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A SIMPLE FORMULA FOR THE DISTRIBUTION OF ENERGETIC SECONDARY BARYONS FROM PROTON-INITIATED COLLISIONS

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A slight generalization of the multiperipheral model developed by authors in UCRL-18275 leads to a simple formula for the energy-angular distribution of the most energetic secondary baryons from proton-initiated collisions. The multiperipheral diagram is that of Fig. 1, corresponding to the production of n pions together with

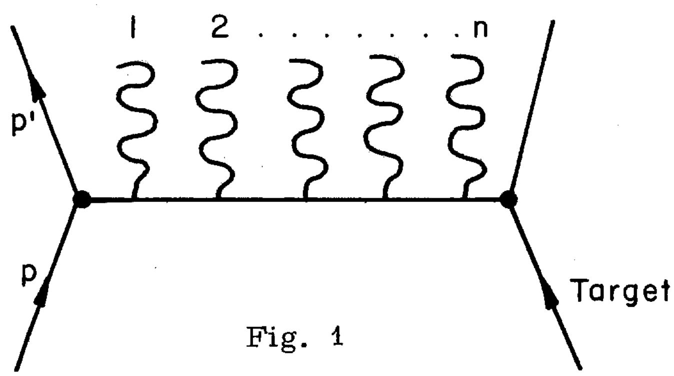

Fig. 1

the energetic non-strange baryon p, which may be either a nucleon or a low-mass resonance.

In the model of UCRL-18275 let us integrate over all variables except the two referring to the left-most link in Fig. 1. These two remaining variables we here call x and t, rather that x_1 and t_1. The differential cross section is then ($e^{X_o} = E_{lab}/M_p$)

$$\frac{\partial^2 \sigma_n^{p \to p'}}{\partial x\, \partial t} \approx \sigma_{tot}^{p \to p'} \left(g^2\right)^n \frac{\left(X_o - x\right)^{n-1}}{(n-1)!}\ \bar{a}\ e^{at}\ e^{-g^2 X_o}, \qquad (1)$$

where we have dropped the M subscript of UCRL-18275, restricting ourselves to the simple model of Sec. E of that paper, where all trajectories are of the "meson" class, none of the Pomeranchuk. We have here inserted a simple exponential t-dependence with a width determined by the parameter a, which may be taken as a constant--or in a better approximation, equal to $a_o + 2\alpha' x$ where $\alpha' \approx 1\ \mathrm{GeV}^{-2}$ is the slope of the average trajectory. Note that g^2 determines both the energy and multiplicity dependence of the cross section. In particular, $\bar{n} = g^2 X_o$. Note also that if we integrate Formula (1) over dx (from 0 to X_o) and over dt (from 0 to $-\infty$), we find

$$\sigma_n \approx \sigma_{tot}^{p \to p'} \frac{\left(g^2 X_o\right)^n}{n!}\ e^{-g^2 X_o}. \qquad (2)$$

Finally, if we sum Formula (2) over all n, we get $\sigma_{tot}^{p \to p'}$ as the energy-independent total cross section for the proton to produce the baryon p'. Alternately we may first sum Formula (1) over n to obtain

$$\frac{\partial^2 \sigma^{p \to p'}}{\partial x\, \partial t} \approx \sigma_{tot}^{p \to p'}\ g^2\ e^{-g^2 x}\ \bar{a}\ e^{at}. \qquad (3)$$

If the x-dependence of a is neglected, we see that the dependence on t and the dependence on x are independent of each other.

The variable t is approximately related to the transverse momentum of the final baryon p' :

$$\left.\begin{aligned} p'^2_\perp &\approx -\left(t - t_{min}\right) \\ \text{where} \qquad -t_{min} &\approx \left(m_{p'}^{\;2} - m_p^{\;2}\right) \frac{s_r}{s} \end{aligned}\right\} \tag{4}$$

if s_r is the square of the invariant mass of all outgoing particles <u>except</u> p' . The longitudinal momentum of the final baryon, either in the lab or c.m. systems, is given by

$$p'_\parallel \approx p_a \left(1 - \frac{s_r}{s}\right) .$$

What we need, finally, is the relation between s_r and x. This is given by

$$\frac{s_r}{s} \approx k\, e^{-x} , \tag{5}$$

where k is a constant near unity that depends on dynamical details of the multiperipheral chain. The model fails for x very close to zero, so we cannot determine k by normalization at x = 0. In terms of $p'_\perp$ and $p'_\parallel$, the distributions are

$$\frac{\partial\sigma}{\partial\left(p'_\perp\right)^2} \sim e^{-a\left(p'_\perp\right)^2} ,$$

$$\frac{\partial\sigma}{\partial p'_\parallel} \sim \left(p_a - p'_\parallel\right)^{g^2 - 1}, \quad \frac{p'_\parallel}{p_a} > 1 - k .$$

The model thus contains four parameters, $\sigma_{tot}^{p \to p'}$, g^2, a_o, and k, but g^2 is a universal constant which has been determined to be approximately 1.5 by fitting overall energy multiplicity data. We may estimate a_o by realizing that the average value of x is $X_o/n + 1$, while the average value of $(t - t_{min})$ is a^{-1}. For n = 3 at 28.5 GeV, and thus $< x > \approx 1$, $\bar{p}_\perp$ for outgoing protons is $\approx$ 0.34 GeV/c. Thus

$$\bar{a} = a_o + 2\alpha' \approx 9 \text{ GeV}^{-2}, \text{ or } a_o \approx 7 \text{ GeV}^{-2}. \tag{6}$$

The value of k can be obtained from the experimental mean inelasticity factor

$$\left\langle \frac{p'_\parallel}{p_a} \right\rangle \approx 0.6 , \tag{7}$$

which from (5) and (6) tells us that

$$k\, e^{-< x >} \approx 0.4 . \tag{8}$$

Now the average n is $g^2 X_o$, so the overall average value of x is roughly independent of X_o:

$$< x > = \frac{X_o}{< n > + 1} = \frac{X_o}{g^2 X_o + 1} \approx \frac{1}{g^2} \approx 0.67 \ . \tag{9}$$

Thus

$$k \approx 0.4\, e^{-0.67} \approx 0.8 \ . \tag{10}$$

The coefficient $\sigma_{tot}^{p \to p'}$ can be determined by measuring at any given energy the total cross section for producing energetic baryons of the type p' .

The spectrum of secondary pions is also well defined in the multi-peripheral model but cannot be so easily expressed as in the simple Formula (1). It could easily be calculated on a numerical basis.