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PROPOSAL TO INVESTIGATE REGENERATION OF NEUTRAL

K - MESONS AT VERY HIGH ENERGIES

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1. Introduction

The regeneration amplitude of K_s mesons is, as is well known, proportional to $[f(0) - \bar{f}(0)]$, where f, \bar{f} are the forward scattering amplitudes for K, \bar{K} on the target used for regeneration. The Pomeranchuk Theorem says that $\text{Im } f \rightarrow \text{Im } \bar{f}$ as the laboratory energy tends to infinity; thus K_s regeneration should vanish asymptotically if that theorem holds, and this regardless of the target material used. Furthermore, the regeneration phenomenon depends on both the magnitude and the phase of $f - \bar{f}$, and thus supplies more information than the comparison of total cross-sections, which depend only on the imaginary parts. From a technical point of view, the comparison of K^0, \bar{K}^0 scattering amplitudes is much easier at ultra-high energies than that of K^+, K^- cross-sections, as no separated are required and the K_s 's are, so to speak, self-identifying.

These interesting aspects of high-energy regeneration have of course not escaped the attention of experimentalists. They were discussed in the IRL Study of 1964/65^[1], and in the NAL Summer Study of 1968^{[2],[3]}, and I. Savin is currently performing a regeneration experiment at Serpukhov.^[4] K. Kleinknecht^[5] offers a discussion of the impact of recent Serpukhov K^+ total cross-section data on high-energy regeneration experiments.

In making this proposal there are several points that we wish to emphasize:

(a) K_s regeneration can be done at a very early stage of NAL's operation. A K_L -beam could be produced long before all the facilities for charged beams are installed, and statistically satisfactory regeneration experiments

could be performed well before the design intensity of 10^{13} protons/pulse is attained. The relevant data are given in Section 6.

(b) K_s regeneration by nuclei (e.g., carbon) is almost as useful as the "fundamental" experiment with hydrogen, and in fact provides unique information (see Section 2). The installation of a sizeable hydrogen target may have to await the completion of real experimental areas, while regeneration experiments on simple targets can be done "in the mud."

(c) The authors of this proposal have the necessary equipment, software, and experience to perform the proposed experiment essentially now. For more comments on this point see Section 7.

2. Summary of Useful Concepts

The regeneration amplitude ρ_0 in a slab of material of thickness L (thin with respect to the K_s mean decay length $\Lambda_s(p)$) is given by

$$\rho_0 = 2\pi i N (f_{21} h/p), \quad 2f_{21} = f(0) - \bar{f}(0) \quad (1)$$

i.e.,

$$|\rho_0| = 7.4 \times 10^{-3} \frac{d}{A} |f_{21}| / p \text{ cm}^{-1}, \quad (2)$$

where d = density in g/cm^3 , A = atomic weight, p = K -momentum in GeV/c , and $|f_{21}|$ is measured in fm . (Note that this thin-regenerator assumption is made throughout the proposal.)

Coherent (transmission) regeneration produces $L^2 |\rho_0|^2 e^{-L/\mu}$ K_s 's per incident K_L , where μ = mean free path for K_s (or K_L) interaction, while diffraction regeneration has a forward cross-section given by $|f_{21}(0)|^2$. The ratio of events due to these two processes into a small angle $\Delta\theta$ about 0° is the so-called Good ratio, R :

$$R = (2\pi/k)^2 LN / \Delta\Omega = 0.74 dL / Ap^2 \Delta\Omega, \quad (3)$$

where L is in cm and $\Delta\Omega$ is in milliradians. R is independent of f_{21} , but depends on L/p^2 . For separating the coherent peak from the diffraction "background" R should be large, i.e., the angular resolution good ($\Delta\Omega$ small). In fact, as the angular dependence of the diffraction peak goes as

$$|f_{21}|^2 \sim \exp[(-0.032p/p_0)^2 \Omega] \quad \begin{cases} p_0 = 6.2 \times 10^{-2} \text{ GeV}/c \text{ (carbon)} \\ p_0 = 14.1 \times 10^{-2} \text{ GeV}/c \text{ (hydrogen)} \end{cases} \quad (4)$$

one must have $\Omega \ll (0.032p/p_0)^2$ in order to extrapolate the diffraction distribution under the coherent(transmission) peak (see Fig. 4 for some examples).

The phase $\varphi_r \equiv \arg \rho$ is measured via the interference of 2π 's from the regenerated K_s 's with 2π 's from K_L 's [6],[7]. With the usual def-

initiation of η_{+-} , one has behind a regenerator ($t = 0$) the proper-time distribution of the 2π intensity

$$I_{+-}(t) = |\eta_{+-}|^2 [e^{-t\Gamma_L} + |\rho/\eta_{+-}|^2 e^{-t\Gamma_S} + 2|\rho/\eta_{+-}| e^{-t(\Gamma_S+\Gamma_L)/2} \cos(\Delta mt - \Phi)] e^{-L/\mu} \quad (5)$$

where

$$\rho = \rho_O L, \quad \Phi = \arg(\rho/\eta_{+-}) = \varphi_r - \varphi_{+-}, \quad \varphi_{+-} = \arg \eta_{+-}$$

$$\Delta m = M_L - M_S = 0.540 \times 10^{10} \text{ sec}^{-1}$$

Note that for thin regenerators $\varphi_r \simeq \arg(f_{21})$. By observing I_{+-} over a suitable t -range, one can obtain both $|\rho|$ and φ_r without absolute rate measurements; it is, however, necessary that $\epsilon(p,t)$, the acceptance of the apparatus, be well understood. We shall refer to this measurement of the regeneration amplitude as the RSW method. On the other hand, one can obtain $|\rho|$ by measuring $I_{+-}(t)$ at early times only, where the interference term is as yet unimportant; this requires an absolute measurement of 2π decays per incident K_L , and a knowledge of the interaction length μ . We call this the ρ^2 -method. In section 5 we discuss how each method is used at various stages of our program.

The RSW method supplies only the phase difference Φ , while for strong interaction physics φ_r is the quantity of direct interest. The quantity φ_{+-} is, however, already known to be $(46 \pm 7)^\circ$ from "vacuum regeneration" experiments [8],[9] and will be even better known before turn-on at NAL. Furthermore, as φ_{+-} is independent of momentum (or else we would really make a great discovery!), the variation of $\Phi(p)$ gives directly the dependence $\varphi_r(p)$ of interest.

At this point it is appropriate to summarize the Regge theory predictions for the p -dependence of f_{21}/p and φ_r . Only the C-odd

trajectories ρ , ω , φ , ... contribute to the difference $f - \bar{f}$, so that one predicts^[10]

$$f - \bar{f} \sim p^{\alpha(0)} \simeq p^{1/2}, \quad |f_{21}|/p \sim p^{-1/2}. \quad (6)$$

In addition Regge theory predicts

$$\arg(if_{21}) = -45^\circ \quad (6')$$

i.e.,

$$\bar{\varphi} = -90.9^\circ$$

No published measurements of ρ in hydrogen exist, but it is well established that the optical model (based on K^+N and K^-N data) describes ρ for complex nuclei well. Accurate measurements exist for carbon^[7] and copper^{[6],[11]} for 2 - 6 GeV/c and these "low energy" measurements already show surprising agreement with (6'):

$$\begin{aligned} \arg(if_{21})[\text{carbon, 4.5 GeV/c}] &= (-43.7 \pm 10.5)^\circ \\ \arg(if_{21})[\text{copper, 2.7 GeV/c}] &= (-50.1 \pm 7.6)^\circ, \end{aligned} \quad (7)$$

where the first result was recalculated using $\varphi_{+-} = (46 \pm 7)^\circ$. In addition, $\rho(p)$ is quite compatible with (6).

The above point is relevant to the planning of the experiment. As the phase appears to agree, even for complex nuclei and at moderate energies, with the "asymptotic" Regge predictions, it should be interesting to measure the p -dependence of $|\rho|$ alone up to 100 GeV as a first check, and only then attack the determination of φ_r .

Without going into detail, it is of interest to note^[10] that regeneration on a $T = 0$ nucleus (e.g., carbon) should be dominated by ω -exchange, while ρ -exchange is supposed to lead for regeneration on protons. Thus, as mentioned above, regeneration on complex nuclei has intrinsic physical

interest.

Finally, a word about electron regeneration. As the K^0 is not a "strictly neutral" particle, regeneration by a virtual photon ($C = -1$) is quite possible. The process has been most recently investigated by Kroll, Lee, and Zumino^[12] who predict for the K^0 charge radius

$$\sqrt{\langle R^2 \rangle} = 0.28 \text{ fm} \quad (8)$$

This prediction is quite consistent with the "null" result of a recent CERN experiment^[13]; viz. $\sqrt{\langle R^2 \rangle} = 0.22 \begin{matrix} + 0.20 \\ - 0.22 \end{matrix} \text{ fm}$.

3. Experimental Arrangement

The experimental set-up (Figure 1) involves a target, regeneration target, various trigger counters, a wire-chamber spectrometer, and special counters for identifying (and suppressing as triggers) $K_{\mu 3}$ and $K_{e 3}$ decays. The design of this set-up is based on long experience in K^0 studies with a wire-chamber spectrometer, and is by now conventional. Whereas our previous work^[8] involved kaons below 8 GeV/c this set-up is designed to study regeneration above 40 GeV/c.

The regenerator, preferably placed within the field of a sweeping magnet, is followed by a veto counter $\bar{1}$ to define the decay region; the latter should be filled with He to keep multiple scattering and neutron-induced triggers to a minimum. The basic 2π -trigger consists of $\bar{1}$ plus one and only one counter in each of the hodoscope banks 2L, 2R, 3L, and 3R. The counters in hodoscope 2 have very thin ($< 1/16''$) scintillators in order to keep multiple scattering low, and thus preserve directional information on tracks prior to their entry into the magnet. Hodoscope 3 could be followed by an optional counter 4, divided into four equal and independent quadrants. By requiring two diagonally opposite quadrants of 4 in the signature, one imposes a mild coplanarity requirement to suppress $K_{e 3}$ triggers.

A shower counter, following the design of Heusch and Prescott^[14], is used in anti-coincidence to suppress events with electrons in the final state, as it would take gas Cherenkov counters of extravagant design to put high pions below threshold. The shower counter, based on actual calibrations, discriminates between electrons and pions with $\sim 1\%$ chance that a pion be mislabelled. This rejection ratio is, in

combination with our spectrometer resolving power (in angle and kaon invariant mass) amply sufficient for our purposes. Furthermore, any failing of the electron shower counter (which can be studied by tagging 2π -events in the K mass peak) leads at most to a loss in 2π -events, and not to a dilution of the signal by unwanted K_{e3} decays. The shower counter has a hole in the region where it is traversed by the neutral beam.

Muon events are suppressed by adding to the signature a veto counter hodoscope 5 located behind a massive absorber (e.g., 4m of Pb). The $\pi - \mu$ discrimination of this simple device, again in conjunction with the spectrometer resolution, is amply sufficient.

4. Properties of the Spectrometer

The spectrometer consists of a magnet (say, 26" vertical aperture, 40" deep, such as the ANL magnet SCM-105 we are currently using) sandwiched between two sets ("tables") of four wire-spark-chambers (crossed-wire, x and y readout) each, so that 8 coordinates are obtained for each track. The first of these is a "split chamber", i.e., its two halves (left/right) are read out independently, to resolve ambiguities.

The critical feature of the spectrometer is its resolution, both in space and momentum. This is governed by the spatial resolution of the individual chambers, and by the separation S between planes (see Fig. 1). While we shall forecast the resolution of the spectrometer to be used at NAL on the basis of the instrument we are currently using at ANL, we shall obviously allow for the fact that one needs [because of the Good ratio, Eq(3)] roughly $20^2 = \underline{400 \text{ times better angular resolution at } 100 \text{ GeV than at } 5 \text{ GeV.}}$

Figure 2 shows an angular resolution distribution obtained by us with a B_4C (equivalent to dense carbon) regenerator at a mean momentum of 3 GeV/c. Note that part of the width of the coherent peak is not due to instrumental resolution, but rather to a finite source size. Figure 3 shows the invariant 2π -mass distribution; its width (6 MeV, FWHM) agrees with that predicted from the resolution of the individual wire planes (0.8 mm, FWHM).

The chambers used to obtain figures 2 and 3 have 24 wires/in. and were separated by $S = 1 \text{ m.}$ By separating them by 20 m we can obtain the required improvement in angular resolution. While such a separation

is quite practical for the proposed experiment, we currently favor a scheme with $S = 10$ m and chambers having 48 wires/in. A smaller S appears to have practical advantages which need no discussion here. This reduced wire-spacing has already been used by others (but not with core readout), and we are currently building a prototype chamber. Figure 4 shows how the angular distributions from regeneration experiments with 100 GeV/c kaons will look for various specified regenerators with the proposed spectrometers; total lepton suppression has been assumed and so-called "inelastic" regeneration (at most of the order of the diffraction component) has also been neglected. The reader is supposed to be surprised by the cleanliness of the coherent peak.

In practice, "total lepton suppression" is not achieved. One eliminates K_{e3} and $K_{\mu3}$ events by (a) selective triggering (see Section 3), (b) selection of events with $M_{\pi\pi}$ near M_K , and (c) reconstructing p_K and seeing whether it points in the direction of the incident beam.

By cutting on $M_{\pi\pi}$ we estimate that in the region of solid angle containing the coherent peak ($\Omega \lesssim 2 \times 10^{-6}$ milliradians) the total lepton contamination is 40% (compared to the η_{+-} [vacuum regeneration] level). However, since the angular distribution of these lepton events is essentially the same as that of the diffraction-plus-inelastic background (see Figure 5) the leptonic events are automatically removed by the subtraction technique used to separate the coherent events.

Despite the fact that contamination of the coherent peak is made negligible by the subtraction, we include in the apparatus (see Section 3) lepton-suppression devices with rejection factors of $\gtrsim 100$ in order to lessen the number of triggers due to unwanted events. With this rejection

factor the leptonic background in the coherent peak is $\approx 0.4\%$ before subtraction.

It remains to know the acceptance, $\epsilon(p, z)$ of the spectrometer for $K_{2\pi}$ events (z = distance of decay point from the magnet center). This quantity, computed by a Monte Carlo program, is plotted in Figure 6a as a function of z for $p = 40$ and 100 GeV/c; Figure 6b shows the total acceptance for $D = 20$ m, as a function of p . The acceptance falls off sharply below 40 GeV/c, but this appears desirable in view of the overwhelming trigger rates which the much more copiously produced "low energy" kaons would cause. The flatness of $\epsilon(z)$ is almost irrelevant for the measurement of $|\rho|$, and is satisfactory for a measurement of ϕ_r by the RSW method (see Section 2). Both the total and differential acceptance will be checked in a vacuum run (see Section 5).

The extraordinary resolution of the spectrometer at lower energies (say 10 GeV) can be exploited to measure the electron regeneration in some high Z material such as Pb. One would of course shorten D for this purpose. The higher resolution would enable one to improve on the recent CERN experiment^[13] but would by no means solve the problem of the inelastic background.

There remains to discuss the event rate at which the spectrometer can operate. Again, we shall make use of our experience at ANL to predict this. Our spark chamber (magnetic core) readout system operates reliably with a block time of 3 msec, i.e., after this time the process of firing, core-scanning, and data transfer to the computer can be repeated. Conservatively, we can expect to collect ~ 100 events/NAL burst, where we of course do not imply that all of these will be good 2π events.

A mere 10^2 events/burst might appear a low rate to the advocates of the more modern proportional (Charpak) chambers, or to people who have fallen under the spell of these enthusiasts. The first point is that (see Section 6 for rate estimates) running time does not seem to be the limitation of the experiments proposed here, and that it is not clear -- especially until a lot more work is done -- why one would like to know, even for hydrogen, the phase φ_r to 1° : Second, large area proportional chambers are built with 2 mm wire spacing, thus giving probably four times worse resolution in each coordinate (16 times in spatial angle) than the conventional chambers proposed here. Third, the cost of large area proportional chambers and associated electronics is so large that the number of such chambers is to date kept at a minimum (say, four per spectrometer). Ambiguities in pattern resolution, losses through inefficiencies thus created, etc. are to be handled by accepting only "perfect events" for analysis. Our experience with regeneration experiments indicates that this fraction will be rather small. What counts is the rate of reconstructed events/burst, and to quote a known saying, "cleanliness is holiness." So we propose to use our old-fashioned chambers, adding perhaps later on Charpak chambers as decision making elements.

Finally, we mention that the analysis of "only" 1.5×10^6 events (15K bursts/day, 100 events/burst) collected in a day constitutes in itself a formidable task. Inasmuch as it is unthinkable (at least financially) for a university-based group to handle this much data or more off-line, we are currently assembling a system that will accommodate on-line reconstruction of two-track events at a rate of at least 20

events/burst (> 300 K /day). This is, briefly, a multi-processor scheme using Data General Supernova CPU's in conjunction with an EMR - 6040 computer.

5. Plan of Experimental Measurements

We propose the experiment as a series of distinct steps, for several reasons. First, since no one has experience in such high energy neutral beams, we will start with an arrangement which is as clean as possible in terms of neutron-induced backgrounds. The excellent mass and angular resolution of the apparatus enables us to eliminate backgrounds due to other K^0 decay modes. Second, we expect to rely on the results of each step to optimize the experimental details of the following steps. Third, this approach affords us the flexibility of performing the later stages of the research in light of the physical interest generated by the earlier, simpler stages. Finally, the initial parts of the experimental program will not involve certain sophisticated equipment such as large hydrogen targets, and so the experiment is expected to be more compatible with the evolving experimental facilities at NAL. It should be emphasized that all the steps in this program are quick checks (see Section 6) except for the last one which will be a precision measurement.

In this section we outline the steps as currently envisaged and in the next section we present estimates of the expected rates for each configuration.

(a) "Vacuum Regeneration" Run: As a test of the operation of the spectrometer, we plan a preliminary run with no regenerator. The CP-violating $K_L \rightarrow \pi^+ \pi^-$ mode will be used to check the acceptance of the 2π decays; the K_{L3} modes, which will be used as monitors in later stages of the experiment, will also be calibrated in this low-background run. At the same time (every other burst, for instance) we will place a carbon absorber in the beam, far upstream from the apparatus, to measure the total cross-section. This information is needed for the regeneration studies

which follow.

(b) $|\rho|^2$ Measurement in Carbon: This initial regeneration test in carbon has two experimental advantages. First, the carbon block is simpler and more easily handled than a hydrogen target. Second, carbon has a rather long collision length and hence give us the smallest background. We will use a moderately thick regenerator (~ 1 m, or 1.5 collision lengths) and a short decay region (15 m, or $3\lambda_s$ at 100 GeV/c) and measure the intensity of regenerated $K_s \rightarrow \pi^+ \pi^-$ decays relative to semi-leptonic K decays at early proper time downstream of the regenerator. It should be noted that by examining the differential distribution of this data we can also obtain information at "low" momentum (~ 40 GeV/c) on φ_r via the RSW method.

(c) φ_r Measurement in Carbon: In order to obtain complete information on regeneration in carbon, we will proceed to a more complete determination of the momentum dependence of the phase of ρ . This is accomplished by the RSW method but with a different arrangement that that used for (b). In order to minimize the neutron-induced background, a shorter regenerator (~ 20 cm) will be used. To study the interference downstream of the regenerator a longer decay volume of 20 m is needed.

(d) Regeneration in Methane: We will next replace the carbon regenerator with one of CH_4 . From this data and the earlier carbon data we can extract the first measure of regeneration from protons. (This is because of the coherent addition of regeneration amplitudes.) Methane has the following advantages. First, it has a high regeneration power ($|\rho| \simeq 10 |\eta_{+-}|$ for a target of 1 collision length = 335 cm). Second, the relative hydrogen density is a factor 1.7 times that in liquid H_2 .

(e) Regeneration in Liquid Hydrogen: The experience gained in the previous steps, plus the value of ρ_{hydrogen} deduced from (b), (c), and (d) will serve to specify the optimum experimental arrangement for this effort. It appears that the statistical power per unit time is greatly enhanced by measuring $|\rho|^2$ and φ_r separately as in (b) and (c).

It is assumed that a hydrogen target of ~ 2 m length will be needed.

6. Rates and Time Estimates

Estimates of kaon fluxes are taken from the Summer Study paper of Smith^[15] and the NAL Technical Manual of Awschalom and White^[16]. Our calculations are based on the assumption of a 10 mrad beam with a nominal solid angle $\Delta\Omega = 1.3 \times 10^{-5}$ milliradians (1.5" x 1.5" at 1200') and 10^{12} interacting protons per burst. For some stages of the experiment such a beam will give excessive rates. We will indicate the amount of extra collimation required for each stage.

All the estimates of regeneration rates are based on the momentum dependence of f_{21}/p given by Eq. 6. Table 1 contains some relevant information on the regeneration properties of different materials.

Table 2 gives the particulars of the set-up for the first four stages and the expected rates for each. We assume that the actual trigger rate is one order of magnitude larger than the $\pi^+\pi^-$ rate quoted. The running time is computed on the basis of 1.5×10^4 pulses/day.

It is seen that 3 - 4 weeks of actual running are needed for the preliminary steps. Beam studies and set-up of the apparatus will depend very much on conditions at NAL. Under the best circumstances 4 weeks would be sufficient.

The rates and estimates for the liquid hydrogen run are not included in the table. Depending on which of the current estimates for f_{21}/p are used, running times of anything up to ~ 6 weeks are indicated for a precision measurement using a LH_2 regenerator of available length (≤ 2 m). The results of the methane run will enable us to predict the running time for liquid hydrogen quite accurately.

7. Qualifications of the Proponents

Our group has long experience in the field. The first wire-chamber experiment ever performed^[17] was done here, we built the first wire-chamber spectrometer specifically for K^0 decays^[18], and we have completed to date two major experiments^{[8],[19]} with that spectrometer. We have the hardware, the software and an exceptionally qualified support staff available.

Our collaborators in the ANL experiments, Professors H. Goldberg and J. Solomon of the Chicago Circle Campus of the University of Illinois, though not yet committed to this specific experiment have indicated their interest in continuing the collaboration.

TABLE 1: REGENERATION PROPERTIES

Material	d [g/cm ³]	α [10 ⁻⁶]*	c [mb]†	Good Ratio [10 ⁶]‡	Coll. Length[cm]§	$ p/\eta $
Hydrogen	0.06	0.25	1.0	15**	375	1.11**
Methane	0.42	0.56	2.83	14	335	10
Carbon	1.55	1.3	10	8	67	8
Boron Carbide	2.45	1.3	10	9	41	5
Copper	8.80	4	37	1.7	13	7

* $d\sigma/d\Omega|_{p=100} \approx e^{-\alpha\theta^2}/2$

† $|f - \bar{f}|/k = c p^{-1/2}$

‡ $L_{reg.} = 1$ coll. length at $p = 100$

§ Coll. lengths known at "low" energy

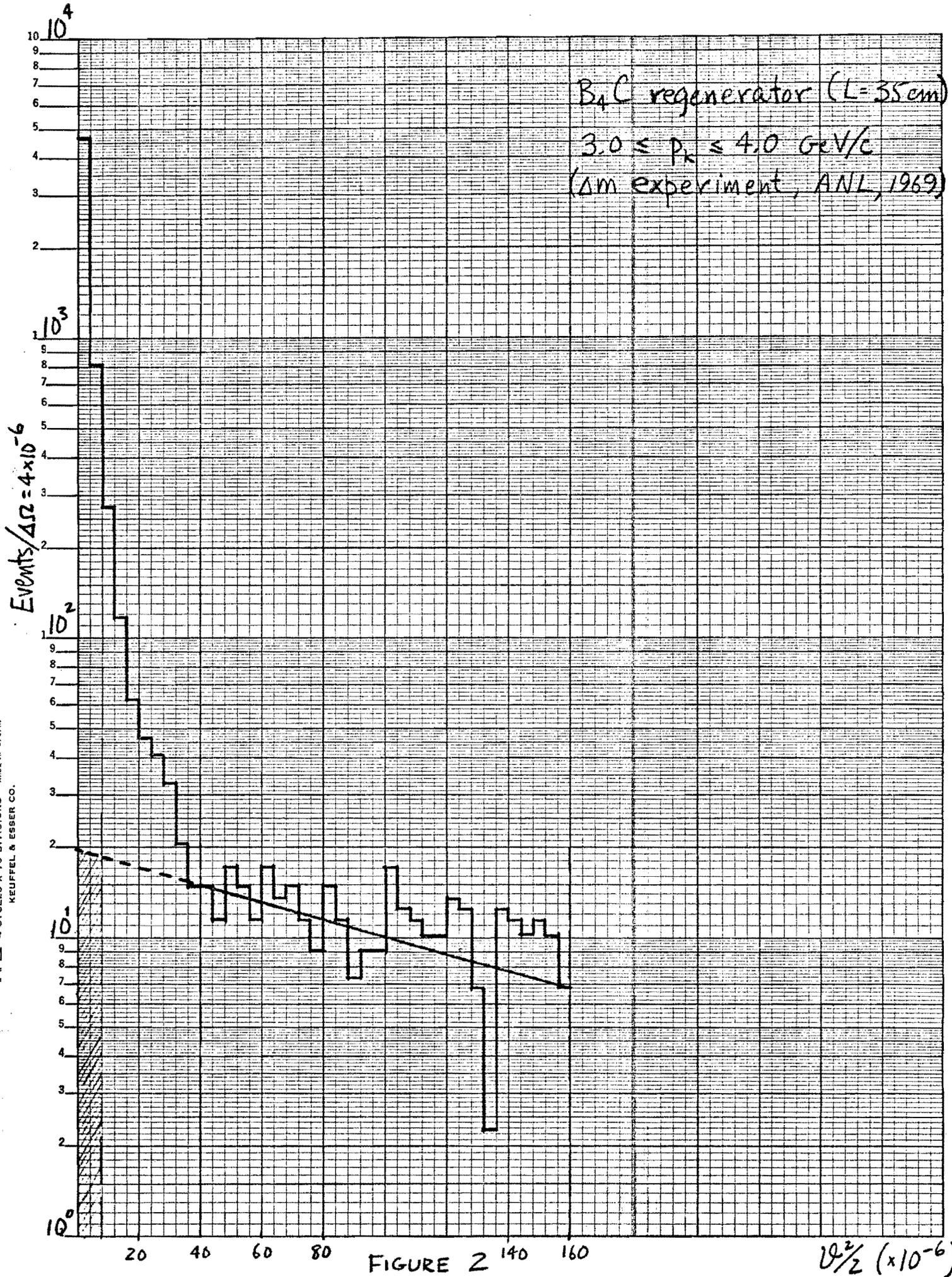
** Computed for a 2 m regenerator.

TABLE 2: RATE AND TIME ESTIMATES

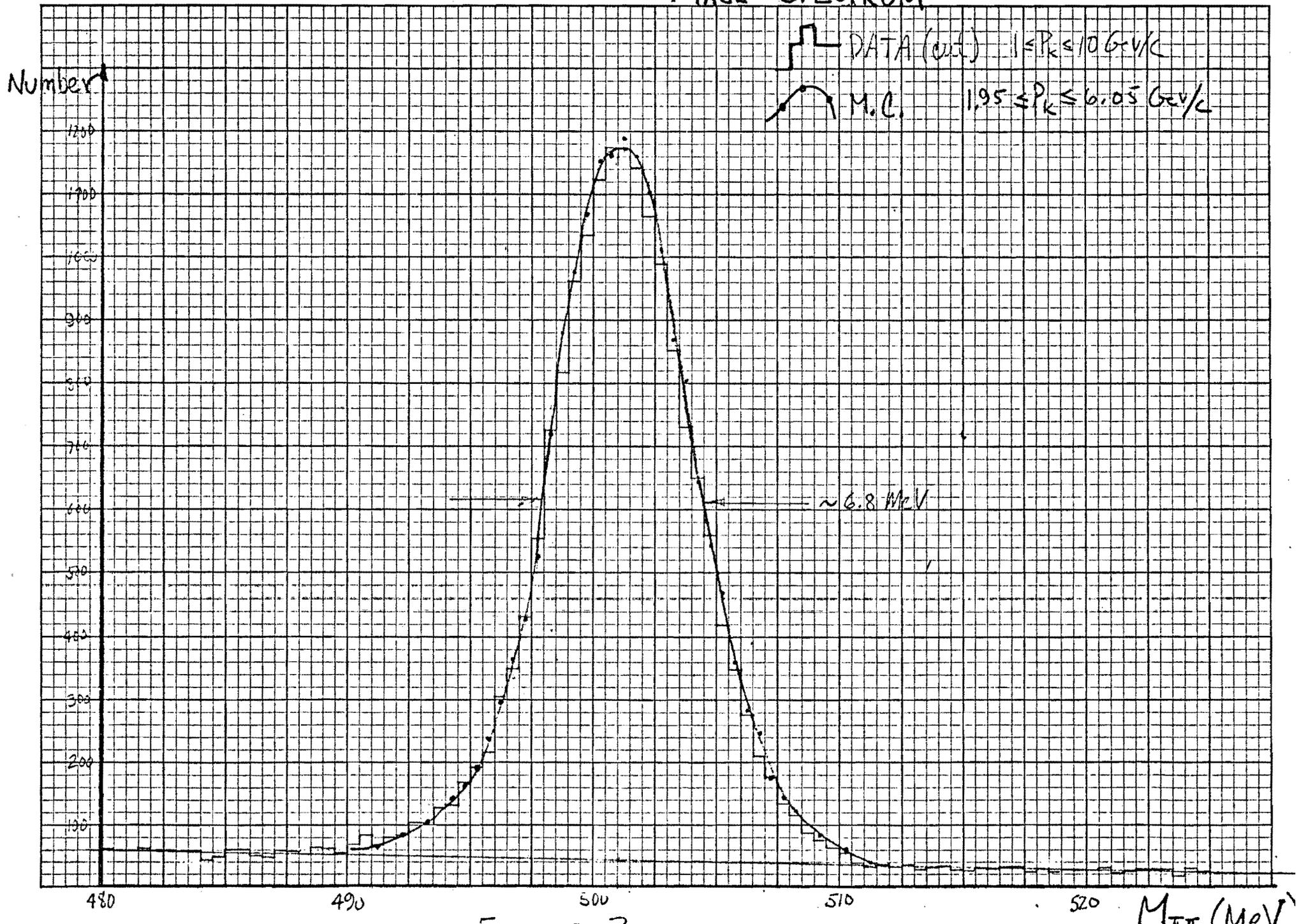
Experiment	Beam $\Delta\Omega$ [mster]; K_L Flux [/sec]	Decay Region [m]	L_{reg} [m]	2π Rate [/pulse]	2π Events Needed	Running Time [d]	Errors
Vacuum Run	0.4×10^{-5} ; 0.2×10^6	40	-	8	10^5	1	$\Delta\epsilon = \pm 5\%$ $\Delta\sigma_t = \pm 2\%$
$ \rho ^2$ Carbon	0.5×10^{-6} ; 0.2×10^5	15	1	8	5×10^4	<1	$\Delta \rho ^2 = \pm 2\%$ at $p = 100$
φ_r Carbon	1.3×10^{-5} ; 0.6×10^6	20	0.2	16	3×10^5	6	$\Delta\varphi_r = \pm 5\%$ at $p = 100$
ρ Methane	1.0×10^{-6} ; 0.4×10^5	40	3.4	16	10^6	20	comparable to Carbon
ρ Hydrogen	(See text, Section 6)						

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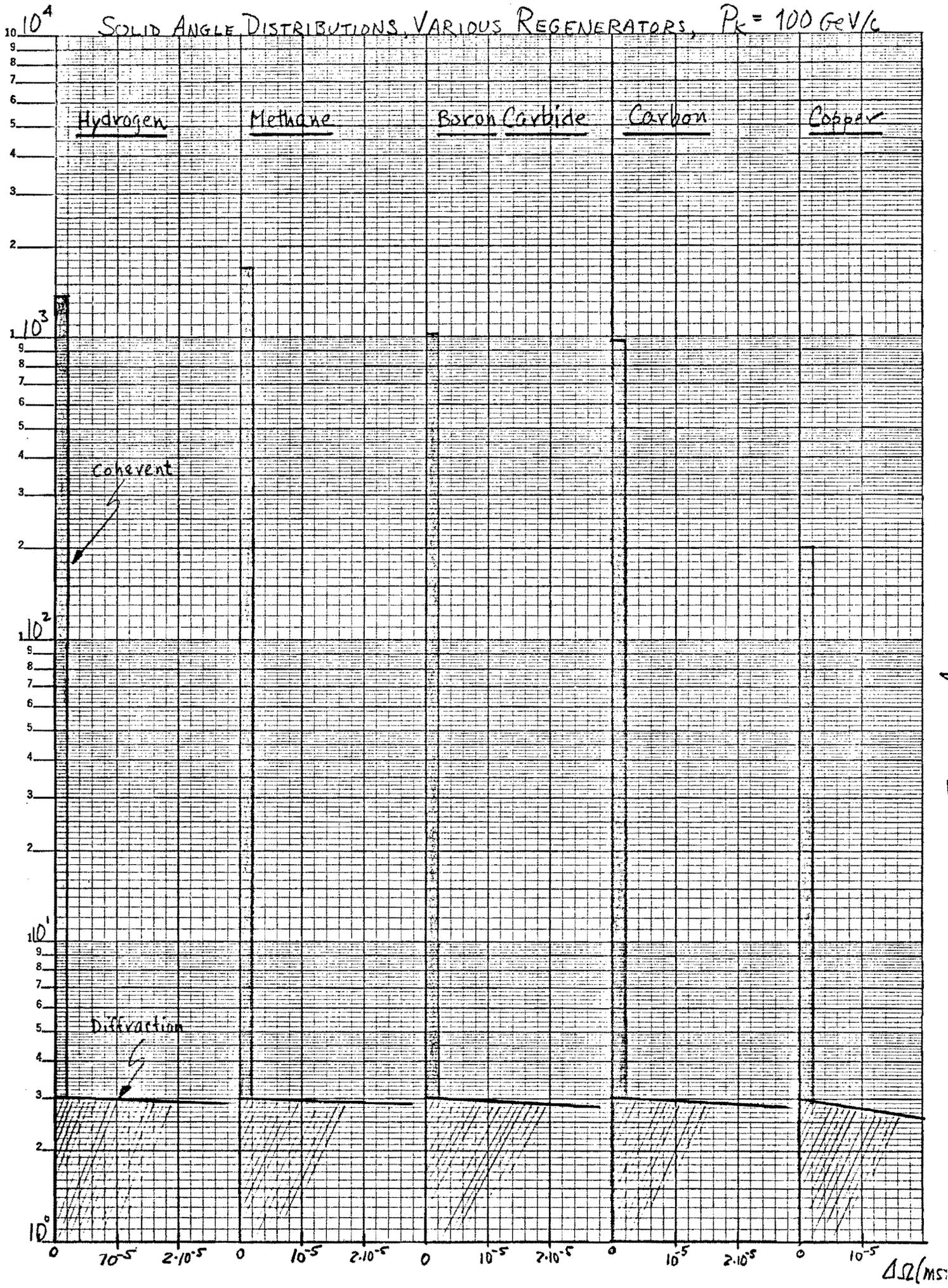
MASS SPECTRUM



SOLID ANGLE DISTRIBUTIONS, VARIOUS REGENERATORS, $P_k = 100 \text{ GeV/c}$

KE SEMI-LOGARITHMIC 46 6010
4 CYCLES X 70 DIVISIONS
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$\Delta N / \Delta \Omega$



Lepton Backgrounds (Carbon Regenerator)

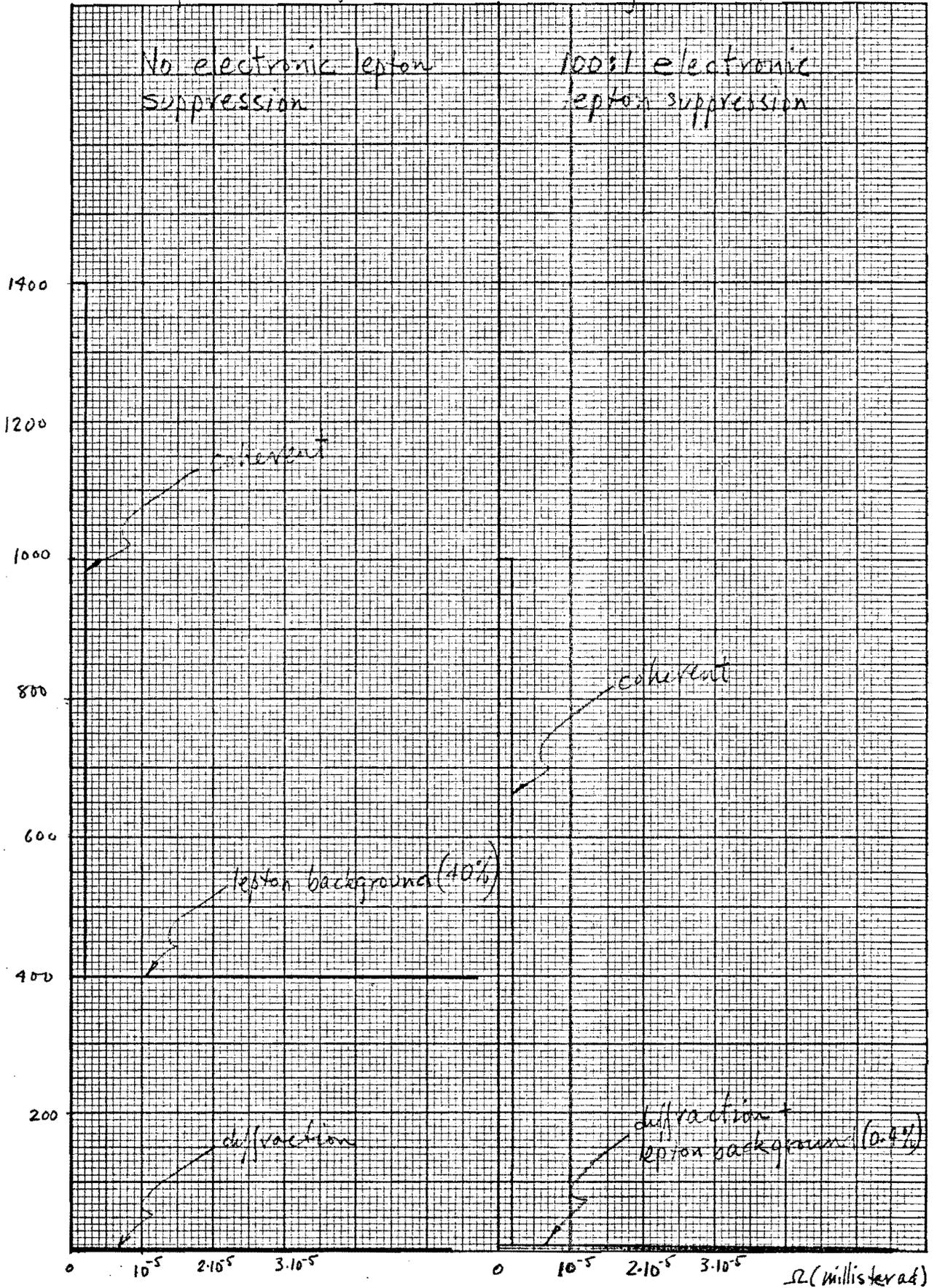


Figure 5

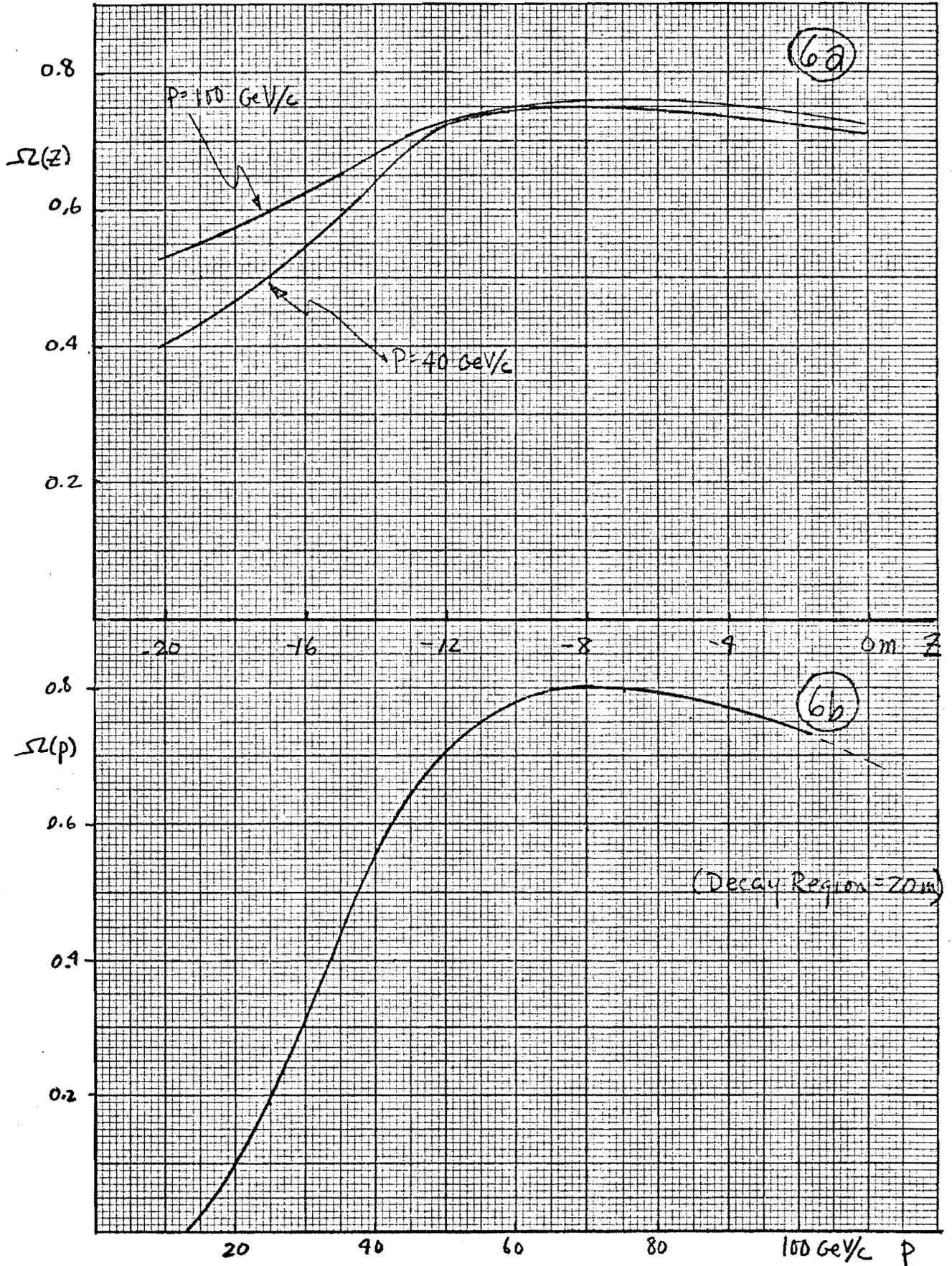


Figure 6