NAL PROPOSAL No. 64

Correspondent: A. L. Read
Experimental Facilities
National Accelerator Lab
Batavia, Ill. 60510

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HADRON SPECTRA FROM HIGH ENERGY PROTON-PROTON INTERACTIONS

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Argonne National Laboratory

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University of Bari

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ABSTRACT

It is proposed to measure the hadron spectra resulting from high energy proton-proton collisions using a single arm focusing spectrometer. These measurements will provide elastic and inelastic P-P cross sections for incident beam energies up to 200 GeV/c and for momentum transfers $|t|$ from 0.01 up to about 10 to 15 $(BeV/c)^2$. In addition, we will obtain yields of pions and kaons produced in the interactions.

Correspondent: A. L. Read, NAL

(Note: Local leaders are underlined thus).

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INTRODUCTION

We are interested in measuring the spectra of protons and other particles emitted from high energy P-P collisions. From these data we will extract cross sections for elastic proton-proton scattering, N^* production and deep inelastic scattering. In addition, we will obtain cross sections for $\pi^{\pm}$, $K^{\pm}$, and antiproton production. This work should be carried out over as wide a range of energy and momentum transfer as possible.

The choice of a suitable proton beam for the experiment is a compromise between intensity and resolution. Since the cross sections decrease rapidly with momentum transfer, the intensity of the incident proton beam must be varied from about 10^{10} or more to 10^6 particles per pulse. On the other hand, the investigation of elastic scattering and inelastic scattering in the resonance region requires a high quality incident beam.

We propose to construct a focusing spectrometer which would view a target in the 2.5 mrad secondary beam planned for Experimental Area #2. The spectrometer is a 200 GeV/c instrument with a momentum resolution of ±0.032% and a solid angle of 4.5μ-sterad. The system permits particle identification up to the highest momentum. Separation of π's, K's, and P's is accomplished by the combined information obtained from a DISC Čerenkov counter which is located in a relatively divergence free section of the optics and four threshold counters located behind the magnets. A shower counter and a hadron absorber have been added downstream to define electrons and muons.

The total cost of the apparatus for this experiment is estimated to be about M$1.7. The detailed costing is given in Section 5. We expect NAL to provide the magnets, cooling water, power, shielding, tunnel housing, hydrogen target, and on-line computer at a cost of about M$1.3.

The particle detectors and beam instrumentation can be constructed and tested before summer 1972, which is the current date for Area #2 turn-on. These items will be provided by users who are contributing in addition to their time about $415K. Assuming that the spectrometer can be installed at the same time as the beam line, we could begin running as soon as the Area is opened for experimentation.

The group making this proposal is interested in all aspects of the work described herein--the design and development of the spectrometer with its associated instrumentation, as well as the performance of the experiment. Each of the outside collaborators has undertaken to build some of the instrumentation at his home laboratory in addition to spending an adequate amount of time on site to carry through his responsibilities.

The ten NAL physicists involved in this experiment expect to contribute to their fullest extent consistent with the fraction of their time that they are encouraged to spend in research activity.

We are requesting 200 hours of machine time to check out the apparatus and 1000 hours to carry out the experiment.

1. PHYSICS JUSTIFICATION

We intend to measure $p + p \rightarrow p + \underline{X}$ as a function of the momentum of the final state proton for a range of incident energies and scattering angles, where $\underline{X}$ is anything. A single arm spectrometer will be used to measure the spectra of the final state protons. This will provide a survey of both elastic and inelastic scattering over a wide range of momentum transfers and incident energies. The spectra naturally break up into three regions of interest.

I. Elastic Peak

The measurements we propose will provide information about the elastic cross-section for incident energies between 200 and 50 GeV and values of $|t|$ from .01 to about $10(\text{BeV/c})^2$. The interest in these results stems from a number of different questions, among which are the following:

a. Does the diffraction peak shrink over the energy range investigated?

b. Does the functional form of $d\sigma/dt$ approach $G_M^{\ 4}(t)$? (G_M is the magnetic form factor of the proton). This hypothesis has been suggested by Wu and Yang (Phys. Rev. 137, B708 (1965) and Abarbanel et.al. (Phys. Rev. 177, 2485 (1969)).

c. What kind of dynamical model best describes the behavior of $d\sigma/dt$? Models such as those of Chou and Yang (Phys. Rev. Letters 20, 1213 (1968); Durand and Lipses (Phys. Rev. Letters 20, 637); and Islam and Rosen (Phys. Rev. 185, 1917 (1969)) provide predictions which can usefully be compared with experiment.

II. Region of Resonance Excitation

By measuring the spectrum with moderately high resolution in the region of missing mass below 3 GeV we will be able to learn something about the energy and momentum transfer dependence of some of the nucleon resonances. Of particular interest are the resonances thought to be excited by Pomeron exchange, such as the N^* (1470) and the N^* (1688). These should be amenable to study up to the highest incident energies. The other resonances, which decrease in amplitude with increasing energy, will be pursued to as high an energy as is possible. The major interest in these measurements is that they will provide information about the high energy behavior of specific channels.

III. Deep Inelastic Scattering

A very likely fate of a high energy hadron colliding with a nucleon is to end up in the broad kinematic region which is denoted deep inelastic scattering. For example, Anderson and Collins (Phys. Rev. Letters 19, 201 (1967) gives $19\pm^{6}_{2}$mb for the non-resonant part of the total p-p cross-section (38.8mb) at 30 GeV. And it is very likely that the fraction of proton interactions that are highly inelastic will increase at higher energies. One of the experimental challenges for the NAL accelerator will be to delineate the regularities of this part of the inelastic cross-section.

The first exploratory investigation should be done with a single arm spectrometer which measures the momentum spectra of final state protons over a range of angles and incident energies. By using an array of Čerenkov counters set to be sensitive to different masses, the momentum distributions of protons, π^+, and K^+ can be simultaneously measured. The spectra of anti-protons, π^-, and K^- can be simultaneously measured with reversed magnetic fields. This type of experiment provides an excellent over-all picture for a relatively small investment in experimental apparatus and machine time. There are some recent theoretical models which make predictions for these spectra after appropriate averages are made over undetected particles. These are the predictions of Chou and Yang based on the hypothesis of limiting fragmentation. (Phys. Rev. 188, 2159 (1969). Feynman's parton picture (Phys. Rev. Letters 23, 1415 (1969) forms the basis of other predictions, and the multiperipheral model (L. Caneschi and A. Pignotti, (Phys. Rev. Letters 22, 1219 (1969)) provides another viewpoint in the interpretation of these data. It will be of great interest to compare the experimental results with these recent speculations.

2. Layout of the Experiment

The layout of the experiment is shown in plan view in Fig. 1. The experiment uses a proton beam on a hydrogen target at the downstream end of the 2.5 mrad high-energy, high-resolution beam (HEHR) in Experimental Area #2. The angle at which the spectrometer detects particles can be varied by changing the angle of the incident beam upon the hydrogen target. This is done over the range 0-50 mrad by the two angle-magnets A_1 and

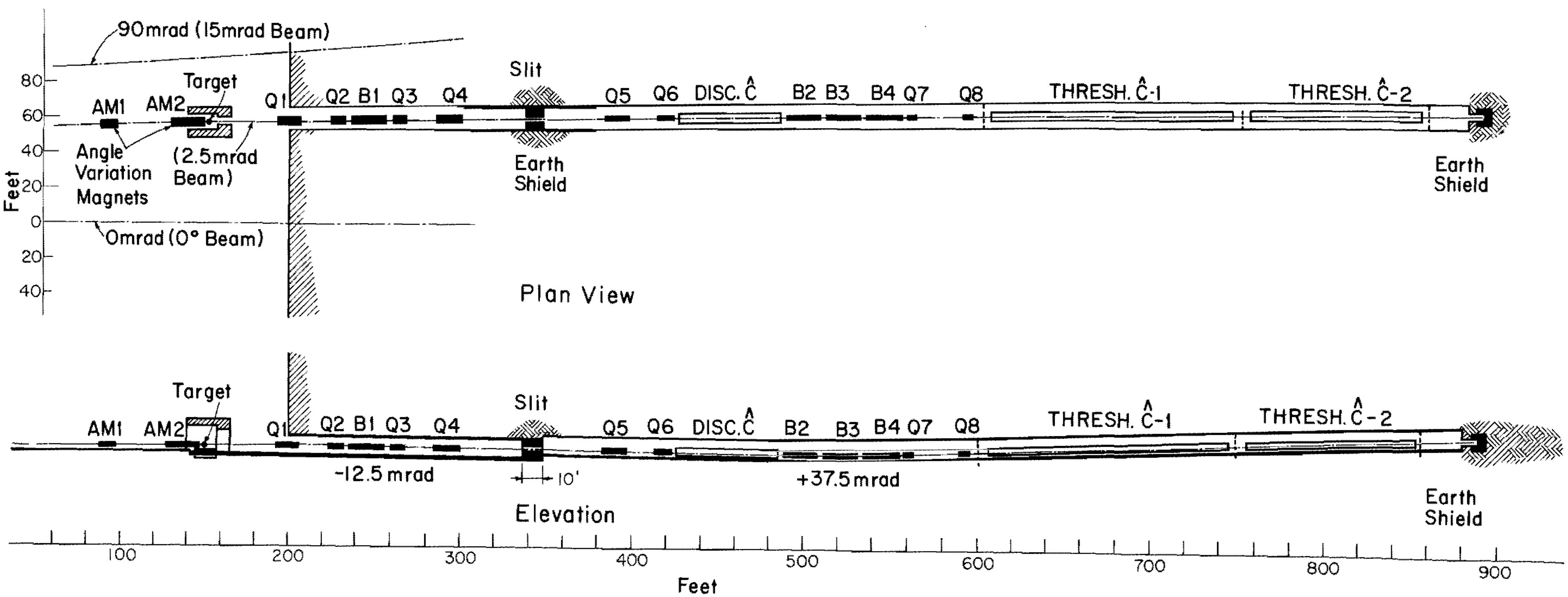

Fig. 1. Plan View of Spectrometer in Area #2.

A_2. The hydrogen target is located 50' from the downstream end of the experiment building which is planned to house the experiments using all of the several secondary beams from Target #2.

The spectrometer is 700 feet long; the 650-foot section of its length which is located downstream from the experiment building could be housed in a structure made from prefabricated main ring tunnel sections. A typical main ring service building with a connecting entrance to the enclosure would be located about half-way down the enclosure for power supply installation. If necessary, a trailer could be placed next to the service building for a counting room.

We have shown in Fig. 1 our preferred location of the experiment, in the 2.5 mrad HEHR beam. The possibility of studying π meson interactions with the spectrometer and the requirements of beam quality make this a more attractive location than in the 1.5 mrad diffracted proton beam in Area #2. The specific location for the spectrometer along the beam line was chosen so that room would be available in the experimental area building for other experiments to be set up along the same beam line and/or around the hydrogen target. Another consideration in the choice was to allow for the possibility of the addition of a second spectrometer arm at a later time.

Another possible location of the spectrometer which we have studied (see Appendix C) is in the External Proton

Beam enclosure which leads to future Experimental Area #3. In this location, we would use a more limited spectrometer design consistent with the layout of enclosures "B" and "D" of the external proton beam; in this case however, the spectrometer, while nominally a 200 GeV/c instrument, would have a limited capacity up to 500 GeV/c. An important advantage of initially placing a spectrometer in the Area #3 beam transfer tunnel is that it would permit an interesting physics program to be started approximately one year before Area #2 is ready in mid-1972. The costs for this alternative are modest since most of the items are paid for out of the construction contract which provides for all the beam tunnels and the beam transfer magnets. The magnets would have to be provided a year earlier, however, at a cost of about $500K to the laboratory's FY-71 construction budget. Over and above this, the cost for the apparatus required to do the proposed research program would be about $800K, approximately half of which would be supplied by outside users. The NAL half (385K) includes $150K for an on-line computer which would be part of the general purpose laboratory equipment which is to be loaned to experiments. This early research program in the proton beam enclosure would be completed before Area #3 is ready for research use and most of the magnets would be left in place to be the transfer elements to Area #3.

3. Running Time Requirements

We plan to cover a wide kinematic range in order to make an adequate exploratory investigation. Our preliminary plan is to collect data at four incident beam energies and a number of production angles. Spectra would be measured from a momentum of about 10% of the incident momentum up to the highest accessible values. The momentum transfer range to be covered is from about 0.10 GeV/c to about 4(GeV/c). The upper limit is set by where the counting rate runs out. At missing masses of 3 GeV or less, and at small and moderate values of t, we will take data in fine steps of momentum in order to observe resonance structure and the elastic peak. The data will be taken in coarse steps at higher values of missing mass. Spectra of negatively charged particles will also be taken in coarse steps.

In Table II, we show the kinematic range which we would like to cover in this experiment.

TABLE II

Incident Energy	Range of Production Angle
200 GeV	0.5mr to 25mr
150 GeV	0.67mr to 30mr
100 GeV	1mr to 37mr
50 GeV	2mr to 50mr

In making the estimate of the running time required, we have used a parametric fit from the results of Anderson et.al., (Phys. Rev. Letters 19, 198 (1967)) who measured p-p inelastic scattering for 10, 20 and 30 GeV/c protons out to momentum transfers of 2 GeV/c. For an estimate of the elastic scattering rates we used the asymptotic model of Abarbanel et.al. We assumed a 20 cm liquid hydrogen target, a maximum beam intensity of 10^{10} protons per pulse, and the specifications of the proposed spectrometer for Area #2. On this basis we estimate that 1000 hrs. of running time will be required to carry out the series of measurements. We would expect to give a progress report after about 500 hrs. of running, along with a reassessment of the additional time needed to complete the experiment.

We also will require at least 200 hours of testing time prior to data taking.

4. Apparatus

I. Spectrometer

As a result of considering the requirements of this proposal and the predicted future uses of a single-arm spectrometer, it was possible to arrive at a list of basic specifications. The specifications of the instrument we propose to build have resulted from these considerations, and are given in Table I. A detailed description of this system, which consists of main ring dipole and quadrupole magnets, is given in Appendix A. This design should be regarded as preliminary. Additional work is required to optimize it fully; however, it closely represents the type of system that can be achieved

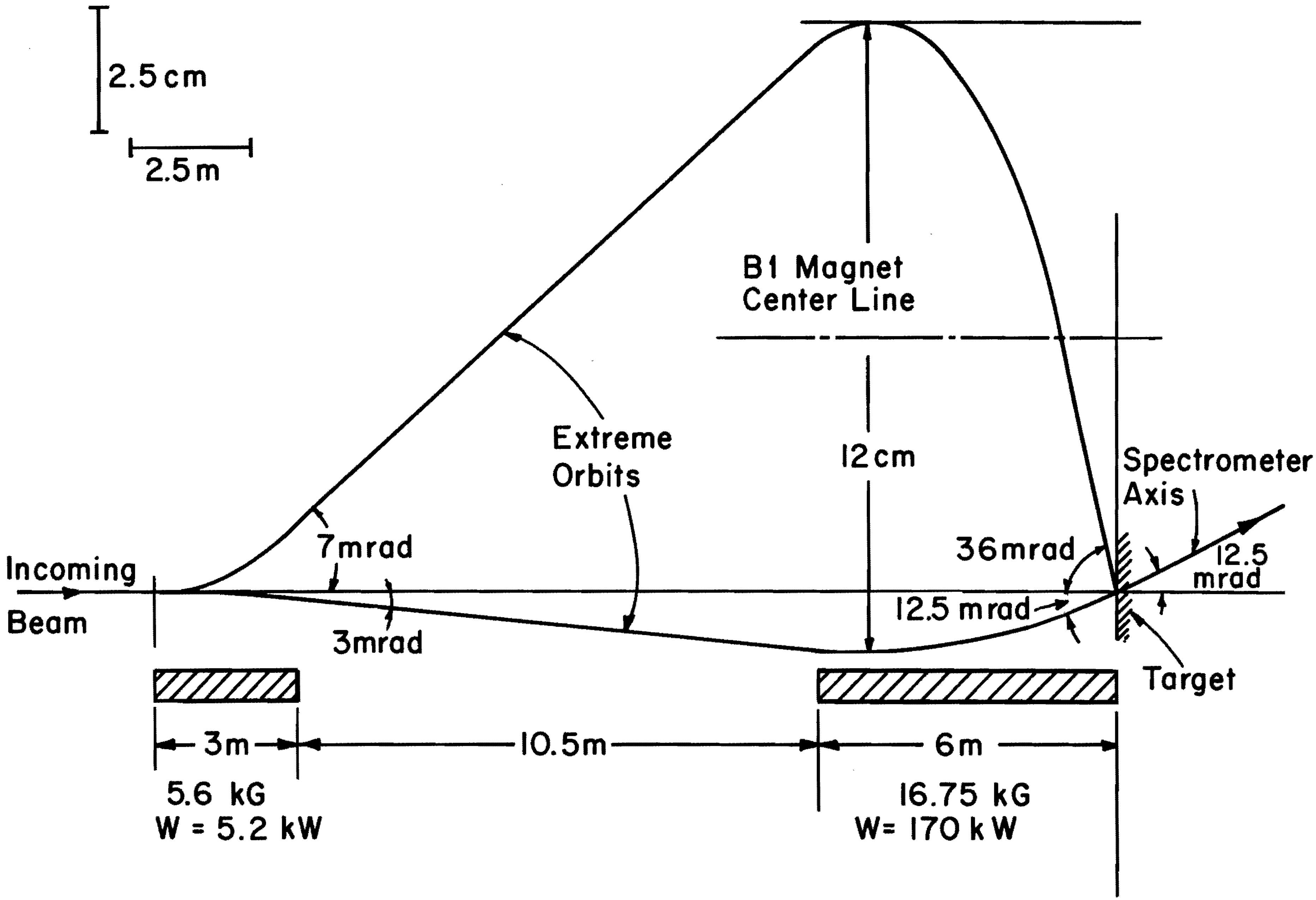

Fig. 2. Scattering Angle Variation System.

beam to provide the required angle variation.

The design of a system for varying the scattering angle is based on a maximum transverse momentum of 5 GeV/c over as wide a range of primary momentum as possible. Two distinct types of limitation occur; namely, the maximum available $\int Bd\ell$ and the transverse aperture in the magnet just prior to the target. In order to achieve the maximum range of momenta and production angles, there is a premium on keeping this magnet short and using as high a field as possible.

The system proposed here consists of one external proton beam dipole magnet, a drift space of 10.5 meters, followed by one main ring B-1 magnet (Figure 2). For the maximum incident momentum of 200 GeV/c, the transverse momentum of 5 GeV/c corresponds to a maximum scattering angle equal to 25 mrad. Thus, the spectrometer axis is aligned at an angle of 12.5 mrad to the incoming beam and the B-1 magnet must then bend ±12.5 mrad. By using 12 cm of the available 12.8 cm B-1 aperture width and centering the magnet aperture as shown in Fig. 2, it is possible to achieve the extreme orbits shown, giving a maximum production angle of 50 mrad. Under these conditions the maximum production angle of 50 mrad will be available up to a primary momentum of 67 GeV/c and above this momentum the maximum angle is limited by the magnetic field available. For example, at 100 GeV θ_{max} = 37.5 mrad and at 200 GeV/c primary momentum θ_{max} = 25.0 mrad.

If the experimental results indicate a need for an increased scattering angle range, the two alternatives are either to insert a fixed bend in the 10.5 meter drift space and to move the B-1 magnet appropriately, or to replace the B-1 magnet with a special bending magnet having a higher peak field and wider aperture, or perhaps to leave out the inner coils of the B-1 magnet.

III. Detectors

The spectrometer will have two distinct detector systems. One system, composed of wire plane detectors and scintillation counters, will be used to determine the production angle and momentum of the detected particle. The second system, consisting of DISC Ĉerenkov counters and UV threshold Ĉerenkov counters, will be used to determine the identity of the detected particle.

(i) Focal Plane Detectors

The spectrometer will produce a momentum focus in the vertical direction beyond the last quadrupole doublet. Because in this system no sextupoles are being used to correct 2nd order aberrations, the momentum focal plane will make a very shallow angle with respect to the optic axis. This shallow angle precludes the placing of any type of detector array within and parallel to the focal plane. Rather, a particle's momentum and production angle will have to be measured by determining the particle trajectory in the region of the focal plane and calculating where the trajectory crosses the momentum plane.

Three wire plane spark chambers, placed perpendicular to the optic axis, will span the entire focal plane. The dimensions of these wire chambers will be about 15 cm by 15 cm and each will consist of 3 separate wire planes. Wire planes are chosen over scintillation hodoscopes since they will provide the better spatial resolution in the determination of the particle momentum and angle. The poorer time resolution of the wire chambers should not be a great problem as backgrounds are expected to be low in the focal plane region. We will however use proportional wire chambers if these prove to be technically adequate for our application.

Scintillation counter planes in front of and behind the focal plane will serve as a trigger counter telescope. A fast coincidence between signals from these planes will define the passage of a particle through the detection system and provide a master trigger for the circuitry. In addition, there will be coarse scintillation counter hodoscopes for momentum and angle measurements just downstream from the focal plane. The momentum hodoscope will provide a momentum resolution $\Delta p/p$, somewhat in excess of $\pm 0.15\%$. In addition to providing a useful redundancy in detection, this hodoscope will provide readily accessible on-line information which will be very helpful in making decisions during the measurements.

(ii) Detectors for Particle Identification.

In the studies of hadron spectra it is essential to be able to identify the detected particle. The best instruments for providing mass separation at momenta above 100 GeV/c are the DISC Ĉerenkov counter and the UV threshold Ĉerenkov counter. The most taxing requirement is to be able to separate pions from K mesons at the highest momenta. For a mean Ĉerenkov angle of 20 mr at 200 GeV/c the angular separation of the pion and the K meson Ĉerenkov angles is 0.14 mr. The resulting limit on the angular divergence of the particle trajectories within the DISC places stringent requirements on the design of the spectrometer. This angular separation also places heavy demands on the operation of the DISC, which is discussed in Appendix B. A DISC capable of making this separation presents a formidable challenge. The spectrometer has been designed for the use of a DISC, and we expect to develop a counter of this type that will be capable of such a separation above 100 GeV/c.

Our second tool to provide π-K separation above 100 GeV/c is the UV threshold Ĉerenkov counter developed at Serpukhov. Four of these counters will be placed between the last quadrupole and the momentum focus. At 100 GeV/c an absolute pressure of 70mm of Hg. for hydrogen gas is required to put K mesons at threshold, in which case 80 meters constitute 1.2×10^{-3} radiation lengths and a negligible amount of multiple scattering. The Prokoshkin group (IHEP preprint 69-63) have obtained

$$N_e = 160\ \theta^2 L$$

where N_e is the number of photoelectrons in a 56 UVP photomultiplier, θ is the Ĉerenkov angle in radians, and L is the active length in centimeters. With the counter set to K threshold

$$N_e = 3.6\left(\frac{L}{10}\right)\left(\frac{100\ \text{GeV/c}}{p}\right)^2$$

for pions, giving about 30 photoelectrons from 80 meters of counter at 100 GeV/c. At 200 GeV/c, this becomes 7 photoelectrons, leading to a probability for no signal of about 10^{-3}.

In order to distinguish between hadrons, muons, and electrons we plan to install a total absorption counter which will contain about 7 interaction lengths (1 meter) of Fe plates interleaved with scintillators. By observing the shower development, we can distinguish electromagnetic showers, strong interactions, and minimum ionizing particles. The lateral size of the total absorption counter will be about 20 cm x 30 cm. This counter should have an energy resolution for hadron showers of better than 25% (FWHM).

IV. Beam Flux Determination

The incident beam flux monitors must handle from 10^6 to about 10^{10} particles per second. For total flux determinations of instantaneous rates below 10^7 per second a simple scintillator telescope in the incident beam is sufficient. As an additional independent monitoring system, we plan to use a high pressure gas Čerenkov counter. For instantaneous rates above 10^7 per second the high pressure gas counter will be modified so that the total anode current of the phototube can be measured. A suitably designed system would yield an anode current of about 0.05 ma for a beam intensity of 10^{10} per second. With a correction for dark current this device can be used over the entire range required, including absolute calibration against a counter at 10^7 particles per second. The calibration constant is sensitive to proton velocity and is expected to vary by about 4% in a known way for protons over 50 GeV/c. As a second incident beam monitor, above the range where individual beam particles can be counted, we will install a thin scattering target (about 0.1 g/cm^2) in the beam. In counter telescopes set off to the side of the beam we would expect a counting rate of about 100 per second for an incident beam of 10^6/second, and a counting rate of 10^6/second for an incident beam of 10^{10} protons/second.

V. Measurement of Beam Phase Space at the Target

The spectrometer resolution is predicated on the knowledge of the beam phase space at the target. We need to install sufficient instrumentation to be able to determine the beam size and divergence at the scattering target.

The spectrometer requires a beam size not larger than 1 mm x 2 mm, and a horizontal divergence not exceeding 10^{-4} radians. To measure the beam phase space we would install

2 pairs of 0.1 mm diameter tungsten wires which are moved through the beam both vertically and horizontally. In the region between the pairs the beam size can be determined to about 0.1 mm FWHM and the divergences to about 5×10^{-5} radians. In addition, the incident beam angle can be determined with similar accuracy.

VI. Hydrogen Target

We plan to use a 20 cm long liquid hydrogen target. The length of the target along the beam line is circumscribed by the requirements on the angular divergence in the DISC counter and the angular resolution of the spectrometer. We plan to have an empty replica of the hydrogen target for empty target measurements. A remotely controlled mechanism will place either the full or empty target into the beam line. The target complex will also have a ZnS screen that can be placed in the beam line for the purpose of checking beam alignment.

VII. Computer Needs

In order to collect, buffer, and store the data from the spectrometer system an on-line computer is required. These functions must be done with adequate bandwidth so that the data are accepted at the expected rates. In addition to these functions, it is also highly desirable to have adequate storage and computation capability to accomplish at least logical checks of the data, a small amount of additional data reorganization or formatting and a minimum level of histogramming capability.

For the spectrometer if we assume a maximum of

64	magneto-strictive wire chamber scalers	64 words
128	counters and other ON/OFF bits	8 words
16	analog to digital signals	16 words
8	counter scalers	8 words
	Miscellaneous data	8 words
	Total words (16 bits assumed)	104 words

then we expect 104 words/trigger. Assuming an average of 30 triggers/pulse with an instantaneous peak rate of three times that, then a buffer of 9600 words of a 16 bit or larger word size machine is required. This would imply a core memory requirement of about 32,000 words (16 or 18 bits) with access times in the vicinity of 1 microsecond, giving adequate data and histogram buffer areas and also system and user program area. Transfer time through a direct memory connection should take less than 500 μsec per event, which is more than adequate for this event rate.

In addition to this memory configuration, the computer should have:

a. direct data connection capability
b. interrupt system
c. CRT display
d. hard copy output capability
e. a slow card reader
f. large computer-compatible tape drive of high quality. One such tape unit is necessary and it is assumed that the laboratory will have spare tape drive units available to all research groups while they are running experiments.

g. A program and data storage medium, such as another tape (compatible or not) or a disk.

A system such as a PDP15 or Sigma 3 integrated directly into the experimental set-up will satisfy these requirements.

This equipment must be available and operational at the start of the experiment test run. It would not be possible to acquire data and make the necessary on-line diagnostic checks without such an on-line computer system.

5. Cost Estimate, Division of Responsibility and Schedule

I. Cost Estimate

Activity		Estimated Cost to NAL K$	Estimated Cost to User Group K$	User Group Providing Funds
A)	1) Structure including one service bldg., shielding cover and dirt beam stop	250		
B)	Special shielding for the experiment	50		
C)	Additional utility lines	50		
D)	Front end magnet system including power supplies	75		
E)	Spectrometer magnets including DC power supplies, regulators and controls	400		
F)	On-Line computer and related equipment	150		
G)	NAL engineering and technician support for above activities (1/2 ME, 1/2 EE, 1 MT, 1 ET, 1 DD, for a year)	60		
H)	Hydrogen target and associated equipment	50		
I)	Wire spark chambers and associated interface equipment		50	Cornell
J)	Beam monitors, trigger and other counters and fast electronics		100	MIT
K)	Threshold Ĉerenkov counters		50	ANL
L)	Disk Ĉerenkov counters		50	CERN
M)	Installation	200		
N)	Contingency		165	Bari, Brown CERN, Cornell, MIT
	Sub-total	1285K$	415K$	

Grand Total..........1700K$

II. Division of Responsibility

		Responsible Research Group	People Involved
1.	Building and Facilities including shielding, beam dumps, hydrogen target, installation of apparatus and interfacing with NAL Beam Transfer, Experimental Facilities, Plant Engineering, Operations and Safety Groups.	NAL	M. Awschalom R. Juhala A. L. Read P. J. Reardon of NAL
2.	Spectrometer Design and Construction.	NAL	R. Billinge R. Peters J. Schivell of NAL
3.	Wire spark chambers and associated interface equipment.	Cornell	B. Gittelman E. Loh of Cornell
4.	Counter Hodoscopes, trigger and other counters and fast electronics.	MIT	J. Friedman H. Kendall L. Rosenson of MIT R. Lanou of Brown L. Guerriero of Bari
5.	Disc Ĉerenkov counters.	CERN	G. Cocconi and 2 or 3 others of CERN R. Juhala and T. White of NAL
6.	Threshold Ĉerenkov counters	ANL	R. Diebold of ANL
7.	On-Line-computer.	NAL	A. Brenner of NAL
8.	Data Analysis.	All Groups	Appropriate people as required
9.	On-Line computer Software.	NAL ANL MIT	Appropriate people as required

at NAL. These MIT collaborators have agreed to have the full time equivalent of three physicists and 1 programmer ready to take full research time responsibility for the instrumentation of this experiment beginning November 1, 1970. A similar situation exists with the Rosenson group at MIT, the Lanou group at Brown and the Guerriero group at Bari. The above groups estimate that all of the instrumentation can be ready and installed and individuals will spend such time at NAL as is required to effect coordination and installation.

G. Cocconi has indicated that he will personally take an active role in getting this experiment ready for running.

B. Gittleman and E. Loh have indicated that they will take the responsibility for building at Cornell all of the wire spark chambers and associated interface equipment required to instrument the spectrometer.

R. Diebold and his ANL group will build the four threshold counters at Argonne and also contribute to the development of Data Analysis programs. The close proximity of ANL to NAL makes this aspect of the collaboration most efficient, particularly in the areas of the final design of the spectrometer and the Čerenkov counters.

IV. Schedule

If this proposal is approved, we plan to have the spectrometer installed and ready for beam trials by the summer of 1972, or two years from now. Working with the NAL Experimental Facilities Section we would hope to have all of the interfaces with the Area 2 Experimental Hall resolved to the point where the enclosure to house this spectrometer could be included in the Request for Proposal for the construction of Area 2.

The same is true for the additional utilities which are required. With the size of the group and the fact that these facilities responsibilities are to be undertaken by the NAL people involved in the experiment, we believe that our schedule can dovetail with that of Area 2 so that the design of the spectrometer facility can be integrated with that of Area 2 and all its services.

The same point of view can also be taken with regard to the secondary beam design. The elements for the spectrometer can be purchased at the same time as those for the secondary beam and should further study on the secondary beam elements indicate that magnets other than main ring or proton beam-transfer magnets are preferable, then it may also be that these elements are equally or more suitable as spectrometer elements. The installation of both the secondary beam and the spectrometer could be done by the same people in sequence.

V. Conclusion

We believe that our installation schedule can dovetail with that of the facilities, services and secondary beam elements of Area 2. This group is ready to go, and is committed to match the Area 2 schedule completion. We plan to have the spectrometer and all of the instrumentation built and installed simultaneously with the 2.5 milliradian secondary beam if this proposal is approved in the fall of 1970.

APPENDIX A

SPECTROMETER DESIGN FOR AREA #2

Table A-1 lists the parameters of the 200 GeV/c spectrometer; the optics are shown in Fig. A-1. There are two stages, the first with a 12 mrad bend followed by a collimator to stop particles which are off momentum by more than a few percent. The second stage has a parallel section for differential Ĉerenkov counters, followed by about 36 mrad of bend; the particles are then focused onto various detectors. The first three quadrupole doublets have identical properties; the last one has less strength, giving some magnification.

The production angle is varied by steering the beam near the target. For good resolution the spectrometer bends are orthogonal to the production angle.

The spectrometer has been designed around main ring magnet elements. Their costs are known and their properties well understood; these are listed in Table A-2. Other magnets specially designed for this purpose might better optimize the spectrometer, but a broad range of interesting physics can be done using this spectrometer. Since the main ring magnets require considerable power and high current at peak field, we plan to run these elements at less than maximum field. The spectrometer has been designed so that all dipole and quadrupole magnets are connected in series.

The spectrometer itself is a well instrumented beam transport system. The design shown in Fig. A-1 is 216 meters

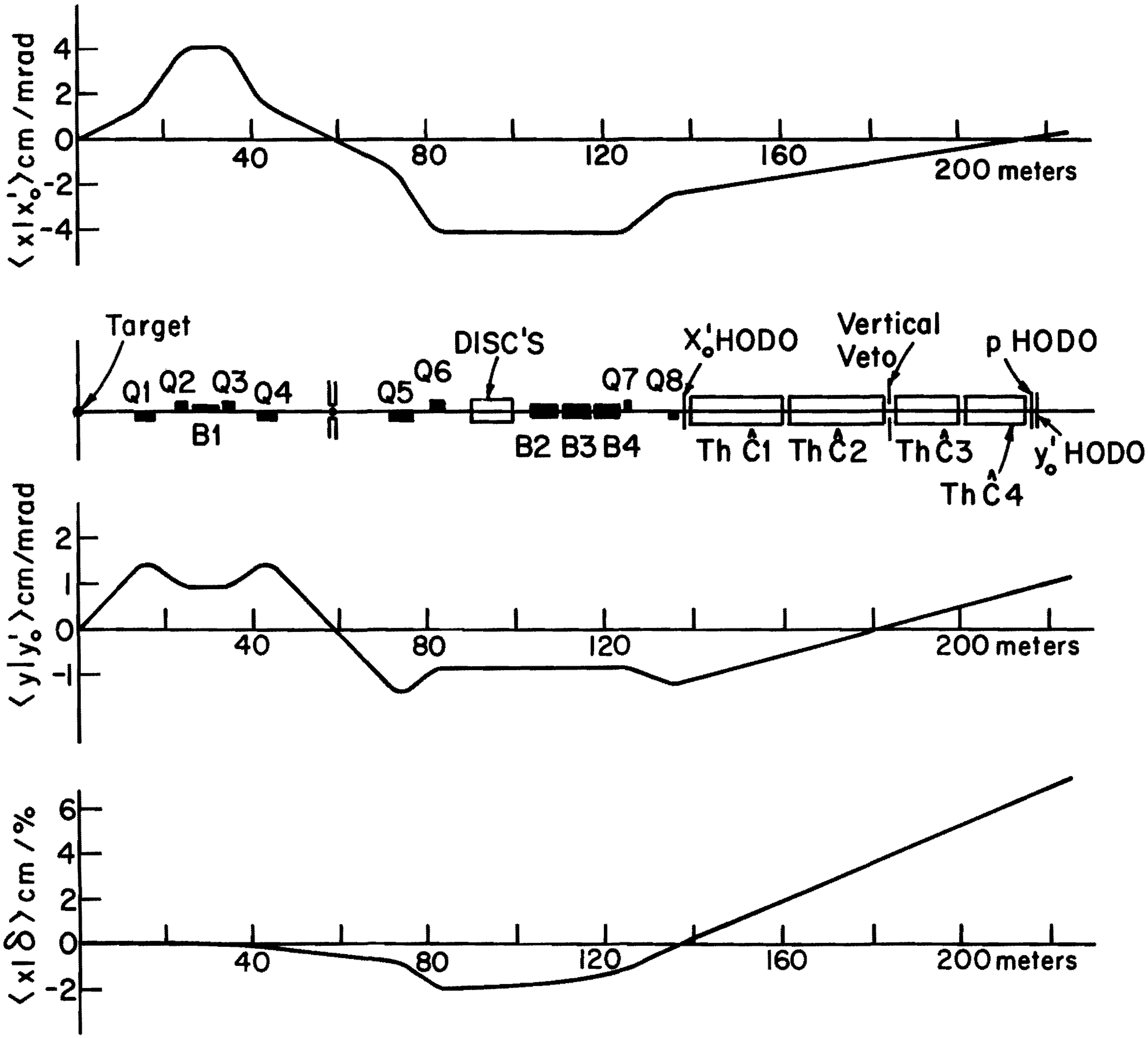

Fig. A-1. Spectrometer and Optics Details.

long and requires 0.9MWatts at 200 GeV/c.

The following sections discuss the parameters in more detail.

1. Initial beam spot size. $x_o = \pm 0.05$ cm (vertical), $y_o = \pm 0.1$ cm (horizontal).

The momentum resolution of the spectrometer depends critically on x_o. At finite production angles the projected target length in the nonbend (y) direction of the spectrometer is $L_{tgt} \cdot \theta_{prod.}$ For a 10 cm target, ± 0.1 cm corresponds to a production angle of 20 mrad ($-t = 16$ GeV^2 at 200 GeV/c). The divergence of the beam at the differential Čerenkov counters is mainly determined by y_o; for example, at 200 GeV/c good π-K separation will require $|y_o| < 0.1$ cm (see below).

2. First stage cleanup. $\Delta p/p = 2\%$.

The first stage is symmetric with point-to-point imaging in both planes with unit magnification. The bending magnet in this stage gives a dispersion of 0.51 cm/% at the cross-over where there will be massive collimators defining both x and y. To calculate the fuzziness of the slits we consider a particle at 1 mrad and assume that one meter (8 absorption lengths, 250 radiation lengths) of heavymet (tungsten alloy) will effectively stop the particle and its interaction products. This leads to each edge having a fuzzy region of about 1mm. A 1 cm slit should thus have relatively well defined edges; this would give a 2% momentum bite (4 GeV/c). Chromatic aberrations (the $\langle x|x_o'\delta\rangle$ term) will smear out the edges of the cut in δ leaving a flat δ acceptance over 63% of the nominal bite. Depending on rates the collimator could be opened wider (the pass band of the spectrometer is 5% FWHM). In the y direction a 0.5 cm slit should clean up pole face scattering from the first stage.

3. Solid Angle. $\Delta\Omega = 4.5\mu ster$; $\Delta x_o' = 2.2$ mr (vertical), $\Delta y_o' = 2.6$ mr (horizontal).

There is a very direct trade-off between solid angle and the various resolution requirements. In order to maximize the counting rate and t range accepted by the spectrometer, a large solid angle is clearly desirable. For fixed-aperture magnets, however, a large solid angle implies short distances, reducing the momentum resolution for a given beam spot size, as well as making the beam divergence larger at the DISC Ĉerenkov counter. We believe the present design to be a reasonable compromise between the various requirements.

4. Angular Resolution. $\sigma_{x_o'} = \pm 0.07$ mrad (vertical); $\sigma_{y_o'} = \pm(0.07$ to $0.14)$ (horizontally).

A measurement of the angle in the bend plane, x_o', is required for the determination of the production angle near 0^o; it is also needed to correct the momentum measurement for chromatic aberrations (see below). One meter after the exit of the last quadrupole there is a δ crossover where x does not depend on δ to first order:

$$x = -2.7x_o - 2.42x_o' + .15x_o'\delta$$

$$x_o' = -\frac{0.41\ x + 1.12x_o}{1 - .06\delta}$$

where the units are cm, mrad, %.

We assume the use of a detector with ±0.1 cm bins; if this resolution is combined with the uncertainty $\sigma_{x_o} = \pm 0.05$ cm and corrections are made for the chromatic aberrations, then $\sigma_{x_o'} = \pm 0.07$ mrad. The chromatic aberrations for the worst case ($x_o' = 1.1$ mrad, $\delta = 1\%$) make a difference in x_o' of 0.07 mrad; this correction is easily made if the high precision is required.

A parallel-to-point focus in the y plane exists 80 meters downstream of the last quadrupole:

$$y = 0.86y_o' + 0.32\ y_o\delta - 0.05\ y_o'\delta$$

$$y_o' = \frac{1.15\ y - .37y_o\delta}{1-0.06\delta}$$

Taking $\sigma_y = \pm 0.1$ cm gives $\sigma_{y_o'} = \pm 0.12$ mrad. At 200 GeV/c this corresponds to an uncertainty in transverse momentum of ±24 MeV/c. If σ_y can be reduced to ±0.03 cm, the contribution to $\sigma_{y_o'}$ is then only ±0.035 mrad; this is comparable to the worst case contribution from the unmeasurable second-order term in the numerator: $|0.37y_o\delta| \leq 0.04$ mrad. A correction can easily be made for the small chromatic aberration term in the denominator.

5. Second stage cleanup.

The y envelope has a waist approximately 47 meters after the exit of the last quadrupole:

$$y = 3.8y_o - 0.21y_o'\delta.$$

All legitimate trajectories are within 1/4 inch of the central trajectory and veto counters can be used to reject those particles outside these limits. Such unwanted particles could come from interactions of the beam halo at the target or from pole-face scattering.

6. Parallelism at Differential Ĉerenkov Counters.

$\sigma_{x'} = \pm 0.13$ mr, $\sigma_{y'} = \pm 0.11$ mr.

At the center of the drift space left for Ĉerenkov counters

$$x' = .24x_o + .12\delta - 0.063x_o'\delta$$

$$y' = 1.1y_o - 0.06\ y_o'\delta$$

For $\delta = 0$, $\sigma_{x_o} = \pm 0.05$ cm, and $\sigma_{y_o} = \pm 0.1$ cm the angular spread becomes ±0.01 and ±0.11 mrad in the x and y directions, respectively. For $\delta = 1\%$, $\sigma_{x'} = \pm 0.13$ mrad.

A DISC counter operating at 17 mrad Ĉerenkov angle will have a separation of 0.3 mrad between the π and K light at 150 GeV/c. The average spread in angle of ±0.12 mrad is somewhat greater than the ±0.1 mrad required for good πK separation, but the DISC should still be capable of some πK discrimination up to the full 200 GeV/c. Good πK separation in this region will require the combined use of both DISC and threshold counters.

7. Momentum resolution. $\sigma_\delta = \pm 0.032\%$

At the momentum focus (80 meters downstream of the last quadrupole)

$$x = 3.2x_o + 6.6\delta - 0.94x_o'\delta$$

$$\delta = \frac{.15x - .49x_o}{1 - .14x_o'}$$

Uncertainties of $\sigma_x = \pm 0.1$ cm and $\sigma_{x_o} = \pm 0.05$ cm combine to give $\sigma_\delta = \pm 0.029\%$. Taking $\sigma_{x_o'} = \pm 0.10$ mrad (the uncertainty in x_o' if δ information is not used) the chromatic aberration gives a contribution to σ_δ of $< \pm .014$.

At 200 GeV/c the total uncertainty in the spectrometer momentum measurement is about ±65 MeV/c, somewhat less than half the 150 MeV/c separation between elastic and inelastic scattering. For inelastic scattering (e.g., $pp \to px$) at small t, the resolution on the missing mass is

$$\sigma_{M_x} = \frac{M_p}{M_x}\,\sigma(p_o - p)$$

where $p_o - p$ is the difference between initial and final proton momenta. An uncertainty in the final momentum of 65 MeV/c gives $\sigma_{M_x} = \pm 40$ MeV at the second resonance (1520 MeV), compared to its natural width of about ±60 MeV. Figure A-2 shows results of the CERN group obtained with a single-arm spectrometer at 19.2 GeV/c. Even at the largest momentum transfer ($-t \approx 6$ GeV^2) they were able to obtain a good elastic signal in spite of the relatively large inelastic cross section at large momentum transfers. Their resolution of about ±60 MeV/c was just barely adequate to separate the second and third resonances, however.

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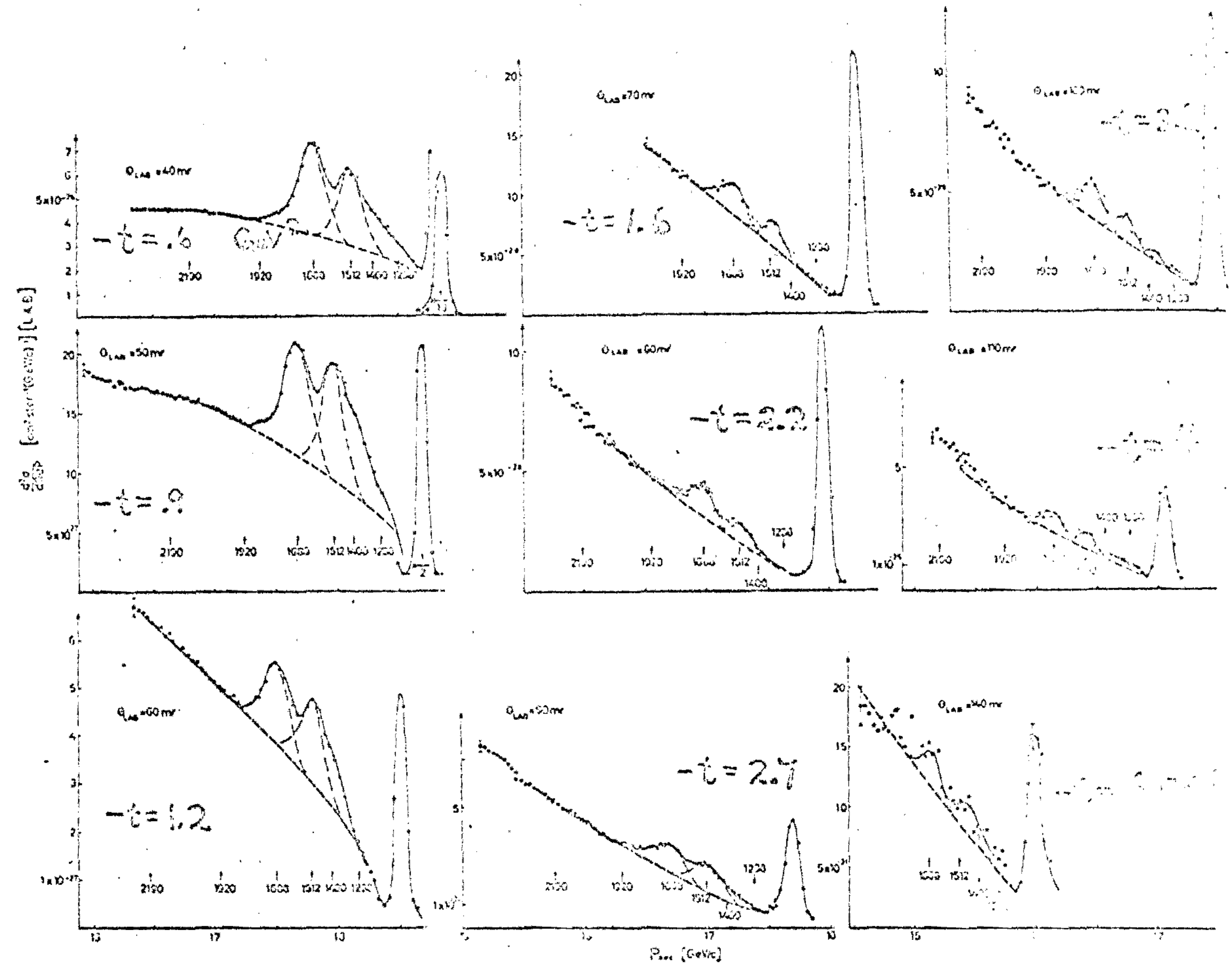

Figure A-2. Proton spectra obtained by J. V. Allaby et.al., Phys. Rev. Letters <u>28B</u>, 229 (1968).

TABLE I. Design Parameters of Focusing Spectrometer for Area 2

p_{max} (GeV/c)	200
ΔΩ (μster)	4.5
Angular Range (mrad)	0 to 50
Angular Acceptance (mrad)	
Bend Plane	±1.1
Production Plane	±1.3
Angular Resolution (mrad)	
Bend Plane	±0.07
Production Plane	±(0.07 to 0.14)
Angular Dispersion (cm/mrad)	0.83
Momentum Dispersion (cm/%)	
Cross-over	0.51
p-Hodoscope	6.6
Momentum Resolution	±0.032%
Angular Spread at DISC (mrad)	±0.12
Bend Direction	Vertical
Threshold cntr length (meters)	80
Over-all length (meters)	220
Number of magnets	
Bend (6m each)	4
Quads (half 2.1m and half 1.3M)	14
Power at p_{max} (MWatts)	0.9

TABLE II. Parameters of Magnets Used for Spectrometer Design

	Bends (B1)	Quads (2.1M)	Quads (1.3m)
Nominal Aperture (inches)	1.5 x 5	2 x 5	2 x 5
Useable Aperture assumed (inches)	1.2 x 4	1.6 x 4.5	1.6 x 4.5
Maximum useable field assumed	13.6 kG	5 kG/inch	5 kG/inch
Current at maximum field (amps)	3500	3500	3500
Power at maximum field (kW)	75	40	26

APPENDIX B

DIFFERENTIAL ACHROMATIC ČERENKOV (DISC) COUNTER FOR NAL 200 GeV BEAMS

1. The number of protons produced by Čerenkov radiation is

$$\frac{dN}{dK} = 2\pi L \alpha \sin^2\theta \quad K = \frac{1}{\lambda} .$$

With the best phototubes, the accepted light-band ($2200 < \lambda < 5000$ Å, $\Delta K = 2.5 \times 10^4$ cm^{-1}) has an average efficiency $\bar{\Sigma} = 0.07$, taking into account the losses in the optical system of a DISC (see Duteil, et.al, CERN, 68-14). The total number of photoelectrons per unit length is then

$$\frac{N}{L} = 2\pi \alpha \Delta K \bar{\Sigma} \theta^2 = 80 \theta^2 \text{electrons cm}^{-1} .$$

If eight phototubes collect the light along the circumference, the average number of photoelectrons per tube $\bar{N}$ must be such that the eightfold coincidence has a probability of occurring of at least 2/3, i.e.

$$(1 - e^{-\bar{N}})^8 = 0.66.$$

This gives $\bar{N} = 3.0$, $N = 8$, $\bar{N} = 24$, and for the length L of the Čerenkov counter

$$L = \frac{0.30}{\theta^2} \text{ cm.} \qquad (1)$$

2. From the relation giving the Čerenkov angle

$$\cos\theta = \frac{1}{\beta n}$$

differentiation gives

$$tg\ \theta = \frac{d\beta}{\beta} + \frac{dn}{n} .$$

When dealing with high-energy particles

$$\gamma = \frac{E}{n} >> 1, \quad \beta = \sqrt{1 - \frac{1}{\gamma^2}} = 1 - \frac{1}{2\gamma^2}$$

and small Čerenkov angles ($\theta << 1$), the index of refraction of the gas in the counter is close to unity ($n - 1 << 1$) and the first relation can be simplified:

$$\cos\theta = 1 - \frac{\theta^2}{2} = \frac{1}{\left(1 - \frac{1}{2\gamma^2}\right)[1 - (n - 1)]} = 1 - \left[(n - 1) - \frac{1}{2\gamma^2}\right]$$

which gives

$$(n - 1) = \frac{\theta^2}{2}\left[1 - \frac{1}{(\gamma\theta)^2}\right] . \qquad (2)$$

For a gas, the relation between n and the pressure P, in atmospheres, is

$$(n - 1)_p = (n - 1)_{p=1} \times p$$

and the pressure needed in the counter is

$$P = \frac{\theta^2}{2(n - 1)_{p=1}}\left[1 + \frac{1}{(\gamma\theta)^2}\right] \text{atm.} \qquad (3)$$

3. At fixed n, i.e., fixed gas pressure and temperature

$$\frac{d\beta}{\beta} = \theta\ d\theta .$$

At fixed energy (or momentum) E, two particles of mass m_1 and m_2 have

$$\Delta\beta = \frac{m_1^2 - m_2^2}{2E^2},$$

and in principle the separation of the two masses requires a circular slit which, after chromatic corrections, accepts light, satisfying the relation:

$$\theta\ \Delta\theta = \frac{m_1^2 - m_2^2}{2E^2} .$$

In practice, the effective separation, when the wanted particles (K-meson) are a small percentage of the unwanted ones (π meson), must be at least <u>three</u> times as large, and the maximum energy resolved is given by the relation

$$\theta\ \Delta\theta = \frac{m_1^2 - m_2^2}{6E^2}\ . \qquad (4)$$

TABLE B-1

	$m_1^2 - m_2^2$
d-p	2.66 GeV^2
p-K	0.63 "
p-π	0.86 "
K-π	0.228 "
π-μ	0.083 "

4. A practical lower limit for the angular width of the slit is

$$\Delta\theta = 10^{-4} = 0.1\ \text{mrad} = 20\ \text{sec. of arc.}$$

This represents also the maximum beam divergence accepted by the counters at maximum resolution.

First let us see what is the limitation introduced by the aberration of the spherical mirror that focuses the Ĉerenkov light on the slit:

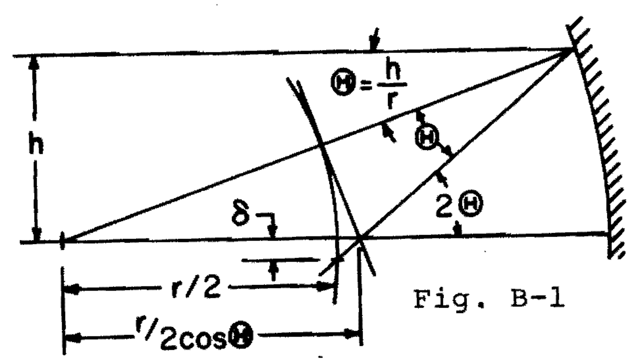

Fig. B-1

h = width of light beam

$r = 2f$ - radius of curvature

δ = spherical aberration

= width of image at focal plane

$$\delta = \left(\frac{r}{2\cos\theta} - \frac{r}{2}\right).2.\theta = \frac{r}{2}\ .\left(\frac{1}{\cos\theta} - 1\right).\ \frac{2h}{r} \approx \left(1 + \frac{\theta^2}{2} - 1\right) = \frac{h^3}{8f^2}\ .$$

The confusion angle produced by spherical aberration is then

$$\Delta\theta_{sph} = \frac{\delta}{f} = \frac{h^3}{2f} \quad . \qquad (5)$$

Since $h \approx 2f\theta, \Delta\theta_{sph} = \theta^3$, and the condition

$$\Delta\theta_{sph} < 10^{-4}$$

gives as an upper limit for the Čerenkov angle

$$\theta < 10^{-1.33} = 45 \text{ mrad},$$

a condition not difficult to meet in our case.

5. The chromatism of the Čerenkov radiation should be corrected because otherwise, at constant β, the resolution of Eq. (4) cannot be reached:

$$\theta\ \Delta\theta_{chrom} = \frac{\Delta n}{n} = \frac{\Delta n}{n-1}\ \frac{n-1}{n} = \frac{1}{V}\ (n-1) \quad (n \approx 1) \ ,$$

where $V = (n-1)/\Delta n$ is Abbe's number of the gas evaluated for a pair of wavelengths ($\lambda_1 = 2800$, $\lambda_2 = 4400$ A) representative of the band accepted by the photomultipliers.

Using expression (2) one obtains

$$\Delta\theta_{chrom} = \frac{\theta}{2V}\left[1 + \frac{1}{(\gamma\theta^2)}\right] . \qquad (6)$$

This equation also shows that the chromatic angle is not constant, but changes as γ changes.

TABLE B-2

Substance	$V = (n - 1)/\Delta n$ $\lambda_1=2800$ $\lambda_2=4400$	$n - 1$ $\lambda=3600$
SiO_2	17.5	0.474
NaCl	9.2	0.580
KCl	8.3	0.524
H_2	16.0	$1.44 \times 10^{-4} \times P$
He	55	$0.35 \times 10^{-4} \times P$
CO_2	19.3	$4.62 \times 10^{-4} \times P$
A_{iv}	21.4	$3.01 \times 10^{-4} \times P$
SF_6		$7.06 \times 10^{-4} \times P$

6. The chromatism of the Čerenkov light can in part be corrected with an axicon (i.e., a circular prism). In the original DISC, the axicon is composed of two elements glued together, one of SiO_2, the other of NaCl, that converge the blue and the ultraviolet light on the slit without any over-all deflection. For the large and long-focus mirrors needed at our energies it is possible to use a single SiO_2 element that, notwithstanding the slight over-all deviation of the light toward the axis, does not create any interference with the accepted beam.

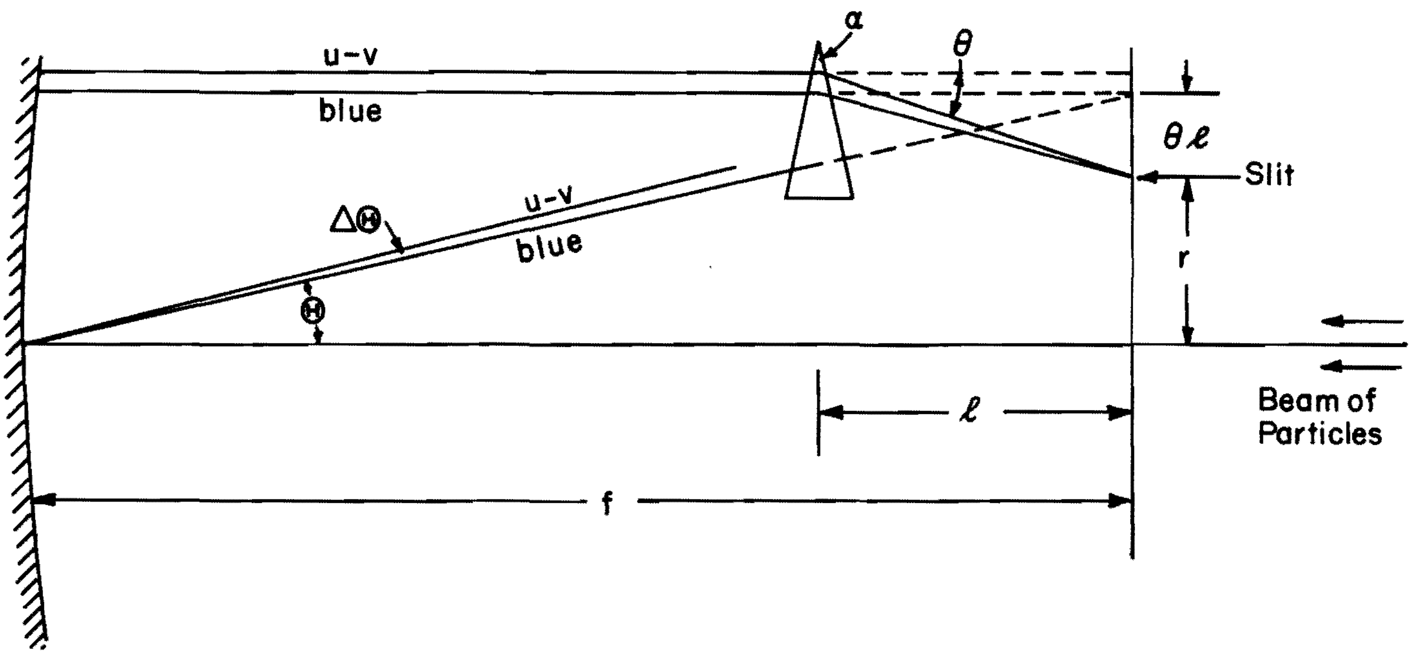

Fig. B-2.

For a prism at minimum deviation

$$n = \frac{\sin(\alpha + \theta)/2}{\sin \alpha/2} \underset{\text{small angles}}{} 1 + \frac{\theta}{\alpha}, \qquad \theta = \alpha(n - 1)$$

$$\frac{d\theta}{\theta} = \frac{dn}{n - 1} \text{ and } \Delta\theta = \theta \frac{\Delta n}{n - 1} = \frac{\alpha(n - 1)}{V}$$

where $V = (n - 1)/\Delta n$ is the Abbe' number of the glass (see Table 2).

The achromatism is obtained when

$$\Delta\theta \ \ell = \Delta\theta_{chrom} f,$$

consequently

$$\alpha = \frac{V}{n - 1} \frac{f}{\ell} \Delta\theta_{chrom}$$

$$\theta = V\frac{f}{\ell} \Delta\theta_{chrom}. \qquad (7)$$

Since $\Delta\theta_{chrom}$ depends on γ [Eq. (6)] once α is fixed, when γ changes, ℓ should be changed in order to satisfy Eq. (7). In that case also θ changes, and since the radius of the circular slit r is fixed, the change of ℓ implies a slight change of θ, the Čerenkov angle.

In practice, a fixed position of the prism could be satisfactory over a limited range of γ, and this is the solution described below.

7. Example for the 200 GeV beam

The most demanding doublet is $K\pi$, and Table 3 shows the values of $\theta\Delta\theta$ needed.

TABLE B-3

Energy (GeV)	$\theta\Delta\theta$ for $K - \pi$ [Eq. (4)]
200	0.95×10^{-6}
180	1.15×10^{-6}
150	1.70×10^{-6}
100	3.8×10^{-6}

The instrument can be optimized for 150 GeV K mesons ($\gamma = 300$). Then $\theta\Delta\theta = 1.7 \times 10^{-6}$. With $\Delta\theta = 10^{-4}$, $\theta = 17$ mrad, and [Eq. (1)], the length of the counters is $L = 0.3/\theta^2 = 10^3$ cm = 10 m. These parameters determine all the others given in Table 4.

When particles with γ different from 300 are analyzed, the chromatism is not fully corrected. However, provided

TABLE B-4

Ĉerenkov angle..............................	$\theta = 1.7 \times 10^{-2} = 17$ mrad
Slit aperture	$\Delta\theta = 10^{-4} = 0.1$ mrad
Slit width..................................	$f\Delta\theta = 1.0$ mm
Focal length of mirror f = L.................	$f = 10^{3}$ cm = 10 m
Diameter of accepted beam....................	$2\rho = 10$ cm
Diameter of mirror = $2\rho + 20f$................	$2R = 44$ cm
Chromatism of Ĉerenkov light $\gamma=\infty$ .... [Eq. (6)] for air	$\Delta\theta_{chrom} = 3.97 \times 10^{-4}$
$\rightarrow \gamma=300$............	$\Delta\theta_{chrom} = 4.12 \times 10^{-4}$
Distance of prism from focal plane...........	$\ell = f/10 = 10^{2}$ cm
Height of light beam on mirror = $\theta f + 2\rho$......	$h = 27$ cm
Minimum height of prism = $h\ \ell/f$..............	$h_m = 2.7$ cm
Central radius of axicon $\theta(f - \ell)$..........	$\delta = 15.3$ cm
Angle of SiO prism [Eq. (7)]................	$\alpha = 0.152 = 8.75^{o}$
Mean deviation produced by prism.............	$\theta = 0.072 = 4.15^{o}$
Distance of slit from axis = $\theta f - \theta\ell$.........	$r = 98$ mm
Air pressure $\gamma=\infty$; $1 + 1/(\gamma\theta)^{2}$ = 1.0000........ [Eq. (3)]	$P = 0.478$ atm
$\gamma=300$ 1.0385........	0.496
$\gamma=200$ 1.0865........	0.520
$\gamma=100$ 1.348.........	0.643
$\gamma=50$ 2.39..........	1.140
$\gamma=100$: Chromatism of Ĉerenkov light.......	$\Delta\theta'_{chrom} = 5.35 \times 10^{-4}$
Width of beam at focal plane.....	$f(\Delta\theta' - \Delta\theta) = 1.23$ mm
$\gamma=50$: Chromatism of Ĉerenkov............	$\Delta\theta''_{chrom} = 9.5 \times 10^{-4}$
Width of beam....................	$f(\Delta\theta'' - \Delta\theta) = 5.4$ mm

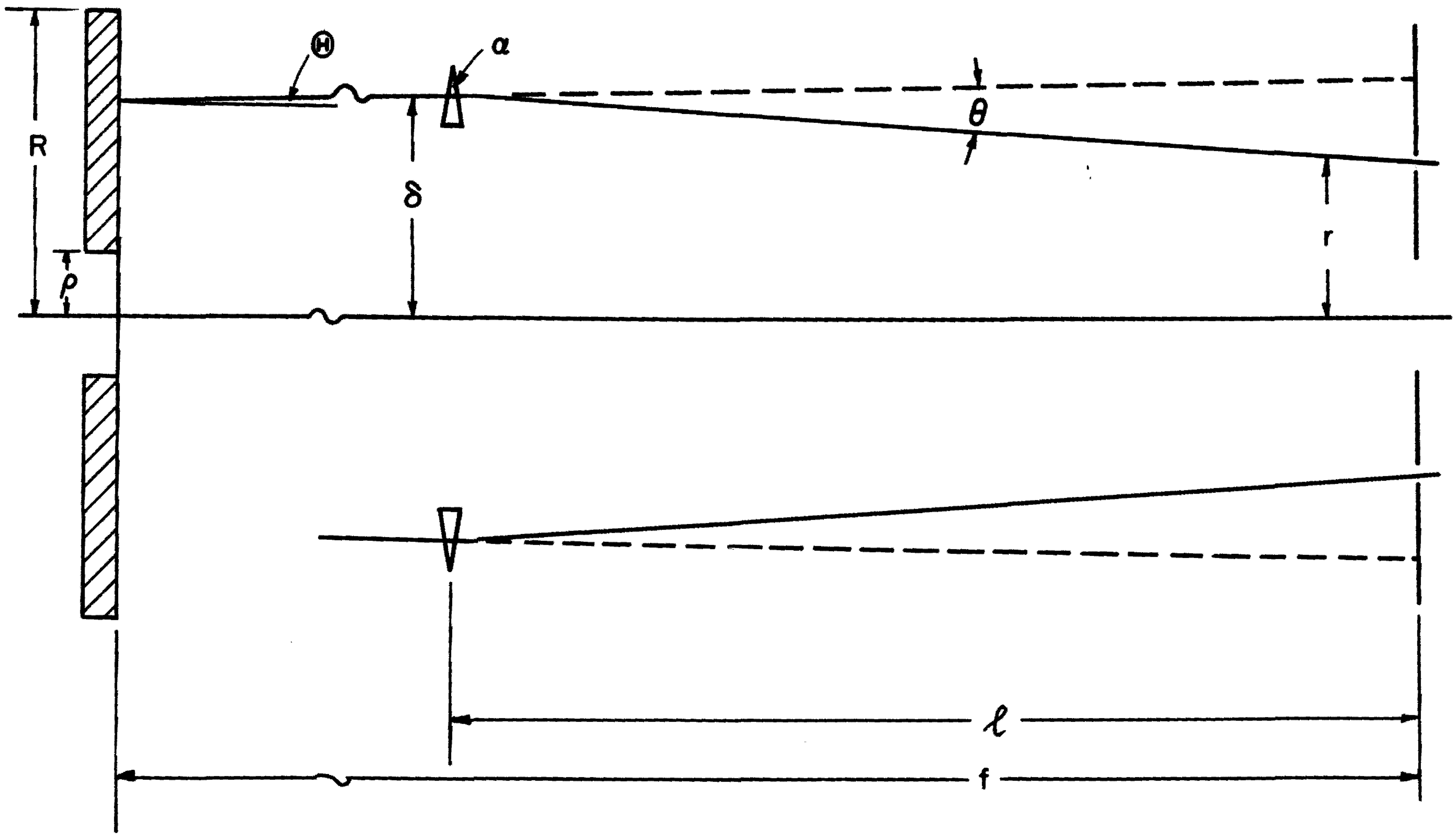

Fig. B-3

$\gamma > 100$, the resolution of the instrument remains almost unchanged, as can be seen from the example given in the table.

At $\gamma = 50$, the Čerenkov light is distributed, at the focal plane, over a ring $\sim$ 6 mm wide and the slit should be opened accordingly. This is the case for 50 GeV protons (or antiprotons), but then the pK doublet is separated at the focus by 74 mm. $\gamma \approx 50$ should be considered the lowest value of γ at which the instrument can be useful.

8. Comments on the proposed center

a) The alignment of the counter within 0.1 mrad is delicate. Also, the particle beams should be of adequate quality.

These requirements, however, should be met for any high-resolution DISC, independent of its length.

b) The great length implies a rather large mirror ($2R \approx 50$ cm) of long radius of curavture ($2f = 20$ m), and a structure of adequate rigidity.

c) The gas pressure inside the counter, never greater than few PSI above atmospheric pressure, only demands a container that can be evacuated. It allows thin (10 cm) windows for the beam. It is also possible to use the phototubes without any window in front, a gain in light collection.

d) The width of the slit should be adjustable from the outside, over a range from 1-6 mm. It is the only movable part of the counter. The light collection after the slit can be helped by mirrors and light guides. It is assumed that all of the light eventually reaches the eight phototubes.

e) At fixed β(or γ, or E), $dn/n = \theta\Delta\theta \approx 10^{-5}$. This implies that the index of refraction of the gas inside the counter can be measured and monitored with a precision $dn/n \approx 10^{-8}$, if reproducibility is wanted. A refractometer placed inside the counter itself is necessary, to avoid pressure and temperature differences. The refractometer used by Meunier et.al., consisting of an interferometer with a laser beam and an electronic system reading the number of fringes, performed very well and seems to provide the best solution. Unless it can be found on the market, this part of the project constitutes by itself a serious enterprise.

f) Astigmatism of the prism: only the central ray crosses the prism at minimum deviation, θ. Rays coming from the extreme of the light cone, i.e., at an angle $\Delta\phi = \theta/2$ to the central ray, are deflected by $\theta + \Delta\theta$:

$$\Delta\theta = \Delta\phi \left(\frac{\partial\theta}{\partial\phi}\right) + \frac{\Delta\phi^2}{2} \left(\frac{\partial^2\theta}{\partial\phi^2}\right)$$

$\partial\theta/\partial\phi = 0$ at minimum deviation (See Born and Wolf, p. 178).

$$\frac{\partial^2\theta}{\partial\phi^2} = 2\ \mathrm{tg}\phi \left(1 - \frac{\mathrm{tg}^2\psi}{\mathrm{tg}^2\phi}\right) \xrightarrow{\phi = n\psi \approx \alpha/2 << 1} \alpha \left(1 - \frac{1}{n^2}\right)$$

and, at the focal plane, the width of the image due to prism astigmatism is

$$\Delta r_{ast} = \ell\Delta\theta = \frac{1}{8}\ell\theta^2 \alpha \left(1 - \frac{1}{n^2}\right).$$

With the figures of Table 4 one obtains

$$\Delta r_{ast} = 2.9 \times 10^{-4}\ \mathrm{cm} = 2.9 \times 10^{-3}\ \mathrm{mm}.$$

a completely negligible quantity compared with the 1 mm minimum width of the slit.

This is true, however, only for particles moving along the optical axis.

Off-axis particles give rise to astigmatism that for an axicon can become serious. This point should be analyzed more deeply.

According to Meunier, a solution involving not an axicon but a curved-face corrector (a lens with a hole) could be more advantageous and simpler to build.

APPENDIX C

Possible Spectrometer System for Use in Area #3 Proton Beam Enclosures

The principal advantage of this alternate proposal is that it allows a sound physics program to be started about one year earlier than would be possible in Area #2. It also enables measurements to be made up to momenta of 500 GeV/c, but at the expense of a smaller solid angle, viz, about one microsteradian. This system would have a resolution $\frac{\Delta p}{p} = \pm 0.03\%$. The layout of the spectrometer in Area #3 proton beam enclosures is shown in Figure 1C.

The costs for this project are modest since many of the costs are covered by proton tunnels and beam transfer elements which are to be provided anyway, although they would have to be purchased earlier than had originally been planned. The cost estimate for this alternate is 800K, 415K of which would be provided by the User collaborators and 150K of which is for an on-line computer which would be on loan to this research effort. The costs to NAL in FY-71 for stepping up the installation of the two long beam transport channels to bring a 500 BeV beam to Area #3 would be approximately 500K including utility lines and installation.

APPARATUS

In the optics of this system, the point-to-parallel focus is made with a doublet, each element of which consists of 2 Main Ring 84-in. quadrupoles. By simply reversing

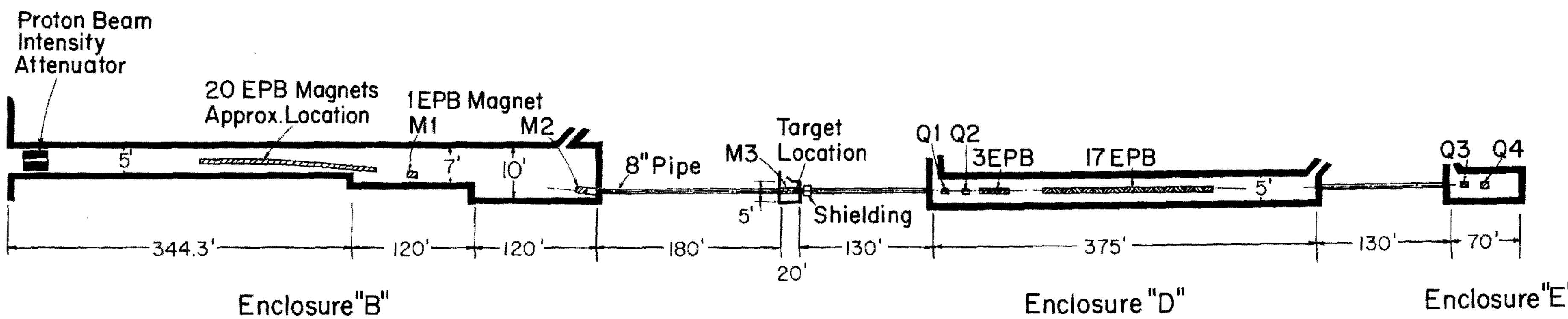

Fig. C-1 (a). Spectrometer in Proton Beam Enclosure #3.

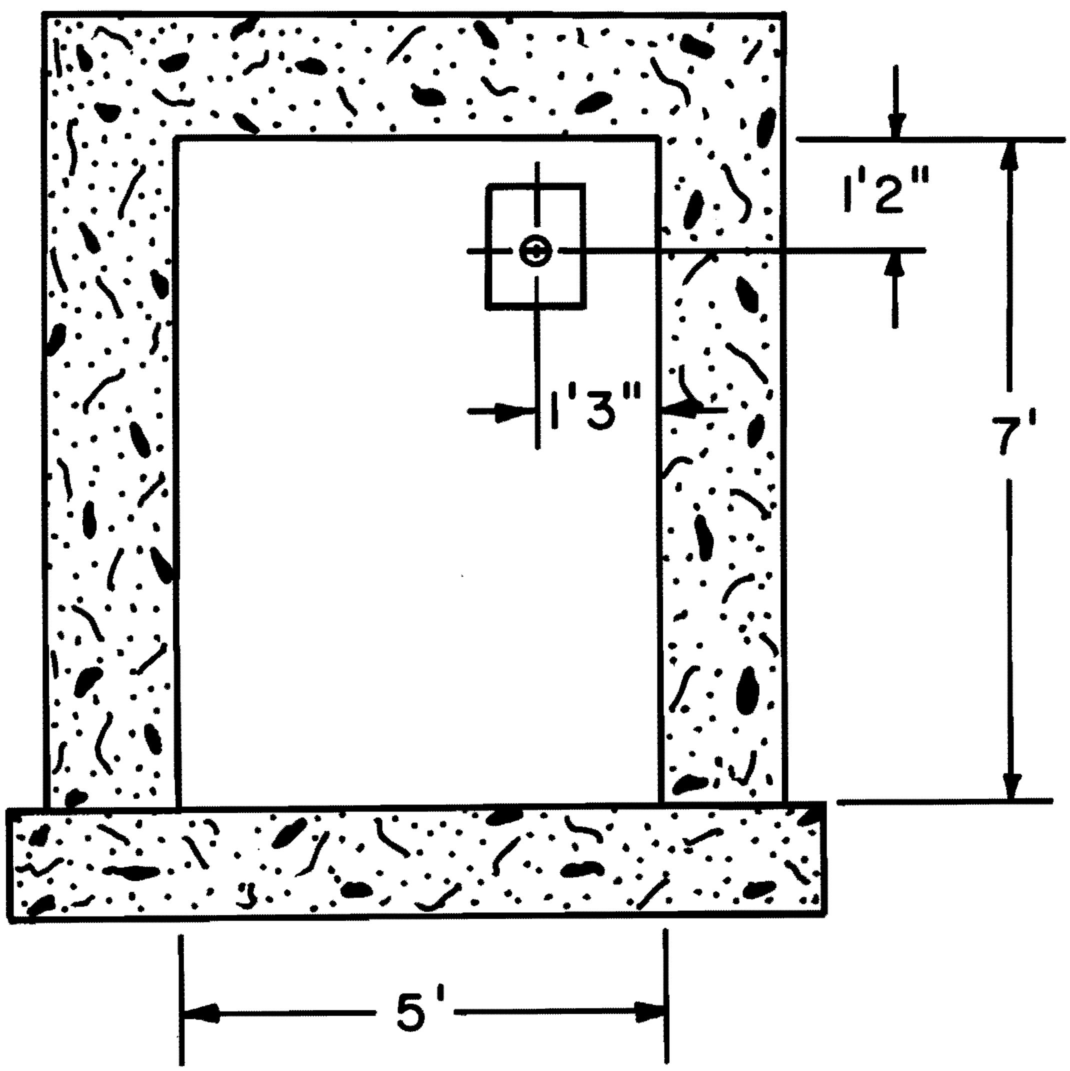

Fig. C-1 (b). Cross Section of Enclosure D, Looking Downstream.

the current leads, we can run with the doublet either FD or DF. Assuming, for the present, a sufficiently small vertical spot size (2 mm total height), we choose the DF mode because of its larger angular acceptance and better momentum resolution.

The rear lens focuses in the bend plane only and can be made with 2 Main Ring 84-in. quadrupoles.

Below is a summary of the magnets needed:

Type	Number	Field (or Gradient)
Quad, aperture 4.0 cm x 11.7 cm 2.13 m length		(for secondary momentum of 500 GeV/c)
in Qd	2	-2.83 kG/cm
in QF	2	+2.50 kG/cm
Bending aperture 3.8 x 8.9 cm length 3m	20	14.0 kG
Quad length 2.13m	2	+3.00 kG/cm

The acceptance is determined as follows. The aperture limit in the bend (x-) plane is the good field width of the bending magnets, $x = \pm 4.5$ cm. This implies $\theta_o = \pm 0.57$ mrad, with our focal length $f_x = 75.8$ m. (The quads at the end of the spectrometer give a matched aperture). The y aperture limit is determined by the front D quadrupoles, which give $y = \pm 2.0$ cm. The y acceptance is $\phi_o = \pm 0.5$ mr. The solid angle acceptance is then $\Delta\Omega = 0.90$ μsr. Calculations for running in the FD mode give $\theta_o = \pm 0.8$ mr, $\phi_o = \pm 0.25$mr, $\Delta\Omega = 0.63$μsr.

(The θ_o acceptance for this mode is limited by the rear quadrupoles. It could be increased by making wider quadrupoles and/or reducing the momentum acceptance from the ±1% taken here.)

The resolution of the system has been determined by a second-order calculation. It has been assumed that one would measure x at planes (1) and (2) to better than ±0.5 mm. The angle θ observed is given by:

$$\theta = -0.13\ x_o + 0.504\ \delta + 0.020\ \theta_o\delta,$$

where $x_o = \pm 0.05$ cm and the other units are mrad and percent. Without measuring θ_o, one has a momentum resolution of $\delta_{min} = 0.078\%$ (FW), and with θ_o measurement, one can achieve $\delta_{min} = 0.025\%$ FW. The position x_1 is given by

$$x_1 = 0.45\ x_o + 7.58\ \theta_o + 1.65\delta + 0.13\ \theta_o\delta,$$

therefore one has a resolution in θ_o of θ_o min = 0.04 mr FW. The position y_1 is given by

$$y_1 = -1.63\ y_o + 3.42\ \phi_o + 0.17\ \phi_o\delta,$$

and with $y_o = \pm 0.1$ cm, ϕ_o min = 0.1 mr FW. Note that one can measure ϕ_o with, strictly speaking, only one y hodoscope.

Triggering on certain momentum bins is done in the focal plane behind the rear quadrupoles.

The angular spreads of the particles through a DISC counter behind the first 3 bending magnets are ±0.08 mr in the bend plane and ±0.03 mr in the non-bend.

A summary of the above (DF mode) spectrometer properties, as well as those of the FD mode, is given below.

	DF	FD
δ_{min} (no θ_o measurement)	0.07%	0.09%
δ_{min} (with θ_o measurement)	0.026%	0.06%
$\theta_{o\ min}$	0.04mr	0.15mr
$\phi_{o\ min}$	0.10mr	0.02mr
at DISC		
θ	±0.08mr	±0.08mr
ϕ	±0.03mr	±0.02mr

The instrumentation that would be used with this spectrometer is the same as that for the Area #2 spectrometer, and the collaborators will assume the same responsibilities as are described in the basic proposal. The variation of the scattering angle would be accomplished in the same way.

A technical problem for which we do not present a solution is the method of varying the intensity of the incident proton beam in tunnel #3. This depends in detail on the intended configuration of beam transfer elements in the region of the convergence of the beam lines. Once this is known, a compatible solution can be given.

Cost Estimate, Schedule, and Division of Responsibility

In order to obtain a starting point for the purposes of making a cost estimate and schedule, it is assumed that this alternate proposal will be approved on or about September 1, 1970. If that is the case, we believe that installation of the apparatus in Sections B and D of the beam transfer enclosure going to future Area 3 (See Fig. C) can begin in the spring of 1971 and that the entire system can be available for research by the fall of 1971, to make proton on proton studies up to incident proton beam energies of 500 BeV/c. A requirement of this approach is that NAL install in the proton beam transfer tunnel arrangement the proton beam transfer elements required to bring a 500 BeV/c proton beam to Area 3 a year or so ahead of the schedule now contemplated. The costs for doing this are not included in the costs for this experiment as this is part of the facilities and apparatus that will be provided by NAL prior to the completion of the construction contract. The costs for the magnets and power supplies required in the front end for production angle variation as well as any special beam dumps or shielding which are required for the experiment are included in our cost estimate. The cost of an on-line computer of the Σ3 class is included in the cost of the experiment but it is recognized that this is a laboratory owned facility that will be loaned to other groups as well, once this experimental program is completed.

Cost Estimate in Thousands of Dollars

Activity		Estimated Cost to NAL	Estimated Cost to user	Responsible User Group
A)	Building modifications (See Fig. 1-C)	20		
B)	Special shielding	20		
C)	Primary beam intensity attenuator	20		
D)	Additional utility lines	50		
E)	Front end magnet system including power supplies	75		
F)	On-line computer	150		
G)	Design construction and assembly effort on items A through G	50		
*H)	Hydrogen target and associated equipment		50	Bari
I)	Proportional wire spark chambers and associated interface equipment		50	Cornell
J)	Beam monitors, triggers proportional counters and fast electronics		100	MIT
K)	Threshold Ĉerenkov counters		50	ANL
L)	Disk Ĉerenkov counters		50	CERN
M)	Installation		50	Brown
N)	Contingency		65	ANL, Cornell MIT
	Sub-total	385	415	
	Grand-total	800		

* Funds to be provided by Bari and target to be constructed at NAL.