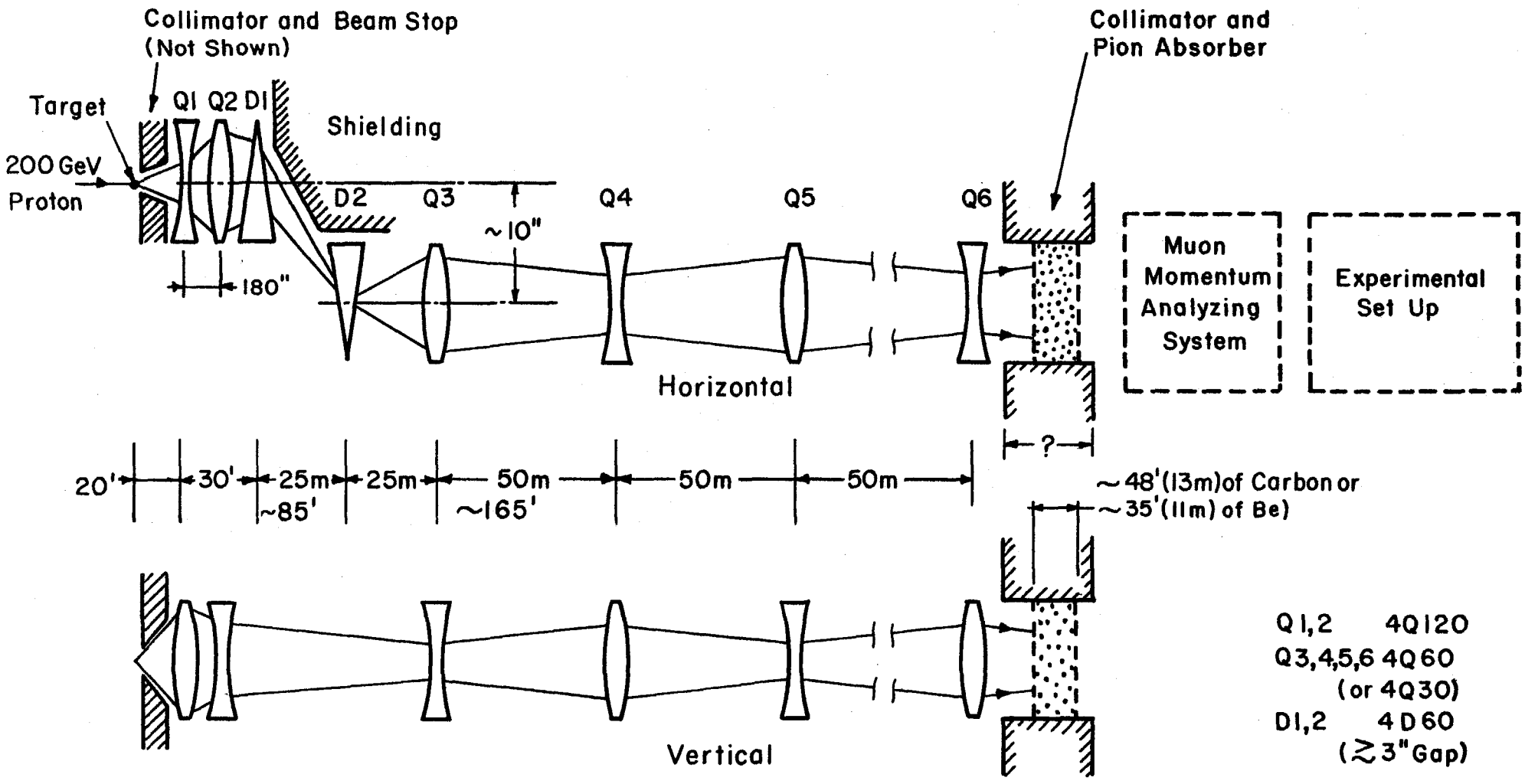


Fig. 8. Final muon focusing system.



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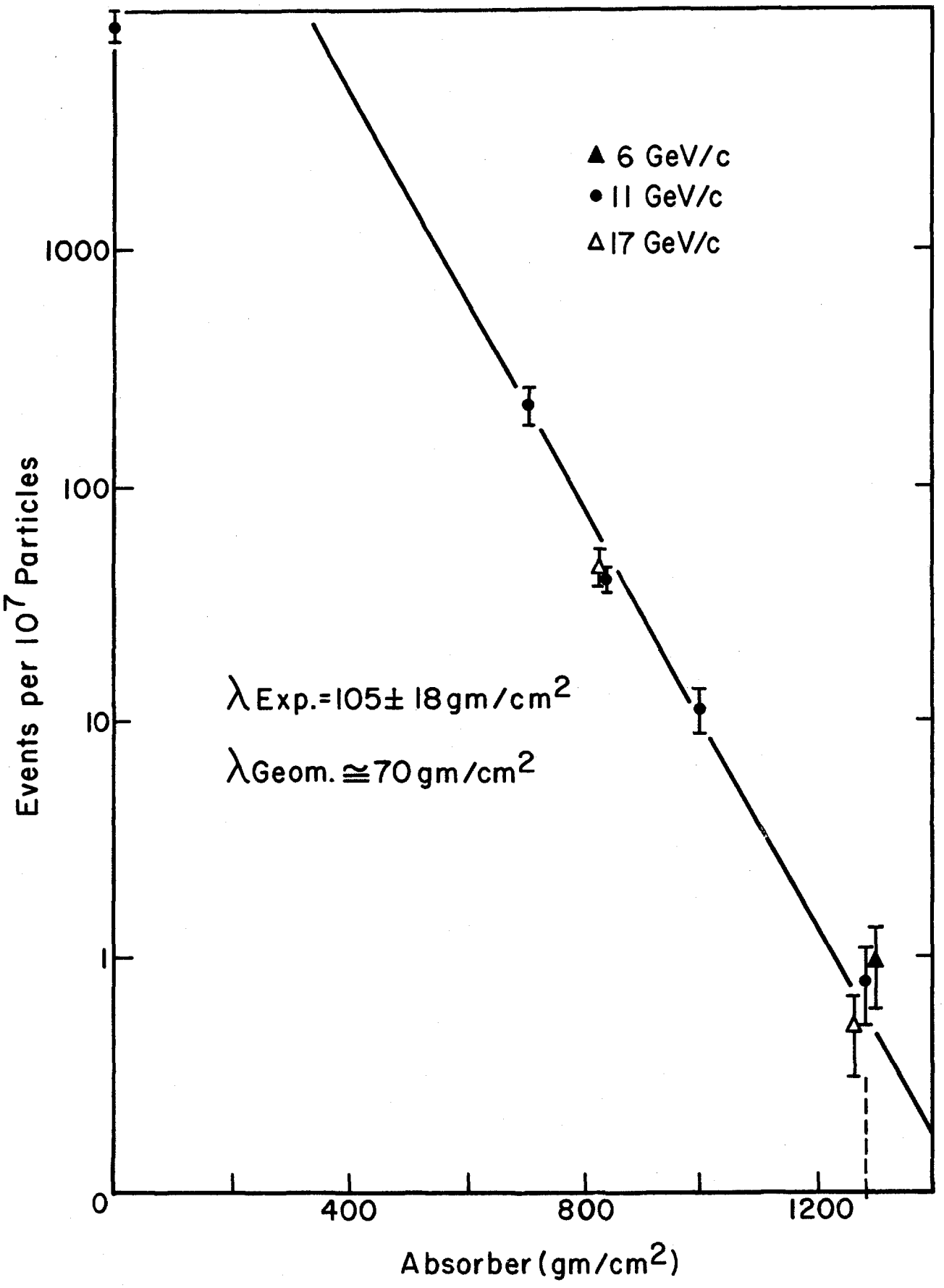


Fig. 9. Results of AGS experiments on attenuation of high-energy pions.