NAL PROPOSAL No. 270

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FTS/Comm: 8-253-1000

THE STRUCTURE FUNCTIONS OF NEUTRON AND PROTON AND QUARK MODEL COMPARISONS

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23 November 1973

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Abstract

We request pictures from a run on the deuterium filled 15 foot bubble chamber; we want a total of 10^{19} protons on target equally divided between ν-enhanced and $\bar{\nu}$-enhanced settings for the horn. We also request a sample of 25,000 events from the hydrogen run to establish analysis procedures and test methods for neutrino energy estimates, separating neutral current events, and hadron jet analysis. The purpose of the run is to measure structure functions and compare neutron and proton with respect to hadron charge distributions, neutral current interactions, and structure function differences. We expect the experimental results to answer definitive questions about quark models, hadron deexcitation and the structure of the weak currents both charged and neutral.

I. Introduction

Developments in the past year or so have opened new dimensions in the study of neutrino interactions. These developments not only give heightened urgency to the pursuit of this physics, this may lead as well to a rethinking and reordering of priorities in the NAL bubble chamber program. Taken chronologically the new physics is:

1) The possible manifestation of quark signature in the hadronic end products of deep inelastic electron scattering.

2) The probable discovery of neutral currents.

Continuing chronologically, the results of experiments at SLAC (Dakin et al PRL 31 786 (1973)) show that a large +/- ratio is observed in the hadronic end products of deep inelastic electron scattering on protons (see Fig. 1). This is particularly true if one looks at the highest momentum hadrons ($X_F = p_{||}/p_{||\ max}$ large) and at the highest q^2. A similar but smaller indication is shown also when the neutron is the target. In view of the current successes of quark models one is tempted to explain the results qualitatively by a picture that the high momentum hadrons reveal their quark parentage. The parents are struck by a virtual photon in proportion to $(\text{charge})^2$. Possibly the ultimate +/- ratio available from the valence quarks of the proton is

$$2 \times \left(\frac{2}{3}\right)^2 / 1 \times \left(\frac{1}{3}\right)^2 = 8/1$$

The results with the neutron are, in a way, more dramatic.

$$+/- \;=\; 1 \times \left(\frac{2}{3}\right)^2 / \; 2 \times \left(\frac{1}{3}\right)^2 \;=\; 2/1$$

That such large ratios are not seen is compatible with the presence of "core" quarks which give charge symmetries. Since neutrino reactions are not biased toward low q^2 from the photon propagator such charge ratios may have great diagnostic power measured not only on the proton but on the neutron as well. The equivalent ratios for neutrinos on valence quarks would be infinite since a simple quark model would always have an accelerated p quark for ν events and n-quark for $\bar{\nu}$ events - a very dramatic result.

An illustration of the paucity of information on neutron versus proton cross-sections is the ratio

$$\sigma(\nu n)/\sigma(\nu p) \;=\; 1.8 \pm 0.3^{*}$$

This was measured in a propane filled bubble chamber and is dominated by systematic errors. A knowledge of this number is fundamental; the ratio of n quarks between neutron and proton is 2, a tantalizingly simple result. On the other hand the simplest quark model fails to explain the ratio of neutron to proton structure function at large $x = q^2/2M\nu$; it would be the sum of the squares of the valence quark charges, namely, 6/9 to 9/9 i.e. 2/3. Instead it seems to be measured as $\sim 1/3$. It would be very interesting to see what neutrino reactions show in this region. We would argue, then, that it is extremely important to start a neutrino program on the simple targets, protons and neutrons with the ability to see the final state clearly. The hydrogen runs will do half this job; however, deuterium runs should follow very shortly.

* G. Myatt and D. H. Perkins PL 34B 542 (1971)

The probable presence of neutral currents has opened up the physics of neutrinos considerably. To begin with those experiments designed explicitly to look for neutral currents will probably command less urgency in the future. However, the quantitative measurements of neutral current interactions will command more attention. As an example we cite the interaction term between the neutral current, Z^o, and the hadronic current a la Weinberg

$$\nu \bar{\gamma} (1 + \gamma_5) \nu \left(\bar{J}^{(3)} - \sin^2\theta \, \bar{J}^{em}\right)$$

$\theta \equiv$ Weinberg angle

$J^{(3)} \equiv$ 3rd component of a weak isospin hadronic current

$J^{em} \equiv$ electromagnetic hadron current

The important point is that the weak current couples partly to the quark charge and partly to the electromagnetic charge. There is then no simple relation between neutron and proton cross-sections. With charge currents we expect and experiments indicate that the structure functions, $F_i(x)$ in the cross-section

$$\frac{d^2\sigma^{\nu\,\bar{\nu}}}{dxdy} = \frac{G^2 M E_\nu}{\pi} \left[(1-y)\, F_2^{\nu,\bar{\nu}} + y^2\, x\, F_1^{\nu,\bar{\nu}} \mp y\,(1 - \tfrac{y}{2})\, x\, F_3^{\nu\,\bar{\nu}}\right]$$

$$x = q^2/2M\nu \qquad\qquad y = \nu/E_\nu$$

are related through isospin symmetry by

$$F_i(\nu p) = F_i(\bar{\nu} n)$$

This is no longer necessarily true with neutral currents. We no longer expect equalities between the F_i's as appears to be true for charge currents. This makes it important to measure both neutron and proton cross-sections; indeed some of the most important physics may come out of this comparison. We are, in effect, coupling to both quark and electromagnetic charge; we need two different mixtures (of quarks?) to have a measure of their relative effects.

The isospin breaking may also be reflected in charge current interactions; this is characteristic of some models that accommodate neutral currents without $\Delta S = 1$ transitions. In a particular quark model (see Ref. below) transitions can occur through charge currents between p and λ quarks though proportional to $\sin^2\theta_c$ ($\theta_c \equiv$ Cabibbo angle). The amusing aspect is that the neutron-proton difference may appear in the strange particle production events which though small ($\sim$1%) are definitely measurable in our proposal.

It is true that measurement of neutral current events suffers from lack of kinematic knowledge when using a broad beam. There are, however, some simple results derivable from quark models such as the yield ratios of neutral to charged current events for ν and $\bar{\nu}$ (R and $\bar{R}$ - see Ref. below). These have been measured in the Gargamelle experiment (though coarsely) and give numbers in agreement with a simple quark model. Indeed a test of the model is a definite relation between R and $\bar{R}$. (Much of the above physics was taken from three preprints by L. Sehgal Preprints Physikalisches Institut Technische Hochschule Aachen, Aachen W. Germany, June 1973.)

There are, of course, other results available to a deuterium run which will make it worthwhile. We will study the "elastic" event on the neutron,

$$\nu + d \rightarrow \mu^- + p + p \text{ (spectator)}$$

this has the particular merit of being fully analyzable. A comparison with elastic electron or μ scattering is suggested. Particular channels such as Δ or strange particle production are of interest since they are measurements of the isotopic spin and hypercharge changing character of the weak currents.

II. Proposal

A. General

We propose to measure neutrino and anti-neutrino interactions on deuterium in the 15 foot Bubble Chamber with sufficient events to:

1. Measure the F_i's for the neutron; we will, of course, have concurrent data on the proton.
2. Analyze the hadronic final states in deep inelastic events for their expected diagnostic value in revealing quark content both by electric charge ratios and hypercharge ratios.
3. Pick out and analyze the neutral current events to get a comparison of their cross-section with respect to charged current events and determine how the hadronic final state is different.
4. Study of quasi-elastic, π-Δ production, and strange particle production.

The run length is determined by the desire to obtain a statistically significant number of events in each energy bin at a high enough $x(= q^2/2M\nu)$ to assure that we are interacting primarily with "valence" quarks. We have arbitrarily selected the region $0.5 < x < 1.0$. This limits

us to 13% of the events. We have used the structure functions from the quark model of Kuti and Weisskopf;* these are in agreement with the experimental results of the Cal Tech group.** In addition, the leading hadron charge effect was seen in inelastic electron scattering+ by examining the highest energy hadrons - $X_F = p_{||}/p_{||}$ (max) > 0.5. This further cuts down our sample. We have assumed the X_F distribution seen in hadronic collisions which is compatible with electron scattering data. We have also assumed a logarithmic increase in particle multiplicity with hadronic c-of-m energy. Our estimated event count is given in Table 1. We took a fiducial volume 2.8m in diameter and 3m in length.

The events found at $X < 0.5$ are still of great interest and we estimate (as shown in the table) that events in the bins $0.3 < X < 0.4$ and $0.4 < X < 0.5$ have a comparable or larger number of events as those in $0.5 < X < 1.0$. In these two bins a model such as Kuti and Weisskopf's gives a valence to core quark ratio of 3.5 and 4.5 respectively.

In addition to events from the deuterium run we request 25,000 events in our fiducial volume from a hydrogen run. The purpose of this would be to establish that our methods of analysis are correct and to get preliminary results on the physics of hadronic deexcitation. The analysis of proton events would insure that we had a smooth running analysis system and that the deuterium data would be analyzed expeditiously. In the same spirit we would like some events from the early hadron runs to enable us to check programming changes necessary for the MIT-PEPR system with 15 foot Bubble Chamber film.

* J. Kuti and V. F. Weisskopf Phys. Rev. D4 3418 (1971)

** Barish et al PRL 31 565 (1973)

\+ Dakin et al PRL 31 786 (1973) and its back references

B. Summary of Requests

1) As soon as possible – a sample of events from the 15 foot bubble chamber.

2) 25,000 events from either a ν or $\bar{\nu}$ broad band hydrogen run.

3) Film from a 5×10^{18} 350 GeV/c proton run with deuterium filling and horn set for ν's.

4) Film from a 5×10^{18} 350 GeV/c proton run as in 3) but for $\bar{\nu}$.

Table 1A - ν events (5×10^{18} 350 GeV/c protons and horn)[1]

Neutrino Energy Bin (Gev)	Events: Total	With $X > 0.5$; $X_F > 0.5$[3] Total	Proton[2]	Neutron	Neutron (Spectator Visible)	Analyzable[4]
10 - 20	15,000	1050	350	700	210	∿ 320
20 - 40	27,000	2700	900	1800	600	∿ 680
40 - 80	19,000	2400	800	1600	540	∿ 460
> 80	8,500	1200	400	800	260	∿ 190

Table 1B - $\bar{\nu}$ events (5×10^{18} 350 GeV/c protons and horn)[1]

Neutrino Energy Bin	Total	No. of Events $X > 0.5$ $X_F > 0.5$[3] Total	Proton[2]	Neutron	Neutron (Spectator Visible)	Analyzable[4]
10 - 20	2,500	180	120	60	20	∿ 50
20 - 40	4,500	450	300	150	50	∿ 110
40 - 80	3,000	390	260	130	43	∿ 70
> 80	1,500	210	140	70	27	∿ 30

1 We use the spectrum calculated by F. A. Nezrick (3 Jan. 1973) in NAL Publication "Neutrino Area Operations Package" - Fiducial volume; Diam. 2.8m Length 3m.

2 We assume $\sigma(\nu n) = 2\sigma(\nu p)$ - Myatt and Perkins PL 34B 542 (1971) and also $\sigma(\bar{\nu}p) = 2\sigma(\bar{\nu}n)$

3 Estimated events in the X bins .3-.4 and .4-.5 can be obtained by multiplying the numbers in this section by 1.45 and 1.00 respectively.

4 These numbers represent a lower limit; see discussion in Sect. III.

III. ANALYSIS OF EVENTS

A. Introduction

The problem of how the neutrino events may be analyzed separates into two parts, the analysis of the charged current events and the identification of neutral current events. The former presents a more difficult problem because of the interest in obtaining charge ratios for the hadronic final states of the deep inelastic events which requires a complete identification of the particles. In the case of the neutral current events, as will be discussed, this is not necessary. A determination of the neutrino energy must be made for the charged current events using the energies of the observed particles, and we will discuss the effect on this determination of multiple neutral particles.

B. Charged Current Events

$$\left.\begin{array}{ll} \nu + p \rightarrow \mu^- + p + \pi^+ + N\pi & \text{1A} \\ \quad\;\; \rightarrow \mu^- + n + \pi^+ + \pi^+ + N\pi & \text{1B} \end{array}\right\} Q_h = 2$$

$$\left.\begin{array}{ll} \nu + n \rightarrow \mu^- + n + \pi^+ + N\pi & \text{2A} \\ \quad\;\; \rightarrow \mu^- + p + N\pi & \text{2B} \end{array}\right\} Q_h = 1$$

$$\left.\begin{array}{ll} \bar{\nu} + p \rightarrow \mu^+ + p + \pi^- + N\pi & \text{3A} \\ \quad\;\; \rightarrow \mu^+ + n + N\pi & \text{3B} \end{array}\right\} Q_h = 0$$

$$\left.\begin{array}{ll} \bar{\nu} + n \rightarrow \mu^+ + p + \pi^- + \pi^- + N\pi & \text{4A} \\ \quad\;\; \rightarrow \mu^+ + n + \pi^- + N\pi & \text{4B} \end{array}\right\} Q_h = -1$$

The charged current events of interest will fall into one of the eight subcategories given above. Here $N\pi$ signifies additional pions whose total charge is zero. The analysis of these events must distinguish the μ and the proton from the pions, neutrino from antineutrino events, and neutron from proton events.

The μ will uniquely be determined by its charge in event types 1A, 1B), 2A), 3B) and 4B) if there are no additional pions. For higher multiplicities,

there is the possibility of additional charged pions, but the quark model cross section prediction given in Section 1 indicates that in at least 50% of the events, the μ will be the outgoing particle with the largest momentum for ν events. For $\bar{\nu}$ events this percentage is even higher since present data indicate a y distribution in agreement with $(1 - y)^2$. A Monte Carlo simulation using this cross section and appropriate assumptions about the hadronic final state (for details see Appendix) indicates that for the class of events of special interest ($x > 0.5$), the μ^- and the group of hadrons are either spacially resolved with the hadrons grouped carrying appreciable transverse momentum opposite to that of the μ or the μ is the highest momentum particle. If the μ is identified as the particle of maximum transverse momentum, providing $p_\perp > 1.0$ GeV/c, this results in a correct identification 93% of the time. Eighty per cent of the events mis-identified on this basis are two hadron final state events. The μ assignments may, of course, be checked experimentally by looking for secondary interactions of the particles selected as μ. Since the pion interaction length in deuterium is 4.8m, the probability of interaction in an average track length of 2.3m is 38%, permitting a good check of the identification.

Once the μ is identified, the events may be separated into the four types by the net charge of the hadrons, indicated by Q_h in equations 1A) through 4A). There is, of course, the question of whether there will be confusion with the spectator proton in neutron events. Figure 2 gives the momentum distribution for the spectator proton calculated from the Hulthen wave function and for comparison, the momentum distribution obtained at SLAC for the protons from the μp scattering events with large q^2. The

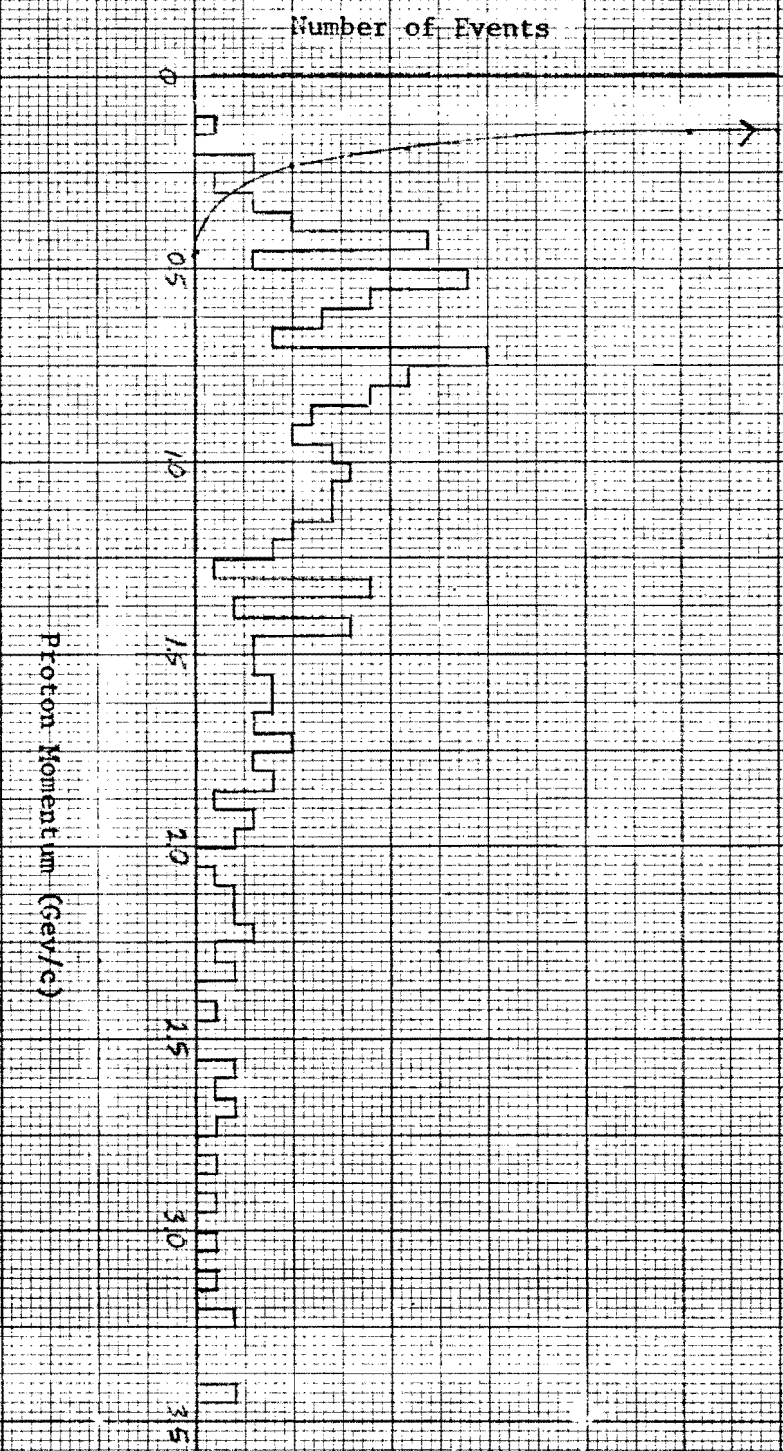

Figure 2: Comparison of the momentum spectrum for the spectator proton calculated from the Hulthen wave function (curve) and protons observed in μp deep inelastic scattering (histogram for $3.24 < s < 6.25$ $(GeV)^2$ and $q^2 > 0.5$, private communication, J. Ballam, SLAC, 1973).

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overlap between the two distributions is seen to be small and should be even less in the case of neutrino events for $x > 0.5$. Thus, protons with momentum < 400 MeV/c may be taken to be spectator particles. This hypothesis will be checked, using type 2E) and 4B) events where both the spectator particle and outgoing baryon are protons. Low momentum (<1 Gev/c) π mesons can be distinguished from protons by $\frac{dE}{dx}$. Observation of a spectator proton can, of course, be used to confirm the assignment of an event as a neutron event. From the Hulthen distribution it is estimated that in $\sim$50% of these events the spectator will have momentum $p \leqq 80$ MeV/c, range <1mm, and will not be visible. In $\sim$20% the spectator will have momentum, $80 \text{ MeV/c} \leqq p \leqq 120$ MeV/c, range < 5mm, and although visible, will not be measurable. In the remaining $\sim$30% the spectator momentum will be measurable and will be used as a check on the fitting procedure described in the next paragraph. In summary, we expect to separate neutron from proton interactions very well, and to check our method of separation in about half our events.

In addition to the above separation we expect that some fraction of the events will be fully "analyzable" in the sense that we can check the identities of all particles and calculate the neutrino energy unambiguously. We include in this category events where proton $-\pi^+$ ambiguity is not resolved which should represent a small fraction of the events. The outgoing proton in reactions 1A), 2B), 3A) and 4A) will, if $x > 0.5$, usually be too energetic to be identified by its range or ionization. The Monte Carlo simulation indicates that few protons will be observed with momentum less than 1 GeV/c. Our model treats π's and nucleons on an equal footing.

This assumption is probably incorrect; it is difficult to accelerate baryonic charge. Realistically, we expect the protons to be of lower average momentum than the π-mesons. Studies based on the Monte Carlo simulation indicate that some of the events involving a single neutral particle will only permit a unique kinematic interpretation. Since these results are very model dependent, we are not including such cases in our estimates of analyzable events. In this category, we only include events where one can obtain one to three constraint fits with a good χ^2. The 3C fits will be obtained for events where all outgoing particles are charged. The 2C fits correspond to events where the outgoing baryon is a neutron which then interacts in the bubble chamber. Since the neutron interaction length is 3.0 meters, there will be a 54% probability of interaction for an average path length of 2.3 meters. The 1C fits correspond to events where the single neutral is a π^o, a decay photon of which produces a pair in the bubble chamber. Since the radiation length in deuterium is 9m, there is a 35% probability that one decay photon will produce a pair in 2.3 meters and a 5% probability that both will. In the kinematic fitting procedure, an unseen spectator will be included by some assumption, tested empirically, such as assigning it a momentum of 50 ± 50 MeV/c and the same direction within large uncertainties as the non-spectator baryon.

The energy of the incident neutrino will, of course, be obtained from these kinematic fits. However, since we anticipate being able to also include some classes of events where there is an unseen single neutral particle, it is useful to point out that the error in the determination of the neutrino energy from the presence of a second neutral particle has an average value of $\sim$10% calculated on the basis of our Monte Carlo simulation (see Appendix).

c. Neutral Current Events

$$\nu + p \rightarrow \nu + \text{hadrons} \qquad Q_h = 1 \qquad 5)$$

$$\nu + n \rightarrow \nu + \text{hadrons} \qquad Q_h = 0 \qquad 6)$$

$$\bar{\nu} + p \rightarrow \bar{\nu} + \text{hadrons} \qquad Q_h = 1 \qquad 7)$$

$$\nu + n \rightarrow \nu + \text{hadrons} \qquad Q_h = 0 \qquad 8)$$

The neutral current events should be identifiable by their imbalance of outgoing transverse momentum. As discussed in the Appendix, our Monte Carlo simulation has indicated that for large lepton angles, a statistical separation is possible between neutral current events and charged current events with unseen neutral hadrons. The neutron events can be distinguished from the proton events by the total hadronic charge (Q_h). However, it is not possible to obtain the incident neutrino energy for the events in the neutral current sample.

D. Number of Useable Events

Table 2 gives an estimate of the percentage of events that will give 3C fits and 0C calculations as a function of neutrino energy, based on the experimental average multiplicities in hadron interactions and the following assumptions: 1) the probabilities that the final baryon will be neutron or proton are equal; 2) the charge multiplicity in neutrino interactions is the same as in hadron interactions for a given s; 3) $\bar{n}_{\pi^0} = \bar{n}_{\pi^+} = \bar{n}_{\pi^-}$; 4) n follows a Poisson distribution. It is seen that it will be possible to analyze between 16% and 32% of the events as 3C, 2C (neutron star) or 1C (pair from π^0 photon) events.

Table 2

Probability that an event will contain no π^0, ($P\,0\pi^0$), or one π^0, ($P1\pi^0$) as a function of neutrino energy, E_ν. This assumes equal probability for proton and neutron, that the charged pion multiplicity (n equal twice the neutral pion multiplicity (n_{π^0}), and that the multiplicity can be represented by a Poisson distribution. The last columns give the expected fraction of events with 3C fits, the fraction which are 0C events which will definitely be analyzable because of a neutron star or π^0 decay photon conversion, and the total fraction which are analyzable.

E_ν (GeV)	$\bar{s}$	P_{lab}	$\bar{n}_{ch}$	$\bar{n}_{\pi\pm}$	$\bar{n}_{\pi^0}$	$P_{(0\pi^0)}$	$P_{(1\pi^0)}$	Total 3C	Total 0C	Analyzable 0C	Total Analyzable
10-20	7.9	3.1	2.6	2.1	1.05	.35	.37	.18	.36	.15	.33
20-40	1.5	7.0	3.2	2.7	1.35	.26	.35	.13	.30	.12	.25
40-80	29	14.4	3.7	3.2	1.60	.20	.32	.10	.26	.10	.20
80-160	58	30	4.6	4.1	2.05	.13	.27	.07	.19	.09	.16

1. XVI International Conference on High Energy Physics, 3, 318 (1972).

IV. ANALYSIS: EFFORT AND MANPOWER

From Table 1 it is seen that we anticipate $\sim$81,000 events from our requested neutrino and antineutrino exposures in deuterium in our chosen fiducial volume. In addition, we request 25,000 events from neutrinos in hydrogen. The combined sample of 106,000 events represents a scanning and predigitizing effort of $\sim 10^4$ machine-hours. Using our full strength of 6 image plane digitizers, this implies approximately one year to complete the scanning and predigitizing. Since the bulk of the events will be measured by PEPR, the measurements would be completed almost simultaneously with the predigitizing. Those events rejected by PEPR would be measured on one of our three film plane digitizers.

The participants in this experiment would include eight Ph.D. physicists and two or more graduate students as well as technical support staff.

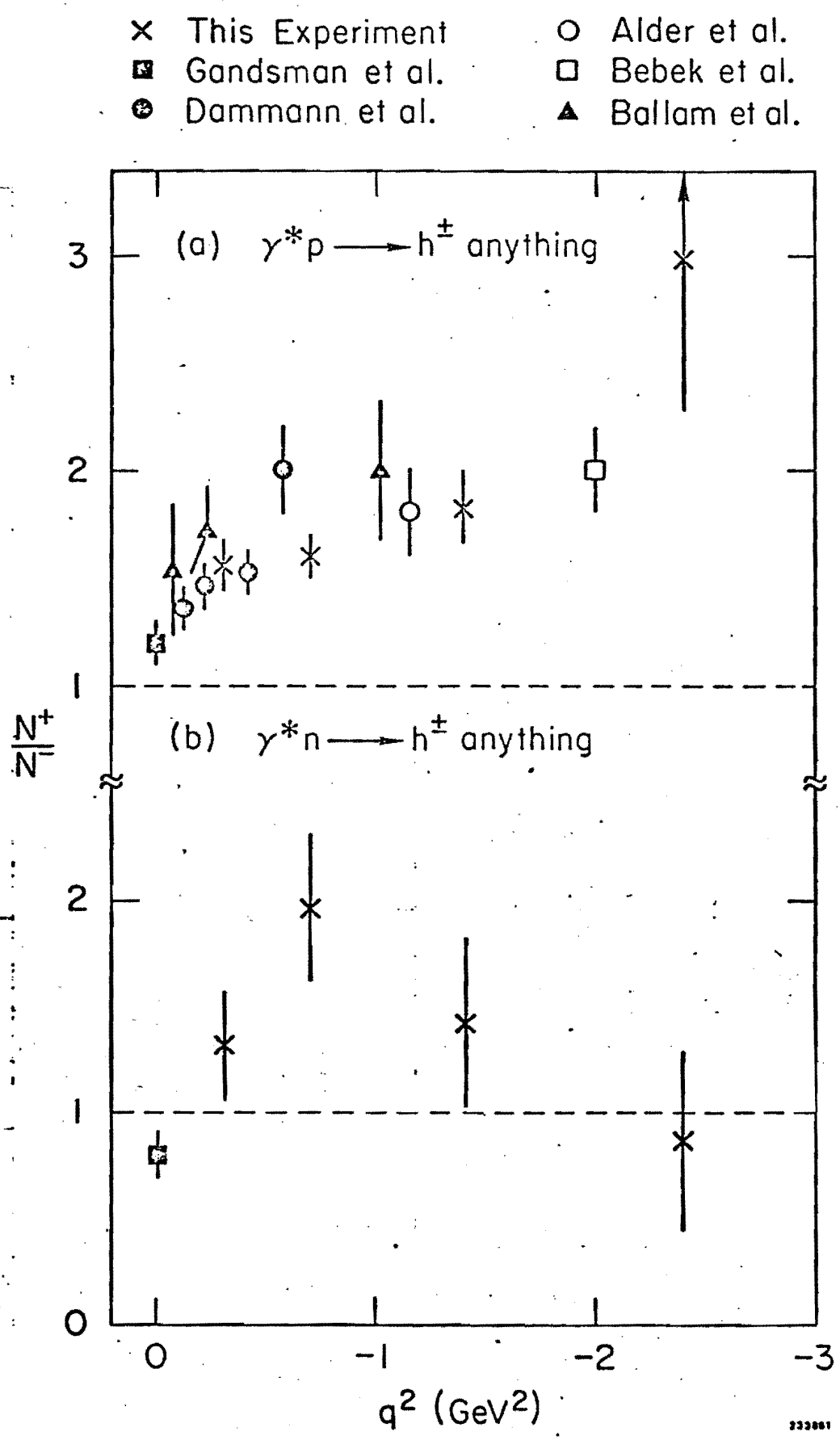


Fig. 1 - Results from Dakin et al PRL 31 786 (1973) showing the hadron charge

APPENDIX

In order to check some of the problems we may have in analyzing events we have generated some Monte Carlo events to simulate the expected events. The following assumptions were used:

1. $\frac{d\sigma}{dxdy} \propto (1 - x)^3$ (flat y dependence)

2. We assume multiple π production with a Poisson distribution about an $\bar{n}$ and one neutron or proton in the final state.

$$\bar{n} = 2\, S^{0.28} + .25$$

$$S \equiv (\text{hadronic c of m energy})^2$$

3. The particles distribute in the momentum according to
   a) $e^{-6\, p_\perp^2}$, transversely to the total hadronic momentum
   b) phase space, longitudinally

These assumptions are in agreement with deep inelastic electron scattering.

This program is the GENIS program of C.E.R.N.

We have checked the reliability of the method of determining the μ-meson by identifying it with the particle of maximum transverse momentum, provided $p_\perp > 1.0$ Gev. A scatter plot of $p_\perp$ for the hadrons is shown in Fig. A-2 for events with $x > 0.5$. It is clear from the figure that this method of selection works quite well (93% of the time). The events shown are a sample of 821 for $\varepsilon_\nu = 25$ Gev. This selection mislabels only 4% of the events with $x > 0.5$ for 25 Gev neutrinos. In 80% of the mis-identified cases there is a two body final hadron state which would not be of interest to us for charge ratios. In any event this method can be checked experimentally by looking for nuclear interactions which occur 38% of the time for the hadrons leaving the chamber.

If the EMI is operating we can use it as a check. We note that the EMI is particularly useful for small x events where the μ comes out at small angles. The region of interest to us (large x) gives large angle μ's where the EMI is less efficient. Thus our selection method offers a system complimentary to the EMI.

We have also tested a scheme by which we estimate the neutrino energy. Having picked the μ-meson by the criterion above the events will normally not be balanced in observed transverse momentum because of missing neutral hadrons. We make the assumption that the missing neutrals come off at an angle equal to that of the total observed hadronic momentum. This enables us to calculate the missing longitudinal momentum. Assuming this is carried by π^o's we calculate the missing energy and thus the neutrino energy. A plot of the error scatter is given in Figure A-1. This error spread is quite acceptable. In particular it falls off from its peak at a much faster rate than the spectrum of events plotted against ε_ν so that the feeding of low ε_ν events into the scarcer high ε_ν bins is minimal. The plot represents events for ε_ν = 25 Gev. Again, these assumptions can be checked by looking at those events where π^o γ's convert in the chamber. We have also checked this method for events of all x. The percentage error distribution is shown in Figures A-1b, c, and d for neutrino energies of 25, 50 and 100 Gev respectively.

The most challenging problem is the separation of neutral from charged current events. We make use again of the absence, this time, of the large transverse momentum carried by the outgoing lepton. In every event we find the plane perpendicular to which momentum balances and includes the maximum absolute transverse momentum; this is close to the lepton-hadron collision plane. We plot the two total momenta

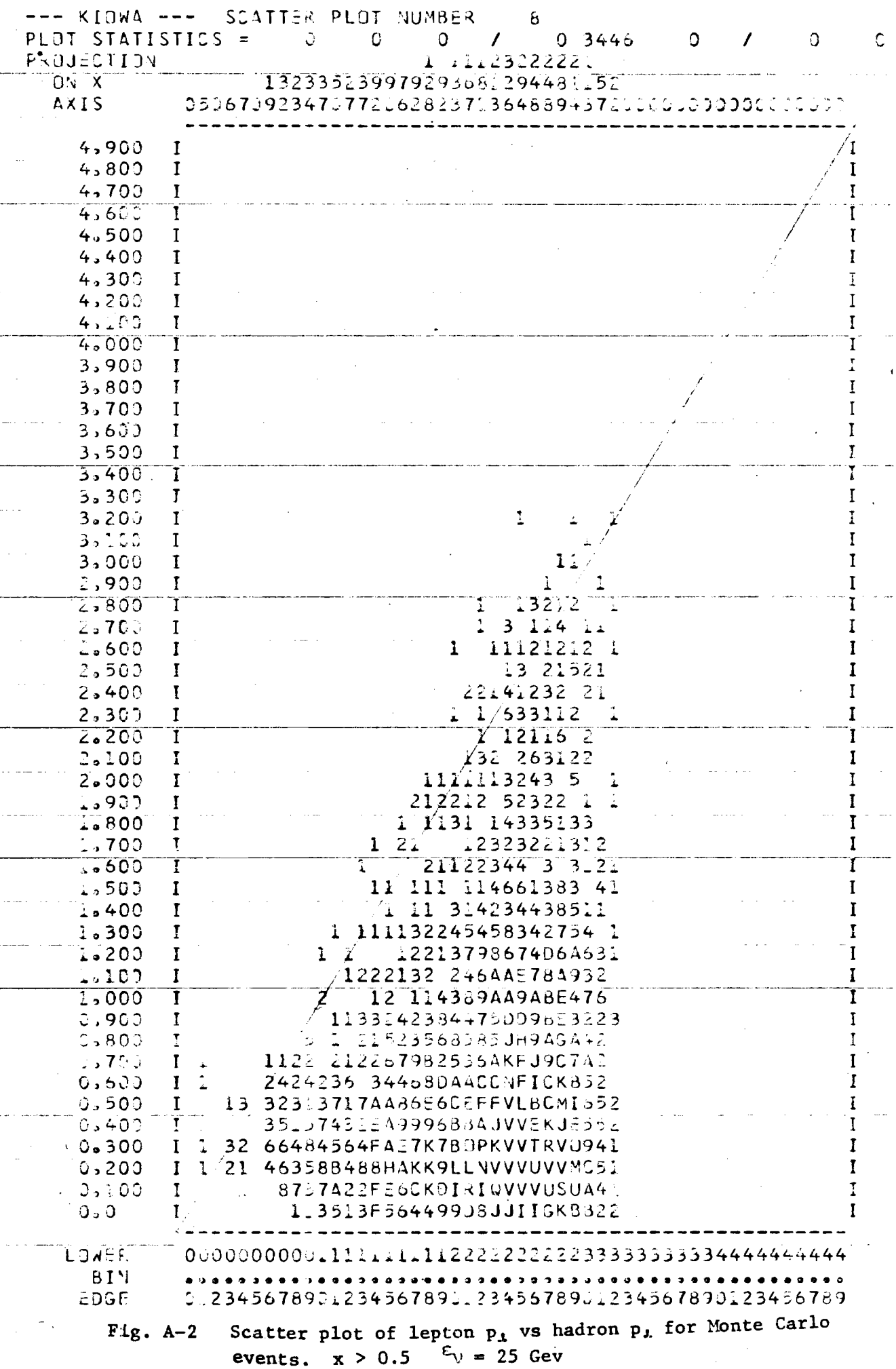


Fig. A-2 Scatter plot of lepton $p_\perp$ vs hadron $p_\perp$ for Monte Carlo events. $x > 0.5$ $\varepsilon_\nu = 25$ Gev

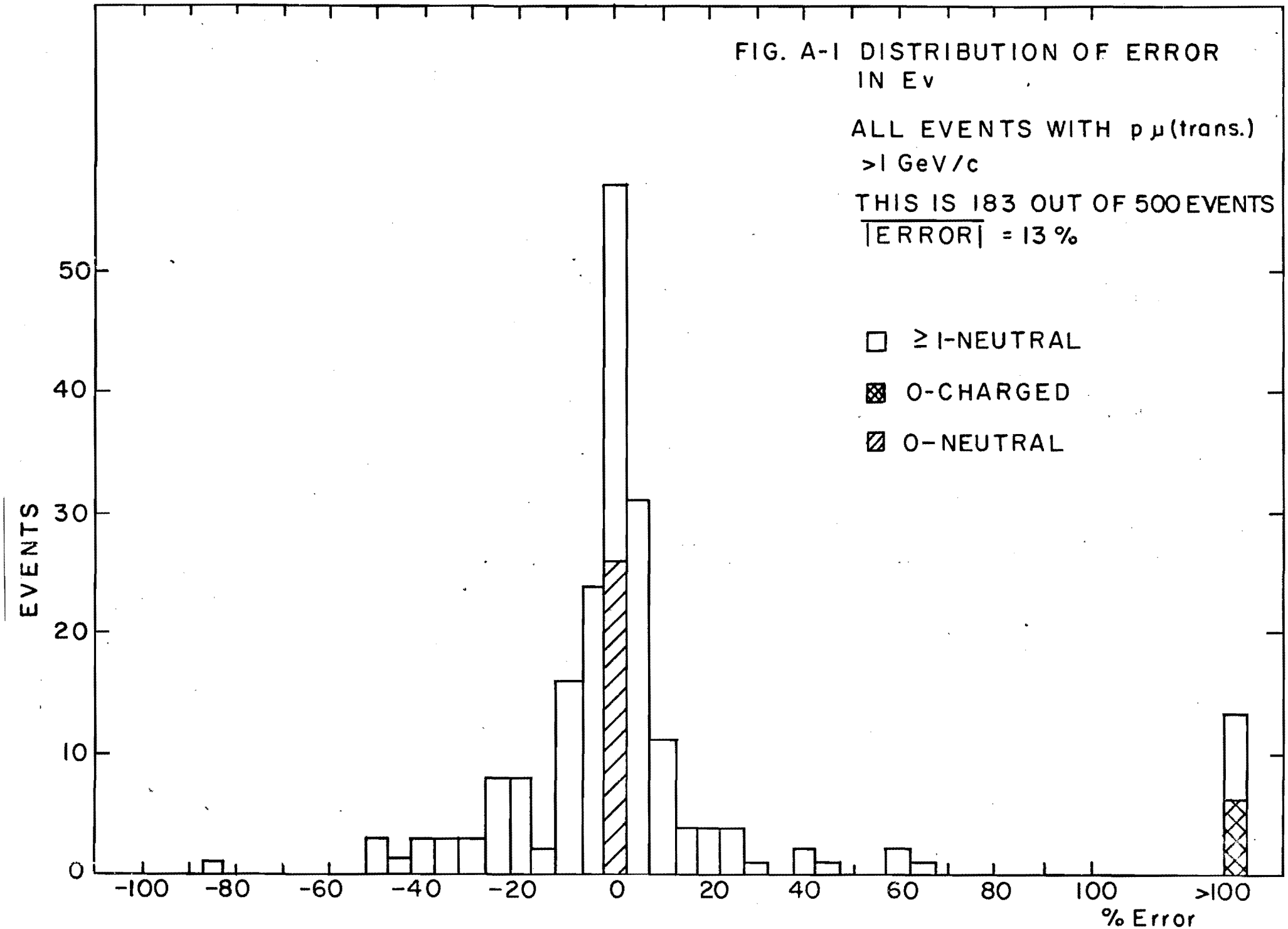


FIG. A-1 DISTRIBUTION OF ERROR IN Ev
ALL EVENTS WITH pμ(trans.) >1 GeV/c
THIS IS 183 OUT OF 500 EVENTS
|ERROR| = 13 %
≥ 1-NEUTRAL
0-CHARGED
0-NEUTRAL
EVENTS
0
10
20
30
40
50
-100
-80
-60
-40
-20
0
20
40
60
80
100
>100
% Error

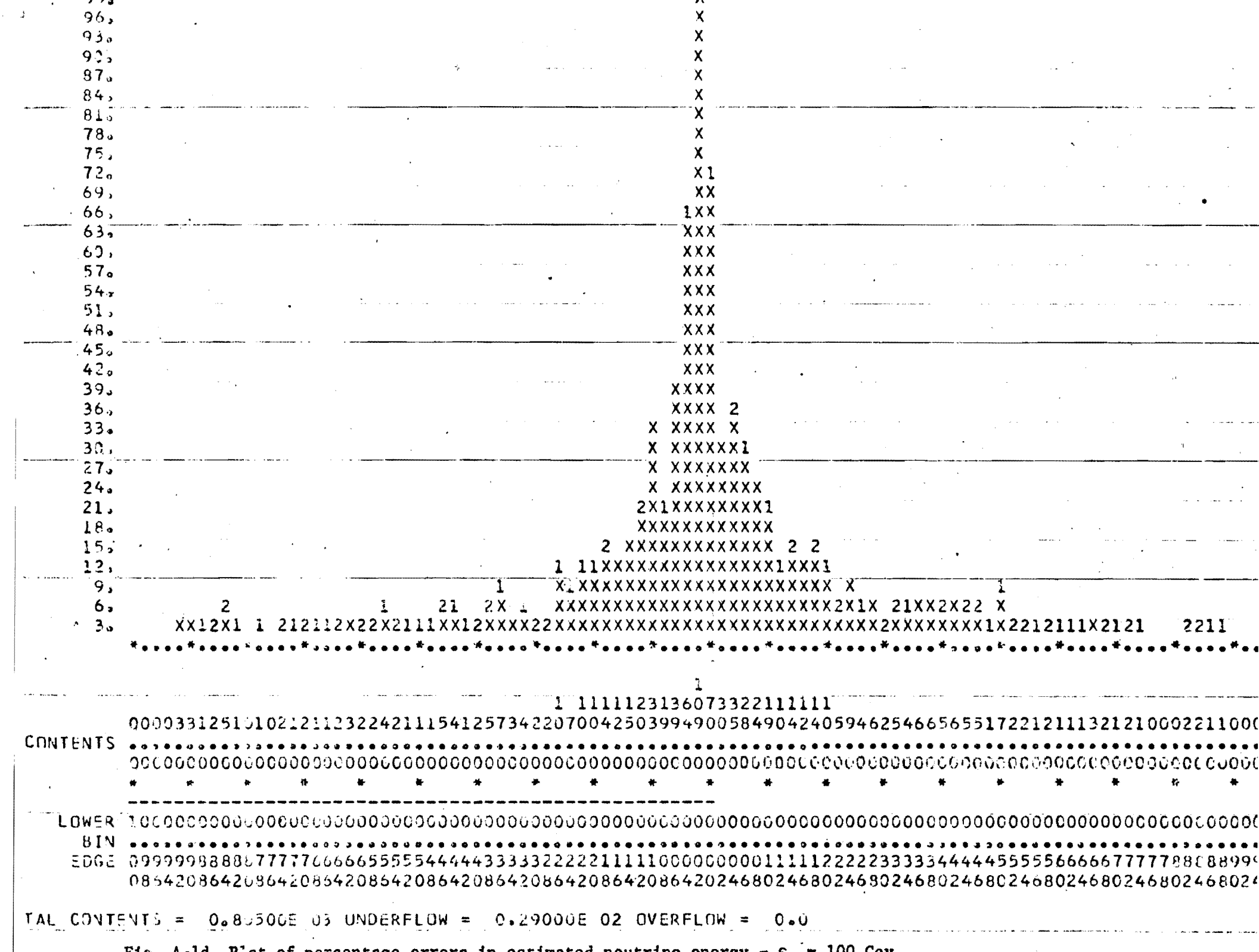


Fig. A-1d Plot of percentage errors in estimated neutrino energy – ε_ν = 100 Gev.

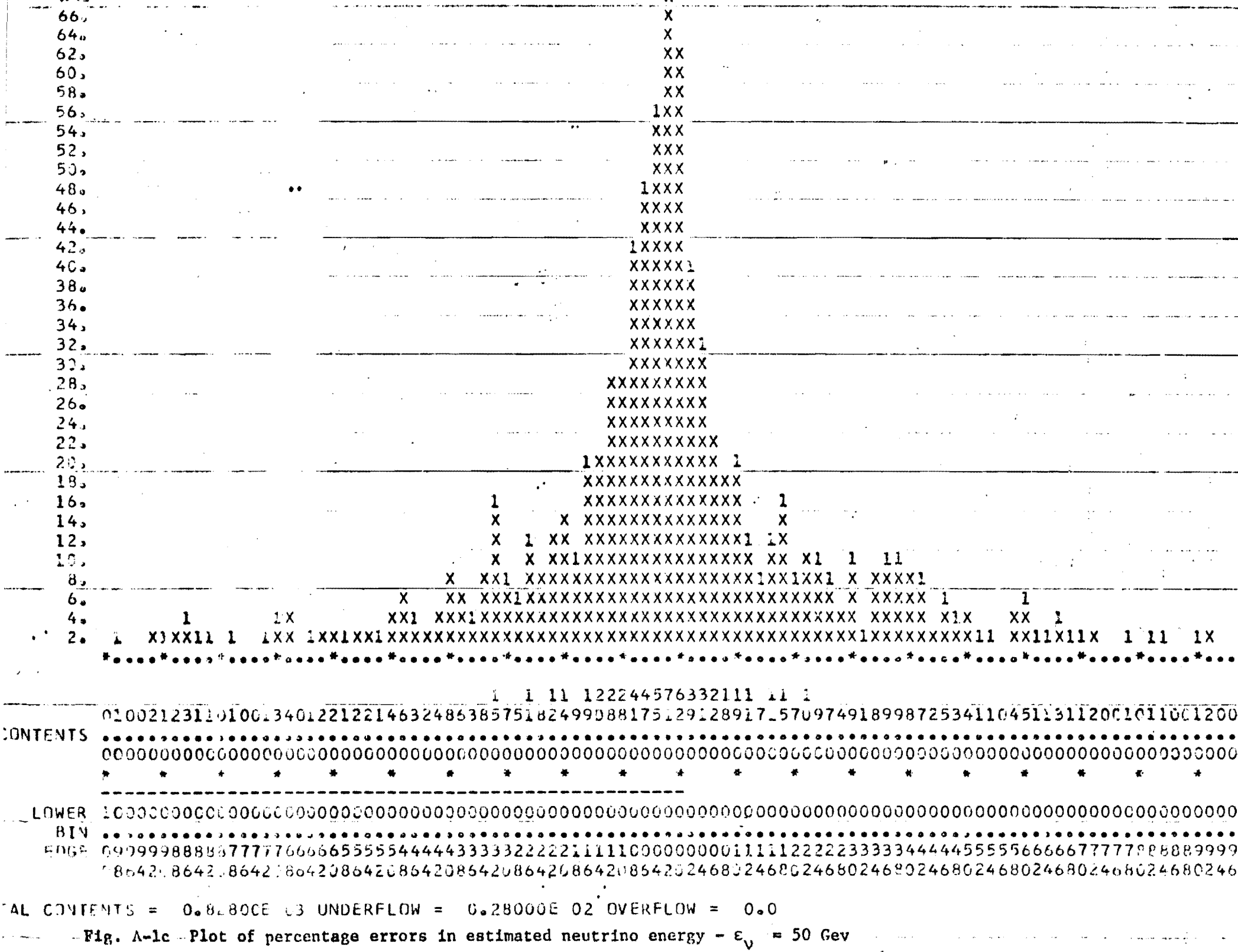


Fig. A-1c Plot of percentage errors in estimated neutrino energy - ε_ν = 50 Gev

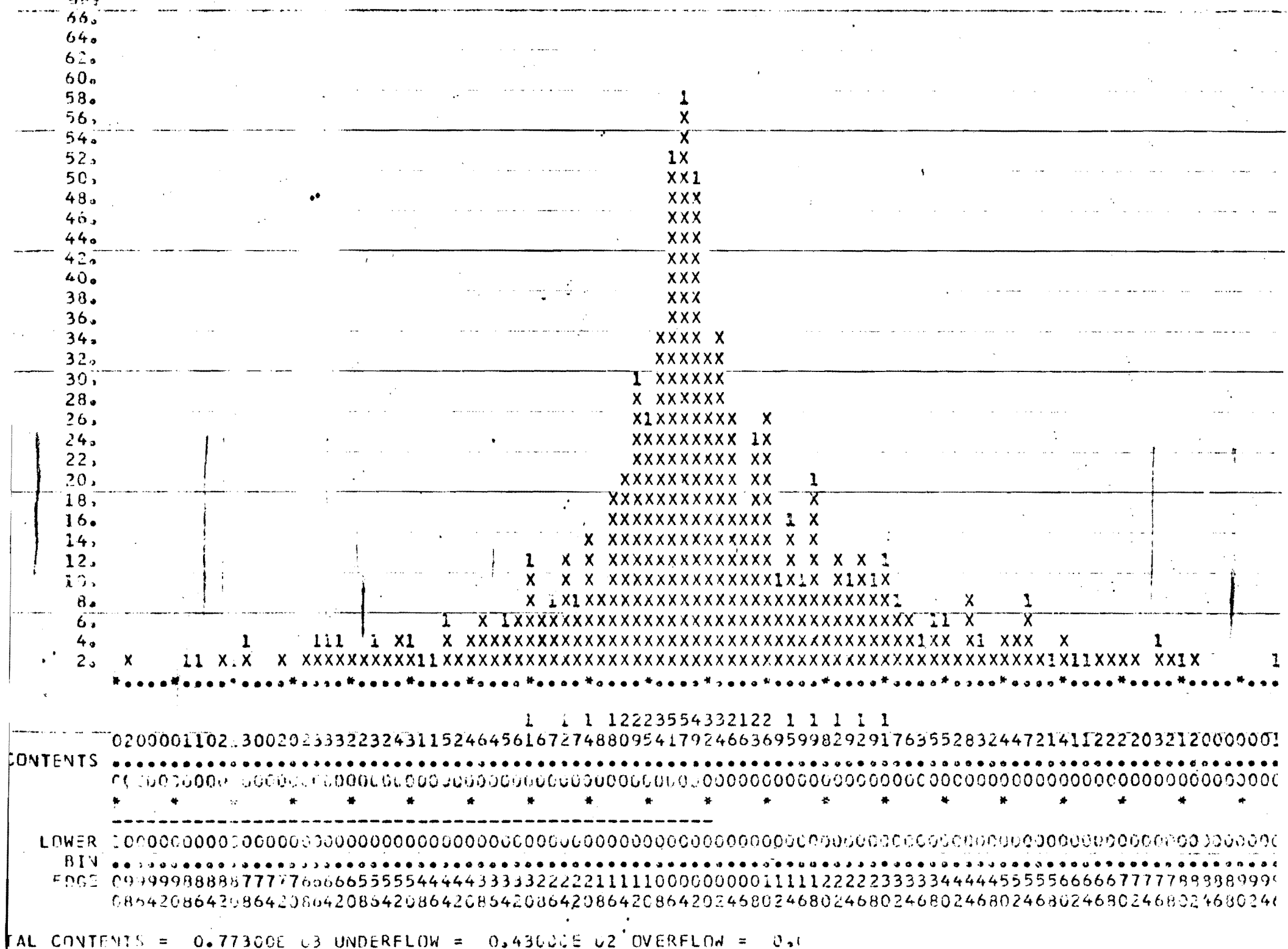


Fig. A-1b Plot of percentage errors in estimated neutrino energy – ε_ν = 25 Gev

observed on both sides of the ν-axis in this plane. This is shown in Figures A-3 and A-4. The first includes the outgoing lepton, the second does not. The plots show a graphic difference between the two types which are clearly separable in a statistical way. In the high x regions one could assign them on an event by event basis. Naturally, the EMI provides a positive identification in some cases particularly for small lepton angle and therefore small x which can check our separation method. Our method operates at large x where the EMI becomes less efficient and so is complimentary to it. The separation is still complicated by lack of knowledge of the incoming neutrino energy; the nice separations seen in Figures 3 and 4 are smeared by weighting over the total neutrino spectrum. Neutral current events are not amenable to our energy estimation scheme.

Finally, for amusement, we have drawn some typical high x-events to illustrate their visual drama (see Figure A-5). To see such large angle collisions at high energies is the most convincing demonstration of the point like quality of the contents of proton and neutron. These events also illustrate why our method of lepton identification works.

In conclusion our Monte Carlo program has generated a spectrum of recoil protons for events with $x > 0.5$ (Fig. A-6). Note the happy kinematic result that there are no low momentum recoil protons ($p < 0.5$ Gev/c). This is somewhat expected since high x implies high hadron momentum but low hadronic excitation energy. We are thus able to identify low proton momenta as the recoil spectator with little ambiguity in such events.

SCATTER PLOT NUMBER 1

PLOT STATISTICS = 0 125 71 / 0 801 3 / 0 0 0

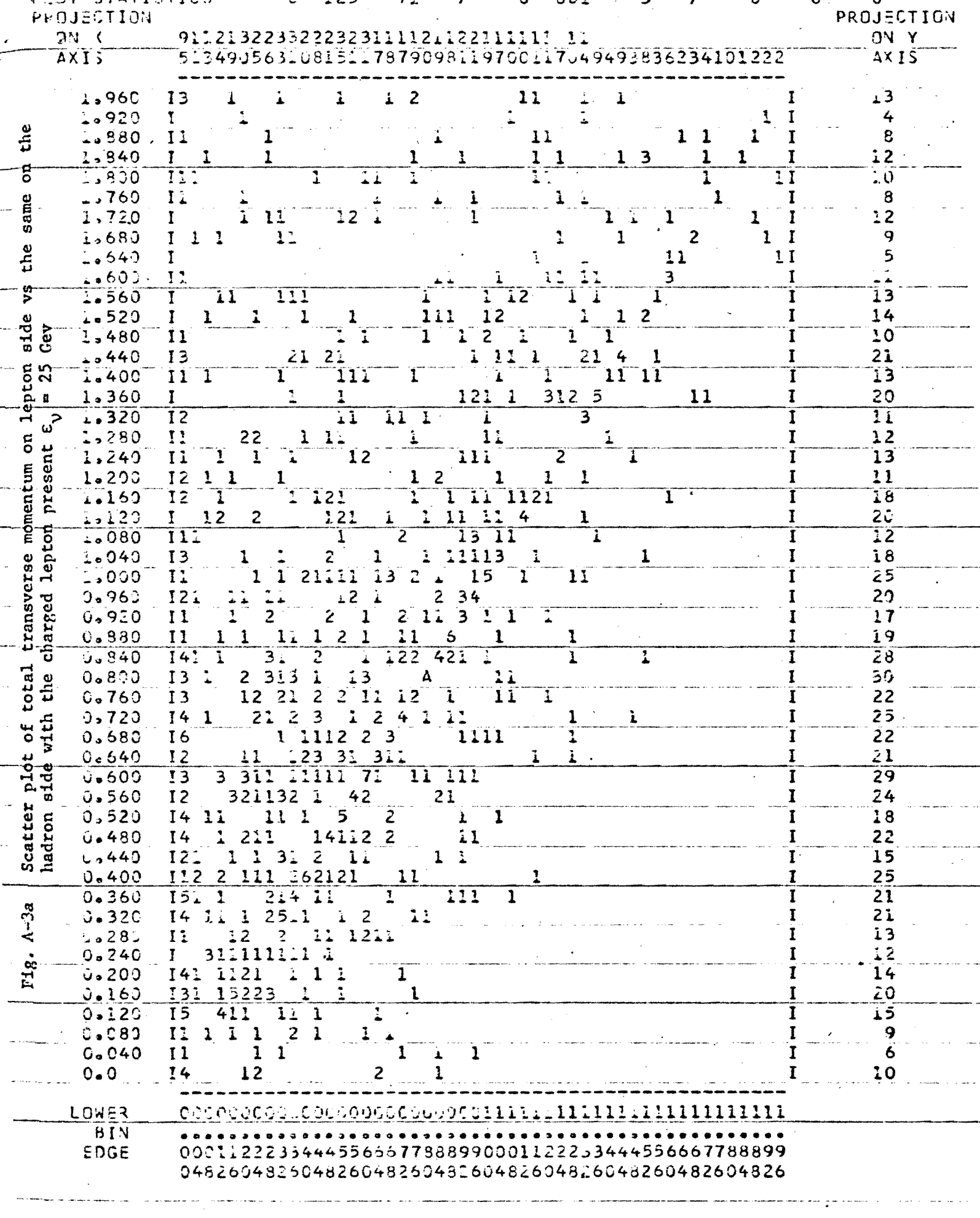


Fig. A-3a Scatter plot of total transverse momentum on lepton side vs the same on the hadron side with the charged lepton present ε_ν = 25 Gev

```
--- KIOWA ---   SCATTER PLOT NUMBER    3
PLOT STATISTICS =    0   23   10   /    0  967    0   /    0    0    0
PROJECTION  1                                                          PROJECTION
  ON X      0325454444333332211221 1  11  1                             ON Y
  AXIS      3160055351667309 2.2545509298649613457536342403121          AXIS
            ---------------------------------------------------
  3.920   I     1                1   1 1      2            1  1I        8
  3.840   I1                 1          1   1      11        1 I        7
  3.760   I1                             2              111 I        6
  3.680   I1          1  1           1    111              1   I        8
  3.600   I211   1             1     1 12          1 1 1      I       13
  3.520   I              11     1     1   1   1 1   3         I       10
  3.440   I1                   11 1         1    1     23      I       11
  3.360   I     1       1                        121     1      I        7
  3.280   I1        1 111                 1   1        2        I        9
  3.200   I2          1            1 1  1          11         I        8
  3.120   I          1   21   11 11    1111 1   11            I       15
  3.040   I   1  1 1   1    2      1  2           2 1           I       12
  2.960   I1         1    11                 1 1  22           I       10
  2.880   I                11      1 11  1 111 11    1         I       12
  2.800   I2 1   1     1 21          31  1   1 2               I       16
  2.720   I3 11   11            1.   1 11  11 3                I       17
  2.640   I             1  13 11 2 11   3 1  1111              I       19
  2.560   I 1          1    11   1   1  1  13                  I       11
  2.480   I       1  1   1     111       1   1                  I        8
  2.400   I21 11 1  1 1    1 11 211       4                    I       19
  2.320   I1   11       121 2  1431  1 113                      I       24
  2.240   I2     1   12 2      122  1 1321                      I       21
  2.160   I11          111   121 1 112 2  1                     I       17
  2.080   I     1111     11   1 111 13 1   1                    I       16
  2.000   I1   1      11  1 3    1  41  1                       I       15
  1.920   I1         1   12  1111   3                           I       12
  1.840   I1   22     2 12 3  12  3 1                           I       20
  1.760   I1    1  3.2      212412  2  1                        I       23
  1.680   I          12     1 11 8   2                         I       16
  1.600   I11 1 1 1    121511  4                                I       20
  1.520   I312  21   3 23  11  1 2                              I       22
  1.440   I  11 1 1 32 3  2 31       1                          I       19
  1.360   I1 12  11 2 1 153111   1 1                            I       23
  1.280   I11  1 1 111421116  1                                 I       23
  1.200   I21  111  1 25 16  1 1                                I       23
  1.120   I5 11 32 2113134  2 1  11                              I       32
  1.040   I2 123211  335411 1 1                                 I       31
  0.960   I  11 53632213213  1                                  I       34
  0.880   I7  1231333 6121       1                              I       34
  0.800   I5   34322 732  1 1                                   I       35
  0.720   I3111325530 1 1                                        I       39
  0.640   I322  4167 1111                                        I       30
  0.560   I8121215361231121 1                                    I       39
  0.480   I3113215242 11                                         I       26
  0.400   I52213731432 2                                         I       35
  0.320   I21247 425 1                                           I       28
  0.240   I63115.3 2       1                                     I       23
  0.160   I8243 311 21                                           I       25
  0.080   IA52313 2                                              I       26
  0.0     I33  3   1                                             I       10
            ---------------------------------------------------
 LOWER    0000000000000111111111111222222222222333333333333
  BIN     ••••••••••••••••••••••••••••••••••••••••••••••••••
 EDGE     0012345678890122345667890012344567889012234566789
          0864208642086420864208642086420864208642086420864208642
```

Fig. A-3b Scatter plot of total transverse momentum on lepton side vs the same on the hadron side with the charged lepton present ε_ν = 50 Gev

```
--- KIOWA ---   SCATTER PLOT NUMBER    5
PLOT STATISTICS =    0   10    3   /    0  925    0   /    0    0    0
PROJECTION                                                          PROJECTION
  ON X        7435554443322222111 21 11 11          1                 ON Y
  AXIS        2691554.392186345494925832631597854956141431C00211      AXIS
              ---------------------------------------------------
   5.880  I                      1                      2      11I      5
   5.760  I     1                     1   1 1  1   1            I      6
   5.640  I        1  1              1          1 2   1    1    I      8
   5.520  I  11         1    1  1         1 1   1               I      8
   5.400  I 1  11       1      1    1        12                 I      9
   5.280  I        1                    1        1 1       1    I      5
   5.160  I  1             1     1  11      1 1 1    1 1        I     10
   5.040  I 1 1                  1       12  1   11 12          I     12
   4.920  I                 11        11 1      1               I      6
   4.800  I 1                         21        1 12            I      9
   4.680  I1          1  11        1   31 2        1            I     12
   4.560  I     1  1  1    1            1  1    115 1            I     14
   4.440  I1 1 1          11111   11 1      1                    I     12
   4.320  I 1       1         31  11 1 1 1111   1               I     15
   4.200  I        2      1        1    1  1 5  1               I     12
   4.080  I1   11  1     1  1                 1                  I      7
   3.960  I    2              11                                I      4
   3.840  I      1      1 11 1111212  11   31                   I     19
   3.720  I1  111      11  1   12  11  1 111 11                 I     18
   3.600  I3                1      1   1 11 1                   I      9
   3.480  I3 1    1   1    1   1   21 122  1                    I     17
   3.360  I                1 1 113 1                            I      8
   3.240  I 121      11  1  1          4 1                      I     13
   3.120  I 11       1 1  1 111 1 21  121                       I     16
   3.000  I21   1 1 1      1 121  1 3  111                       I     18
   2.880  I11  1   1         21 11 1                            I     10
   2.760  I   11 2  112131211 2 4                                I     23
   2.640  I1    1  131 2 1141   2212                            I     23
   2.520  I1    1  11 11  1 11 1221                              I     15
   2.400  I 1  11 131421 211    21 1                             I     23
   2.280  I1      1 1  2   1  22 2                              I     12
   2.160  I     11 3  114331132                                 I     24
   2.040  I4  11  31212   32511 1                               I     28
   1.920  I311 1 31 22 31323   11                               I     28
   1.800  I1     1232224 3321                                   I     26
   1.680  I   411   11 3 4                                       I     15
   1.560  I  124312 31417 1                                      I     30
   1.440  I211 3  2255 4212                                      I     30
   1.320  I3 31242114453 1                                       I     34
   1.200  I52 444211o311                                         I     34
   1.080  I4 2223534933 1                                        I     41
   0.960  I 325225582  2                                         I     36
   0.840  I2114343932 1  1                                       I     34
   0.720  I646485431 1 1                                         I     29
   0.600  I32443B2  1   1                                        I     31
   0.480  I9622732211                                            I     36
   0.360  I452923111  1                                          I     29
   0.240  I552 1111                                              I     16
   0.120  I22231 1                                               I     11
   0.0    I353111 1                                              I     15
              ---------------------------------------------------
  LOWER       0000000011111111222222223333333334444444455555555
   BIN        ........................................................
  EDGE        01234678902345689012456780123467890234568901245678
              02468024680246802468024680246802468024680246802468
```

Fig. A-3c Scatter plot of total transverse momentum on lepton side vs the same on the hadron side with the charged lepton present ε_ν = 100 Gev

```
--- KIOWA ---   SCATTER PLOT NUMBER     2
PLOT STATISTICS =     0     0     0   /     0  926    74   /     0     0     0
PROJECTION                                                                  PROJECTION
  ON X        9112132233222323222221222211111 111 1 1                         ON Y
  AXIS        6346937653292734111049013190343835063714874645343 7             AXIS
              ----------------------------------------------------
   1.960   I                                                          I        0
   1.920   I                                                          I        0
   1.880   I                                                          I        0
   1.840   I                                                          I        0
   1.800   I                                                          I        0
   1.760   I                                                          I        0
   1.720   I                                                          I        0
   1.680   I                                                          I        0
   1.640   I                                                          I        0
   1.600   I                                                          I        0
   1.560   I                                                          I        0
   1.520   I                                                          I        0
   1.480   I                                                          I        0
   1.440   I                                                          I        0
   1.400   I                                          1               I        1
   1.360   I                                                          I        0
   1.320   I                                                          I        0
   1.280   I                                                          I        0
   1.240   I                                                          I        0
   1.200   I                              1     1                      I        2
   1.160   I                        1 1             2                  I        4
   1.120   I                                                          I        0
   1.080   I                                                          I        0
   1.040   I                                                          I        0
   1.000   I              1         1   11                             I        4
   0.960   I                1                           1              I        2
   0.920   I1                             11       1                   I        4
   0.880   I               1       1     1                             I        3
   0.840   I      1  1                   1                             I        3
   0.800   I1        11   1        1                                   I        5
   0.760   I  1             11     1          1  1       1             I        7
   0.720   I       1 1       1  2 1    1      1                       I        8
   0.680   I1 1            1 11       11  1          1    1            I       10
   0.640   I2           1 2     2     1 1  1      1                   I       11
   0.600   I2  2     11  2 11  1 2      1       1                     I       15
   0.560   I       11111  1   1 1 3 21111 1  1                        I       19
   0.520   I2   12       1 12 2 11 1          1     1                I       16
   0.480   I1 1  1 113 11  1 111111   1 1     11 1                  I       22
   0.440   I41   1 1  131111 2 1  21  1   111           1            I       26
   0.400   I3   3111 111131 11112  2 2  1        1                   I       29
   0.360   I2  2 12113 42111   131 122        11   2                I       35
   0.320   I5 11  234321 3 21 1 1  22  112 1                         I       39
   0.280   I5   142 1 1  243 2  121 111             1       1        I       34
   0.240   I12   1 2321  1  12 12 1  1   1 2   2                 2I       28
   0.200   I72 2 42221 11 121 111111  1         1       1            I       37
   0.160   I412 322 14212  33 1121 111  111  1    1        1   1I       45
   0.120   I9  3411 23 312111212 1111112 1 211 1 2  12   1   1I       58
   0.080   I5322 3321111 5 3321   412  2 2 111 1            111I       56
   0.040   IA  311 11 12  231  1 1 14 111   11  1  1        11 I       42
   0.0     IV46B6AABD9BA9E7897785495689177735563647255354412 12I      342
              ----------------------------------------------------
  LOWER       0000000000000000000000000011111111111111111111111111
    BIN       ..........................................................
   EDGE       0001122233444556667788899000112223344455666778889 9
              0482604826048260482604826048260482604826048260482604826
```

Fig. A-4a Scatter plot of total transverse momentum on lepton side vs the same on the hadron side with the lepton momentum removed ϵ_ν = 25 Gev

--- KIOWA --- SCATTER PLOT NUMBER 4

PLOT STATISTICS = 0 0 0 / 0 990 10 / 0 0 0

```
PROJECTION                                                                  PROJECTION
  ON X        8323454444333333221122111 2  11  11                              ON Y
  AXIS        6260055351667310212746500399649610459647453410322 1              AXIS
              ---------------------------------------------------------
   3.920   I                                                          I      0
   3.840   I                                                          I      0
   3.760   I                                                          I      0
   3.680   I                                                          I      0
   3.600   I                                                          I      0
   3.520   I                                                          I      0
   3.440   I                                                          I      0
   3.360   I                                                          I      0
   3.280   I                                                          I      0
   3.200   I                                                          I      0
   3.120   I                                                          I      0
   3.040   I                                                          I      0
   2.960   I                                                          I      0
   2.880   I                                                          I      0
   2.800   I                                                          I      0
   2.720   I                                                          I      0
   2.640   I                                                          I      0
   2.560   I                                                          I      0
   2.480   I                                                          I      0
   2.400   I                                                          I      0
   2.320   I                                                          I      0
   2.240   I                                                          I      0
   2.160   I                                                          I      0
   2.080   I                                                          I      0
   2.000   I                                                          I      0
   1.920   I                                                          I      0
   1.840   I                                                          I      0
   1.760   I                                                          I      0
   1.680   I                                                          I      0
   1.600   I                        1                                 I      1
   1.520   I                                                          I      0
   1.440   I                                                          I      0
   1.360   I                              1                           I      1
   1.280   I         1                                                I      1
   1.200   I            1   1                                         I      2
   1.120   I             111        1                                 I      4
   1.040   I1  1           2                                          I      4
   0.960   I  1   11  1122     11                                     I     11
   0.880   I1    11 1     3   1  1         1          1               I     11
   0.800   I    1 2    121     11   1  1                              I     11
   0.720   I  1221 221 21 11 2  1 1                                   I     20
   0.640   I111  1412121 1 221111   2    1                            I     28
   0.560   I22  141222 222 21  111 1 1         1                      I     30
   0.480   I5 2214671111322211    1111  1                             I     47
   0.400   I11112524255444441 2 34   1  12   1                      1I     65
   0.320   I722734464711 321221 21  11221   1    11                  I     72
   0.240   I833444A366236 2331 2261 1421    12   1    1              I     95
   0.160   IC41266138165421143134122 3 231   2 1                  1 I     96
   0.080   IH553822513424 23 3 3133 11  21112122112 1                I     98
   0.0     IVF98CNB9FEFB7GAC8765776659438A657412831725241 321 I    373
              ---------------------------------------------------------
LOWER         0000000000000111111111111222222222222222333333333333
  BIN         ..................................................
 EDGE         0012344567889012234566789001234456788901223456678 9
              0864208642086420864208642086420864208642086420864 2
```

Fig A-4b Scatter plot of total transverse momentum on lepton side vs the same on the hadron side with the lepton momentum removed ε_ν = 50 Gev

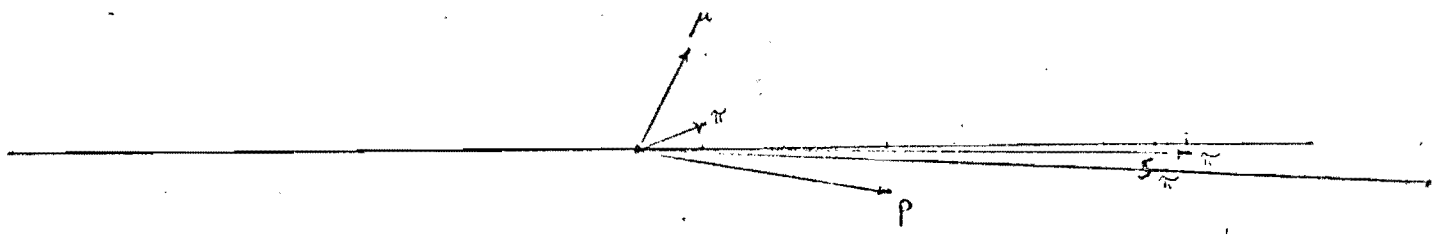
```
--- KIDWA ---   SCATTER PLOT NUMBER    6
PLOT STATISTICS =     0     0     0   /    0  935    3   /    0    0    0
PROJECTION                                                                PROJECTION
  ON X      6435554443322222211121 11 11          1                        ON Y
  AXIS      669155403931963554940359326315978549571415410C0221             AXIS
            ----------------------------------------------------
   5.880  I                                                          I      0
   5.760  I                                                          I      0
   5.640  I                                                          I      0
   5.520  I                                                          I      0
   5.400  I                                                          I      0
   5.280  I                                                          I      0
   5.160  I                                                          I      0
   5.040  I                                                          I      0
   4.920  I                                                          I      0
   4.800  I                                                          I      0
   4.680  I                                                          I      0
   4.560  I                                                          I      0
   4.440  I                                                          I      0
   4.320  I                                                          I      0
   4.200  I                                                          I      0
   4.080  I                                                          I      0
   3.960  I                                                          I      0
   3.840  I                                                          I      0
   3.720  I                                                          I      0
   3.600  I                                                          I      0
   3.480  I                                                          I      0
   3.360  I                                                          I      0
   3.240  I                                                          I      0
   3.120  I                                                          I      0
   3.000  I                                                          I      0
   2.880  I                                                          I      0
   2.760  I                                                          I      0
   2.640  I                                                          I      0
   2.520  I                                                          I      0
   2.400  I                                                          I      0
   2.280  I                                                          I      0
   2.160  I                                                          I      0
   2.040  I                                                          I      0
   1.920  I                                                          I      0
   1.800  I                                                          I      0
   1.680  I                1                                         I      1
   1.560  I                                                          I      0
   1.440  I                                                          I      0
   1.320  I                1       1  1                              I      3
   1.200  I       1  1  2 1                                           I      5
   1.080  I        212  1                                             I      6
   0.960  I 1 122 12113                                               I     14
   0.840  I 1 3233 224132 11     11 1                                 I     31
   0.720  I2123213 233 1 1 1  1         1   1  1                      I     29
   0.600  I2 41263433211222 11 11  1  1 11         1                  I     47
   0.480  I174443A43442 131. 12 22 21 1  1     1                      I     69
   0.360  I7537593A37271213625 2 2311111 1       11                   I    103
   0.240  ICA4B8555431664343222 2  3 11431  2 1   1  11     1  I      122
   0.120  IB44475344542 5223 1  2441111112 22  11 111 11      1 I     100
   0.0    IVHIHNLDAJHC99DA9D8A75D44773552555552645921321   111I     399
            ----------------------------------------------------
  LOWER     000000000111111112222222233333333344444444555555555
   BIN      ....................................................
  EDGE      012346789023456890124567801234678902345689012456 78
            124680246802468024680246802468024680246802468024 68
```

Fig. A-4c Scatter plot of total transverse momentum on lepton side vs the same on the hadron side with the lepton momentum removed ϵ_ν = 100 Gev

Large Angle Events ($\theta_\mu \gtrsim 20°$), Get: $\left(\frac{S}{115}\right) \sim 7\%$ ($x \gtrsim .5$)

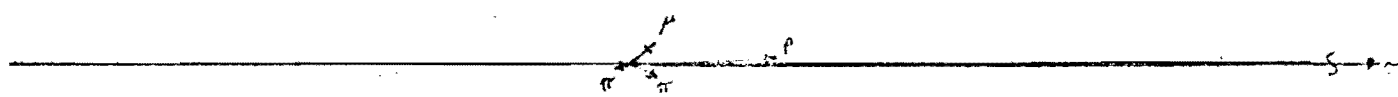
$\theta_\mu = 57°$ $E_\mu = 1.4$ GeV $E_p = 3.3$ GeV

$\theta_h = 2.8°$ $x = .7$ $y = .9$



$\theta_\mu = 45°$ $E_\mu = 0.3$ GeV $E_p = 2.0$ GeV $x = .1$

$\theta_h = 0.5°$ $y = 1$



$\theta_\mu = 28°$ $E_\mu = 3.8$ GeV $E_p = 7.8$ GeV $x = .6$

$\theta_h = 4.8°$ $y = .8$

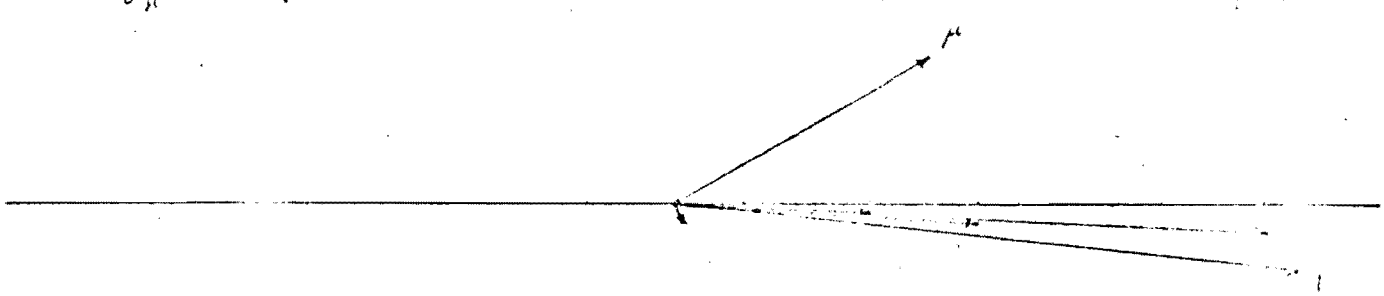
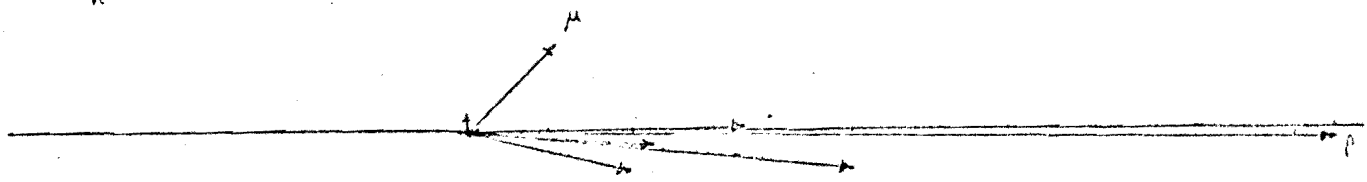


Fig.A5a Some typical momentum plots for events with x > 0.5

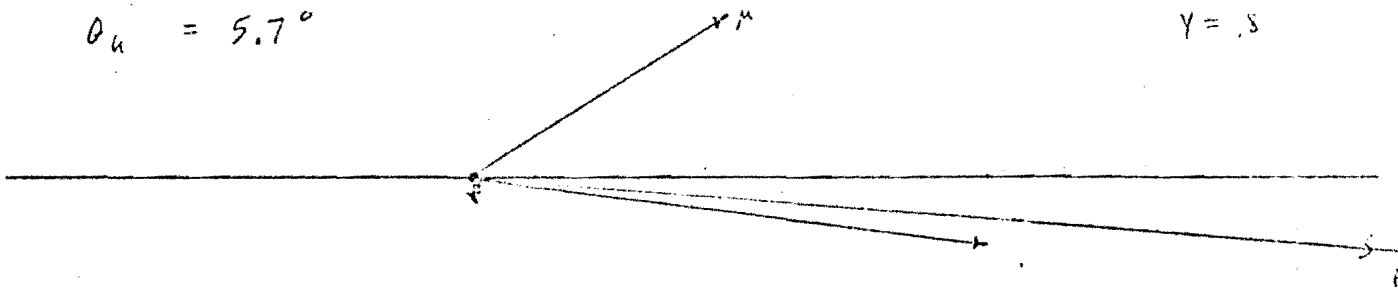
$\theta_\mu = 45°$ $E_\mu = 1.6$ GeV $E_p = 11$ GeV $X = .5$

$\theta_h = 2.7°$ $Y = .9$



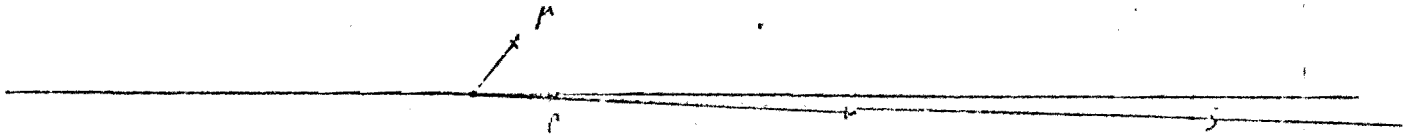
$\theta_\mu = 33.5°$ $E_\mu = 3.9$ GeV $E_p = 15$ GeV $X = .8$

$\theta_h = 5.7°$ $Y = .8$



$\theta_\mu = 52°$ $E_\mu = 0.9$ GeV $E_p = 1.4$ GeV ($P \cong 1.1$ GeV)

$\theta_h = 2°$ $X = .6$ $Y = 1$



$\theta_\mu = 20°$ $E_\mu = 7.5$ GeV $E_p = 16$ GeV $X = .7$

$\theta_h = 8°$ $Y = .7$

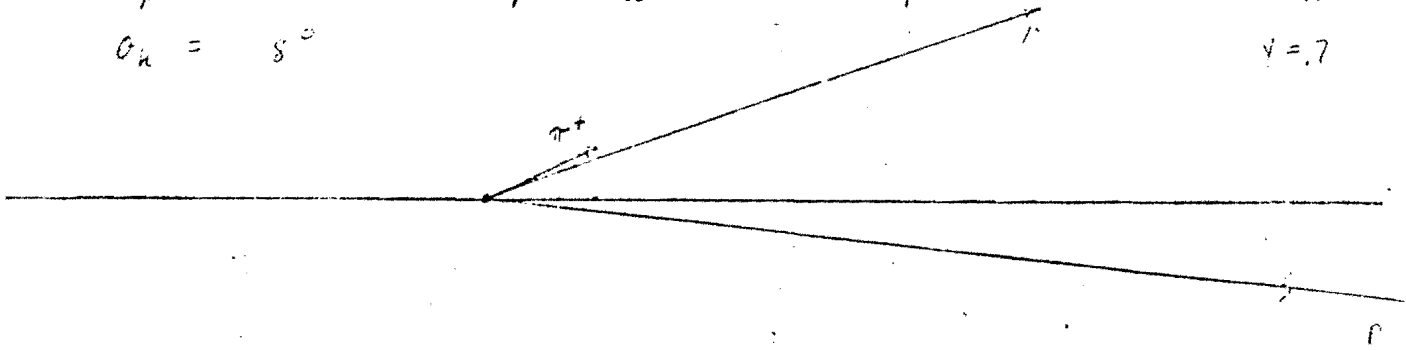
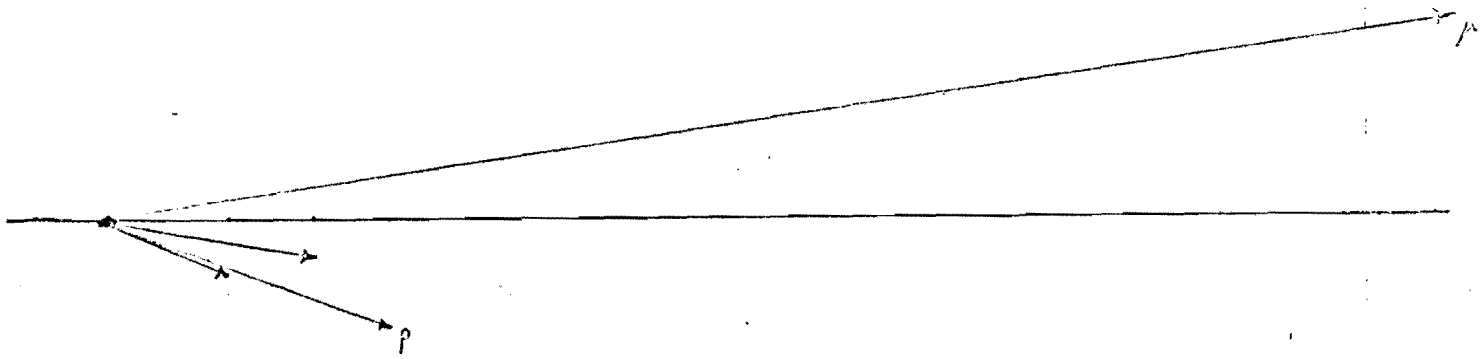


Fig A-5b

Small Angle Events ($\theta_\mu < 20°$) with $x \geq .5$ $\left(\frac{13}{115}\right) \sim 11\%$

$\theta_\mu = 8.7°$ $E_\mu = 17$ GeV $E_p = 4$ GeV $x = .7$

$\theta_h = 17.7°$ $y = .3$



$\theta_\mu = 8.5°$ $E_\mu = 16$ GeV $E_p = 3$ GeV $x = .5$

$\theta_h = 14°$ $y = .4$

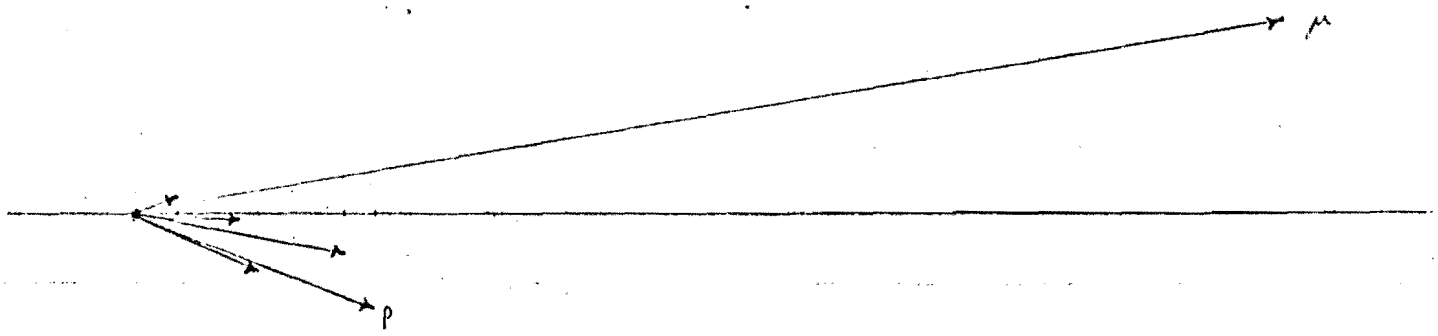


Fig. A-5c

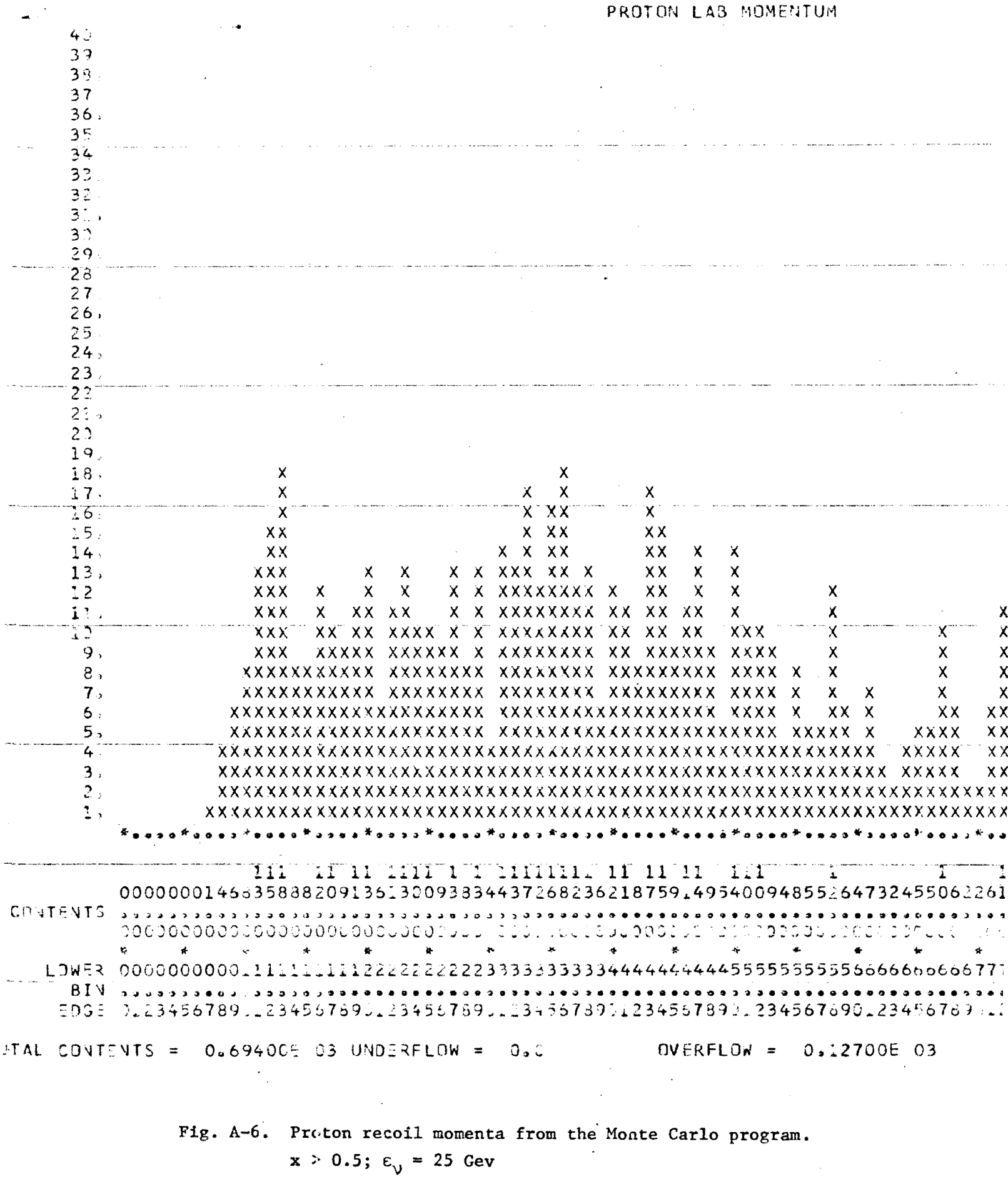


Fig. A-6. Proton recoil momenta from the Monte Carlo program.
$x > 0.5$; $\epsilon_\nu = 25$ Gev