Proposal for a New Measurement
of the Magnetic Moment of the $\Lambda^0$ Hyperon

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Abstract

The discovery of a polarization in inclusive $\Lambda^0$ production from beryllium by 300 GeV protons makes possible the remeasurement of the magnetic moment of the $\Lambda^0$ hyperon to a precision of better than 1% in 160 hours of running in the M-2 beam.

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I Introduction

The magnetic moments of the baryons are of intrinsic interest as static properties of elementary particles. The moment is often written in terms of the dimensionless spin vector by the equation

\[ \mu = g \frac{e\hbar}{2mc} \mathbf{S}, \]

where \( g \) is called the "g factor", and \( m \) is often taken to be the proton mass, in which case \( e\hbar/2mc \) is the proton magneton. Charged point-like spin 1/2 fermions should have \( g = 2 \), and neutral spin 1/2 fermions should have \( g = 0 \). Such particles have pure Dirac moments. Departures from this rule are referred to as anomalous magnetic moments, and are attributed for the baryons to currents which contribute both to the moment and to the finite size of the particles.

The proton g factor (\( g = 5.58 \)) has been obtained to great precision (4 parts in \( 10^7 \)) by measuring the ratio of the Larmor frequency to the cyclotron frequency in a constant magnetic field. The neutron has a magnetic moment of the same order of magnitude as the proton (\( g = -3.8 \)), also known to about 3 parts in \( 10^5 \), by combining magnetic resonance with reflection of polarized neutrons off magnetized mirrors. The nucleons have sizeable anomalous moments, which should be explainable in terms of the structure of the particles - how they are built out of fundamental point-like constituents.

The quark model and the symmetry group \( SU_3 \) is one promising model of the point-like constituents of the baryons. This model
relates the magnetic moments of p and n to other baryons in the octet, namely $\mu_\Lambda$, $\mu_+^\Sigma$, $\mu_-'^\Sigma$, $\mu_0^\Sigma$, $\mu_-'^\Xi$, $\mu_0^\Xi$ and the transition moment $\Sigma \rightarrow \Lambda$, which enters in the radiative decay of the $\Sigma^0$. Of these seven possibilities, $\mu_-'^\Lambda$, $\mu_-'^\Sigma$, $\mu_-'^\Xi$ and $\Sigma \rightarrow \Lambda$ transition moment have been measured. The lambda magnetic moment is the most accessible to experiment, because it is neutral, has a long lifetime for a hyperon, and has a strong asymmetry in the decay $\Lambda \rightarrow p\pi^-$. The present world average value for the $\Lambda$ moment is

$$\mu_\Lambda = (-.672 \pm .061) \frac{e\hbar}{2m_p c}. \tag{2}$$

Two high statistics experiments contribute to this average: a spark chamber experiment with 3868 events,$^1$ and an emulsion experiment with 1300 events.$^2$

The simple quark model predicts

$$\mu_\Lambda = -.96 \frac{e\hbar}{2m_p c},$$

almost 5 standard deviations above the world average. Various broken $SU_3$ models predict lower values, in agreement with the observations.

A more precise measurement of the $\Lambda^0$ magnetic moment would be a challenge to the various models of broken unitary symmetry, especially if accompanied by a more precise measurement of the $\Sigma^0 \rightarrow \Lambda^0$ transition moment, which will be the subject of another proposal. The value of $\mu_\Lambda$ stands on its own independent of various models of baryon structure, however, as a fundamental static property of an elementary particle. It is in this spirit that an
improved measurement of $\mu_\Lambda$ is proposed.

II Experimental Method

The formula for the motion of the spin of a neutral particle in a magnetic field $\hat{B}$ in the laboratory is

$$\frac{d\hat{S}}{dt} = \hat{S} \times \hat{\Omega},$$

where the vector $\hat{\Omega}$ about which the spin precesses is

$$\hat{\Omega} = g\frac{e}{2m}(\hat{B} - \frac{\gamma - 1}{\gamma} \frac{\hat{B} \cdot \hat{p}_\Lambda}{\gamma p_\Lambda^2} \hat{p}_\Lambda).$$

Here $\hat{p}_\Lambda$ is the three momentum vector of the hyperon in the laboratory. Note that if $\hat{B}$ and $\hat{p}_\Lambda$ are parallel, $|\hat{\Omega}| = |g\frac{e}{2m}B|$, which reduces the angle of precession substantially for very large values of $\gamma$. If, on the other hand, $\hat{B}$ and $\hat{p}_\Lambda$ are chosen perpendicular, so that $\hat{S}$ precesses in a plane perpendicular to $\hat{B}$ which contains $\hat{p}_\Lambda$, then $|\hat{\Omega}| = \frac{g}{2m}\hat{B}$, and

$$\frac{d\omega}{dt} = g\frac{e}{2m}B.$$  

gives the rate of change of the angle $\omega$ between $\hat{S}$ and $\hat{p}_\Lambda$. Figure 1a shows the relevant vectors. For a magnetic field of length $l$, and hyperons of velocity $c\beta$, the total precession angle is given by

$$\Delta\omega = \frac{ge}{2m^2c^2}Bd\ell$$

At FNAL energies, $\beta \approx 1$ to a good approximation ($\Delta\beta \approx \frac{1}{2}\frac{\Delta P}{P}$), so that $\Delta\omega$ is a constant, independent of hyperon energy, which depends on the desired factor $g$ and on the field integral $\int Bd\ell$. If $m$ is the proton mass, then $e/mc = 18.3^\circ$ per Weber/meter. Equation
(2) gives $g/2 = -0.67$, so that for $\Delta \omega = 45^\circ$, say, $\int B \, d\ell = 3.6$ webers/meter, or 18 kg and 2 meters long.

The direction of the spin vector $\vec{S}$ can be measured through the asymmetry in the decay $\Lambda^0 + p\pi^-$. If $\vec{p}_p$ is the momentum of the final proton in the $\Lambda^0$ rest frame, then

$$dN(\vec{p}_p) = \frac{1}{2} (1 + \alpha \vec{S} \cdot \vec{p}_p) \, d\cos \theta_p. \quad (7)$$

Figure 1b shows this geometry. The best value of $\alpha = 0.647 \pm 0.013$. \(^3\)

The necessary ingredients of a precision measurement of $g_\Lambda$ are:

a) A polarized high flux $\Lambda^0$ beam.

b) A spin precession magnet (a 10 foot dipole would serve nicely).

c) A $\Lambda^0 + p\pi^-$ detector after the precession magnet which has good acceptance and sensitivity to the decay distribution in the hyperon center of mass.

Figure 2 summarizes the data on $\Lambda^0$ polarization in inclusive production by 300 GeV protons $p + \text{Be} \to \Lambda^0 + \text{anything}$. These data were taken in the E-8 neutral hyperon beam. Geometrical biases were eliminated by precessing the $\Lambda^0$ spin in the magnetic field of the sweeping magnet, and taking data with both polarities of field. \(^4\) The observed polarization maximizes around a production angle of $8^\circ$ with an $\alpha_{P_\Lambda}$ value of

$$\alpha_{P_\Lambda} = 0.10 \pm 0.015, \quad (8)$$

for a momentum range $150 \text{ GeV/c} \leq p_\Lambda \leq 210 \text{ GeV/c}$. The $\Lambda^0$ flux in this momentum range into the E-8 solid angle is $1.5 (\Lambda^0 + p\pi^-)/10^6$ interacting protons in the beryllium target.
Figure 3 shows the geometry for the proposed experiment. Steering magnets will deflect the proton beam vertically onto a beryllium target. Data will be taken with both signs of the production angle (± 8 mr). Two magnets are shown in which the hyperon spin will precess: the sweeping magnet, which eliminates charged particles from the neutral beam; and the external precessing magnet placed immediately downstream of the sweeper channel output. Both magnets are shown with vertical magnetic fields. There are two ways to avoid this apparent duplication, both of which have disadvantages. The effect of the sweeper on the spin of the \( \Lambda^0 \) could be eliminated by making \( \mathbf{S} \) and \( \mathbf{B} \) parallel - i.e. produce the \( \Lambda^0 \)'s in the horizontal plane. The problem here is that for the incident proton beam coming from the left and the sweeper polarity set to bend protons to the left the beam is partially transmitted through the collimator, making the background strongly dependent on the polarity. On the other hand, the precessor could be eliminated, and the magnetic moment measured by the rotation angle in the sweeper alone. Three problems arise: a) The excitation of the sweeper cannot be varied at will because it is a radiation shield; b) The \( \int \mathbf{B} \mathbf{d}l \) for the sweeper is difficult to measure to high precision because of its length (5.4 meters) and small aperture (4 mm); and c) The effective path length of the \( \Lambda^0 \) in the sweeper field is difficult to know precisely - it is assumed that all \( \Lambda^0 \) are born at the 15 cm long beryllium target, but this would have to be studied in detail. Hence the introduction of two magnets in a row, one of which is easily mapped and varied. Data can be taken in such a way that the precession angle of the \( \Lambda^0 \) spin in
the sweeper field can be eliminated. On the other hand, the $|Bd|l$ in the sweeper is about twice the maximum $|Bd|l$ in the precessor, so that greater accuracy in the measurement of the magnetic moment could be achieved from the precession angle in the sweeper provided that the problems listed above could be overcome.

III Running Requirements

The flux of $\Lambda^0$ with the polarization given by Eq. (8) at a proton flux of $10^8$/pulse, which is easily achievable in the M-2 line, is $150$/pulse. The best E-8 trigger is 60% efficient for $\Lambda^0 \rightarrow p\pi^-$ detection; the rest of the triggers are $K_S^0$ decays, neutron interactions in the vacuum windows, $\gamma \rightarrow e^+e^-$ conversions, and various other small effects. This would imply a trigger rate of $250$/pulse, which can be handled by the PDP-II buffer. Assuming a 13 sec period for the accelerator cycle the yield would be polarized $\Lambda^0 \rightarrow p\pi^- = 4 \times 10^4$/hour. (9)

Referring to Fig. 1b, the decay of a particular $\Lambda^0$ event can be written as an element in a likelihood function

$$L_i = \frac{1 + \alpha P_A (\cos \omega \cos \alpha_i + \sin \omega \sin \alpha_i \cos \phi_i)}{\int_{\text{acceptance}} (1 + \alpha P_A (\cos \omega \cos \alpha + \sin \omega \sin \alpha \cos \phi)) d\omega}$$

(10)

The magnetic moment information is contained in the angle $\omega$:

$$\omega = \omega_0 \pm \frac{e}{2mc} |Bd|l,$$

(11)

where the sign refers to a reversal of the field in the precessing magnet. The angle $\omega_0$ depends of the motion of the spin in the magnetic sweeping channel. By taking data with both polarities of the sweeping field, both polarities of the production angle, both polarities of the precessing magnet, and both polarities of
the analyzing magnet (16 in all), the angle $\omega_0$ as well as all possible geometrical biases in the apparatus in the measurement of the asymmetry can be eliminated. These various reversals also afford numerous internal cross checks. Without going into all the details, the error in $\omega$ can be calculated from the two components a and b obtained from the two dimensional likelihood calculation, and defined such that

$$a_{\rho A} = \sqrt{a^2 + b^2}$$

and

$$\omega = \arctan \frac{b}{a}.$$  \hfill (12)

The sensitivity of the experiment is set by $a_{\rho A} \approx .10$. The absolute error in a or b is of the order of $\delta a \approx \sqrt{3/N}$ for N events. To estimate the precision, let the term $g \frac{e}{2mc} \int B dl = \frac{\pi}{4}$, so that $b = a$. A short calculation then gives the standard deviation

$$\delta \omega \approx 10 \sqrt{\frac{3}{N}}.$$  \hfill (13)

From the measured yield, 160 hours of data would give

$$N = 6.4 \times 10^6 \Lambda^0, \text{ or } \delta \omega \approx 6 \text{ mr},$$

or a standard deviation in $\omega$ of .75%.

The following requests are made to perform this experiment:

1) A 10 foot 1" gap main ring bending magnet must be installed immediately downstream of the neutral hyperon sweeping magnet.

2) Pitching and restoring magnets must be installed upstream of the hyperon production target to allow $\pm 8$ mr production angles in a vertical plane.

3) 160 hours of running with a good spill and $\geq 3 \times 10^{12}$
protons on the Meson Lab target are required for a < 1% measurement of $\mu_\Lambda$.

One question remains - the energy of the proton beam on the Meson Lab target. The diffracted proton beam in M-2 must operate at the same energy as the primary beam. At this time the polarization has only been measured at 300 GeV. There is no reason to believe that the polarization will vanish at 400 GeV, but since the phenomenon was not expected in the first place, one cannot be sure. In any case this matter will be experimentally settled in the near future.
References


Figure Captions

1a. Rotation of the spin vector. $\hat{S}$, $\hat{B}$, and $\hat{p}_\Lambda$ are mutually perpendicular. The angle $\Delta \omega$ shown is positive for a negative anomalous moment at $t = 0$, as in the $\Lambda^0$ hyperon case, and its magnitude is given by Eq. (6).

1b. Decay of the $\Lambda^0$ in its rest frame. The decay asymmetry is given by Eq. (7).

2. $\langle \alpha P_A \rangle$ vs. $p_\perp$ for inclusive $\Lambda^0$ production by 300 GeV protons on beryllium. The polarization is essentially independent of $X = P_H/P_{\text{max}}$.

3. Proposed geometry. The proton beam will be displaced vertically by $\pm 7 \text{ cm}$ and then restored to the beryllium target to vary the production angle. The sweeping magnet and precession magnet will both have vertical fields. Hyperons which decay downstream of the precessor will be detected by the E-8 spectrometer. The ability of this spectrometer to measure the $\Lambda^0$ spin direction has already been demonstrated. A plan view of the motion of a $\Lambda^0$ spin vector in the two magnets is shown below. The angles are approximately correct, assuming $g/2 = .65$. 
PROTON ASYMMETRY

IN $\Lambda^0 \rightarrow p \pi^-$

VS

$\Lambda^0$ TRANSVERSE MOMENTUM

$\langle \alpha p_\perp \rangle$

$p_\perp$ GeV/c