MUON-PROTON INELASTIC SCATTERING

Abstract

This experiment will examine muon-proton inelastic scattering for virtual-photon energies of 10 to 110 Gev and for $|q^2|$ values of 0.2 to 20.0 $(\text{GeV}/c)^2$. The virtual-photon total cross sections $\sigma_T + \epsilon\sigma_S$, or the equivalent expression in W_1 and W_2, will be measured over this range of virtual-photon energies and q^2 values. Some separation of σ_T and σ_S, or equivalently W_1 and W_2, will be made. The multiplicity, momentum spectra and angular spectra of the charged hadrons produced in this reaction will be measured. Some channels such as $\mu + p \rightarrow \mu + p + \rho^0$ will be isolated and completely analyzed. The experiment uses a hydrogen target, wire spark chambers and an analyzing magnet of conventional design.

Experimenters

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Collaborative Nature of This Proposal

This experiment is proposed as a collaborative effort with Harvard University and National Accelerator Laboratory physicists. Preliminary discussions have been carried out with Dr. Thomas Kirk representing Harvard University and with Drs. Taiji Yamanouchi and John Sculli representing NAL. In discussions by letter with Dr. Kirk it was agreed that the time was too short to submit a combined proposal before June 15. However it was agreed that this will definitely be a collaborative effort and that the separate proposals will be rewritten into a joint proposal at a later date.

Also in discussions with Dr. Leon Lederman representing Columbia University, it was agreed that there would be a maximum amount of collaboration in the setting up of a muon beam and in the design and fabrication of equipment for the muoproduction of W experiment and for this muon-proton inelastic scattering experiment. In particular, with respect to equipment, the same analyzing magnet will be used in both experiments.

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II PHYSICS JUSTIFICATION

1. Introduction

The purpose of this experiment is to study muon-proton inelastic scattering for virtual-photon energies of 10 to 110 Gev and for $|q^2|$ values of 0.2 to 20.0 $(\mathrm{Gev}/c)^2$ - the higher end of the q^2 range being attained only in the lower two thirds of the virtual-photon energy spectrum. The following information would be obtained:

1. The virtual-photon total cross section $\sigma_T + \epsilon\sigma_S$, or the equivalent expression in W_1 and W_2, will be measured over the entire range of virtual-photon energies and q^2 values.
2. Over a restricted range of these variables, we will be able to separate σ_T and σ_S , or equivalently W_1 and W_2.
3. The momenta and angles of charged hadrons produced in the interaction will be measured, if the angles are less than 60 to 100 mrad. with the direction of the incident muon.
4. The momentum and angular spectra of the negative pions will be particularly useful because the contamination of negative kaons and antiprotons will be small. The spectra of the postive pions and the fast protons will not be separated.
5. The angle and momentum of slow protons making angles larger than 45° degrees with the direction of the incident will be measured.
6. The channel $\mu + p \rightarrow \mu + p + \rho^0$ will be isolated and these events will be completely analyzed.
7. Channels consisting of only charged hadrons and the final muon are also susceptible to analysis, if the particles fall within the angular regions specified in 3. and 5.

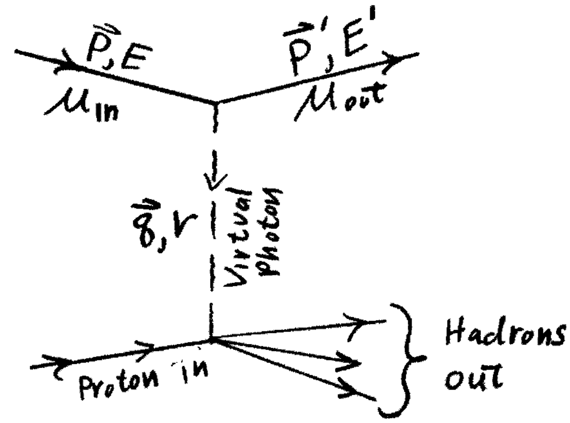
The experiment requires a muon beam with a maximum energy of at least 120 Gev and with an intensity of at least 10^6 particles per pulse. Beam intensities up to 10^7 per pulse may be used if the background event rate is not too high. The apparatus consists of wire spark chambers, a medium large aperture bending magnet and a small hydrogen target. A relatively small aperature bending magnet will also be required to measure the individual momenta of the incident muons.

Many details of this experiment have been described in an N.A.L. Summer Study Report[1] by M.L.Perl. This proposal is a combination of the two kinds of experiments described in that report. Since the report was written we have acquired large amounts of data from the SLAC muon-proton inelastic scattering experiment[2], and from comparable electron experiments at SLAC[3] and DESY[4]. These experiments show that the inelastic cross sections are higher than were expected. Therefore in this proposal the data rate and the q^2 range attained is larger than predicted in that report. One major change of emphasis from that report is that there is now much greater interest in the momentum and angle spectra of the hadrons averaged over all of the hadronic channels. This new interest is taken into account in this proposal. Finally there are some errors in the equations as reproduced in that report; the equations in the Theoretical Section of this proposal should always be used in preference to those in the report if they differ.

2. Theoretical Concepts

A. Kinematic Parameters

The adjacent diagram shows the relevant kinematic parameters on the usual assumption that the muon vertex is limited to one photon exchange.



$\vec{p}$ and E are the laboratory momentum and energy of the incident muon.
$\vec{p}'$ and E' are the laboratory momentum and energy of the final muon.
$\vec{q}$ and ν are the laboratory momentum and energy of the virtual photon.
m is the muon mass, M is the proton mass and M' is the invariant mass of total hadronic system produced in the interaction.
q^2 is the square of the four-momentum transferred to the virtual photon and in our metric is defined by $q^2 = (E - E')^2 - (\vec{p} - \vec{p}')^2$. q^2 is always negative. Also:

$$q^2 = -2EE' + 2pp'\cos\theta + 2m^2 \qquad (1)$$

where θ is the scattering angle in the laboratory system between the incident and final muon. We also note that the laboratory energy of the virtual-photon is:

$$\nu = E - E'$$

The invariant mass M' is defined by:

$$M'^2 = (\nu + M)^2 - (\vec{q})^2 = 2\nu M + M^2 + q^2$$

It has become conventional to use the quantity K defined by:

$$K = (M'^2 - M^2)/2M = \nu + q^2/2M = \nu - |q^2|/2M \qquad (2)$$

K is the energy that a real photon must have if it is to produce the same invariant mass M' as is produced by a virtual-photon with properties $\vec{q}$ and ν.

B. Observed Cross Sections and Derived Cross Sections or Form Factors

If the incident muon energy is fixed and the only experimental observation is of the momentum and angle of the scattered muon, we measure the double differential cross section $d^2\sigma/d\cos\theta\, dp'$. This differential cross section can be written as a function of just two independent functions[5] in a number of ways; two ways have become conventional. We shall also use the slightly different form of differential cross section, namely $d^2\sigma/dq^2dK$.

One set of functions $\sigma_T(q^2, K)$ and $\sigma_S(q^2, K)$ are defined by:

$$d^2\sigma/dq^2dK = \frac{\alpha K}{2\pi|q^2|\ p^2}\left[\left[1 - \frac{2m^2}{|q^2|} + \frac{2EE' - |q^2|/2}{\nu^2 + |q^2|}\right]\sigma_T + \left[\frac{2EE' - |q^2|/2}{\nu^2 + |q^2|}\right]\sigma_S\right] \quad (3)$$

α is the fine structure constant. The units of σ_T and σ_S are μbarns, of energy is Gev and of momentum is Gev/c. Thus the units of the differential cross section are μbarns/(Gev/c)^{2}Gev.

σ_T and σ_S may be thought of as the total cross section for the interaction of transversely polarized and scalar virtual-protons with protons, respectively. The kinematic factors which multiply these cross sections may then be considered as virtual photon fluxes. The transversely polarized virtual-photon flux is Γ_T and that for the scalar photons is Γ_S, where

$$\Gamma_T = \left(1 - \frac{2m^2}{|q^2|} + \frac{2EE' - |q^2|/2}{\nu^2 + |q^2|}\right)\left(\frac{\alpha K}{2\pi|q^2|\ p^2}\right) \quad (4)$$

$$\Gamma_S = \left(\frac{2EE' - |q^2|/2}{\nu^2 + |q^2|}\right)\left(\frac{\alpha K}{2\pi|q^2|\ p^2}\right) \quad (4')$$

There is an arbitrariness in these definitions; they are only unambiguous in the limit $q^2 \to 0$, where $\sigma_S(q^2, K)$ becomes zero and $\sigma_T(q^2, K)$ becomes equal to the total cross section of real photons on protons. Those real photons have energy K.

It is convenient to write Eq. 3 in the form:

$$d^2\sigma/dq^2dK = \Gamma_T\sigma_T + \Gamma_S\sigma_S = \Gamma_T\left[\sigma_T + \epsilon\,\sigma_S\right] \quad (5)$$

Here $\epsilon = \Gamma_S/\Gamma_T$ and is thus the ratio of the fluxes.

Another description[6] of the differential cross section uses the inelastic form factors $W_1(q^2, K)$ and $W_2(q^2, K)$ as defined in the equation:

$$d^2\sigma/dq^2dK = \frac{2\pi\alpha^2}{|q^2|\ p^2}\left[W_1\left[1 - \frac{2m^2}{|q^2|}\right] + W_2\left[\frac{2EE' - |q^2|/2}{|q^2|}\right]\right](\hbar c)^2\ 10^{30} \quad (6)$$

The W's have the dimensions of inverse energy and the factor $(\hbar c)^2\ 10^{30}$ is explicitly inserted to make the units of $d^2\sigma/dq^2dK$ μbarns/(Gev/c)^2(Gev).

The relations between σ_T, σ_S and the W's are:

$$W_1 \quad (\hbar c)^2\ 10^{30} = (K/4\pi^2\alpha)\ \sigma_T \tag{7}$$

$$W_2 \quad (\hbar c)^2\ 10^{30} = (K/4\pi^2\alpha) \quad \left|\frac{|q^2|}{\nu^2 + |q^2|}\right|\left|\sigma_T + \sigma_S\right| \tag{8}$$

If the square of the muon mass m^2 is small compared to q^2 and if m is small compared to ν, some of the equations simplify as follows. These are the equations used in electron-proton inelastic scattering, where the electron mass can always be neglected.

For m = 0

$$q^2 = -2pp'\ (1 - \cos\theta\ )$$

and Eq. 6 can be rewritten as:

$$d^2\sigma/dq^2dK = \frac{4\pi\alpha^2 p' \cos^2(\theta/2)}{|q^4|\ p} \quad \left(\quad 2W_1 \tan^2\ (\theta/2) + W_2\right) \tag{9}$$

a form generally seen in papers on electron-proton inelastic scattering. The only simplification in Eqs. 3, 4, 5 is that the $2m^2/|q^2|$ term is set to zero in Eqs. 3 and 4, and p = E and p' = E'.

Finally we consider the case where both EE' and ν^2 are large compared to q^2, and where m can be neglected. Then $\nu \approx K$. Eq. 3 becomes

$$d^2\sigma/dq^2dK = \frac{\alpha\ p'}{\pi|q^2|\quad pK}\left|\left(\frac{K^2}{2pp'} + 1\right)\sigma_T + \sigma_S\right| \tag{10}$$

where $K \approx p - p'$

In all of this analysis σ_T and σ_S, or equivalently W_1 and W_2, are independent functions of q^2 and K. They may of course be regarded as functions of some combinations of q^2 and K, such as $2M\nu/q^2$ and ν for example. However they are independent of the incident momentum p; the dependence of the differential cross section on p is taken up completely in the explicit kinematic factors.

3. The Experimental Situation at Present

One extensive muon-proton inelastic scattering experiment has been carried out at SLAC[2] and several extensive electron-proton scattering experiments have performed at SLAC[3] and DESY[4]. Inelastic electron work was done earlier at CEA[7]. Muon-nucleus inelastic scattering experiments have been performed at SLAC[2] and BNL[8]. An extensive electron-nucleus inelastic scattering experiment has just been completed at SLAC. In all of these experiments only the final lepton was detected.

We will present next a brief, rough summary of the results of these experiments. Our purpose is only to obtain formulas for extrapolation to higher energies; this summary should in no way be considered definitive or critical. Possible differences in behavior between the muon and electron are ignored.

Most of the muon data and a large part of the electron data has ϵ greater than 0.7. For this data it is convenient to use the combination $\sigma_{exp} = (\sigma_T + \epsilon\sigma_S)$, and to consider this combination not very different from $(\sigma_T + \sigma_S)$. The experimental measurements of σ_{exp} can be represented by the simple equation

$$\sigma_{exp} = S_o(K)/(\,1.0 + R(K)|q^2|\,) \tag{11}$$

This equation holds over the entire K and q^2 range, which has been measured, to $\pm 30\%$. $S_o(K)$ is the total cross section for real photons on protons at laboratory energy K. R(K) does not vary much with K; at low values of K, R is about 1.5, and for K above 1.5 Gev, R seems to vary between 2.0 and 3.0. The units of R are $(\mathrm{Gev/c})^{-2}$.

The form of Eq.11 was predicted by Sakurai[9] using vector dominance ideas. The prediction was only intended for smaller values of q^2, not too far from the square of the rho mass. It is surprising that Eq.11 holds to $|q^2|$ values of 4 or 5 $(\mathrm{Gev/c})^2$, even more so because the accompanying prediction of the ratio σ_S/σ_T is wrong at large q^2[3]. The range of theoretical validity of this prediction has been discussed by Sakurai.[9]

The analysis presented above, does not apply to very low K values where nucleon isobars are produced[10]. The production of these isobars is much more dependent on q^2, the form factors behave in roughly the same way as the elastic form factors. We will not pursue this subject because the proposed experiment does not make useful measurements in the isobar region- the momentum resolution is too poor. From the theoretical point of view there is also no value in trying to measure isobar production by virtual photons at NAL. The only purpose in studying isobars at high energy is to be able to achieve very large q^2 values. But the muon intensity at NAL is too small to yield significant data at high q^2 values compared to what has been and what can be achieved at SLAC with the electron spectrometers. Therefore this discussion and this proposal are confined to the continuum region of the inelastic spectrum, that is the region which has a K value too large to produce isobars.

The analysis of the electron experiments in the continuum region has emphasized the quantity νW_2. Bjorken[11] predicted that νW_2 should be a function of the combination $\omega = \nu/|q^2|$ when both ν and $|q^2|$ are very large. This has been found to be true over a wide range of K and q^2 values. Furthermore for ω values of 1.5 to 7 Gev^{-1}, νW_2 is a constant equal roughly to 0.3.

There is a simple relation between νW_2 and σ_{exp} when ν is large compared to $|q^2|/2M$ and $|q^2|/\nu$. Eq.8 reduces to

$$\nu W_2(\hbar c)^2\ 10^{30} = \left[|q^2|/(4\pi^2\alpha)\right]\sigma_{exp}$$

if for ϵ close to 1.0 we use the approximation $(\sigma_T + \epsilon\,\sigma_S) = (\sigma_T + \sigma_S)$. This can be rewritten as:

$$\sigma_{exp} = \left[\nu W_2 4\pi^2\alpha(\hbar c)^2\ 10^{30}\right]/|q^2| \qquad (12)$$

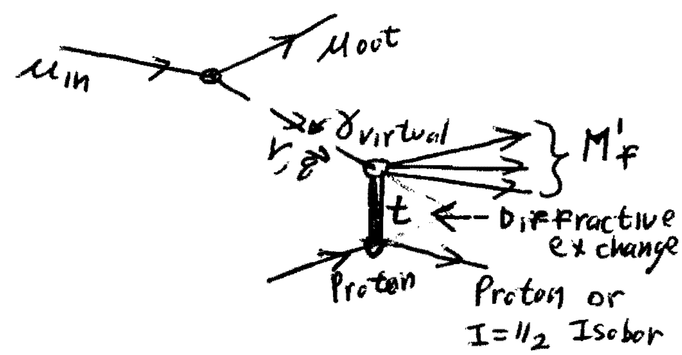
In the limit of $R|q^2|$ large compared to 1.0, Eq.11 is the same as Eq.12. Thus with the restrictions we have listed for ν and q^2, the experimental verifications of Eq. 11 and of the Bjorken prediction are really the same observation.

Fortunately at NAL energies the limitations on ν and $|q^2|$ set in the last paragraphs are easily met, thus there is no conflict between the vector dominance and the νW_2 predictions;and we can use Eqs. 11 and 12 to predict the event rate for this experiment. Of course we do not know if the basic theoretical ideas, upon which the predictions are based, will hold at NAL energies; this is one of questions which the experiment is designed to answer.

Some separation of σ_T and σ_S has been carried out in the electron experiments. The observed ratio σ_S/σ_T is never greater than 1.0. For $|q^2|$ values greater than 1.0 $(\text{Gev}/c)^2$, it varies between 0.0 and 0.8 and is not clearly dependent on K. The errors in the measurements are large and much work remains to be done at the energies attainable at SLAC and DESY. However it is clear that no large ratios like 2.0 or 3.0 have been observed. Therefore in our design for the NAL expriment we are not expecting any large **values** for σ_S/σ_T, and are prepared to ascertain values for that ratio in the range of 0.2 to 1.0. If the ratio is larger it will of course be detected.

4. Theoretical Speculations and the Hadronic Spectrum

A remarkable feature of the inelastic scattering observations is that compared to the elastic form factors, σ_{exp} is such a moderate function of q^2. σ_{exp} behaves like $|q^2|^{-1}$ while the elastic form factors behave like $|q^2|^{-4}$. Furthermore this behavior is almost independent of K. (Of course we are restricting our discussion to the continuum region.) There have been two different explanations for this behavior. Harari[12] has speculated that the hadronic production at the proton vertex is mainly diffractive. The large four-momentum carried by the virtual photon (q^2) can be carried off by the production of very forward mesonic states; the four-momentum transferred to the proton can remain small. We call t the square of the four-momentum transferred to the proton. (See the adjacent diagram) There is then a minimum value of t called t_{min}. t_{min} depends upon q^2, ν and the invariant mass of the forward mesonic states called M'_f.



$$|t| > |t_{min}| = (M'^2_f + q^2)^2/(4\nu^2) \quad (13)$$

Suppose ν is 10 Gev and M'_f is 2 Gev, than for a $|q^2|$ of 4 (Gev/c)^2, t need only be larger than .16 (Gev/c)^2. Thus the four-momentum transferred to the proton can be much smaller than the four-momentum carried by the virtual photon. It is the four-momentum absorbed by the proton which directly influences the proton form factor. In inelastic scattering this absorbed four-momentum (t) can be small. Conversely in elastic lepton-proton scattering, all the four-momentum carried by the virtual photon must be absorbed by the proton, hence the elastic form factors are directed influenced by q^2.

Another view has been put forth by Bjorken[13] and has been developed by Feynman[14] and by Drell and his collaborators[15]. They regard the proton as being made up of point like particles called partons. At high ν and high q^2 the virtual photon scatters directly from these partons. Since the partons are point particles they have no form factor dependence on q^2. The only dependence on q^2 comes from the parton distribution in the proton, and this gives a slow dependence on q^2.

Very recently Harari[16] has described a possible connection between these two views. He sees the diffractive production of mesons by virtual photons as involving the creation of quark-antiquark pairs. Then to quote Harari, "The possiblity of a photon-induced production of quark-antiquark pairsin the hadron in one frame is translated into the interaction of the photon with an infinite sea of such pairs in a different Lorentz frame (as viewed by the parton model)".

At present there are no measurements of the hadronic spectrum to test these speculations. Some of the authors of this proposal have proposed a spark chamber experiment at SLAC[17] to measure the hadronic spectrum from inelastic electron scattering. A lower energy experiment of the same type is now being run at DESY.[18] These experiments will produce data that will help in the planning of the NAL experiment, but the questions which we are asking about the hadronic spectrum at 10Gev will have to be asked again at 100 Gev.

First we have the general question of how the hadronic spectrum produced by virtual photons on protons will compare with the spectrum produced by pions or protons on protons. We know the latter spectrum has some clear characteristics. The transverse momentum of the hadrons averages 300 or 400 Mev/c and is almost independent of energy. The multiplicity of hadrons increases as the logorithm of the energy. Two body and quasi-two body processes have strong forward peaks if Pomeranchuk or meson exchange is permitted. Will these properties persist with virtual photons as the projectile? Of course these questions cannot be completely answered with the first experiment. But this experiment will produce a complete negative pion spectrum in angle and momentum and will also give statistics on the multiplicities of charged particles. We will also look for forward enhancements in mass analogous to the A_1 and Q enhancements.

Second we will specifically analyze the channel $\mu + p \rightarrow \mu + p + \rho^0$ This channel is diffractive and according to Harari's speculations, the strength of this channel and its q^2 behavior whould be independent of K in the 10 to 100 Gev range. At present the only extensive studies of this channelhave been with real photons; where the production is definitely diffractive and amounts to 10% of the total cross section. As discussed by Dieterle[19] and others, the channel $\mu + p \rightarrow \mu + p + \rho^0$ has other very useful properties. By studying the production and decay distribution of the ρ^0, the contributions of the transversely polarized and scalar photons can be separated. Vector dominance makes a definite prediction for some of the behavior of the transversely polarized photons, buthas nothing to say about the scalar photons. Thus there is much interesting data in the study of this channel by itself.

Third we will make a preliminary studyof the slow proton spectrum. From the diffraction model we expect that a majority of the high q^2 interactions will produce slow protons, the proton either being excited to a slow isobar or not being excited at all. Apparently at this time, the parton model does not make any definite predictions about the proton spectrum, but it would seem to predict some enhancement in the production of fast protons. However in this experiment there is no separation of protons and positive pions. Therefore we will have to depend on the slow proton spectrum to indicate the best direction for further measurements of the proton spectrum

5. Aspects of Muon-Proton Inelastic Scattering Not Included in this Proposal

We have not attempted to do very small q^2 inelastic scattering in this design. The study of muon-proton inelastic scattering in the range of $|q^2|$ values of 0.02 to 0.2 $(Gev/c)^2$ can be used to extrapolate to $q^2 = 0$ to give the real photon proton cross section. We do not propose to this because the triggering on very small angles of scatter like 0.5 mrad is very inefficient in the present design and will lead to very large background problems. Secondly we have not yet made an evaluation of the virtues of this method of measuring the real photon cross section versus the method which used real photons such as described by Toner.[22] Experience at SLAC and DESY shows that _if_ an adequate photon beam is available the direct measurement is superior.

No attempt is being made in the proposed experiment to compare muon-proton and electron-proton inelastic scattering to look for muon-electron differences. First, because we would have to use muon beam energies of less than 20 Gev so that comparisons could be made with the SLAC electron data; and this would eliminate all the new, very high energy physics in the experiment. Secondly a very extensive comparison is in the final stages using the results of the SLAC muon experiment[2] and another very extensive experiment will soon be run at Brookhaven[23]. The results of these two experiments must be in hand before further comparison experiments are designed.

III EXPERIMENTAL ARRANGEMENT

6 The Muon Beam

At this time no definite position or configuration for the muon beam can be selected. The very general question appears to be undecided of whether the beam should be built in conjunction with a neutrino facility or whether it can be built in conjunction with a hadron beam in Area 2. Therefore we will simply use the parameters of the type of beam designed by Yamanouchi[20] without specifying its location or configuration. As summarized in Table 1 of Christenson's report[21] the following parameters are feasable for a 100 Gev beam. We have designed the experiment for 120 Gev but with less flux than estimated for the 100 Gev beam by Yamanouchi and Christenson. Thus the additional energy will not be a problem.

Muon beam momentum	80 to 120 Gev/c
Momentum spread	±5%
Angular spread	±2 mrad.
Beam size	10 cm by 10 cm
Flux	10^6 per pulse

The amount of flux which can be used depends upon the resolving time of the wire spark chambers used in the experiment. There are wire chambers in the muon beam after the hydrogen target, which detect both the scattered muons and the associated hadrons. We would like to use proportional wire chambers in these positions but there are two unresolved problems. First some of the wire chambers are quite large, and the expense of the amplifiers for proportional chambers may be excessive. Second, we do not know if we can get the desired spatial resolution of 0.35 mm per chamber with proportional chambers. Therefore we have taken the conservative alternative and planned for magnetostrictive wire chambers, with perhaps proportional chambers in the beam regions. With this choice a flux of 10^6 muons per pulse would seem to be the maximum that can be used.

7. The Experimental Apparatus

A. Measurement of the angle and momentum of the incident muon.

Fig. 1 shows the section of the apparatus which measures the angle and momentum of the incident muon. Mb is a conventional beam bending magnet, 2 meters long, with a clear aperture of 10 cm by 10 cm and with a field of 22 kg. 1,2,3 and 4 are proportional chambers with a 10 cm by 10 cm sensitive volume and a spatial resolution of 1.0 mm. The distances between 1 and 2 and between 3 and 4 are both 10 m, so that the angles into and out of Mb are known to ±.14 mrad. The momentum of the incident muon is determined to ±1.8%. This portion of the apparatus can handle fluxes greater than 10^6 per pulse because proportional chambers are used here.

H1 and H2 are scintillation counter hodoscopes which determine the transverse postions of the muon to ±0.5 cm. Thus the angle of incident muon is determined to ±1 mrad in each transverse direction by these hodoscopes. This information is used for trigger selection.

B. The hydrogen target and associated counters.

The liquid hydrogen target is 75 cm long and 15 cm in diameter. The length was selected in the following way. If only muons were being detected, a much longer target could be used. But in this length target each hadron produced has a 4 % chance of interacting in the hydrogen before it leaves the target. Such interactions can cause confusion in measurements of the hadron spectrum, in analysis of specific channels and in measurements of multiplicity. If on the average 3 charged hadrons are detected, there will be a 12 % correction; larger corrections caused by a longer target seem unacceptable.

The first set of counters associated with the target are the veto counters, designated by V in Fig. 2. These counters have holes to let the main beam through and are simply intended to veto muons in the beam halo. Such counters were found very important in the SLAC experiment for reducing false triggers.

The second set of counters marked C in Fig. 2 are used to detect charged particles at large angles to the beam which do not enter the spark chamber 5. There are 16 C counters around the sides of the target and 4 C counters at the back end. The C counters are designed to measure the multiplicity of the charged particles not seen in the spark chambers. They do not measure the angles of these particles except in a very rough way. The total charged particle multiplicity will be the total counts in the C counters plus the charged particles detected in chamber 5. It is clear that too high a flux would also make this measurement difficult.

The third set of counters marked P are 3 m from the target and subtend an angle of from 45 to 90 degrees from the mean beam line. However they only subtend an arc of 10 degrees about the beam axis. Their purpose is to sample the slow proton spectrum and they are not expected to be useful in the general analysis of an event. We expect to time the protons to .25 nsec. using these counters and the C counters. In addition to identifying slow protons and roughly determining their momentum, the P counters will also measure their angle with the beam to 5 degrees.

C. The analyzing magnet and associated spark chambers.

The magnet Ma should have a clear aperture of at least one square meter and the product of particle path length and magnetic field should be about 20 kgauss meters. A particular magnet cannot be selected now because we do not know what magnets will be built at NAL. Some possible magnet designs have been discussed by Perl[1]. This magnet should be the same magnet as is being requested by the NAL-Columbia group for their muoproduction of W experiment. They are proposing a magnet with a 2 m by 1 m aperture and a 2 m deep field. The Harvard proposal which will be a collaboration with us is based on a 60 in. by 30 in. aperature and a 82 in. deep field. Our design of the experiment is based on this same size, using the Columbia design as the nominal size.

The angles of the particles leaving the hydrogen target and entering the magnet are determined by the wire chambers 5 and 6. These chambers each have a spatial precision of 0.35 mm. They are 5 m apart resulting in an angular precision of $\pm$0.1 mrad. Chambers 7 and 8 are similar to 5 and 6, but larger. They are also 5 m apart and provide an angular precision of $\pm$ 0.1 mrad.

The acceptance of the apparatus is basically fixed by the aperture of Ma. It will be $\pm$ 120 mrad in the vertical plane and $\pm$ 60 mrad in the horizontal plane. However to avoid making the chambers 7 and 8 too large, the vertical acceptance will be further limited by counters to $\pm$100 mrad.

D. Muon identification.

To identify the muon we use a method which has proven to be very successful in the SLAC muon experiment. A muon is identified by its ability to pass through 1.5 m of iron without interaction. The path of the muon is detected by chambers 8,9 and 10. The muon may bend no more in the iron than can be accounted for by multiple scattering.

E. Trigger Hodoscope

The trigger hodoscope consists of three planes of scintillation counters T1, T2, and T3. The muon beam passes through the central counters in T1 and T2, but the central region of T3 does not have counters. Thus an unscattered muon will not be detected in T3. T3 is also behind 1.5 m of iron. Scattered muons will reach T3, but hadrons will interact and the interaction products will be absorbed before T3. Thus a count can be obtained in T3 only from a scattered muon.

The hodoscopes T1,T2,T3 are connected to a fast logic system which makes the following demands on the counts from the T counters. The counts from T1 and T2 must be such that they can come from a particle that makes an angle of at least 5 mrad with the incident beam direction. The associated count in T3 must come from a counter in T3 so that the particle detected by T1 and T2 had a momentum greater than 10 Gev/c.

Of course when forward hadrons are produced there will be several counts in both T1 and T2. Therefore the T1T2T3 logic set up to detect a scattered muon of greater than 10 Gev/c momentum can certainly produce false triggers. For example there might be a hadron at greater than 5 mrad. and a very small angle scattered muon which combine to give the desired trigger. We do not contemplate setting up a very complicated fast logic system to sort out these false triggers. Rather we depend on the wire chamber information being sorted out in a small buffer computer.

F. Triggering Method

The trigger requirement is $H1H2T1T2T3\overline{V}$. The T1T2T3 counts must meet the restrictions discussed in the last section. There must be only one count in H1 and only one count in H2. The H1H2 combination must define a beam particle whose angle is less than 2.0 mrad with respect to the beam direction.

There is no comparison of the angle defined by H1H2 and the angle defined by T1T2. Therefore the angle between the incident and scattered muon has a minimum which varies between 3 and 7 mrad.

G. Precision of momentum determination.

If we assume that 20 kgauss meters is available to bend the particles passing through Ma, then for a momentum p' (in Gev/c) the bending angle, called α, is given by $\alpha = 600/p'$ mrad. p' will vary from 10 to 110 Gev/c leading to values of α ranging from 5.4 to 60 mrad. The total error in α will be $\pm$ 0.14 mrad. Thus the error in momentum will be $\pm$ 2.3(p'/100) %. The 20 kgauss meter parameter is conservative, a 2 m deep magnet might give 30 kgauss meters, reducing the momentum uncertainty by one third, which would be desirable for the larger p'.

8. Event Rate for Measurement of σ_{exp}

To estimate the event rate we have used predictions of Eqs. 11 and 12, accepting the theoretical speculations that this behavior will persist in the 100 Gev range. It is also necessary to assume some average value for $S_o(K)$, the total cross section of real photons on protons at energy K. From 6 to 16 Gev this cross section decreases slowly from 118 to 113 μbarns[21]. We assume the slow decrease will not accelerate and use an average value for S_o of 110 μbarns over the K range of 10 to 110 Gev. Thus we use the equations

for $|q^2| < 1.0\ (\text{Gev}/c)^2$ $\qquad \sigma_{exp} = 110/(1.0 + 2|q^2|)\ \mu\text{barns}$ (14)

for $|q^2| > 1.0\ (\text{Gev}/c)^2$ $\qquad \sigma_{exp} = 110/(3|q^2|)\ \mu\text{barns}$ (15)

The event rate is based on a 75 cm long hydrogen target, 10^6 muons per pulse and 800 pulses per hour. We have assumed the acceptance for muons to be 2/3. Some muons are lost because the combination of their scattering angle and their momentum puts them at the center of the T3 hodoscope where there are no counters.

There is a cut off for low q^2 events because there is only a known acceptance for the muons when their angle of scatter is greater than 7 mrad.

There is a cut off for large q^2 events because the maximum angle of scatter of the muons is about 80 mrad. This cut off eliminates high q^2 events at large K values. However the rates are so low in this region that this cut off does not cause a loss of useful data.

Table I presents the number of events per hour. These are events in which the scattered muon is detected. Most but not all of the events will have accompanying hadrons. We are designing for 200 hours of effective running time at 120 Gev and 100 hours at 80 Gev. Table I shows the event rate for the 120 Gev data. Multiplying the numbers in the table by 200 will thus give the expected statistics, for the 120 Gev data. The 80 Gev data will have about one third that number of events.

TABLE I

Incident muon momentum = 120 Gev/c

Numbers in the table are events per hour. Multiply by 200 to get the statistics for the experiment.

The absence of an entry indicates that the region is outside the low q^2 or high q^2 cut offs.

K interval (Gev) q^2 interval $(Gev/c)^2$	10 to 30	30 to 50	50 to70	70 to 90	90 to 110
0.2 to 0.5				20	7.5
0.5 to 1.0	110	42	21	11	3.9
1.0 to 2.0	66	24	12	6.1	2.3
2.0 to 4.0	31	12	5.7	2.9	1.1
4.0 to 6.0	10	3.9	1.9	1.0	.36
6.0 to 8.0	5.4	2.0	1.0	0.5	
8.0 to 12.0	5.4	2.0	1.0	0.5	
12.0 to 16.0	2.7	1.0	0.5		
16.0 to 20.0	1.6	0.6			

9. Background Considerations

False or useless triggers will occur in the following cases.

1. A Mixed Hadron-Muon Trigger. This is described in Section 7 E. A hadron gives a count in T1 and T2, this count when combined with a small angle muon scattered into T3 gives a false trigger.
2. Elastic Events. Elastic events cannot be used because the energy resolution is not good enough to separate elastic and very low K inelastic events.
3. Low K Inelastic Events. The problem is the same as with elastic events; furthermore these events are not useful because better data can be obatined with electrons at SLAC
4. Low q^2 Inelastic Events. The cut off at 7 mrad is not sharp; muons which scatter with as little as 3 mrad may be detected by T1T2T3.
5. Beam Halo Muons. There is less than one useful event per million muons. The stray muons in the beam halo which give the trigger described in Section VI F, must be reduced to less than 5 or 10 per million beam particles.

The output from the wire chambers must be examined in a small on-line computer to eliminate most of these background events. Type 1 can be mostly eliminated by seeing if there is a connected muon track through chambers 5 through 10 in the plane perpendicular to the plane in which the particles bend. Type 5 can be eliminated by tracing back the muon from chambers 6 and 5 to see if it intersects the fiducial volume in the target. Some of Type 4 and some of Type 2 can be eliminated by checking the scattering angle of the muon. But most of Types 2, 3 and 4 will have to be put on tape for off-line processing.

An estimate of these backgrounds is given in Appendix A.

10. The Separation of σ_T and σ_S

We propose to run two thirds of the time at 120 Gev/c and one third of the time at 80 Gev/c; and to separate σ_T and σ_S at K= 60 Gev. Using Eq.10 we find that for the same Δq^2 bin the ratio of the number of events at 120 Gev/c (called $N(120,60,\Delta q^2)$) to the number of events at 80 Gev/c (called $N(80,60,\Delta q^2)$) is:

$$r = \frac{N(120,60,\Delta q^2)}{N(80,60,\Delta q^2)} = \frac{(2/3)(1/2)\left[1.25\,\sigma_T + \sigma_S\right]}{(1/3)(1/4)\left[2.125\,\sigma_T + \sigma_S\right]} \qquad (16)$$

Suppose we can measure this ratio r to $\pm 5\%$, than Table II gives the error in the ratio σ_T/σ_S.

TABLE II

σ_S/σ_T	Error in σ_S/σ_T
0.2	+0.2,-0.2
0.5	+0.3,-0.2
1.0	+0.5,-0.4
2.0	+1.0,-0.6
5.0	+3.6,-3.1

Given the normaliziztions problems in a high intensity muon experiment we cannot expect to measure the ratio r to better than $\pm$ 5%. Furthermore Eq.16 shows that the number of events in the 80 Gev/c data will be about one half of the number in the 120 Gev/c data for K = 60 Gev. Looking at Table I we can see that for 200 hours of running at 120 Gev/c(and 100 hours of running at 80 Gev/c) the highest $|q^2|$ (for K = 50 to 70 Gev) at which we can measure r to $\pm$ 5% is 4.0 $(\mathrm{Gev/c})^2$.

The situation is not very different at other K values. In general we expect to get the degree of separation indicated in Table II for the K interval of 30 to 50 and the K interval of 50 to 70 Gev, with $|q^2|$ values up to 4.0 $(\mathrm{Gev/c})^2$. An increase of event rate would be most valuable for this separation. However we feel that to expect to use substantially more than 10^6 muons per pulse for the first experiment of this type at NAL would be unduely optimistic. The limitation on event rate we expect will be the Background events and triggers discussed in Section 9; only experience can show how to reduce this background. We expect that in future experiments the muon beam and the event rate will be increased by a factor of 10 or 20 with much better separation of σ_T and σ_S.

II. Measurement of the Hadronic Spectrum and of the Multiplicity

The average acceptance of the magnet for charged particles from the hydrogen target is about $\pm$ 70 mrad. For a transverse momentum of 0.3 Gev/c, particles with longitudinal momenta greater than 4.0 Gev/c will be accepted. Even for a high (according to present ideas) transverse momentum of 0.9 Gev/c, particles with longitudinal momenta greater than 12 Gev/c will be accepted. Thus most of the charged hadrons will pass through the system and their angles of production and their momenta will be measured, unless the distribution of transverse momenta is much broader than we now expect. But even in that case the difference between the observed spectra and the expected spectra will be obvious.

The acceptance for charged particles which do not go through the magnet is much larger, about $\pm$ 150 mrad. While we will not measure the momenta of these particles we will measure their production angle.

The combination of the chambers 5 and 6 and the C counters ensures that all charged particles produced in the target will be detected and the charged particle multiplicity will easily be determined as a function of K and q^2.

The spectrum of slow protons at angles larger than 45 but small than 90 degrees will be sampled by the counters P. Protons with less than 200 Mev/c will stop in the target, and protons with momenta greater than about 700 Mev/c will not be separated from pions. But we will obtain a measurement of the rough angle and momentum spectrum between these limits and this will tell us if there is anything unexpected that merits further study in a later experiment.

12. The Channel $\mu + p \rightarrow \mu + p + \rho^0$

The study of this channel has been discussed in the report by Perl[1] and the theoretical implications have been discussed by Dieterle[19]. We expect that the mass resolution on the rho meson will be about 9 Mev. The uncertainty in the transverse momentum of a particle is $\pm 10^{-4} p$ where p is the momentum of the particle in Gev/c. The particles will have momenta from roughly 10 to 100 Gev/c leading to uncertainties in the transverse momenta of 1 to 10 Mev/c. Thus by measuring the muon and the two pions we will be able to get a good measurement of the transverse momentum given to the recoil nucleonic system.

The measurements of the muon and the two pions by themselves cannot definitely determine the mass of the recoil nucleonic system and hence cannot insure that just a proton recoiled. This is because we always have an uncertainty in the longitudinal momentum of several Gev/c. The further determination of the channel depends upon the use of the C counters. If only a proton is produced in addition to the muon and the rho meson, there are two possibilities. The proton may be very slow and may stop in the hydrogen target; then there will be no count in the C counters. The other possibility is that the proton produces a count in the C counters. But then the direction of the proton will be roughly given by the C counters, and that direction must agree with the direction predicted by the measurements on the muon and pions.

Preliminary calculations indicate that if the rho meson production is about 10% of the total cross section the background contamination from other channels will be small. If the rho meson production is less than 2% of the total cross section, we will only be able to measure the rho production cross section, but we will not be able to study angular distributions in production and decay. As discussed in Section IV we expect a 10% cross section and a lower cross section will be a great surprise and will be of great interest.

13 Running Time Required

200 hours using 120 Gev/c muons and 100 hours using 80 Gev/c muons are requested

IV APPARATUS

The National Accelerator Laboratory would be expected to provide those parts of equipment normally supplied by an accelerator laboratory. This would comprise the magnets and shielding for the muon beam, the hydrogen target and the analyzing magnet. The analyzing magnet has been discussed is Section 7. As discussed in that section the same magnet is required by the NAL-Columbia muoproduction of W experiment. The magnet would have an aperture ranging from .75 m by 1.5 m to 1 m by 2 m. The depth would be 2 m. The product of the particle path length and the field should be at least 20 kgauss meters.

The spark chambers, scintillation counters, special electronic equipment and electronic interfaces would be provided by the collaborating experimental groups.

The situation with respect to two types of equipment is still not clear. First will NAL provide the conventional modular electronic equipment for counter logic? Second, will there be on-line computers of any size available at NAL for use with the wire chambers? Or should the experimenters plan to provide their own computing devices for the chambers?

Appendix A.

In this appendix we give some quantitative estimates of the backgrounds in the experiment which are relevant to the triggering efficiency. These backgrounds are given in terms of the cross section per hydrogen atom in microbarns. For comparison the total cross section for the useful events which comprise Table I is

$$\sigma_{\text{good inelastic}} = 0.29 \text{ microbarns.}$$

We now consider each of the backgrounds listed in Section 9, page 18. The low angle cutoff is the crucial element in all these calculations. We have taken a pessimistic estimate of the low angle cutoff and assumed that the collection efficiency for the muons decreases from 0.67 at 7 mrad to 0.0 at 3 mrad. This is based on the simple trigger system described in Section 7E and 7F. If more constrained triggering is used the backgrounds can be decreased. For example, in this design no attempt is made to use the counter logic to compare the incident muon angle with the scattered muon angle.

Background From Muon-Proton Elastic Scattering

We have used the elastic form factors as found in muon and electron experiments. We find

$$\sigma_{\text{background elastic}\,\mu+p} = 0.15 \text{ microbarns}$$

Background From Muon-Electron Elastic Scattering

We include here the false triggers from the scattered muon, which reaches a maximum angle of 4.8 mrad.(This background becomes very large if one tries to do muon-proton inelastic scattering at very low q^2, unless special provision is made to veto these events.) We do not find any false triggers from the scattered electrons.

$$\sigma_{\text{background elastic}\,\mu+e} = 0.69 \text{ microbarns}$$

Background From Small K Inelastic Events

We include here those inelastic events whose K is less than 10 Gev and whose scattering angle, for the muon, is greater than 7 mrad. These are a large background. I do not see any way to veto these events, because it is a question of distinguishing, by counters,a 120 Gev/c muon from a 110 Gev/c muon through the difference of their bending angles in the analyzing magnet.

$$\sigma_{\text{background small K inelastic}} = 1.26 \text{ microbarns}$$

Background From Low q^2 Inelastic Events

We include here those inelastic events of any K value whose muon scattering angle is between 3 and 7 mrad. We find

$\sigma_{\text{background low } q^2 \text{ inelastic}} = 0.24$ microbarns

Background From Mixed Muon-Hadron Triggers

This is discussed in Section 7E. The exact calculation depends on the beam phase space and some knowledge of the hadron spectrum. We have made a'hand'calculation based on our muon experiment at SLAC. We are now preparing a computer program to make a more detailed calculation. The design of the trigger counter sizes and placement is closely related to this calculation, and both in turn depend on the muon beam design. Therefore we will wait for the final beam design before proceeding with the computer calculation.

$\sigma_{\text{background mixed muon-hadron trigger}} = 0.8$ microbarns

The Total Background

Adding up the five components of the background we find

$\sigma_{\text{background total}} = 3.1$ microbarns

Thus the triggers from background events will be 11 times the triggers from good events. Of these triggers, 1.3 microbarns can be eliminated in a small on-line buffer computer by testing the continuity of the muon track in the stereo view or by calculating the scattering angle of the muon . However 1.8 microbarns will have to remain on the tape with the .29 microbarns of good events; the good events being separated by the full analysis program.

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Initial Section of Experiment – For Measurement of Incident Muon Angle and Momentum

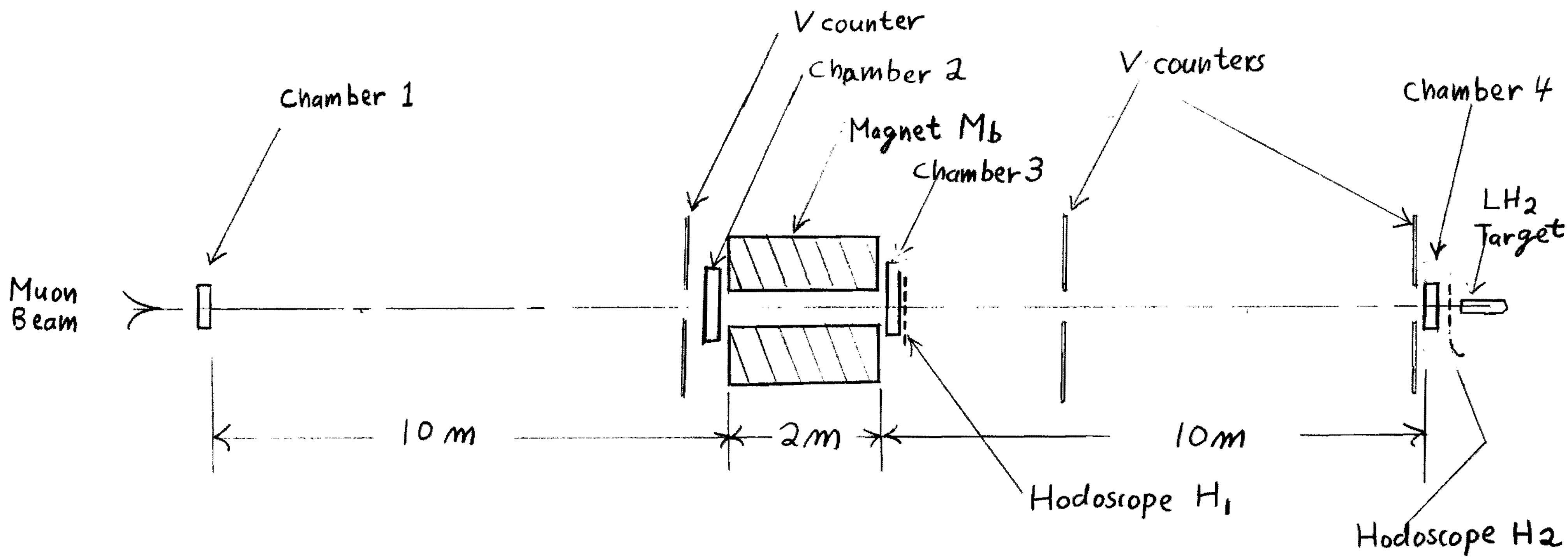

Fig. 1

Scale 1/2" ≈ 1 meter

Main Apparatus — Vertical Section

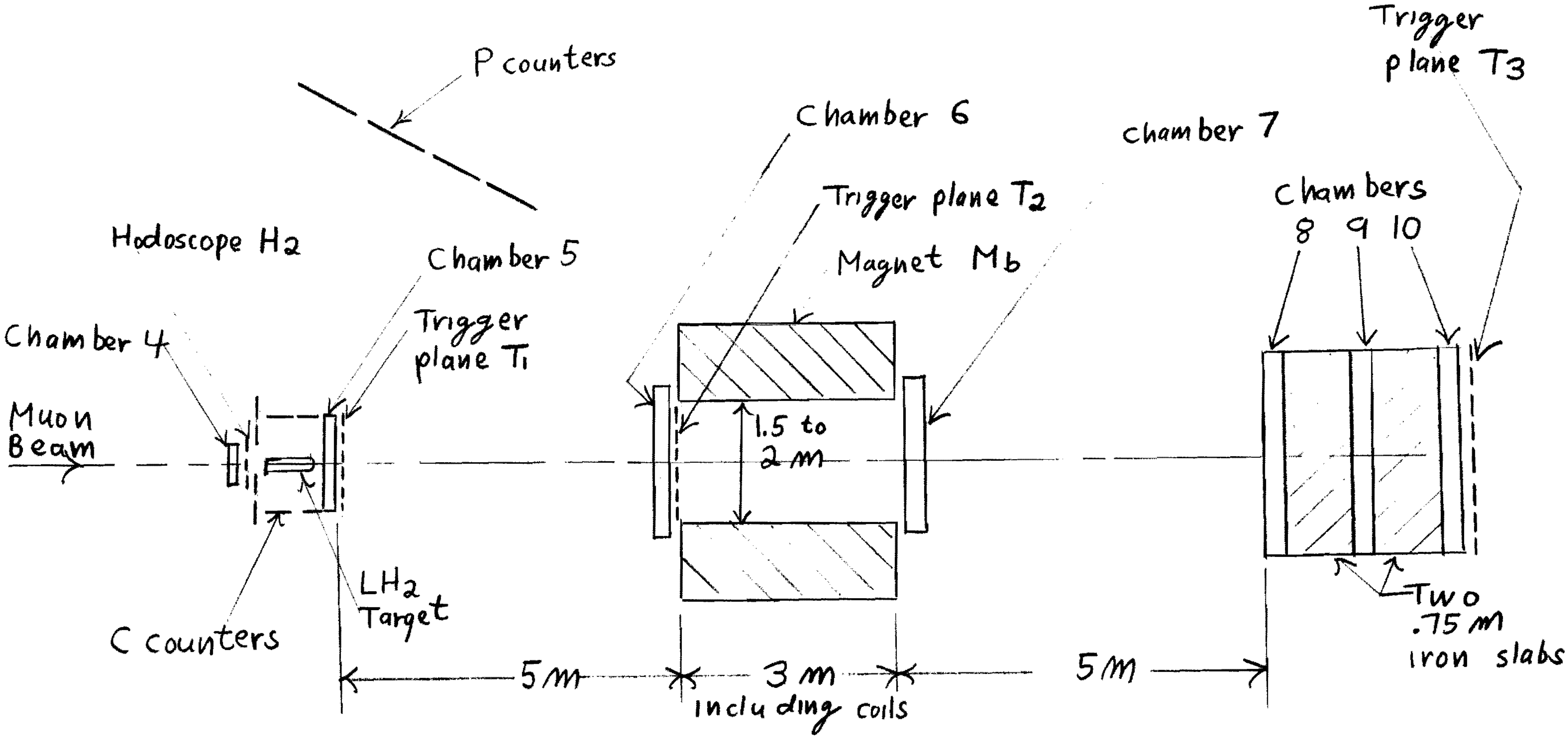

Fig. 2

Scale ½" ≈ 1 meter