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AN EXPERIMENTAL STUDY OF THE SSC MAGNET APERTURE CRITERION

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SUMMARY

The field quality required of the main bending magnets is a critical issue in the design of the SSC. The field quality specification used in the Conceptual Design Report is based on the imposition of bounds to the departure from linear behavior in the oscillation of single particles about their closed orbits. The specification is physically reasonable, and has the advantage that it can be applied to the SSC designs using any of a number of existing computer codes. The criterion contains several parameters. In view of the cost, schedule, and operational implications of the collider design for HEP, it is important that serious attention be given to the values assigned to these parameters. This experiment is part of that effort.

If the betatron oscillations of a particle in a synchrotron are linear, then the oscillation amplitude will be a constant of the motion. If there is no coupling between the two transverse degrees of freedom, the projections of the amplitude on the horizontal and vertical planes will each be invariant. A plot, from turn to turn, of one projection versus the other will yield a single point. Nonlinearities in the magnetic fields will lead to gradual (on the scale of a betatron oscillation wavelength) changes in the magnitude or orientation of the transverse amplitude. The single point of the turn-by-turn plot will develop, in general, into an area. The fractional excursion in length or orientation of the amplitude is required, according to the criterion, to be less that 10% for projected amplitudes up to 5 mm both horizontally and vertically. Are these figures reasonable, too tight, or too loose?

The Tevatron exhibits excellent linear behavior. Though the fractional deviation defined in the preceding paragraph has not yet been measured directly, calculations predict a result at the 1% level for oscillation amplitudes on the 5 mm scale. In brief, this proposal calls, first, for a verification of that prediction, second, for the introduction of nonlinear field components sufficient to yield amplitude perturbations of up to 20%, and, third, for measurement of the correlation between increased amplitude perturbation and performance degredation. It is particularly important to relate tracking calculations to beam measurements. Additionally, it is quite possible that a more direct or effective measure of the deviation from linear behavior will be found as a result of these studies.

Little in the way of new equipment is needed. The nonlinear magnetic elements were built into the Tevatron from the outset; cables, power supplies, and controls are needed to complete their commissioning. The principal impact on Fermilab and the HEP program will be the accelerator time required to develop the techniques and to carry out the measurements. The time request amounts to 30 shifts with beam, allowing for the 50% availability of beam on study shifts.

An Experimental Study of the SSC Magnet Aperture Criterion

A. Scientific Proposal and Justification

A–1. Statement of Purpose

The SSC is the leading proposal for the next research facility by the U. S. high-energy physics community. Other than the extrapolations associated with its size, no accelerator physics consideration has been found to be qualitatively different from those encountered in the past. The challenge in the SSC design lies in the fact that all design parameters need to be optimized in performance and cost at a much more detailed level than before. A critical parameter in the SSC design optimization is the magnet aperture, specified by the inner winding diameter of the coil package. In the SSC Conceptual Design Report (CDR),[1] extensive studies of the aperture issue were performed. A detailed algorithm for aperture evaluation was devised and studied; this led to the conceptual design coil winding diameter of 4 cm. Owing to the great importance of this result for our community, an opportunity is hereby sought for its experimental study at the Tevatron.

In order to optimize the SSC design, the aperture must be chosen (a) to minimize the cost and (b) to provide a high confidence level in achieving or exceeding the design performance goal. If the chosen aperture is too large, the design will not be cost effective. Too small an aperture will cause difficulties in operation, leading to commissioning delays or lower achieved luminosity. The value chosen for the aperture in the CDR is based upon accelerator theory, past experience, and extensive computer simulations. The aperture study in this proposal is important in order to evaluate the design as well as to improve our understanding of accelerator aperture determination.

In the SSC the effective aperture will be determined by non-linearities in the magnetic confinement field. Several key assumptions were made in establishing the magnet physical aperture used in the CDR.

1. The needed aperture can be determined through a combination of calculation from first principles and projection from past experience.
2. Use of the concept of "linear aperture"[1–4] permits a conservative determination of the aperture that will be achieved with a given accelerator configuration and magnetic field error distribution.
3. Current particle tracking algorithms are capable of determining the "linear" aperture with sufficient accuracy.
4. Magnetic field errors for SSC magnets can be extrapolated from Tevatron and CBA experience.

Assumption 4 has been addressed in the laboratory and, as discussed in the CDR, found to be conservative. The other assumptions have been examined in detail both for internal consistency and for conformance to experience. Nevertheless, as has been emphasized in

the CDR and by the DOE Review Committee of the CDR,[5] it is important, if possible, to obtain direct experimental confirmation because of the high stakes. To be useful this determination must come in time to influence the "final" magnet design.

Experimental address of the points in question consists of careful measurements of particle motion as a function of time under the influence of certain non-linearities which can be introduced into the Tevatron lattice by means of sextupoles already installed. Their strength can be controlled, independent of the other elements, by means of additional power supplies which would need to be installed.

The Tevatron is in several important respects unique as a vehicle for experimental tests of the assumptions:

(a) The magnetic field values, including error multipoles, of every magnet are known to measurement accuracy;

(b) The field is very uniform over a significant volume so that the effects of non-linearities introduced specifically for the purpose of experiment will stand out clearly;

(c) Many more sextupole elements have been installed in the confinement system than are needed for normal operation and are therefore available for manipulation for purpose of experiment (the sextupole component of the SSC field is believed to be the principle determinant of its aperture);

(d) It is a cascaded synchrotron complex with essentially the same basic system complexity of the SSC;

(e) Being the only existing superconducting synchrotron, it is the only vehicle where the influence of this technology on operating characteristics can be studied.

Current schedule plans for the SSC call for the beginning of construction in late 1987 in order to provide useful beam for the beginning of experiments late in 1995. This time scale is predicated on proceeding with the magnet production schedule displayed in Appendix A. In this schedule, incorporation of final magnet design changes is envisioned during the model and prototype production of FY 1987. Addition of an aperture change during this period, should it prove necessary, would result in extra delay of perhaps six months provided that the decision could be made in early calendar 1987. Even if construction start is delayed until FY 1989, minimization of the concomitant delay in the "beam on" date requires an aperture determination before the end of 1987. To meet this schedule plan, the aperture experiment data should be available for analysis by the spring of 1987.

A–2. Outline of the Measurements

The particular manifestation of magnetic non-linearities which we wish to study in this experiment has been dubbed (somewhat inelegantly) the "smear". The idea behind the smear can be sketched as follows.

Suppose that the transverse motion of a single particle in a synchrotron is linear and uncoupled, then the Courant-Snyder invariant amplitude,[6] a, defined as

$$a^2 = \frac{z^2 + (\beta_z z' + \alpha_z z)^2}{\beta_z} , \tag{1}$$

where α and β are the usual synchrotron lattice functions and z is either the horizontal or the vertical coordinate, will be a constant of the motion. If plotted on successive turns through a particular location in the ring, the motion will be depicted as a single point in "vertical invariant *vs.* horizontal invariant" coordinates.

In the presence of non-linearities, the bilinear forms defined above are no longer invariants, but they still can be calculated from the results of turn-to-turn measurements, and they will exhibit variations. The result is a blurring of the single point of the linear motion into an area in the vertical-horizontal amplitude plot. Heuristically, the smear quantifies this blurring as the fractional deviation with time in the two degree-of-freedom amplitude.

There are, of course, a variety of ways of calculating the smear consistent with the spirit of the remarks above. For instance, in the case of one degree-of-freedom non-linear motion, a natural measure of smear would use the ratio of the rms amplitude change to the mean amplitude:

$$S = \frac{\sqrt{\langle a^2 \rangle - \langle a \rangle^2}}{\langle a \rangle} , \tag{2}$$

For the two degree-of-freedom tracking studies at the Central Design Group which appear in this proposal, smear is defined as

$$\text{smear} = \frac{\sqrt{2}}{3} \times \max \left(\frac{\hat{a}_x - \check{a}_x}{\sqrt{a_x^2 + a_y^2}} , \frac{\hat{a}_y - \check{a}_y}{\sqrt{a_x^2 + a_y^2}} \right) , \tag{3}$$

where $\hat{a}_x$ and $\check{a}_x$ are the maximum and minimum horizontal amplitudes, and similarly for the vertical amplitudes. The denominator is the average beam amplitude. The rms average has been replaced by a peak-to-peak measurement, which has been scaled to be approximately equal to the rms average. This definition is convenient for calculational purposes, while preserving the intent of the smear. Further discussion of smear calculations appears in Appendix B.

In the absence of linear coupling, a particle with vanishingly small betatron oscillation amplitude will have a smear close to zero. As the amplitude increases, smear also increases due to the magnetic non-linearities. The aperture criterion adopted in the CDR is that the aperture required for routine beam operation must lie within an amplitude which satisfies

the condition

$$\text{smear} < 10\% \ . \tag{4}$$

In other words, the aperture must be large enough to insure that all the beam operational needs such as injection, orbit distortions, finite beam sizes, *etc.*, can be accommodated within the region determined by Eq. 4. The main purpose of the proposed experiments is to verify or to improve this criterion. For the CDR lattice, tuned for the injection optics, with a coil winding diameter of 4 cm and random magnet errors based on Fermilab and BNL experience, the calculated results can be summarized as:[7]

1. The amplitude at which the smear = 10% is (8.2 ± 1.7) mm radius at the maximum beta value in the arc cells;
2. The amplitude needed for routine operations is 5 mm radius at which the smear is (4.8 ± 2.0)%; and,
3. The amplitude needed for long beam lifetime is 3.2 mm radius; at this amplitude the smear is (2.5 ± 0.7)%.

The Tevatron magnets have a relatively large coil diameter of 7.5 cm. This means particle motion is basically linear for the amplitudes that concern the SSC. This is an advantage since then known non-linearities can be intentionally introduced for aperture studies. At an amplitude of approximately 5 mm, the smear due to the magnet errors in the Tevatron is expected to be small, of the order of 1%. The proposal thus consists of first devising a system of non-linearities that will produce more than 10% smear, and then studying the various operational effects under such non-linearities. The result of the experiments will indicate whether the condition given by Eq. 4 is a practical criterion or whether and how it should be improved.

The primary data source for the measurements associated with the smear is the Beam Position Monitor (BPM) system . The Main Ring and the Tevatron have virtually identical BPM systems. Although this proposal is for the Tevatron, the techniques developed here may be of use in understanding the more challenging environment of the Main Ring.

The monitors associated with service buildings E1 and E2 are capable of recording turn-by-turn beam position data for 1024 turns. Simultaneous recording of the signals from two horizontal and two vertical detectors will be used to calculate the four-dimensional transverse phase space position of the beam centroid on each turn. These data will then be converted into a display of a smear diagram or other projections of the phase space found to be useful.

The requisite coherent motion of the beam centroid will be created by firing the Tevatron injection kicker after the beam has been correctly injected and held at 150 GeV. This will produce a horizontal deflection in the beam trajectory. A vertical deflection of the beam can be produced by firing the abort kicker. The present pulse-forming network has a rise time of 2 μsec and a resistive leg which keeps the pulse constant for a long time.

Through a simple modification of the circuit, one can achieve a damped sinusoidal pulse which will give a short train of beam a constant vertical deflection.

Under normal operating conditions, the least count of the digitized BPM signal is 0.16 mm. At the amplitudes of interest to this experiment, this finite resolution can contribute up to 1% to the smear sigma, depending on the calculational methods being used. By reducing the range of the BPM read-out, a factor of two can be gained in the resolution which will be sufficient for the experiment. The existing analysis software also carries out a FFT on the BPM signal and displays a tune spectrum; it is proposed to correlate this spectrum with the observed smear.

A–3. Specification of Magnetic Elements Required

When the Tevatron was constructed, a complete set of sextupoles (normal and skew) for the thirds resonance compensation was installed. These are 64 sextupoles of each type which are capable of an integrated field of 45 kG-inches at one inch radius when excited to the nominal full current of 50 amperes. Initially, cables, power supplies, and the control hardware for these elements were not supplied; they were to be hooked up only if necessary. The distribution of these elements is shown in Fig. 1.

A year and a half ago, eight normal sextupoles, at locations C-32, C-34, C-36, C-38, F-32, F-34, F-36, and F-38, were powered in order to carry out some studies. This proposal requests that eight more normal sextupoles, at locations C-22, C-24, C-26, C-28, F-22, F-24, F-26, and F-28 be commissioned, and that in addition eight skew sextupoles at locations D-12, D-14, D-16, D-18, D-23, D-27, D-37, and D-43 be commissioned. The cable arrangement, power supplies and control cards will be the same as those for the now operating C and F sector sextupoles.

Tracking calculations for normal Tevatron operation conditions have been carried out at 150 GeV in order to determine values of the smear *vs.* amplitude. The Tevatron lattice has been set up to run in the usual fixed-target mode with tunes of $\nu_x \approx 19.4$ in each plane. The multipole content of each Tevatron dipole has been measured and the dipole positions in the lattice are known. The results of the tracking calculation of smear *vs.* amplitude at 150 GeV using these known multipole fields is shown in Fig. 2. At 5 mm amplitude, the smear is about 1.5%. The effects of the magnetic persistent currents have been included in these results.

The intentional non-linearity can be provided by the 16 sextupole and 8 skew sextupole magnets described before. The proposal asks for eight independent circuits (power supplies and associated wiring and controls) to power the sextupoles and four independent circuits to power the skew sextupoles. Tracking calculations for the Tevatron with these sextupole families turned on have been performed for various particle amplitudes. The polarities and strengths of the sextupole circuits are such that at 150 GeV their contribution to the smear is about 20% at 5 mm, as shown in Fig. 3. The smear caused by the skew sextupoles is shown in Fig. 4. When both the sextupoles and skew sextupoles are powered, the smear

is shown in Fig. 5.

In Figs. 3-5, the amplitude is $(\hat{x}^2 + \hat{y}^2)^{\frac{1}{2}}$ where $\hat{x}$ is defined as $\sqrt{x^2 + (\alpha_x x + \beta_x x')^2}$ at $\hat{\beta}_x$ in the arcs, and similarly for $\hat{y}$. All particles are launched with $x' = y' = 0$. Fig. 3 shows the smear *vs.* amplitude with the special sextupoles on and with x motion only, so $\hat{y} = 0$, and the amplitude is just $\hat{x}$. Fig. 4 shows the smear *vs.* amplitude with the special skew sextupoles on and with y motion only, so $\hat{x} = 0$, and the amplitude is just $\hat{y}$. However, for Fig. 5, when both sextupoles and skew sextupoles are on, particles were tracked with $\hat{x} = \hat{y}$, and the amplitude is $(\hat{x}^2 + \hat{y}^2)^{\frac{1}{2}} = \sqrt{2}\hat{x} = \sqrt{2}\hat{y}$. So to compare Fig. 5 with Figs. 3 and 4 for $\hat{x} = \hat{y} = 5$ mm, for example, the corresponding amplitude in Fig. 5 is $\sqrt{5^2 + 5^2} = 7.07$ mm for which the smear is almost 22%. At 5 mm amplitude in Fig. 3 the smear is approximately 16%, and at 5 mm amplitude in Fig. 4 the smear is approximately 8%. Since the x smear in Fig. 3 is larger than the y smear in Fig. 4, the smear in Fig. 5 (which is the maximum of the x smear and the y smear from Eq. 5) is simply the x smear. At 7.07 mm in Fig. 5, the x smear is approximately 22% and the y smear is approximately 16%. So the increase in the x smear from the skew sextupoles is approximately 6%, and the increase in the y smear from the sextupoles is approximately 8%.

With these sources of non-linearities, smear can be controlled by the experimenters up to about 20%. In calculating the smear values above it was assumed that the phase advance between pairs of sextupoles as well as that between sextupole groups was known. Thus all of the effects have been added coherently and are upper values for the smear. If this is not the case in the Tevatron, the observed smear values may be somewhat smaller. The simulations were done with no multipole errors in the Tevatron, and the phase between sextupoles was assumed to be 68°. The smear from the 8 sextupoles in sector C and those in sector F add for the Tevatron with no multipole errors, while in the real machine this may not be the case. A discussion of this issue appears in Appendix B.

The Fourier transform of the motion is another useful diagnostic tool. For a 5 mm x amplitude with all 16 sextupoles fully on at 150 GeV, the peak corresponding to a frequency $2\nu_x$ is almost half as large as the ν_x peak itself, so should be easily measured with sufficient bandwidth. Fig. 6 shows the Fourier transform of the x motion with all sextupoles and skew sextupoles on for an amplitude of 5.7 mm ($\hat{x} = \hat{y} = 4$ mm) for a case in which the tracking calculation gives a smear of 16%. The y motion is shown in Fig. 7. The vertical scale is powers of e with the central peak, which corresponds to linear motion, normalized to 1. Figures 8 and 9 are similar plots for the SSC with a particular random-number seed which yields 17% smear.

The above figures and smear values were the results of tracking done with 400 turns. In order to demonstrate a relation between the calculated short-term smear and a calculated long-term stability, a case corresponding to Fig. 5 has been examined. A particle with a combined x, y amplitude of 7 mm and a synchrotron energy oscillation of $\pm 1.5 \times 10^{-4}$ has been tracked for 1×10^6 turns and has been found to survive.

B. Description of the Experiments

B–1. Preliminary Steps

There are several steps preliminary to the measurements. First, the Tevatron must be set up for the desired operation mode. For all of the experiments herein proposed, the lattice tuning desired is the standard, fixed-target optics. Two different cycle modes are desired:

1. For the initial experiments, Exp's. A and B described below, the cycles should be the standard fixed-target cycles in order to allow repetitive injection.
2. For the first part of Exp. C, the injection studies, the cycle structure should also be the standard fixed-target cycle.
3. The second part of Exp. C and Exp. D must be done under storage conditions in order to see long-time effects.

After setting the appropriate cycle structure and basic optics, the accelerator systems must be prepared such that proton beams of good quality (95% transverse normalized emittance $\leq 15\pi$ mm-mrad, longitudinal emittance = 0.3 eV-sec) are available from the Main Ring and are injected into the Tevatron. The main mode of operation will be to inject about 30 bunches with a single-bunch intensity of $2 - 3 \times 10^9$. These emittance and energy spread requirements are within the range routinely obtained at Fermilab. To verify that the requirements are met, bunch length and wire scanner measurements will be made in the Tevatron. During the course of the experiments proposed, orbit, lifetime, and profile measurements will be performed. To verify that the observations are related to single particle dynamics rather than collective effects the beam intensity must be varied. A factor of 10 variation is possible. The instrumentation for bunch length, bunch profile, lifetime, and orbit measurements will function at the lower limit of the intensity.

Before the intentional non-linearities are inserted, the Tevatron optics must be sufficiently well tuned. For example, the linear horizontal-vertical coupling will be minimized and the linear chromaticities properly compensated by the chromaticity sextupoles. The linear coupling introduces a smear of order $K/(\nu_x - \nu_y)$. Since this smear is caused by linear fields, and not by non-linearities, we plan to minimize it by controlling the coupling constant to $K \lesssim 0.001$ and separating the tunes by 0.2 or more. The chromaticity compensation will be performed to better than 2 to 3 units, which is routinely obtained in the Tevatron operation.

B–2. Experiment A – The Bare Tevatron

The first experiment will be a measurement of the smear as a function of amplitude with no intentional non-linearities. The intent here is to prove that the standard Tevatron has little or no smear at the amplitudes of interest so that the smear observed in later experiments will be known to be caused by the intentionally-introduced non-linearities. To look for the smear, the proton beam is kicked horizontally by the Tevatron injection kicker

and kicked vertically by the abort kicker. The motion of the beam is then followed turn-by-turn by measuring its horizontal and vertical positions at the beam position monitors (BPM's). Horizontal BPM's at locations E-22 and E-24 and vertical ones at locations E-23 and E-25 are chosen for the smear measurements. The data from these four BPM's can be translated to (x, x', y, y') phase space data, which give the smear. Additionally a Fourier analysis of the motion at one BPM will be used as an alternative measure of the smear. The linear coupling and the BPM accuracy are such that the smear measurements have a resolution of about 1%. Filamentation of the beam interferes with the smear measurement; in the bare Tevatron lattice one thousand or more turns are expected before filamentation takes place. The tracking results discussed above indicate that the Tevatron without the intentional non-linearities should have a smear of the order of 1% at 5 mm amplitude. This 5 mm amplitude is adequate for the measurements and is safe regarding magnet quenches. In order to benefit future experiments, measurements will also be taken of the tunes as functions of $\delta E/E$ and betatron amplitude.

B–3. Experiment B – Non-linear Tevatron Lattice and Tracking

With the bare Tevatron optics reasonably defined, the intentional non-linearities are introduced. These non-linearities have much stronger non-linear effects than those in the bare Tevatron. Two types of non-linearities can be introduced. By powering the sextupole circuits and kicking the beam horizontally, the motion will be confined mainly to the horizontal plane. Consequently the experiments should be simpler to perform and the results should be easier to interpret than if the motion were in both planes. By powering the skew sextupoles in addition, horizontal and vertical motions are coupled and the smear will have the more complicated structure expected in the SSC. By providing both types of non-linearities, the SSC aperture issues can be addressed as discussed below.

After examining the motion in the horizontal plane, the skew sextupoles will be turned on and the beam will be kicked vertically in order to determine the coupled-motion behavior. It is envisioned that 2/3 of the time requested for this experiment will be dedicated to the simple, horizontal behavior and the remaining time will be used in the two-dimensional studies.

With the intentional non-linearities introduced, smear as a function of amplitude will be measured for several different amplitudes and sextupole settings. For sextupole and skew sextupole non-linearities, smear is expected to increase linearly with amplitude for relatively small amplitudes. The results are to be compared with the predicted behavior shown in Figs. 3-5. Different settings of the sextupoles and skew sextupoles will be used to check for proper scaling. It is estimated that this experiment will look at the beam behavior at 5 different amplitudes and for 3 or 4 different values of each set of sextupoles. When the intentional non-linearities are at their strongest value, the injected beam is expected to filament in about 100 turns. This should still be adequate for smear evaluation.

B–4. Experiment C1 – Smear/Injection Studies

The aperture criterion is addressed in two further experiments, which are grouped together under the heading of Experiment C. The first addresses the relation of smear with routine beam operation. To do this, the Tevatron will be set up with the intentional non-linearities powered to a strength that simulate the SSC environment, *i.e.* a smear of up to 10% at 5 mm amplitude. Attempts will then be made to inject the beam into the Tevatron under various injection conditions. Tests will be made on injection with betatron amplitude offsets of up to 1.5 mm and energy mismatches of up to 1.5×10^{-4}. In addition to checking the aperture criterion, the experiment can be useful for designing an SSC operation procedure appropriate for a smear of up to 10% even though the smear at amplitudes corresponding to routine operations in the SSC is calculated to be $\leq 5\%$.

Experiment C2 – Smear/Lifetime Correlation Studies

The second part of the experiment addresses the correlation of smear with beam lifetime. A long beam lifetime can be associated with a limit on particle amplitude beyond which the motion is unstable (the admittance). The question is: what is the value of smear at this stable amplitude limit? To perform this experiment, the beam is heated by anti-damping with the super dampers which exist in the Tevatron. Tune values will be carefully chosen in order to avoid low-ordered resonances. As the beam emittance increases, particles are lost when their amplitude reaches the stability limit. By heating the beam, the stability limit can be obtained by measuring the maximum amplitude of the beam particles at the time of sudden drop of beam lifetime. The flying wire scanners will be used for the amplitude measurements. The smear at this amplitude is known from the previous experiments and can be compared with that assumed in the SSC CDR. This experiment will be performed for both sextupole and skew sextupole non-linearities as discussed above. Several different settings of each family of sextupoles will be studied. The beam lifetime will be measured as a function of total smear in order to determine the tolerable limit. In addition, different combinations of sextupole settings giving the same amount of smear will be examined to insure that the results are not dependant on some particular sextupole arrangement.

B–5. Experiment D – Tune Modulation

The final experiment is a study of the effects of tune modulation on particle stability. Tune modulation in the SSC could be caused by the combination of chromaticity and energy oscillations, power supply ripple, and beam collision guidance systems. Controlled tune modulation can be introduced in the Tevatron by using two existing quadrupole circuits originally installed to feed back on the tunes during slow extraction. "Slow" and "fast" circuits can modulate the tune with maximum amplitudes of 0.012 and 0.003, below maximum frequencies of 300 Hz and 2500 Hz, respectively. Preliminary numerical investigations show that these amplitudes, which are significantly larger than the SSC tune modulation specification of 0.001, are expected to dramatically reduce the dynamic

aperture in the vicinity of fifth-order resonances, when the normal special sextupole circuits are fully powered.

An experiment to observe the deterioration of the dynamic aperture in the vicinity of a resonance when modulation is introduced is qualitatively different from the experiments described above. Experiment D is of a more exploratory nature and is designed to improve our general understanding of accelerator aperture determination. However, the past success of a similar but less well instrumented experiment performed at the CERN SPS[8] gives us confidence that the experiment can be made within the time requested.

B–6. Time Request

We have discussed above four experiments A through D. Experiments A through C are of high priority for the SSC design. Our estimate of the time required to carry out the experiments described above is:

Experiment A	Bare Tevatron measurements	5 shifts
Experiment B	Intentional non-linearities	9 shifts
Experiment C	Injection/lifetime studies	10 shifts
Experiment D	Tune modulation	6 shifts

The time request amounts to 30 shifts with beam and allows for the 50% availability of beam during study shifts. The most efficient manner in which to carry out the proposed experimental program is to divide the studies into two different parts: a preliminary amount of time to refine the needed experimental techniques and learn what problems must be overcome, and a second, extended block of time to actually perform the experiments. To this end, we request an initial ten shifts of study time grouped in two-shift blocks during the first running period of 1987 followed by a week of dedicated machine time, perhaps between the CDF and fixed-target running periods.

C. Experimental Impact on Collaborators and Fermilab

C–1. Personnel

1. Spokesman – D. A. Edwards
2. CDG
   (a) A. Chao
   (b) D. Johnson
   (c) S. Peggs
   (d) J. Peterson
   (e) L. Schachinger
   (f) M. Tigner

3. Cornell
   (g) R. Talman
4. Fermilab
   (h) D. Edwards
   (i) D. Finley
   (j) R. Gerig
   (k) M. Harrison
   (l) R. Johnson
   (m) R. Siemann
   (n) M. Syphers
5. LBL
   (o) M. Cornacchia
   (p) G. Lambertson
6. SLAC
   (q) P. Morton

C–2. Equipment and Services

Hardware:

The eight additional sextupoles and eight skew sextupoles need cables from the spool-pieces to the service buildings, power supplies in the buildings, and the necessary control cards in the service buildings. This installation will be performed by Fermilab. These experiments will require the procurement of cables, terminations, control cards, possibly control crates, and power supplies. Additionally, the pulse-forming network for the abort kicker will need to be modified. The hardware work will be carried out by Fermilab.

Software:

Control system application programs are needed to provide the phase space projections and to archive the data. Programming for the initial measurements will be the responsibility of Fermilab. Data analysis after each experiment will be performed by the CDG with help from Fermilab. Final analysis will be done by the CDG.

Accelerator Time:

The proposed experiments conducted on the Tevatron will require dedicated time.

C–3. Proposed Budget:

Budget needs for this experiment are included in the CDG/Fermilab Agreement for SSC work in FY 1987.

APPENDIX A

Dipole Magnet Industrial Plans

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Dipole Magnet Industrial Plans
FY 1986:
        FY86          .        FY86          .        FY86          .        FY86          .        FY86          .        FY86          .
   OCT        NOV     .   DEC        JAN     .   FEB        MAR     .   APR        MAY     .   JUN        JUL     .   AUG        SEP
                       .                      .                      .                      .                      .
                       .                      .                      .                      .          CBD         .Select. for Particip.
                       .                      .                      .                      .      Announcement    .       in Phase A
                       .                      .                      .                      .X         X          .X         X     X----
                       .                      .                      .                 Eval.Board            Contractor          Phase A
                       .                      .                      .                 Appointed          Quals.Submitted        Begins
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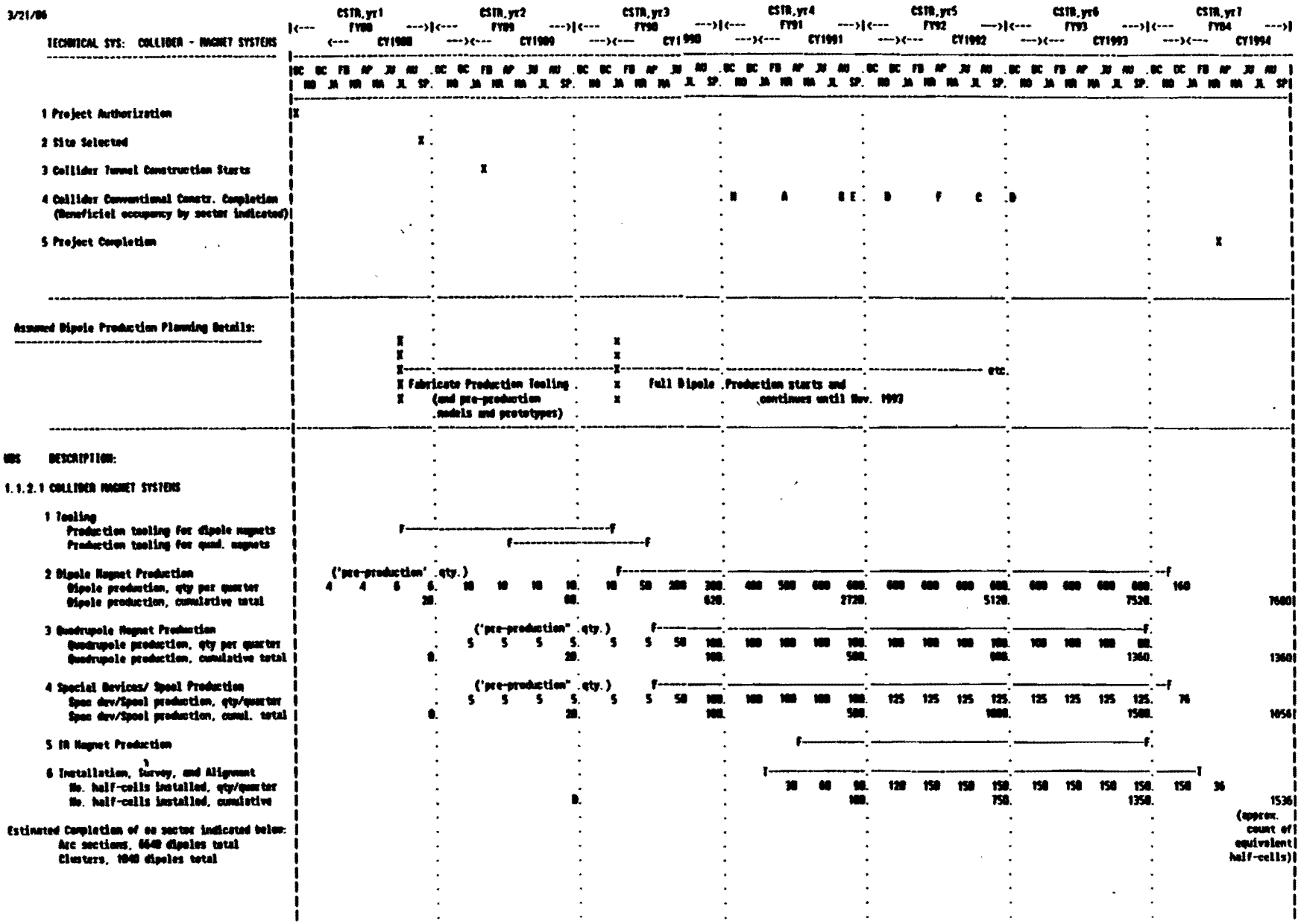
FY 1987:
        FY87          .        FY87          .        FY87          .        FY87          .        FY87          .        FY87          .
   OCT        NOV     .   DEC        JAN     .   FEB        MAR     .   APR        MAY     .   JUN        JUL     .   AUG        SEP
                       .                      .                      .                      .                      .
 Phase A:  Technology Transfer*(See note below)                       .                      .                      .
 ---------------------.-----------------------X                      .                      .XXX - SSC Dipole Magnet
                 |     .                      .                      .                      .       Design Frozen  .
                 X-----------------------------X                      .                      .                      .
            Phase A:  Model Assy (1.8m, 1-ea)                         .                      .                      .
                       .                      .            Receive   .                      .                      .
                       .                      .          Proposals for                      .  Plans, Document Preparation (Manuals, QA,)
                       .     X                .     X        Phase B .          Award Ph.B  .          and Tooling Design.
                       .Draft RFP             . Issue RFP      X     .     X          X----------X-----------------------------------
                       .for Phase B           .for Phase B           Select approx 5      Planning|                .
                       .                      .                      .Most Qualified        .     X-----------------------------------
                       .                      .                      .For Phase B.          .              17m Magnet Assembly (3 ea)
                       .                      .                      .                      .                      .
            <3>        .*Note: Technology Transfer Continues Through Phase B.                .   <4>                .                  <5>

FY 1988:
        FY88          .        FY88          .        FY88          .        FY88          .        FY88          .        FY88          .
   OCT        NOV     .   DEC        JAN     .   FEB        MAR     .   APR        MAY     .   JUN        JUL     .   AUG        SEP
                       .                      .                      .                      .                      .
                       .                      .                      .                      .                      .
 Phase B, Continued:   Tooling Design Continued                       .                      .                      .
 ---------------------------------------------X                      .                      .                      .
                       .                      .                      .                      .                      .
 ---------------------------------------------X                 Phase C                     .                      .
            17m Magnet Assembly (3 ea), Con't.                  Proposals                   .Award Fab.     Production Tooling: Des & Fab
                       .                      .                  Received                    Contract(s)    & Initial Prelim Mag Production
                       .X                     .                   X.         X              .          X-------------------------------->
                   Issue RFP                  .                    .     Selection of        .            Note:
                   for Phase C                .                    .       Phase C           .                Full Scale Production Starts
                       .                      .                    .     Contractors         .                Jan 1, 1990 and continues
                       .                      .                    .                         .                Through Nov 1993.
                       .                      .                    .                         .                      .
   <'N'> Denotes Inputs from Magnet Development Program:                                     .       <6>            .
 1.  1st Long Magnet Test        4.  Start Improved Magnet Design Test                       .                      .
 2.  Multiple Magnet Tests       5.  Multiple Magnet Test                                    .                      .
 3.  Design Improvements         6.  Integrated System Tests                                 .                      .
```

Figure 7.2-1. Collider Ring Magnet Production Plan



APPENDIX B

Analytic Estimate of Smear and Fourier Analysis

Single Sextupole, Pure Horizontal Motion

Successive passages through a single sextupole of strength S are given by

$$\begin{pmatrix} x \\ x' - \frac{1}{4}Sx^2 \end{pmatrix}_{t+1} = \begin{pmatrix} C_0 + \alpha S_0 & \beta S_0 \\ -\gamma S_0 & C_0 - \alpha S_0 \end{pmatrix} \times \begin{pmatrix} x \\ x' + \frac{1}{4}Sx^2 \end{pmatrix}_t \tag{B1}$$

where the total angular deflection due to the sextupole

$$\Delta x' = \frac{1}{2}Sx^2 \quad \left(\doteq \frac{1}{2} \times 0.35(\mathrm{m}^{-2})x^2\right) \tag{B2}$$

has been taken half before and half after the sextupole and where

$$S_0 \equiv \sin\mu_0 \ (\doteq \sin(2\pi \times 0.395) = 0.613) \tag{B3}$$

$$C_0 \equiv \cos\mu_0 \ (\doteq \cos(2\pi \times 0.396) = -0.790) \tag{B4}$$

$$\beta \equiv \text{lattice function } (\doteq 100\mathrm{m}) \tag{B5}$$

$$\gamma \equiv \frac{1+\alpha^2}{\beta} \tag{B6}$$

and α will be assumed to be small. (This is not really true at the proposed sextupole locations but can be partially justified[9]). From Eq. B1 can be derived[10] the relation

$$x_{t+1} - 2C_0x_t + x_{t-1} = \frac{1}{2}\beta S_0 S x_t^2 \tag{B7}$$

After this is solved for x_t the slopes can be obtained from

$$x'_t = \frac{x_{t+1} - x_{t-1} - 2\alpha S_0 x_t}{2\beta S_0} \tag{B8}$$

Linear betatron motion with amplitude a is described by

$$x_t = a\cos\mu_0 t \ (a \doteq 0.005\mathrm{m}) \tag{B9}$$

which satisfies Eq. B7 with zero on the RHS. This can be regarded as a first approximation

to the perturbed motion. A second approximation of the form

$$x_t = a_0 + a\cos\mu_0 t + a_2\cos 2\mu_0 t \tag{B10}$$

can be assumed. It will satisfy Eq. B7 with the RHS approximated using Eq. B9

$$\text{RHS} = \frac{1}{4}\beta S_0 \mathcal{S} a^2 \left[1 + \cos 2\mu_0 t\right] \tag{B11}$$

This yields

$$\frac{a_0}{a} = K\frac{1}{1-\cos\mu_0} \tag{B12}$$

$$\frac{a_2}{a} = K\frac{1}{\cos 2\mu_0 - \cos\mu_0} \tag{B13}$$

where

$$K \equiv \frac{\beta S_0}{8}\mathcal{S} a \quad \left(\doteq \frac{100\times 0.613}{8}\times 0.35a = 2.7a\right) \tag{B14}$$

Substituting into Eq. B8 with $\alpha = 0$ yields

$$x_t' = -\frac{a}{\beta}\sin\mu_0 t - \frac{2a_2}{\beta}\cos\mu_0\sin 2\mu_0 t \tag{B15}$$

The Courant-Snyder invariant (which is no longer invariant) divided by its nominal value is given by

$$\frac{x_t^2 + x_t'^2\beta^2}{a^2} = 1 + \cos\mu_0 t\left[\frac{2a_0}{a} + \frac{a_2}{a} + \frac{2a_2}{a}\cos\mu_0\right] + \cos 3\mu_0 t\left[\frac{a_2}{a} - \frac{2a_2}{a}\cos\mu_0\right] \tag{B16}$$

$$(\doteq 1 + 0.54K\cos\mu_0 t + 2.58K\cos 3\mu_0 t) \tag{B17}$$

The final term gives the dominant variation. With the numbers used so far the amplitude-squared varies by $\pm 3.12 \times 2.7 \times 0.005 = \pm 4.2\%$. Corresponding to this the amplitude varies by $\pm 2.1\%$ which, as "smear", would be quoted as about 1.4%. (This is not precise as the conventional smear refers to a situation in which horizontal and vertical emittances are equal.) The formula used in the routine QANKICK which extracts smear from TEAPOT tracking would yield

$$\text{smear}_x = \frac{(2.58 + 0.54)\times 2.7\times 0.005}{1/\sqrt{2}} \times \frac{1}{2}\times\frac{1}{3}\times 2 = 2\% \tag{B18}$$

Multiple Sextupoles

Since this value of 2% should probably be regarded as too small to be reliably measured, one must attempt to make several sextupoles act constructively. Formulæ for analyzing this are given in Ref. 9, in particular Eqs. 52. Unfortunately the two time-varying terms of Eq. B17 behave differently. We will assume that the strategy is to make the third term add up constructively from the different sextupoles. As a result the second term, which is already small, will become relatively less important and we will ignore it.

Consider a configuration of 4 equal strength alternating-sign sextupoles, situated at points of equal β and separated in betatron phase by 68°. From Eq. 52 one obtains

$$\mathrm{smear}_x(4) = 2 \times \frac{2.58}{3.12} \times [2\cos 12^\circ + 2\cos 36^\circ] = 5.9\% \tag{B19}$$

If instead there were 8 consecutive sextupoles, the result would be

$$\mathrm{smear}_x(8) = 2 \times 5.9 \cos 36^\circ = 9.6\% \tag{B20}$$

It would not be sensible to continue this string to more that 8 sextupoles but one could contemplate wiring up another string of 8 elsewhere in the ring. It would be prudent to regard the betatron phase advance between these two strings as unknown, so the vector could lie between zero and $2 \times 9.6\%$. By empirically trying both signs for the relative polarity and picking the case yielding the larger smear, a value of at least as large as $\sqrt{2} \times 9.6 = 13.5\%$ would be guaranteed.

For the fractional tune which has been used (0.395) it can be said that the behavior is dominated by the 1/3 integer resonance (which "causes" the third term in Eq. B17). Clearly then the smear can be increased by moving the tune closer to 0.333. From Eq. B14 it is also clearly proportional to S and to a, either of which could be increased to give greater smear.

Fourier Analysis and Smear

In the present proposal, as well as in the CDR, smear as defined by Eq. 3 is adopted as the standard measure of the non-linearities of particle motion. A promising alternative parameterization of non-linearities is provided by Fourier analyzing the turn-by-turn transverse beam position monitor outputs. If the peaks of the resulting Fourier spectrum are separable and above the background noise, the amplitudes of these peaks could provide more detailed information of the non-linearities of the motion by allowing its decomposition into the underlying frequency components.

Restating Eq. B10, for purely horizontal motion the transverse displacement at each turn, x_t, $t = 0, 1, 2, ...$ is given by

$$x_t = a_0 + a\cos(\mu_0 t) + a_2\cos(2\mu_0 t) + ... \tag{B21}$$

where $\mu_0 = 2\pi\nu_0$, with ν_0 the betatron tune. The leading non-linear coefficients a_0 and a_2 have been evaluated analytically earlier in this Appendix for a particular sextupole distribution.

If we use Eq. B16 to calculate the maximum deviation of the square root of the Courant-Snyder invariant divided by its nominal value, we get

$$2\left(\frac{a_0}{a} + \frac{a_2}{a}\right) . \tag{B22}$$

If we want to compare directly to our smear definition in Eq. 3 we should multiply our expression by $\sqrt{2}/3 = 0.47$. This almost leaves us with a simple sum of the Fourier coefficients divided by the amplitude of the fundamental. For a more complicated multipole distribution the situation is not so simple analytically, but a sum over the Fourier coefficients is expected to give a measure of the non-linearity of the motion approximately equal to that of the smear.

For two dimensional transverse motion the Fourier sum is more complicated than Eq. B10 with frequencies such as $2\mu_x$, $2\mu_y$, $\mu_x \pm \mu_y$, $\mu_x \pm 2\mu_y$, *etc.* appearing. Now the simple a_0 and a_2 above become the coefficients $a^x(n_x, n_y)$ for the horizontal motion and $a^y(n_x, n_y)$ for the vertical, where n_x and n_y are the small integers which multiply the tunes above. For example, $a^x(2,1)$ is the Fourier coefficient of the frequency $2\mu_x + \mu_y$ in the Fourier analysis of the x motion. It is found empirically that

$$\text{smear} \approx \sum\left[\frac{a^x(n_x, n_y)}{a^x} + \frac{a^y(n_x, n_y)}{a^y}\right] \tag{B23}$$

where the horizontal and the vertical fundamentals and also the linear coupling lines are excluded from the sum. Typical accuracy is ±20%. Leaving the linear coupling terms out of the sum reduces the sensitivity of the experiment to residual coupling. Again Eq. 4 is mainly of intuitive value as it is more interesting and informative to obtain and compare the coefficients individually. This technique is still in its infancy, and needs further development to allow quantitative comparison with the smear.

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Station #
Quad Type

Sector

D	F	A			B			C			D			E			F		
12			X3						S3			X3						S3	
	13	R0			R0			R0			R0			R0			R0		
14			X3						S3			X3						S3	
	15																		
16			X3						S3			X3						S3	
	17	R0			R0		E1	R0		E1	R0			R0		E1	R0		E1
18			X3						S3			X3						S3	
	19	R1			Q1		E1	Q1		E1	R1			Q1		E1	Q1		E1
21																			
	22	R0	X2		R0		E2	R0	S2	E2	R0	X2		R0		E2	R0	S2	E2
23			X4	E4	Q2		E4	Q2	S4	E4		X4	E4	Q2		E4	Q2	S4	E4
	24	R2	X2		Q3		E2	Q3	S2	E2	R2	X2		Q3		E2	Q3	S2	E2
25																			
	26	R0	X2		R0		E1	R0	S2	E1	R0	X2		R0		E1	R0	S2	E1
27			X4	E4	Q4		E4	Q4	S4	E4		X4	E4	Q4		E4	Q4	S4	E4
	28	R1	X2		Q1		E1	Q1	S2	E1	E1	X2		Q1		E1	Q1	S2	E1
29																			
	32	R0	X1		R0		E2	R0	S1	E2	R0	X1		R0		E2	R0	S1	E2
33				E4	Q2		E4	Q2		E4			E4	Q2		E4	Q2		E4
	34	R2	X1		Q3		E2	Q3	S1	E1	R2	X1		Q3		E2	Q3	S1	E2
35																			
	36	R0	X1	E3	R0		E3	R0	S1	E3	R0	X1	E3	R0		E3	R0	S1	E3
37			X4	E4	Q4		E4	Q4	S4	E4		X4	E4	Q4		E4	Q4	S4	E4
	38		X1						S1			X1						S1	
39																			
	42	R0		E3	R0		E3	R0		E3	R0		E3	R0		E3	R0		E3
43			X4						S4			X4						S4	
	44																		
45																			
	46	R0			R0			R0			R0			R0			R0		

Types

Q: normal quad
R: skew quad
S: normal sextupole
X: skew sextupole
E: normal octupole

Groups

R0: Zeroth harmonic coupling

R1,R2: 39th harmonic $\nu_x + \nu_y$

Q1..Q4: $2\nu_x$, $2\nu_y$

S1..S4: $3\nu_x$, $\nu_x+2\nu_y$

X1..X4: $2\nu_x+\nu_y, 3\nu_y$

E1..E2: 39th harmonic octupole extraction

E3,E4: Zeroth octupole harmonic

Figure 1. Tevatron secondary correction package distribution.

Figure 1

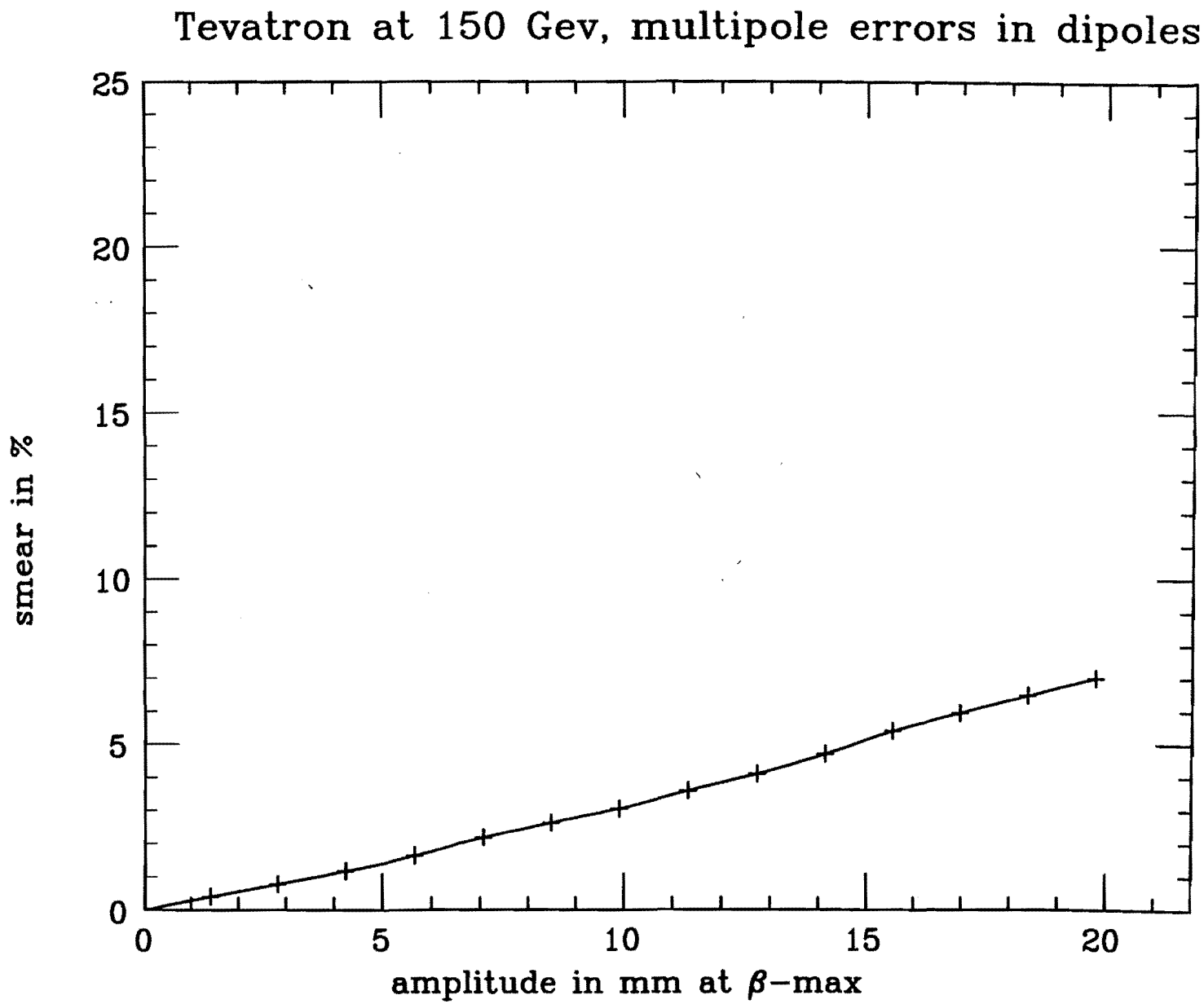

Tevatron at 150 Gev, multipole errors in dipoles
smear in %
amplitude in mm at β−max
0
5
10
15
20
25

Figure 2

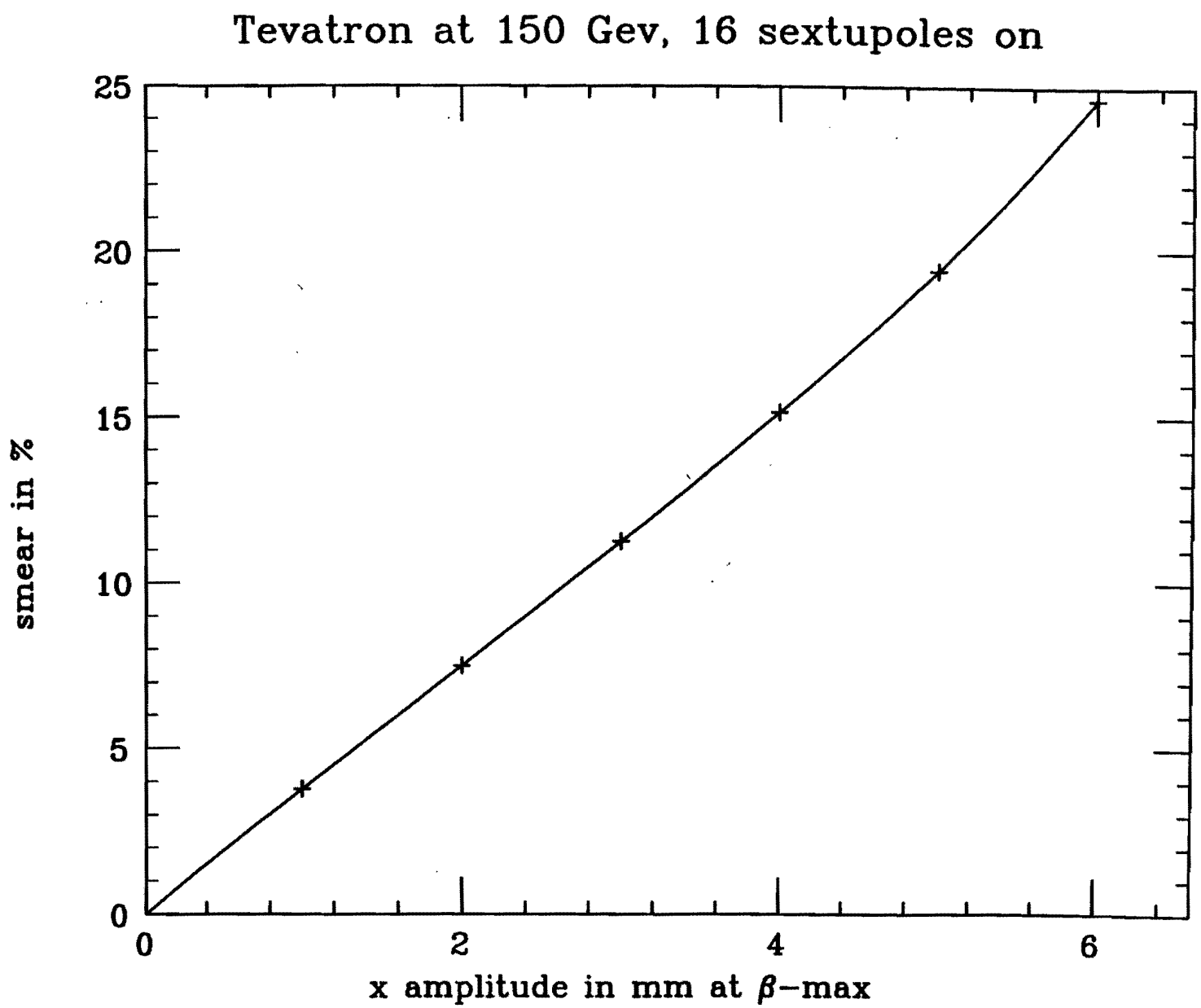

Tevatron at 150 Gev, 16 sextupoles on
25
20
15
10
5
0
smear in %
0
2
4
6
x amplitude in mm at β−max

Figure 3

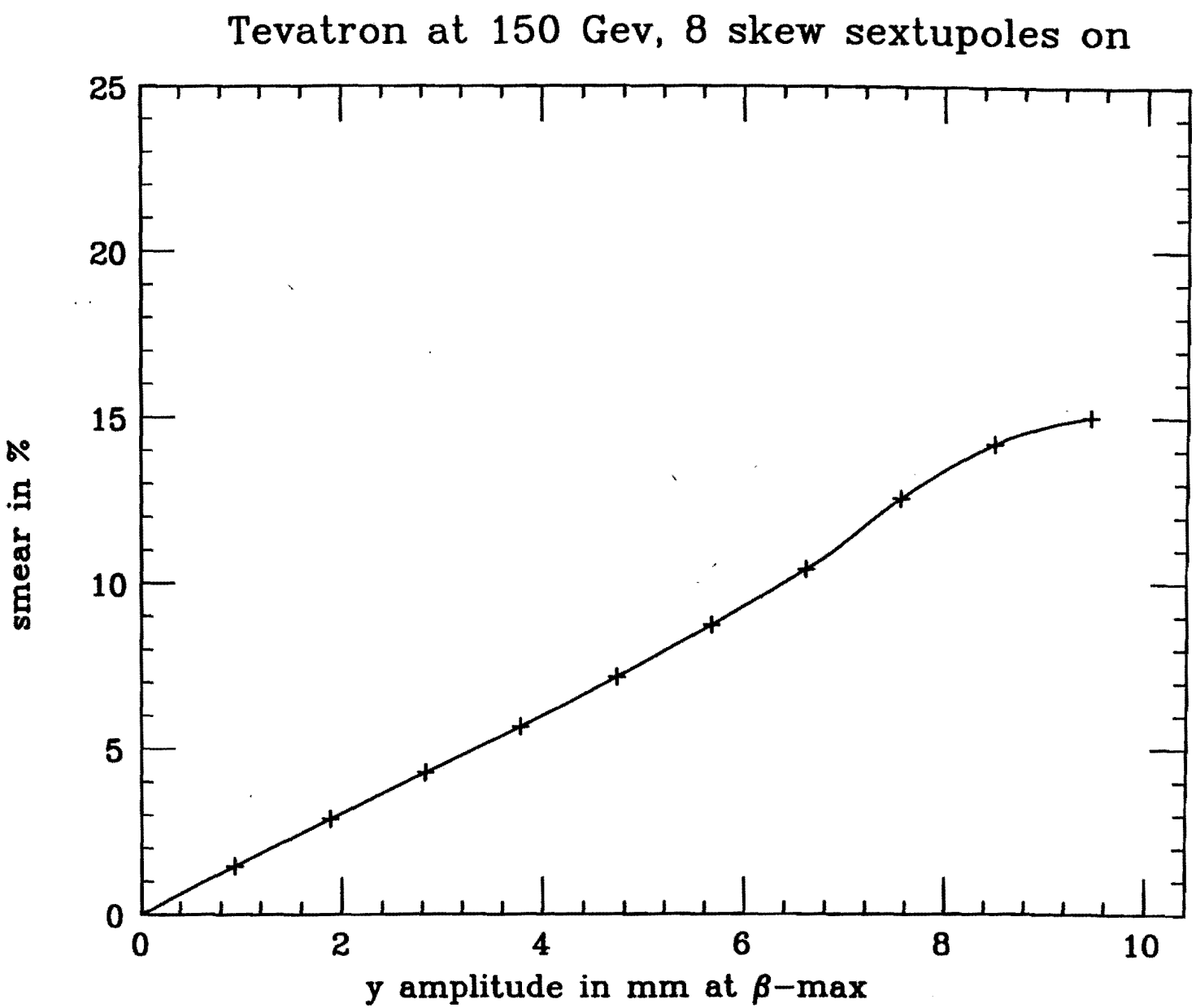

Tevatron at 150 Gev, 8 skew sextupoles on
smear in %
y amplitude in mm at β−max
0
5
10
15
20
25
0
2
4
6
8
10

Figure 4

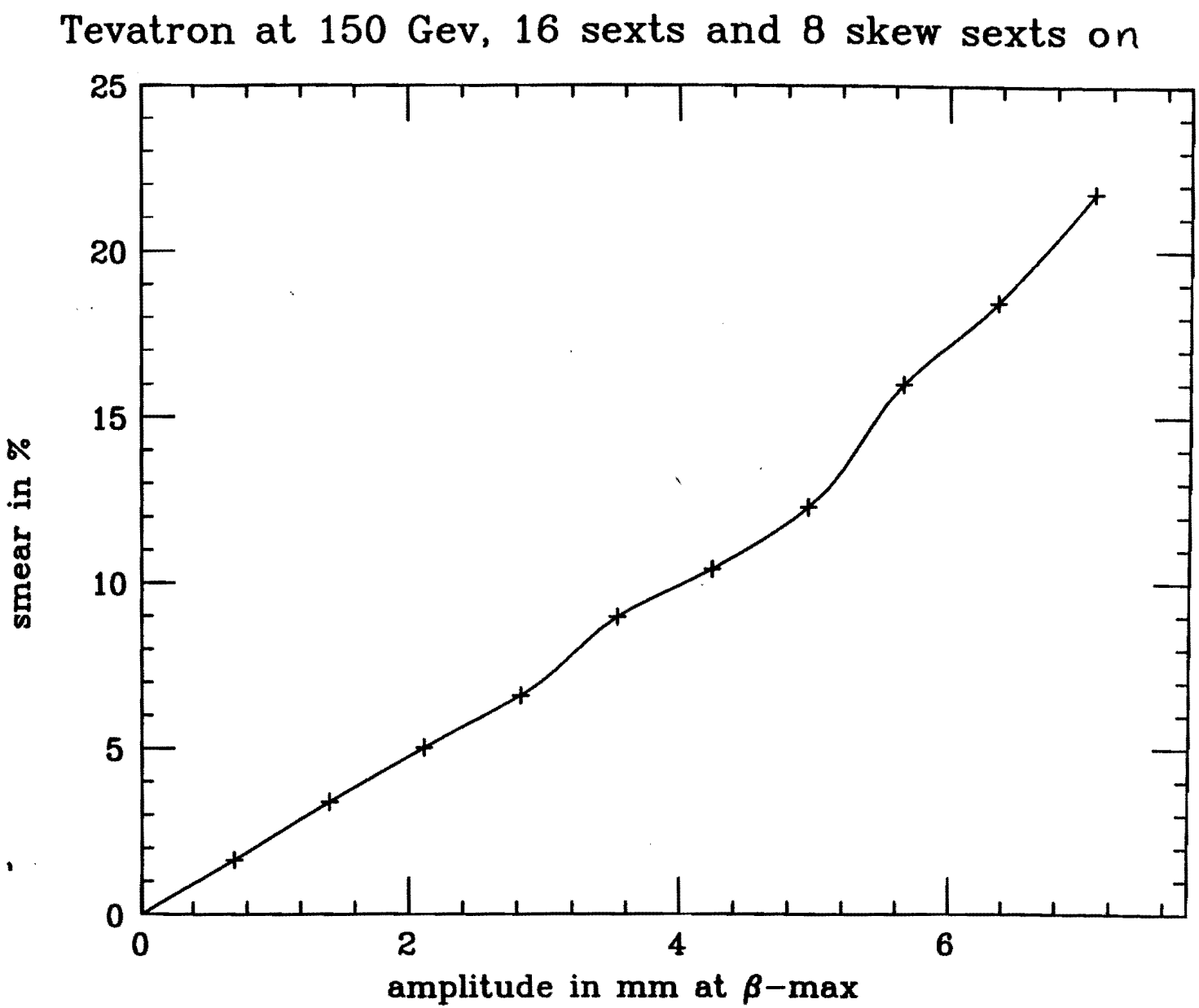

Tevatron at 150 Gev, 16 sexts and 8 skew sexts on
25
20
15
10
5
0
smear in %
0
2
4
6
amplitude in mm at β−max

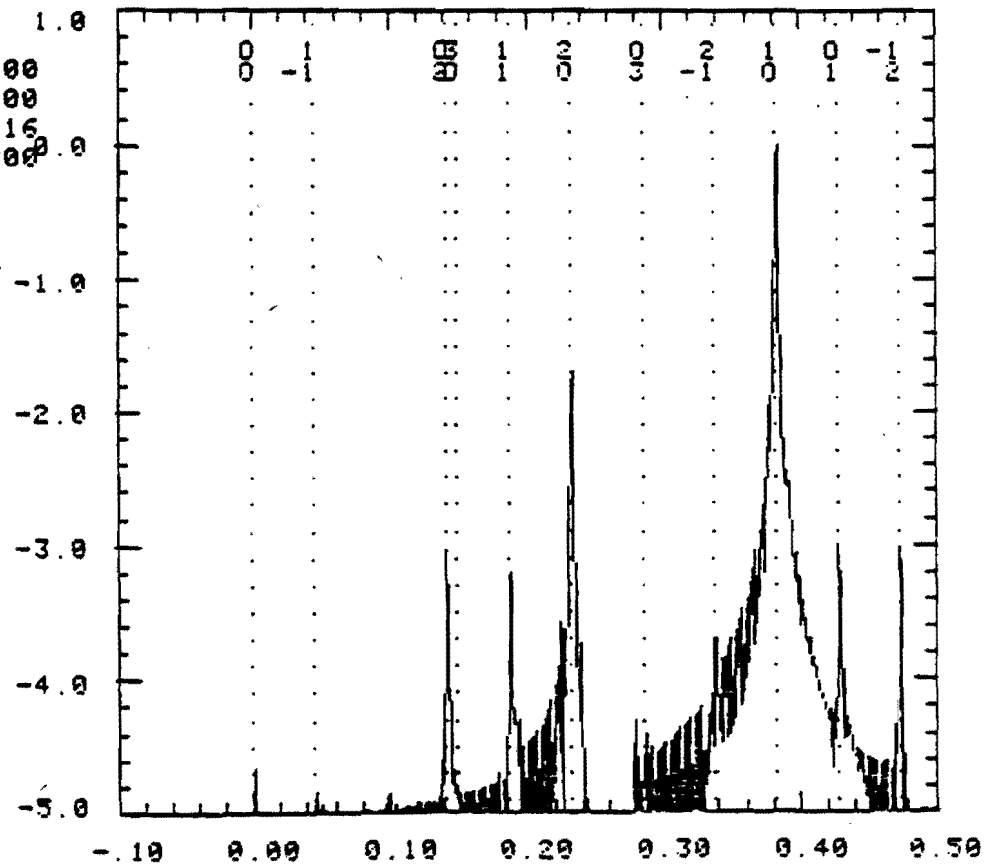
Figure 5

2.00 TRACKING 8/ 6/86 16:33:18 0

TEVATRON LATTICE,FOR TEAPOT, ENTERED BY RT FROM D. JOHNSON, 24 JULY, 1986

X TUNE SPECTRUM

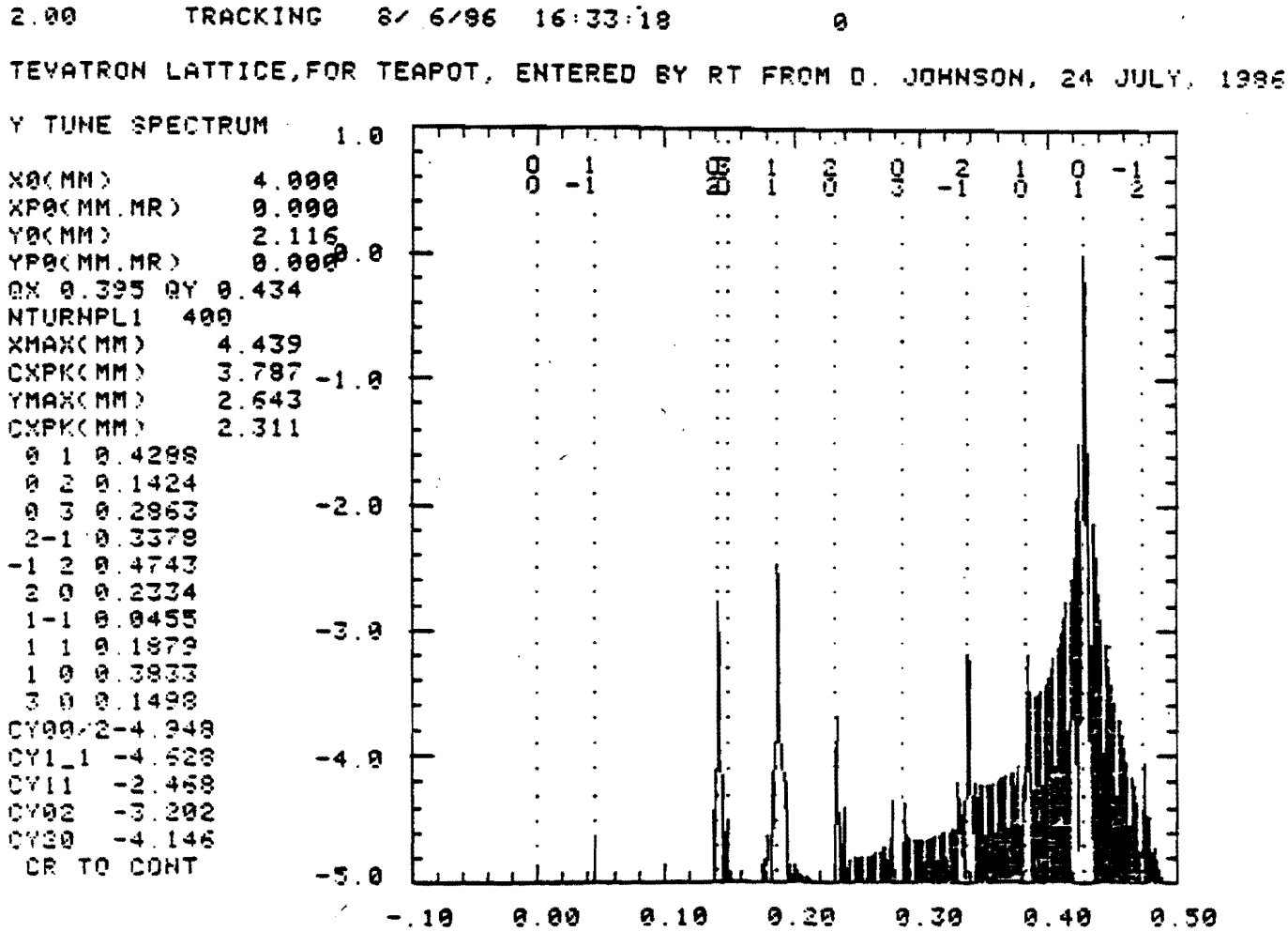
X0(MM) 4.000
XP0(MM.MR) 0.000
Y0(MM) 2.116
YP0(MM.MR) 0.000
QX 0.395 QY 0.434
NTURNPL1 400
XMAX(MM) 4.439
CXPK(MM) 3.787
YMAX(MM) 2.643
CYPK(MM) 2.311
1 0 0.3833
2 0 0.2334
3 0 0.1498
2-1 0.3378
-1 2 0.4743
0 2 0.1424
1-1 0.0455
1 1 0.1879
0 1 0.4298
0 3 0.2863
CX00/2-5.374
CX20 -1.823
CX02 -3.424
CX11 -3.229
CX1_1 -5.706
CR TO CONT



FFT with all sextupole & skew sextupoles on. Smear = 16%

x = y = 4mm

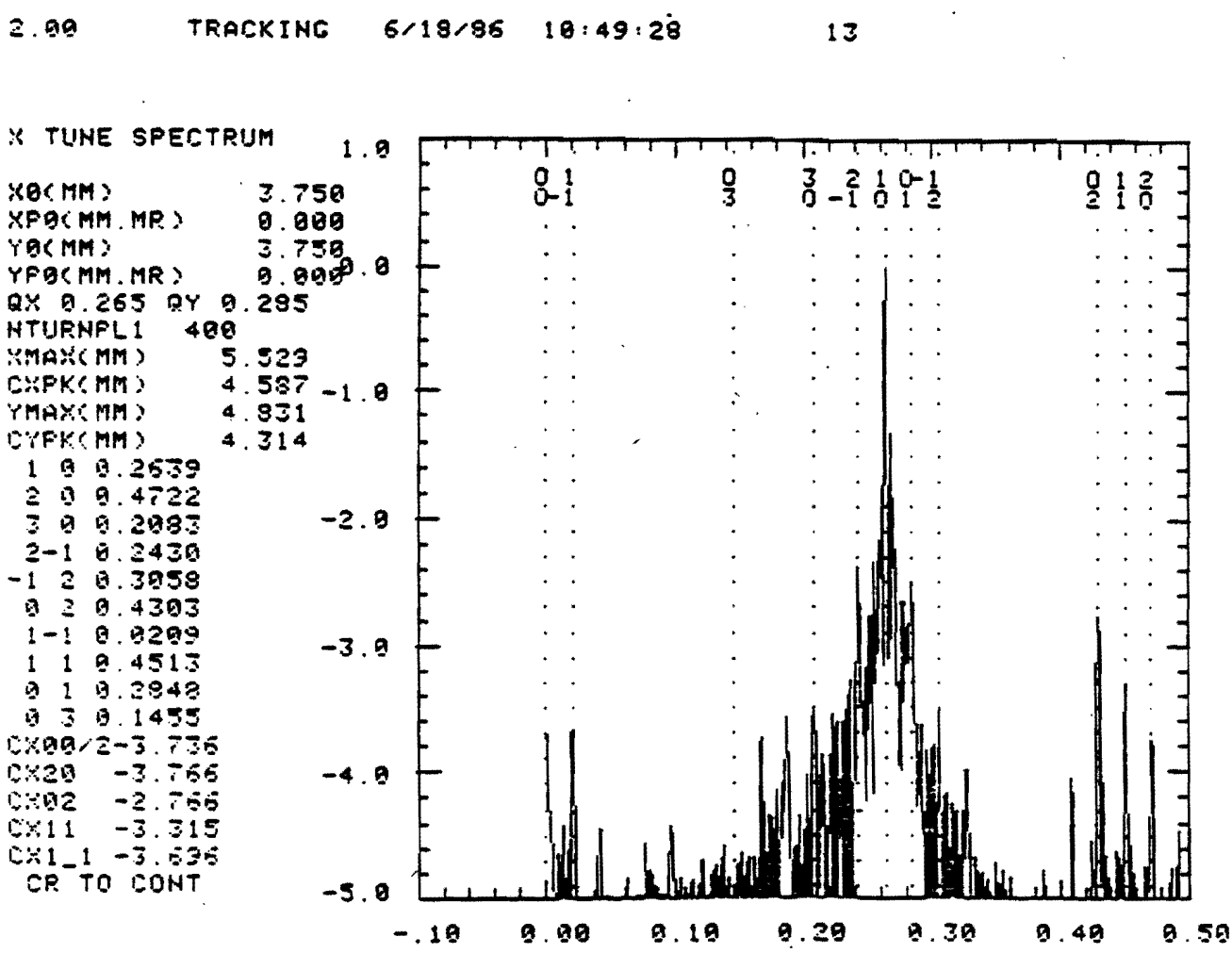
Figure 6



FFT with all sextupole & skew sextupoles on. Smear = 16%

x = y = 4mm

Figure 7



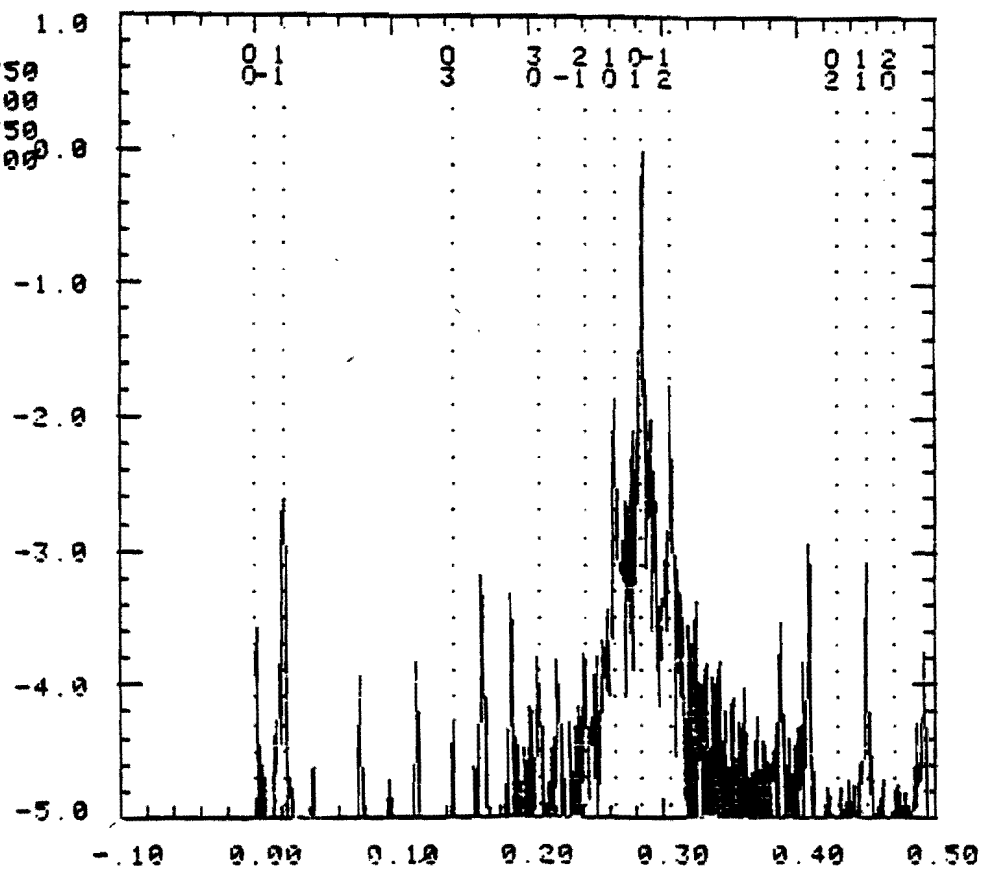
SSC smear = 17%, x = y = 8.8mm

Figure 8

2.00 TRACKING 6/18/86 10:43:28 13

Y TUNE SPECTRUM

X0(MM) 3.750
XP0(MM.MR) 0.000
Y0(MM) 3.750
YP0(MM.MR) 0.000
QX 0.265 QY 0.285
NTURNPL1 400
XMAX(MM) 5.529
CXPK(MM) 4.587
YMAX(MM) 4.831
CXPK(MM) 4.314
0 1 0.2848
0 2 0.4303
0 3 0.1455
2-1 0.2430
-1 2 0.3058
2 0 0.4722
1-1 0.0209
1 1 0.4513
1 0 0.2639
3 0 0.2083
CY00/2-3 925
CY1_1 -2 813
CY11 -3.080
CY02 -6.369
CY20 -4.809
CR TO CONT



SSC smear = 17%, x = y = 8.8mm

Figure 9