

ANALYSIS OF BUBBLE-CHAMBER EVENTS
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In this report we wish to consider the problem of analyzing events which have one π^0 in them. Several papers have pointed out the desirability of measuring charged outgoing particles with sufficient accuracy to determine the presence of missing neutrals. What we wish to analyze is the corresponding question when there is at least one π^0 present.

The type of question we wish to answer: in a four-prong event can I distinguish $\pi^- p \rightarrow p \pi^+ \pi^- \pi^- \pi^0$ from $\pi^- p \rightarrow \pi^+ p \pi^- \pi^- \pi^0$ i. e., can I tell which positive track is the proton? Similarly, can I distinguish $\pi^- p \rightarrow p k^+ k^- \pi^- \pi^0$ from $p \pi^+ \pi^- \pi^- \pi^0$? In events with another neutral beyond the π^0 can I obtain a measure of the mass of the missing neutral $\pi^- p \rightarrow p \pi^- \pi^0 + MN$?

We shall confine our analysis to the problem of energetic π^0 's roughly one GeV or higher because in the relativistic limit we can make simple approximations for the kinematics of the pion. The problem of low-energy π^0 's appears more difficult and will have to be handled with a more detailed analysis.

In Section I we shall make a rough comparison of charged particle and π^0 detection. We shall review the results of the accuracy of determination for charged particles available from several detectors and compare this to π^0 accuracies. In Section II, we shall present the results for the determination of events with π^0 's present. In Section III,

we will summarize briefly the properties of some of the π^0 detection systems and comment on complications in our simplified analysis of Section I. Finally, in Section IV we would like to outline some important problems in detection which we have not considered in the hope that some of these questions may be answered at a later time.

Throughout this report we have drawn principally on the analysis of Trilling (SLAC 5) for the determination of π^0 's and there most of the formulas which we need are derived.

Section I--Comparison of π^0 and Charged-Particle Detection Accuracies

We will wish to consider the accuracy of determination of the longitudinal momentum and the transverse momentum for energetic outgoing tracks. It will appear that a useful measure of the accuracy of π^0 's is $\Delta p/p$ for longitudinal momentum and $\Delta p_t/m_0$ for transverse momentum where m_0 is the mass of the π^0 . The accuracy obtainable for charged-particle tracks in large bubble chambers has been considered by Kramer and Derrick, NAL Summer Study Report A. 1-68-35, for hybrid spectrometers by Fields et al., NAL Summer Study Report A. 3-68-12. In the former report, Fig. 4 gives the accuracy of the $\Delta p/p$. To estimate the accuracy of transverse momentum we have used an estimate for the angular accuracy of $4\epsilon/\ell$ where ϵ is the setting error and ℓ is the length of the track. For the corresponding figure of the hybrid spectrometer we have used the figure of 0.1 GeV uncertainty in longitudinal momentum at high momentum, and we have

estimated as the angular accuracy $2\epsilon/\ell$. This latter estimate improves the accuracy of the transverse momentum substantially: we feel a more detailed estimate including multiple scattering should be made but our estimate is probably more realistic than the figure given by Fields et al. The results are given in Table I.

Table I. $\Delta p/p$ for Charged Particles.

Device	1 GeV/c	5 GeV/c	10 GeV/c	20 GeV/c	100 GeV/c
25-ft chamber ^a	0.0017		0.0017		0.0023
12-ft chamber 40 kG ^a	0.0023		0.0023		0.005
12-ft chamber 20 kG ^a	0.0048		0.0048		0.010
Hybrid	0.007	0.013	0.0015	0.003	0.001

^aTaken from Fig. 4, NAL Summer Study Report A. 1-68-35.

Table II. $\Delta p_t/m_0$ for Charged Particles.

Device/Momentum	1 GeV/c	5 GeV/c	10 GeV/c	20 GeV/c	100 GeV/c
25-ft chamber	0.002	0.011	0.022	0.044	0.22
12-ft chamber 40 kG ^a					
12-ft chamber 20 kG					
Hybrid ^b	0.005	0.025	0.050	0.094	0.155

^aUsing $\Delta p_t \approx 4(\epsilon/\ell)p$ with values of Fig. 4, NAL Summer Study Report A. 1-68-35.

^bFrom NAL Summer Study Report A. 3-68-12, we feel that the multiple-scattering contribution can be reduced using a thinner exit window. A streamer chamber would also reduce the contribution.

In order to obtain similar estimates for π^0 's we will consider two situations, the first in which only the opening angle of the two γ rays is available, the second in which additional information on the momenta of the two γ -rays is available. We will use the formulas given in Appendix

I by Trilling and repeat his notation. First, we introduce appropriate symbols as follows:

β, p, E	=	velocity, momentum, total energy of incoming pion
θ	=	opening angle between the two photon directions
ξ	=	angle between π^0 direction and the line bisecting the angle θ
p_1, p_2	=	momenta of the two photons
θ^*	=	angle between π^0 direction and direction of photon #1 in the π^0 rest system
p_{1m}, p_{2m}	=	measured momenta of the photons
$\Delta p_1, \Delta p_2$	=	rms errors in the measured values of the photon momenta
m	=	π^0 mass

the opening angle θ is then given by

$$\cot \theta / 2 = p / m \sin \theta^*.$$

We see from this that a lack of knowledge of θ^* , when we measure the opening angle only, will produce a large uncertainty in β . Further, any uncertainty in measuring θ will produce a further uncertainty in p . The former can, of course, be overcome by obtaining information on p_1 and p_2 , the momenta of the two photons, the latter presumably is characteristic of the detection system. Since the opening angle θ gets very small at high p (for 10 GeV π^0 the minimum

opening angle is 1.5°) there are difficulties in measuring this angle accurately. Also, because the spatial separation of the showers will be small, there may be difficulties in measuring the momenta of the two photons separately. (At 1 m from the sources the two photons would be only 2.7-cm apart). We, therefore, consider the two sources of error in obtaining an estimate of the error. We define $\Delta p/p$ for this case (no knowledge of p_1 or p_2) as $(p_{\text{calc.}} - p)/p$ where $p_{\text{calc.}}$ is the value calculated for p assuming $\theta^* = 90^\circ$.

The contribution from θ^* is then

$$\left. \frac{\Delta p}{p} \right|_{\theta^*} \approx 1 - \sin \theta^*, \quad (1)$$

using $p_{\text{calc.}} \approx \frac{2m}{\theta}$, $p \sin \theta^* \approx \frac{2m}{\theta}$. If we assume the separation of the γ conversion points is measured with an accuracy 2ϵ and that we are a distance L downstream from the π^0 production point then the contribution from measuring uncertainty in θ is

$$\left. \frac{\Delta p}{p} \right|_{\theta} \approx \frac{p}{2m} \frac{2\epsilon}{L} \approx \frac{p}{m} \frac{\epsilon}{L}. \quad (2)$$

There are similar contributions to the error in the transverse momentum of the π^0 if only the opening angle is measured. The contributions again come from our lack of knowledge of θ^* and the uncertainty in measuring angles. The useful formula is given by Trilling (Eq. 2)

$$\sin \xi = \cos \theta^* \sin \frac{\theta}{2}, \quad (3)$$

or for high π^0 momenta

$$\xi \approx \frac{\theta}{2} \cos \theta^* \quad (4)$$

Defining as the uncertainty in the transverse momentum Δp_t the difference between the momentum along the bisector of the two γ rays and the true momentum we have

$$\begin{aligned} \frac{\Delta p_t}{m} \Big|_{\theta^*} &\approx \frac{p_{\text{calc.}}}{m} \xi = \frac{2m}{\theta m} \frac{\theta}{2} \cos \theta^* \quad (5) \\ &= \cos \theta^* \end{aligned}$$

The uncertainty in Δp_t because of the uncertainty in measuring angles is just

$$\frac{\Delta p_t}{m} \Big|_{\theta} = \frac{p}{m} \frac{\epsilon}{L} \quad (6)$$

From formulas (2) and (6) we see that uncertainties in measuring angles contribute roughly $\frac{p}{m} \frac{\epsilon}{L}$ to $\frac{\Delta p_1}{p}$ and $\frac{\Delta p_t}{m}$, that is the contribution to momentum uncertainties grow with the π^0 momentum and the only way to keep these contributions small is to have very large L for high momenta. The contributions from lack of knowledge of θ^* can be reduced with information from the momenta of the individual γ rays.

When information from the individual γ rays is available,

Trilling has shown that a least-square approach yields (Trilling Eq. 18)

$$\frac{\Delta p}{p} = \frac{\cos \theta^*}{\sqrt{\left(\frac{p_1}{\Delta p_1}\right)^2 + \left(\frac{p_2}{\Delta p_2}\right)^2}} \approx \frac{\Delta p_\gamma}{p_\gamma} \frac{\cos \theta^*}{\sqrt{2}} \quad (7)$$

where $\Delta p_\gamma/p_\gamma$ is the uncertainty in measuring the γ -ray momentum averaged for the two γ rays. Similarly, using Trilling (17),

$$\begin{aligned} \frac{\Delta p_t}{m} &\approx \frac{p \Delta \xi}{m} = \frac{\sin \theta^*}{\sqrt{\left(\frac{p_1}{\Delta p_1}\right)^2 + \left(\frac{p_2}{\Delta p_2}\right)^2}} \\ &= \left(\frac{\Delta p_\gamma}{p_\gamma}\right) \frac{\sin \theta^*}{\sqrt{2}} \end{aligned} \quad (8)$$

We see that the longitudinal and transverse momentum uncertainties measured in terms of the momentum and π^0 rest mass respectively are to roughly one-half the fractional error of γ -ray energy determination. Of course, this holds only as long as the opening angle determination are small compared to this, i. e., as long as $\frac{p}{m} \frac{\epsilon}{l} < \frac{\Delta p_\gamma}{2p_\gamma}$ otherwise the angle determination uncertainty dominates. If we assume $\Delta p_\gamma/p_\gamma \approx 0.2$ and for a large bubble chamber $\epsilon/l \approx 500 \mu/1 m = 0.510^{-3}$ this implies that for $p < 27$ BeV, $\Delta p_\gamma/p_\gamma$ dominates, while for $p > 27$ BeV, angle errors dominate. Assuming that $\Delta p_\gamma/p_\gamma \approx 0.2$, we see $\Delta p/p \sim 0.1$ and, therefore, is much larger than the values given in Table I for charged particles. For $\Delta p_t/m_0$, a value of 0.1 for the π^0 's is again larger than those for charged

particles given in Table II, at 100 GeV/c for bubble chambers the angle uncertainty dominates both the charged and π^0 errors in $\Delta p_t/m$, being $\frac{p}{m} \frac{4\epsilon}{l}$ for charged particles as against $\frac{p}{m} \frac{\epsilon}{l}$ for π^0 's so in this domain the π^0 transverse momentum can be more accurately determined because no curvature problem is involved, however, as mentioned above $\Delta p_y/p_y$ may be extremely difficult to determine.

We have unfortunately not had the time to handle the problem of low-energy π^0 's, nor the question of what accuracies on π^0 determination are available when the total energy and the opening angles are available.

Section II--Analysis Errors in π^0 Events

In this section we wish to reconsider briefly some of the questions concerned with analysis of events involving one π^0 . For simplicity we will assume that $\Delta p/p$ and $\Delta p_t/m$ both are 0.1 and that the π^0 uncertainties dominate the charged particle uncertainty. Then we can make rough estimates, a) whether we can distinguish K^+K^- from $\pi^+\pi^-$ on a pair of tracks, b) whether we can distinguish $K^+\pi^+$ from the interchange π^+K^+ , c) how well we can determine the missing mass in a reaction with one π^0 . Trilling has shown that the distinction in cases a) and b) can be estimated on the basis of the uncertainty in energy balance using the constraint of conservation of longitudinal momentum. Assuming with our results above that the error is dominated by the π^0 we find two terms, ΔE_1 from the uncertainty in longitudinal momentum

and ΔE_2 from the uncertainty in transverse momentum

$$\Delta E_1 = \left(\frac{\Delta p}{p} \right) \frac{p_t^2 - m^2}{2p}, \quad (9)$$

and

$$\Delta E_2 = \frac{p_t}{p} \frac{\Delta p_t}{m} m, \quad (10)$$

where we have used

$$p_t \Delta \theta = \frac{p_t}{p} \frac{p \Delta \theta}{m} m \approx \frac{p_t}{p} \frac{\Delta p_t}{m} m. \quad (11)$$

For our case outlined above we can estimate values of ΔE_1 and ΔE_2 for reasonable values of p_t and p . The difficulty in assessing the usefulness of these estimates comes from the fact that there will be wide variations in p_t and p , and it is only a rough estimate that we obtain. We must look in detail for specific models of π^0 distributions to obtain reasonable distributions for ΔE_1 and ΔE_2 . Our values should be considered as samples. It is clear that for small p_t and large p the parameters ΔE_1 and ΔE_2 decrease (until $\Delta p/p$ increases with p).

Table III. Energy Error ΔE . From Uncertainty
In Longitudinal Momentum, With $\Delta p/p = 0.1$

$p_t \backslash p$	1 GeV/c	10 GeV/c
0.5	0.012	0.001
0.25	0.002	0.0002

If the single π^0 has small transverse momentum or large longitudinal momentum, its contribution no longer dominates the error ΔE_1 , and more careful calculations are required. Similarly, for ΔE_2 using $\Delta p_t/m = 0.1$.

Table IV. Energy Error ΔE_2 , From Uncertainty In Transverse Momentum.

	1 GeV/c	10 GeV/c
0.5	0.0067	0.0007
0.25	0.0033	0.0003

Again for small p_t of the π^0 the contributions of charged particles may be important in ΔE_2 .

II (a). Distinction between K^+K^- is given by

$$\Delta E' = \left(m_K^2 - m^2 \right) \left(\frac{1}{2p_1} + \frac{1}{2p_2} \right) \quad (9)$$

where p_1 and p_2 are the momenta of the two tracks. Hence, we find approximately

$$\Delta E' \approx \frac{0.23}{\bar{p}} \quad (10)$$

Therefore, unless the average momentum of the $\pi^+\pi^-$ or K^+K^- pair is ten times larger than the π^0 momentum, we can still make this distinction, since

$$\Delta E' > \sqrt{(\Delta E_1)^2 + (\Delta E_2)^2} \quad (11)$$

and we still have a significant energy unbalance for the wrong choice.

II (b). Distinction of mass interchange for two tracks.

When two non-identical mass particles have nearly the same momentum it becomes nearly impossible to distinguish which is which. Trilling (p. 48) shows that the effect of interchanging masses on the energy unbalance is

$$\Delta E'' = \frac{m_1^2 - m_2^2}{2} \frac{p_2 - p_1}{p_1 p_2} . \quad (12)$$

For $K\pi$ interchange using $(p_2 - p_1)/p_1 = 0.1$, we find

$$\Delta E'' = \frac{0.23}{2} \frac{0.1}{p} = \frac{0.011}{p} . \quad (13)$$

Comparing with Table III and IV we see that when the average momentum of the pair is larger than the π^0 momentum this distinction is not possible. The $K\pi$ must differ by 0.3 to allow the distinction to be made unless the π^0 momentum is much larger. We must add of course that even in the absence of π^0 's this distinction is difficult.

II (c). Determination of masses of missing neutrals.

In this part we wish to consider the accuracy available for determining a missing neutral if one π^0 is seen, for example,

$$\pi^- p \rightarrow \pi^- p \pi^0 \text{ (MN)}. \quad (14)$$

In NAL A. 3-68-12 the error in the mass squared of the missing neutral is divided into two contributions: from the longitudinal and transverse momentum. In order to avoid confusion with our notation

let us call $E(MN)$, $p_{\parallel}(MN)$, and $p_t(MN)$ the energy, the parallel component of momentum and transverse components of momentum for the missing neutral. Then the contribution from the longitudinal momentum (again assuming the π^0 dominates) is

$$dMM_p^2 = 2 [- E(MN)\beta + p_{\parallel}(MN) \cos \theta - p_{\perp}(MN) \sin \theta] \Delta p, \quad (15)$$

assuming near cancellation of $E(MN)$ and $p_{\parallel}(MN)$ and that β and $\cos \theta$ are also roughly the same for the π^0 . A rough estimate for dMM_p^2 is

$$dMM_p^2 \approx 2 p_{\perp}(MN) p_t \left(\frac{\Delta p}{p} \right). \quad (16)$$

Using 0.5 for the transverse momenta and $\Delta p/p = 0.1$, we find $dMM_p^2 = 0.05$. We can estimate the contribution from the transverse momentum uncertainty of the π^0

$$dMM_p^2 \Big|_{p_t} = -2 [- p_{\parallel}(MN) \sin \theta - p_{\perp}(MN) \cos \theta] \Delta p_t. \quad (17)$$

Again assuming roughly $p \approx p_{\parallel}(MN)$ we can approximate

$$\begin{aligned} dMM_p^2 \Big|_{p_t} &\approx -2 p_{\perp} M \frac{\Delta p_t}{M} \\ &= -2 \cdot 5 \cdot 135 \cdot 1 \\ &= 0.0135. \end{aligned}$$

We see that we have a rough estimate of 0.06 for dMM_p^2 . Since

the MM^2 of the π^0 , $2\pi^0$, η^0 are 0.02, 0.09, 0.25 respectively we see that we cannot make a distinction between one or two extra π^0 but that a missing η can be distinguished. We have been forced to make drastic approximations in these calculations; the results should be taken as rough indication until a complete analysis can be made.

Section III -- π^0 Detection Systems

Several systems for detecting π^0 mesons can be considered. In particular, the possibility of H_2 -neon mixtures in a large bubble chamber has been treated by G. Kalmus in UCRL-16830. Our Fig. 1 summarizes his results for 5 and 20 BeV/c electrons in three possible mixtures varying in radiation length from 50 to 250 cm. We give the uncertainty in the electron momentum determination as a function of the track length of the electron. The results certainly indicate that in such a detector the uncertainty in the γ -ray energy (coming from two electrons) should be less than 20% for a wide range of γ -ray energies.

Trilling in his report in SLAC 5 has evaluated a system of plates in hydrogen. We have evaluated his formulae and find again that for a series of plates 20% uncertainty is available if a 2-m array similar to the one he describes is used.

There are some obvious problems with the above systems which should be investigated in detail. For the neon- H_2 combination it would

be desirable to have an inner interaction region of hydrogen particularly if one wants to examine peripheral interactions where the neon background may be very serious. In both hydrogen-neon and plate arrays we expect a serious number of secondary interactions from charged particles which will make the identification of the source of γ rays difficult. Unfortunately, we have not had the time to study the effect quantitatively; however, some events in FAKE have been run which when analyzed should give some answer to this question. From this point of view a chamber (bubble or streamer) with thin walls and allowing detection of γ rays with high efficiency outside the main volume appears attractive. As mentioned above, for very energetic π^0 's the converters must be far from the chamber if the two showers are to be physically separated. Another possibility is to consider the case where the opening angle is determined with some precision, but only the total energy of the π^0 is determined. This would give rise to a new set of criteria for the accuracy of the momentum determinations.

Throughout we have neglected the important problem of detection of low-energy π^0 's. These should be examined in a FAKE-type program. In general, it appears that if the π^0 detectors are outside the main interaction volume, thin plate arrays offer an interesting possibility. Such thin plates should cover a large fraction of the solid angle.

Section IV -- Other Questions in γ Detection

In this report we have indicated that for events with one π^0

present the uncertainties in measurements of the π^0 will be larger than those of the charged particles for several detector systems. Nevertheless, if a fractional uncertainty in γ -ray momentum of 0.2 can be achieved we have shown in Section II that a determination of masses for outgoing charged tracks can be made. In this analysis we have used only conservation of longitudinal momentum. The results should be checked with detailed fitting. Missing mass determinations are still possible but the errors are found to be rather large. Again the rough results should be checked.

We have not considered the problem of two π^0 's and how well one can distinguish which γ rays should be paired. It appears that for invariant masses of 2 π^0 's large compared to several π masses the distinction should be possible but that for smaller invariant masses the γ rays will be confused. Again this problem should be investigated statistically with a computer. In general the two γ decays of other particles like the η_0 should be easily distinguished from the π^0 as soon as some momentum information is available.

We conclude that it is very desirable at high energy to detect the γ rays with high efficiency to allow the exploration of reactions with one or more π^0 's. We have only outlined some of the questions which must be answered more precisely to design a good working system.

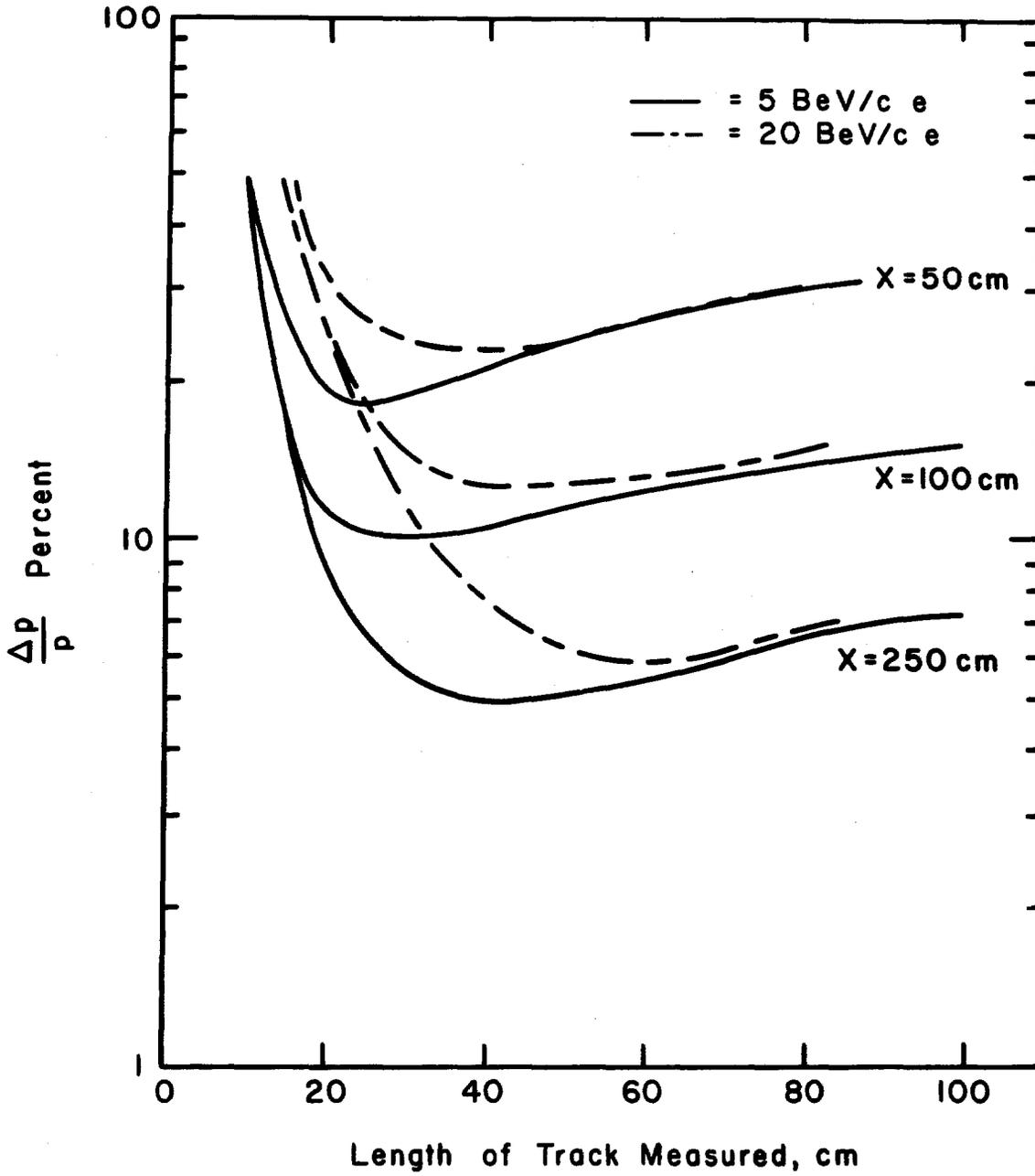


Fig. 1. Error in the momentum determination for 5- and 20-Bev/c electrons, as a function of track length and radiation length X.