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A MEASUREMENT OF THE ELECTROMAGNETIC RADIUS OF THE KAON

AND

THE DIFFERENCE BETWEEN THE PION AND KAON RADII

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A Proposal to the National Accelerator Laboratory

ABSTRACT

We propose an experiment to measure the mean square radius of the kaon to an accuracy of $\pm$ 0.1 fermis^2. The method consists of measuring the differential cross section $\frac{d\sigma}{dt}$ for small t by scattering a beam of 80 GeV/c charged kaons from the atomic electrons in liquid hydrogen. We propose a simultaneous measurement of the difference between the mean square radii of the pion and kaon. Because this difference is proportional to the ratio of the cross sections $\frac{d\sigma}{dt}$ of the pion and kaon, many of the systematic corrections which plague absolute cross section measurements will cancel. The accuracy of this measurement will then be limited only by statistics, and we estimate the error in our measurement of the difference in the mean square radii to be $\pm$ 0.03 fermis^2.

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I. Introduction

Past NAL summer studies have discussed the importance of measuring the electromagnetic radius of the pion and the kaon with pion and kaon beams which will become available at NAL.[1),2),3),4)] Because of the non-availability of pion and kaon targets, such experiments must consist of scattering a high energy pion or kaon beam from the atomic electrons of a material (hydrogen). The kaon is about three times as massive as the pion and because of this fact, the kaon can transfer far less momentum to an electron than can a pion at the same incident energy. Since the accuracy to which one can measure the meson radius depends only on the maximum recoil momentum of the electron, one must use kaon beams at momenta which will be available only at NAL in the near future. We propose to measure the mean-square radius of the kaon to $\pm 0.1 f^2$ using a beam of K^+ and K^- particles at 80 GeV/c momentum.

We also propose to measure the difference between the mean square kaon radius and mean square pion radius. Since any unseparated kaon beam contains a factor of 10 more pions than kaons and since the difference in the mean squares of the kaon and pion radii are directly related to the ratio of the K-e to π-e scattering cross sections, we feel that such a measurement would require very little additional effort beyond the effort required to measure the kaon radius absolutely. In fact, since one is measuring a ratio of cross sections, many systematic corrections which enter into an absolute cross section measurement cancel and to a first approximation, the error in the ratio of cross

sections is purely statistical. This part of the experiment is discussed in section VII. The bulk of this proposal will discuss an absolute measurement of the kaon radius. Most of our conclusions regarding the kaon radius are equally applicable to a measurement of the pion radius. We feel that our experiment is not optimum for an absolute measurement of the pion radius when compared with NAL proposals which propose specifically to measure the pion radius. However, we feel that a measurement of the difference in the mean square radii of these two mesons is unique to this proposal and we discuss it later in this text.

The cross section for the scattering of a point kaon from an electron is given by the Bhabha formula

$$\left(\frac{d\sigma}{dE}\right)_{\text{point } K} = \frac{2\pi m_e r_0^2}{E^2}\left(1 - \frac{E}{E_m}\right) \tag{1}$$

where E = energy of the recoiling electron

m_e = mass of the electron

r_o = classical electron radius

and E_m = maximum recoil energy of the electron and is given by the expression

$$E_m \sim \frac{E_K}{1 + \frac{m_K^2}{2m_e E_K}} \tag{2}$$

where m_K and E_K are the kaon mass and incident energy, respectively. The four-momentum transfer to the electron is $t = 2m_e E$ so that equation (1) may be written as:

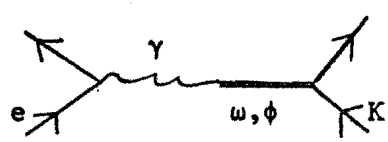
$$\left(\frac{d\sigma}{dt}\right)_{\text{point K}} = \frac{\pi r_0^2}{E^2}\left(1 - \frac{E}{E_m}\right) \tag{3}$$

Furthermore, if one assumes the kaon to have a charge structure, then

$$\left(\frac{d\sigma}{dt}\right)_{\text{measured}} = \left(\frac{d\sigma}{dt}\right)_{\text{point K}} F_K^2(t) \tag{4}$$

$F_K(t)$ is the charge form factor of the kaon. It is the purpose of this experiment to measure $dF_K^2(t)/dt$ at $t=0$, i.e., the slope of the square of the form factor for very small four-momentum transfers to the electron. For small t, $F_K = 1 - \frac{t}{6}\langle r_K^2\rangle$ where $\langle r_K^2\rangle$ is the mean-square radius of the kaon.

Assuming a vector dominance model one can estimate $\langle r_K^2\rangle$ in the following manner. If one assumes that the K-e scattering is mediated by a vector meson (ϕ or ω) as indicated by the graph



,

then

$$F_K(t) = \frac{m_{\omega,\phi}^2}{t + m_{\omega,\phi}^2} .$$

For small t,

$$F_K(t) = 1 - \frac{t}{m_{\omega,\phi}^2}$$

and

$$\langle r_K^2\rangle = \frac{6}{m_{\phi,\omega}^2}$$

or in units of fermis,

$$\langle r_K^2 \rangle = \frac{.23}{m_{\phi,\omega}^2} f^2 \tag{5}$$

where $m_{\phi,\omega}^2$ is in units of $(\mathrm{GeV})^2$. The experimental approach is to measure the differential cross section $d\sigma/dt$ and form the ratio

$$\frac{(d\sigma/dt)_{\text{measured}}}{(d\sigma/dt)_{\text{point K}}} = F_K^2(t) \tag{6}$$

where

$$\left(\frac{d\sigma}{dt}\right)_{\text{point K}}$$

is calculated. This ratio is then plotted versus t.

II. Experimental Considerations

Figure 1 shows the ratio

$$\frac{(d\sigma/dt)_{\text{measured}}}{(d\sigma/dt)_{\text{point K}}} = F_K^2(t) \quad \text{versus } t$$

assuming the kaon radius to be dominated by the ω and ϕ and also a curve assuming the kaon to have the same radius as the nucleon (0.81f). The data points shown are plotted on the basis of 10^5 K-e scattering events for the range of recoil electron energies between 10 and 20 GeV; i.e., between $E_{max}/2$ and E_{max} for the recoiling electron. The corresponding t range is between 0.01 and 0.02 $(\mathrm{GeV}/c)^2$. The assumed beam energy is 80 GeV. The errors assigned to the points are statistical only.

It is clear that in order to obtain the mean-square kaon radius to the desired precision one must make systematic corrections to the data

which must be well understood. The curve which is fit to the data is

$$\frac{(d\sigma/dt)_{measured}}{(d\sigma/dt)_{point\ K}} = 1 - \frac{\langle r_K^2 \rangle}{3} t \equiv y(t) \tag{7}$$

Since the quantity of interest is $\langle r_K^2 \rangle/3$, a question which naturally arises is whether one must measure an absolute cross section to some precision and assume that the curve passes through the point $y = 1.0$ at $t = 0$ or whether one can obtain the slope of the curve from the raw data with only a very crude knowledge of the absolute normalization. A figure of merit related to the question of how well the absolute normalization must be known can be obtained by representing the curve to be fitted by the expression

$y(t) = (1 \pm \delta\varepsilon)(1 - \frac{\langle r_K^2 \rangle}{3} t)$ where $\delta\varepsilon$ is a normalization error and assumed to be momentum independent. This assumption is obviously an oversimplification because $\delta\varepsilon$ reflects uncertainties in the radiative correction calculation (discussed in section VI), wire chamber efficiencies, etc. and some of these uncertainties may be momentum dependent. Using a maximum likelihood technique and assuming 10^5 K-e events over the t range of interest, we compute the error $\delta\langle r_K^2 \rangle$ vs. $\delta\varepsilon$ arising from <u>both</u> systematic and statistical errors and present our results in the table below.

$\delta\varepsilon(\%)$	$\delta\langle r_K^2 \rangle$(fermis^2) for 10^5 K-e events
0	0.03
0.5	0.06
1.0	0.09
5.0	0.17
10.0	0.19

Assuming one knows $\delta\varepsilon$ to 1%, $\delta\langle r_K^2\rangle = 0.10f^2$ and this precision is adequate to separate a nucleon-like radius (0.81f) from a point-kaon by seven standard deviations.

At the lower part of Figure 1 is also plotted the maximum t as a function of incident kaon momentum. It is clear that one should go to as high an incident energy as possible, especially since $\delta\langle r_K^2\rangle \propto \frac{1}{E_{max}\sqrt{N}}$ and $N \propto \frac{1}{E_{max}}$ assuming a constant electron target thickness and beam intensity so that $\delta\langle r_K^2\rangle \propto \frac{1}{\sqrt{E_{max}}}$. However, one quickly reaches a point of diminished return as the kaon energy is increased. The experimental approach is to measure the scattered kaon and recoil electron energy as well as the incident kaon energy to high precision. The signature for an event of interest is an energy balance between the incident kaon and scattered kaon and electron, i.e.,

$$E_K^{in} + m_e = E_K^{out} + E_e \tag{8}$$

and this balance must be assured to 1% in order to eliminate the background resulting from K^+-p collisions. Measuring the final momenta to $\pm 0.5\%$ is required to reject the background and this becomes increasingly difficult as the incident energy is raised. We have chosen 80 GeV/c incident kaon momentum. A beam of this energy (15 mr. beam) will be available in the meson area and represents a good compromise between sufficiently large t_{max} and adequate momentum measurement capability.

III. Apparatus

We propose to scatter 80 GeV/c positive and negative kaons from the medium energy high resolution beam (15 mr. beam) in the meson area from the electrons in

a liquid hydrogen target. The momenta of both final state particles will be measured in the range $10 < p_e < 20$ GeV/c and $60 < p_K < 70$ GeV/c. The directions of the particles into and out of the hydrogen target will be measured by Charpak chambers. A wire chamber spectrometer will measure final state momenta well enough to allow reconstruction of the energy balance to 1%. The energy balance together with an electron detecting shower counter will allow us to reject the strong interaction backgrounds.

We now give a detailed description of the apparatus as it will be used for the K^+e^- scattering measurements. The apparatus is shown in Figure 2.

A. Beam: We plan to use the NAL medium energy high resolution beam, and hope to have $10^5 K^+$ and $10^5 K^-$ per pulse[5] at 80 GeV/c with $\Delta p/p \approx 0.25\%$. This small momentum spread will eliminate the need for a measurement of the incident momentum for each kaon. In order to obtain adequate pion rejection, the beam must contain either 2 DISC counters[6] or an assembly of three threshold counters. We will investigate the use of threshold counters versus DISC counters to achieve pion rejection factors of 10^5 for the pion contamination in the beam. An addendum to this proposal will be submitted to NAL after this study has been completed.

We will use proportional chambers P_1 and P_2 to measure the incident particle direction to 0.1 mr. and to reject more than one incident particle.

B. Target: The target is a 30 cm. long, 5 cm. diameter cylinder of liquid hydrogen (2.1gm/cm^2, 0.03 radiation lengths). Hydrogen was chosen since it has the largest number of electrons/radiation length of any material and because a pure proton target will simplify the strong interaction background

problems. Filling the target with deuterium will provide an additional check of strong interaction processes.

The total amount of material through which the electron passes must be kept to a few hundreths of a radiation length, and every attempt must be made to avoid introducing extra target material at undesirable locations (10 m. of air is 1 gm/cm^2 or 0.03 radiation lengths). Consequently, we intend to put the entire electron flight path in a helium bag (not shown).

C. Anticoincidence Counter: Figure 2 shows the target flanked by pairs of counters A which might be used to reject the strong interaction background processes when one or more particles have a large angle. Monte Carlo calculations indicate that about 95% of the strong interaction background may be eliminated this way (see background section). Unfortunately, the probability for an 80 GeV/c charged particle to produce large angle knock-on electrons which would get out of the target (2.5 cm. of liquid hydrogen) is about 20% for each $1 gm/cm^2$ of target material encountered.[7] The anticoincidence counter, if used, must be a sandwich of counters and absorbers which is not tripped by these electrons and is still effective against hadrons. Our background estimates indicate that such an anticoincidence counter is probably not necessary but we will continue to study its usefulness.

D. Final State Measurements: Proportional chambers P_3 and P_4 will provide a measurement of the particle directions to 0.05 mr. Since the kaon and electron scattering angles are typically 1 mr. and 4 mr. respectively and we only measure the incident direction to 0.1 mr., our knowledge of the scattering angles will only suffice for a rough check of event coplanarity. The main purpose of this measurement is to provide good momentum resolution in the spectrometer

which follows; 0.05 mr. will contribute a 0.14% error to the measurement of an 80 GeV/c particle. The information from P_3 and P_4 will also be used to reject events which have one or more than two charged particles in the final state.

E. Magnet: A total field of 100 kg-m. is required for the spectrometer magnet. The bending may be shared by two 2.0 m, 50 kg. magnets. The horizontal aperture necessary for the second magnet is 1.0 m. if the magnets are offset to take advantage of the great difference in the kaon and electron momenta. The horizontal aperture of the first magnet need only be about 0.5 m. In order to accept a maximum 10 mr. scattering angle, both magnets must have a vertical aperture of 30 cm. We recognize that a 100 kg-meter magnet is a very large and costly magnet and that the entire question of the kind of magnet that will do the job and the cost of such a magnet must be studied by the authors of this proposal. We plan to study this problem in detail and submit a report in the form of a proposal addendum.

F. Spectrometer: The wire chamber spectrometer (W_1 - W_6) shown in Figure 2 will provide momentum resolution to 0.3% in the worst case. The electron arm is purposely closer to the magnet to avoid the necessity of very large chambers. For measurements of the K^-e^- scattering cross section, the hadron arm will be moved to the other side of the spectrometer.

G. Trigger: Scintillation counters S_1 and a large pulse in a lead-plastic shower counter (SH) will indicate detection of an electron. We expect the shower counter to provide about 50:1 electron:hadron rejection and to give a rough measure of the observed electron energy spectrum $(\frac{dN}{dP_e} \propto \frac{d\sigma}{dt})$ as a check on the experiment. Scintillation counters

S_2 will detect charged particles in the hadron arm. An iron wall followed by an anticoincidence counter ($\bar{S}_\mu$) will reject any muon-induced reactions (important for the π-e part of the experiment). The trigger for K^+e^- scattering will be $(\check{C})\cdot(S_1\cdot SH)\cdot(S_2\cdot\bar{S}_\mu)$. Data on π^+e^- scattering at the same four-momentum transfer ($t = 2m_eP_e$) will be taken simultaneously by requiring an appropriate pion signal from the beam Cerenkov counters. The beam will contain mostly π^+, K^+, μ^+ and P. Since the maximum momentum an 80 GeV/c proton can give to an electron is 6.67 GeV/c, the Cerenkov counters in the beam need only reject pions and muons for a K-e event and kaons and muons for a π-e event.

IV. Rates

To calculate the rates, we assume a point kaon. Since we accept all K-e scatters for which the recoil electron has an energy between $E_{max}/2$ and E_{max}, our cross section is given by:

$$\sigma = \int_{E_{max}/2}^{E_{max}} \frac{d\sigma}{dE}\, dE = \frac{2\pi m_e r_0^2}{E_{max}} (1 - \ln 2)$$

Assuming a flux of 10^5 K^+ per pulse and 900 pulses per hour, a liquid hydrogen target 30 cm. in length and E_{max} to be 20 GeV our cross section equals 3.9μb. Our rate equals 4.4×10^2 events per hour. If the beam is reduced in momentum to 40 GeV/c and the magnetic field of the dipole magnet is reduced accordingly, then E_{max} becomes 5.7 GeV and our cross section becomes $\sigma = 13.7\mu b$. Our rate at this reduced energy is 1.5×10^3 per hour. We propose the following assignments of running time:

Beam	Momentum(GeV/c)	Number of hours	Total Number of Events
K^+	80	300	10^5
K^-	80	300	10^5
K^+	40	100	10^5
K^+, K^-	80, 40	100	(Background studies)

V. Backgrounds

The strong interaction background from Kp and πp interactions is one of the most difficult problems for this experiment. In order to produce a 1% measurement of the 1 μb electromagnetic scattering in the presence of the 1000 μb strong interactions we should have a rejection of at least 10^{-6} for any one channel. The problem is aggravated by our ignorance of the mechanisms for the various reactions at 80 GeV/c, since the rejection for a particular channel may depend critically on the differentials of the cross section.

Since we measure the magnitude of all momenta to a few tenths of a percent, the requirement of 99% elasticity for an event is the most important factor in our ability to reject the strong interaction backgrounds since they tend to have high final state multiplicities and many particles share the energy. Below, we give a detailed discussion of the backgrounds for the K^+e^- scattering measurements. The only important initial state is K^+p.

Two body final states with a negative particle are excluded by charge conservation.

For three body final states, charge conservation requires that the third particle be neutral. A likely candidate is $K^+p \rightarrow K^+p\pi^0$ followed

by $\pi^0 \to \gamma e^+ e^-$ or $\pi^0 \to \gamma\gamma$, with γ conversion in the target. In a thin target, these mechanisms have comparable probability. We have not been able to find a measurement of $\sigma_{tot}(K^+p \to Kp\pi^0)$ above 5 GeV/c and we take 1mb as an upper limit. Our background estimate is based on a Monte Carlo calculation using only phase space at 80 GeV/c. The results are presented in Table 1.

As mentioned in the apparatus section, we have considered the use of an anticoincidence counter around the target to reject large angle hadrons. We do not believe it necessary to use such a counter. However, if used, the effect of this counter would be to reduce trigger rates and background contamination in the final sample of data by a factor of about 20.

There are many possible four body final states. We have considered $K^+p \to K^+p\pi^+\pi^-$ in detail using a Monte Carlo calculation based on the results of Berlinghieri et al.[8] The total cross section was found to be 0.5 mb at 12.7 GeV/c and $d\sigma/dt$ was exponential in the momentum transfer to the kaon. The results of this calculation are found in Table 1.

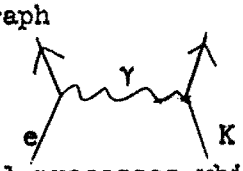
We have not yet considered in detail the effects of higher multiplicity final states. From those channels considered it appears that we can accept contributions from many individual channels since none of the possible channels are expected to be more important than those we have considered.

We have also included the background from the beam in Table 1, i.e., the decay $K^+ \to \pi^+ + \pi^0$ with a subsequent decay of the π^0 to simulate a Ke event. Both the Dalitz decay and the 2γ decay followed by conversion are considered.

We believe that a good test of the strong interaction background in our K^+e^- scattering data can be obtained by taking some data with every magnet in the beam and experiment reversed. We will then have a sample of events which simulate $K^-e^+ \to K^-e^+$. Since there are no positrons in the target and the K^+p and K^-p cross sections are approximately equal, we will have a measure of the strong interaction background.

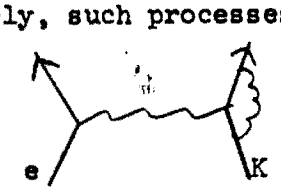
VI. Radiative Corrections

While one wishes to measure the single photon exchange process indicated by the graph



(OPE) ,

there are additional processes which contribute to the same final state, namely, such processes as



,

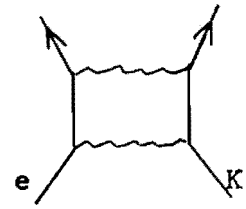
, etc.

These additional processes are referred to as "elastic processes".[9] If $\left(\frac{d\sigma}{dt}\right)_{OPE}$ is the process of interest, then the elastic processes contribute a fractional amount $\frac{\alpha}{\pi}\delta_1$ to the measured cross section or

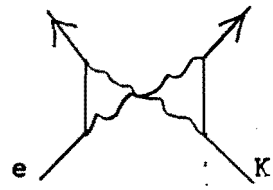
$$\left(\frac{d\sigma}{dt}\right)_{meas.} = \left(\frac{d\sigma}{dt}\right)_{OPE} \left[1 + \frac{\alpha}{\pi}\,\delta_1\right].$$

There are further processes which contribute to $\left(\frac{d\sigma}{dt}\right)_{meas.}$ and they are taken in order below.

The class of processes which involve the exchange of two photons, e.g.,

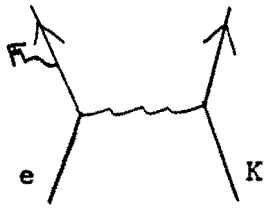


,



, etc.

contribute a fractional amount $\frac{\alpha}{\pi}\,\delta_2$ to the measured cross section. The processes which involve the emission of a real photon as indicated by the diagrams



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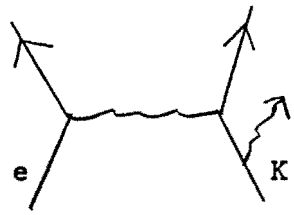

, etc.

contribute a fractional amount $\frac{\alpha}{\pi}\,\delta_3$ to the measured cross section. Although this process is distinguishable from both the elastic and two-photon exchange processes, the emitted photon is not detected and the events which would otherwise satisfy the energy balance criterion but which radiate a photon and hence do not satisfy the energy balance make this correction necessary.

There are two additional radiative corrections which must be applied. The first correction is due primarily to real bremsstrahlung emitted as the recoil electron passes through the H_2 target. The maximum energy which an electron of momentum E_{max} = 20 GeV can radiate and still fall within the energy balance criterion is clearly the momentum resolution of the electron arm or $\Delta p = 20 \times 0.3\% = 60$ MeV/c. The fraction of events that will be lost because of radiation of photons of energy greater than 60 MeV in a liquid hydrogen target of 30 cm. length is given by the expression

$$\delta_4 = 0.015 \int_{60\ \mathrm{MeV/c}}^{20\ \mathrm{GeV/c}} dk/k$$

where 0.015 is the fractional radiation length of 15 cm. of liquid hydrogen. This integral thus yields $\delta_4 = -0.087$.

Finally, a radiative correction must be made for ionization losses as the particles of interest traverse the hydrogen target. While the average losses are small ($\sim$ 1 MeV/gram), sometimes the kaon or recoil electron will transfer sufficient energy to an electron in a single collision to result in the event not satisfying the energy balance. This is a small effect and can be calculated exactly.

Taking the above corrections into account, the measured cross section is given by the expression

$$\left(\frac{d\sigma}{dt}\right)_{\text{measured}} = \left(\frac{d\sigma}{dt}\right)_{\text{OPE}} \left[1 + \frac{\alpha}{\pi}(\delta_1 + \delta_2 + \delta_3) + \delta_4\right].$$

We have estimated δ_1, δ_2, and δ_3 for K^+-e scattering at the maximum t of 0.02 $(\text{GeV/c})^2$ to be -59, +23 and +21 respectively. These numbers are to be regarded as very approximate since the prescription of Kahane[9] is valid for π-e scattering and some terms containing m_π^2 have been dropped. It is nevertheless useful to estimate our correction from this assumption and we estimate

$$\left(\frac{d\sigma}{dt}\right)_{\text{meas.}} = \left(\frac{d\sigma}{dt}\right)_{\text{OPE}} [1 - 0.12] = 0.88\left(\frac{d\sigma}{dt}\right)_{\text{OPE}} .$$

Assuming our knowledge of the experimental parameters to be exact, Professor Anthony Hearn of the University of Utah informs us that the radiative correction to our experiment can be calculated to 1/2% of itself with a reasonable effort. In fact, he has agreed to perform the calculations

in the event this experiment is approved. Although it is probably adequate to calculate the corrections to order α^3, the necessity for a higher-order calculation will be investigated.

VII. A Measurement of the Difference Between the Mean Square Kaon and Pion Radii

The apparatus will simultaneously accept all π-e scattering events for which the recoil momentum of the electron is between 10 and 20 GeV/c. One can then directly obtain the ratio $(\frac{d\sigma}{dt})_K/(\frac{d\sigma}{dt})_\pi$ for $0.01 < t < 0.02$ $(\mathrm{GeV/c})^2$. This ratio can then be related to the difference in the mean square radii of the kaon and pion by the expression

$$\frac{(d\sigma/dt)_K}{(d\sigma/dt)_\pi} \simeq \frac{F_K^2(t)}{F_\pi^2(t)} \simeq \frac{1 - \frac{t}{3}\langle r_K^2\rangle}{1 - \frac{t}{3}\langle r_\pi^2\rangle} \simeq 1 - \frac{t}{3}[\langle r_K^2\rangle - \langle r_\pi^2\rangle]$$

The π-e cross section over the range of t accepted is 10 μb while the K-e cross section is 3.9 μb. The pion beam intensity is a factor of 10 greater than the kaon intensity. We therefore expect 25 times as many π-e events as K-e events in our experiment. Assuming the error in the ratio $(\frac{d\sigma}{dt})_K/(\frac{d\sigma}{dt})_\pi$ to be the statistical error associated with 10^5 K-e events, we have

$$\delta[\langle r_K^2\rangle - \langle r_\pi^2\rangle] = \frac{3}{t}(0.3\%)$$

Taking the root-mean-square value of t to equal 0.0128 $(\mathrm{GeV/c})^2 = 0.32$ fermis^{-2}, we estimate the error in the difference of the mean square radii to be

$$\delta[\langle r_K^2\rangle - \langle r_\pi^2\rangle] = 0.03\mathrm{f}^2$$

Assuming the radius of the pion as given by the ρ dominance value or measured in another experiment to be 0.63 fermis, and the radius of the kaon to be the ϕ dominance value of 0.45 fermis, we can separate these two values by six standard deviations. If the kaon is dominated by the ω, then the radii are equal, and the ratio of cross sections is unity for all t values.

Our assumption that all systematic corrections will cancel is certainly valid for uncertainties in the length of the hydrogen target and counter and spark chamber inefficiencies. It is not valid for corrections which are different for π-e and K-e scattering such as the radiative corrections and background subtraction. This kind of correction must obviously be well understood if we are to achieve the desired accuracy in the difference of the mean square radii.

VIII. Apparatus Requirements

The following table lists the apparatus to be provided by the University of Pittsburgh and that required of NAL.

To be provided by University of Pittsburgh	Required of NAL
Proportional counters and associated readout system	Liquid hydrogen target capable of being filled with liquid deuterium
Wire spark chambers and associated readout system	100 kilogauss-meter magnet (presently under study)
Scintillation and shower counters	Fast electronics for trigger logic
PDP-15 Computer for on-line data acquisition	Camac modules and crates
Interface for PDP-15 Computer	Trailer for housing electronics

In addition to the above apparatus, NAL and/or the University of Pittsburgh must provide Cerenkov counters for tagging kaons and pions in the beam. The type of counter to be used, whether threshold or differential, has not been decided upon as yet. This matter is at present under study.

References

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BACKGROUNDS

Process	Production Probability in Target/Kaon	Trigger Probability/Kaon	Probability of Contributing to Data after Energy Balance Cut
$K^+e^- \rightarrow K^+e^-$	5×10^{-6}	5×10^{-6}	5×10^{-6}
$K^+p \rightarrow K^+p\pi^+\pi^-$	6.4×10^{-4}	7.4×10^{-8}	1.3×10^{-10}
$K^+p \rightarrow K^+p\pi^o$, $\pi^o \rightarrow e$	1.28×10^{-3}	3.8×10^{-9}	7.1×10^{-13}
$K^+ \rightarrow \pi^+\pi^o$, $\pi^o \rightarrow e$	5.0×10^{-4}	8.8×10^{-9}	2.4×10^{-11}

Table 1

Figure Captions

Figure 1. The ratio of the measured cross section to the kaon point cross section for different kaon radii. The ratios are plotted as a function of t for an 80 GeV/c incident kaon beam. Points from a hypothetical sample of 10^5 events are shown. Also shown on the lower part of the figure is a nomograph giving the maximum possible four-momentum transfer (t_{max}) for different incident kaon momenta.

Figure 2. The apparatus as it will appear for the K^+e^- and π^+e^- part of the experiment. The trigger will be $(\check{C}_{\pi \text{ or } K})\cdot(S_1\cdot SH)\cdot(S_2\cdot\bar{S}_\mu)$. To measure the K^-e^+ and π^-e^+ cross sections, the hadron and electron arms will appear on the same side of the beam line. A sample magnet design with two magnets whose centers are offset from the beam line is shown.

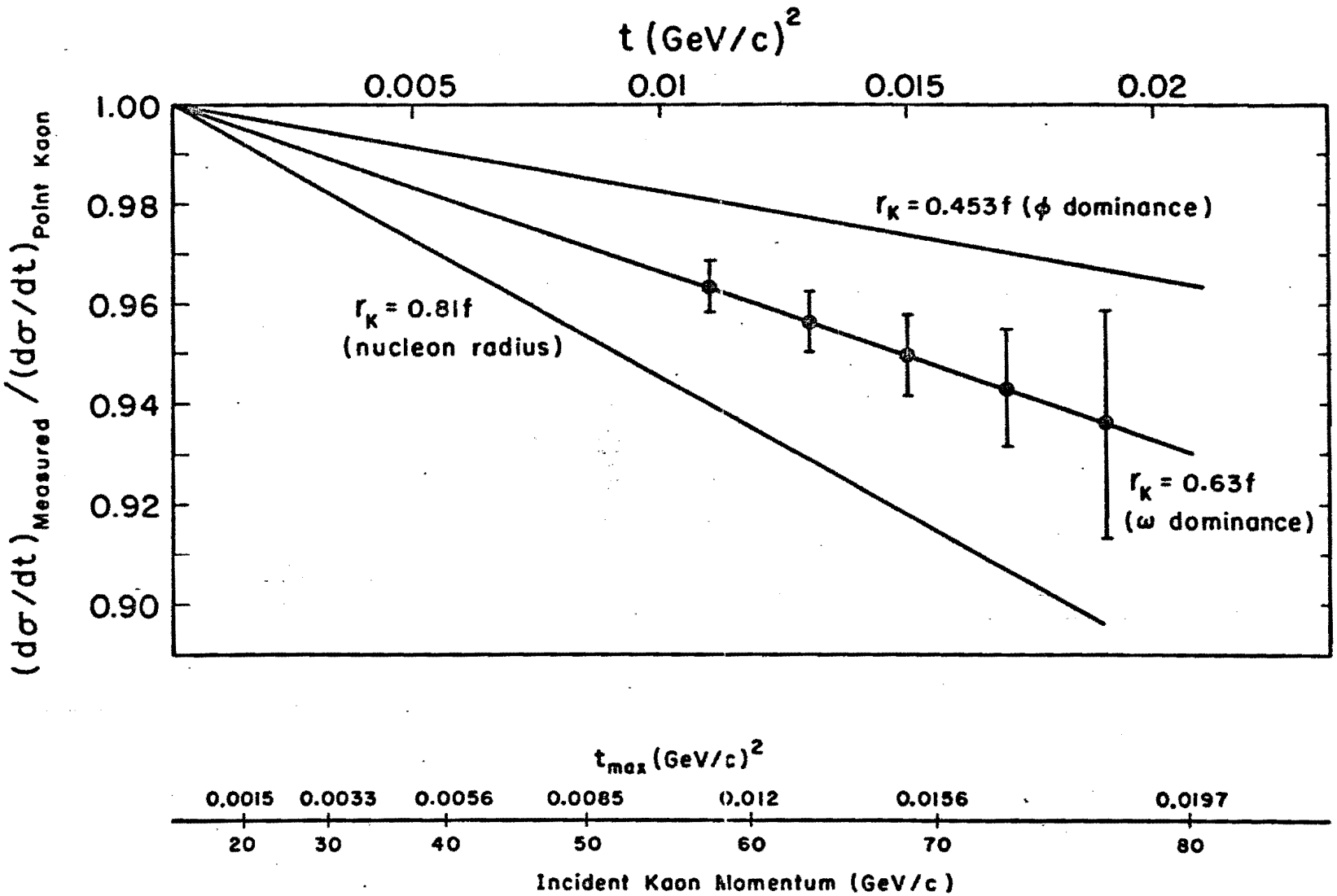
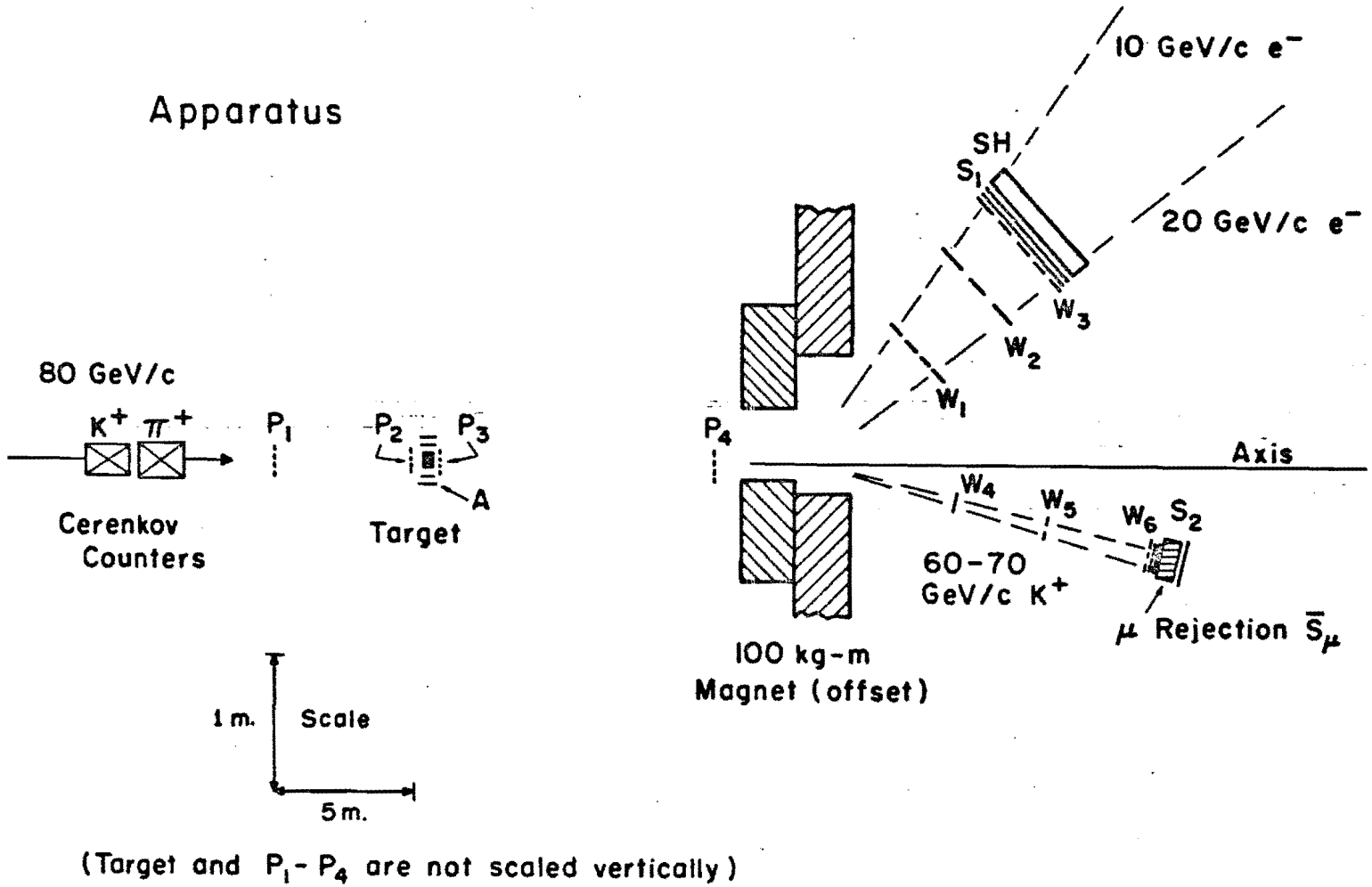

t (GeV/c)^2
0.005
0.01
0.015
0.02
1.00
0.98
0.96
0.94
0.92
0.90
$(d\sigma/dt)_{Measured}/(d\sigma/dt)_{Point\ Kaon}$
r_K = 0.453f (ϕ dominance)
r_K = 0.81f
(nucleon radius)
r_K = 0.63f
(ω dominance)
t_{max} (GeV/c)^2
0.0015
0.0033
0.0056
0.0085
0.012
0.0156
0.0197
20
30
40
50
60
70
80
Incident Kaon Momentum (GeV/c)

Figure 1



(Target and $P_1 - P_4$ are not scaled vertically)

Figure 2