FERMILAB-FN-1023-E

WEAK INTERACTION PHENOMENA:

W-S MODEL OF ELECTRO-WEAK INTERACTIONS

WEAK DECAYS OF QUARKS AND LEPTONS

CHARMED PARTICLE LIFETIMES

THE TAU LEPTON LIFETIME

CHARMED BARYON POLARIZATION IN NEUTRINO INTERACTIONS

By

Steven Michael Errede, B.Sc., M.Sc., Ph.D.

* * * * * * * * * * * * * *

August, 1981

TABLE OF CONTENTS

LIST OF FIGURES

Page

INTRODUCTION

This compendium of seemingly random collection of works was generated in the preparation of my thesis and also from diversionary investigations of physics associated with E-531 during the past couple of years.

While the bulk of this material was deemed inappropriate for inclusion in an already lengthy thesis, it appears here in physical form, so as not to be lost in the depths of my (poor) memory, and the sands of time.

I. THE (EXTENDED) WEINBERG-SALAM MODEL of ELECTRO-WEAK INTERACTIONS

The Weinberg-Salam model of electo-weak interactions is a spontaneously broken, non-abelian gauge theory, believed to correctly describe the weak and electromagnetic interactions of quarks and leptons.

In the "standard" (and extended) W-S model with six quarks and six leptons, the quarks and leptons are arranged in (weak) left-handed isodoublets and right-handed iso-singlets of SU(2) [22-24]. There are three (known) "generations" n = 1, 2, 3 of quarks and leptons, where n is the generation number (a principal quantum number?). For each quark in a particular generation n, a corresponding lepton is observed:

	LEPTONS n=1	LEPTONS n=2	LEPTONS n=3	QUARKS n=1	QUARKS n=2	QUARKS n=3
$T_3=+1/2$: $T_3=-1/2$:	$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L$	$\begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L$	$\begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}_L$	$\begin{pmatrix}u\\ d\end{pmatrix}_L$	$\begin{pmatrix}c\\ s\end{pmatrix}_L$	$\begin{pmatrix}t\\ b\end{pmatrix}_L$
$T=T_3=0$:	e^-_R	μ^-_R	τ^-_R	u_R d_R	c_R s_R	t_R b_R

(The right-handed iso-singlet states for the three neutrino types are not observed in nature.)

Mixing is observed between the d^n (d, s, and b) quarks (no such mixing has been observed for the u^n(u, c (and t)) quarks). The origin of mixing between the d^n quarks is not fully understood at the present time. The d, s, b mass eigenstates are related to the weak eigenstates d' (d', s', b') by a unitary transformation, the unitary matrix is known as the Kobayashi-Maskawa (K-M) matrix V and is given by [187]:

$$\begin{pmatrix}d'\\ s'\\ b'\end{pmatrix} = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 + s_2 s_3 e^{i\delta} & c_1 c_2 s_3 - s_2 c_3 e^{i\delta} \\ -s_1 s_2 & c_1 s_2 c_3 - c_2 s_3 e^{i\delta} & c_1 s_2 s_3 + c_2 c_3 e^{i\delta} \end{pmatrix} \begin{pmatrix}d\\ s\\ b\end{pmatrix}$$

where $c_i = \cos\theta_i$, $s_i = \sin\theta_i$, $i = 1,2,3$
δ = CP violating phase

In the limit $\theta_2 = \theta_3 = \delta = 0$, $\theta_1 = \theta_c$, the Cabibbo angle. No mixing occurs for $\theta_1 = \theta_2 = \theta_3 = \delta = 0$.

The central values for the matrix elements of the K-M matrix are given in Refs.[188-192].

As a consequence of the fundamental relationship (or symmetry) between quarks and leptons (which is at present also not fully understood as to its origin), mixing is expected to occur between the ν^n (ν_1, ν_2, ν_3) mass eigenstates, if the neutrinos are not massless particles. The ν^n (ν_e, ν_μ, ν_τ) weak eigenstates are related to the mass eigenstates by a unitary matrix, similar in form to that for the d^n quarks although the mixing angles for the ν^n leptons need not be identical (but may indeed be related to) those for the d^n quarks, depending on whether or not the mechanism responsible for mixing (a new "super-weak" interaction?) is the same for the d^n as for the ν^n.

The interaction Lagrangian is of the form:

$$\mathcal{L}_{INT} = e\left[A^\mu J_\mu^{EM} + \frac{1}{\sin\Theta_W\cos\Theta_W} Z^\mu J_\mu^N + \frac{1}{\sqrt{2}\sin\Theta_W}(W^{+\mu} J_\mu^c + W^{-\mu} J_\mu^{c\dagger})\right]$$

The fundamental symmetry between quarks and leptons is reflected in the structure of the charged weak (V-A) current, which is of the form:

$$J_\mu^c = (\bar{\nu}_1 \bar{\nu}_2 \bar{\nu}_3)(U_{LEPTON})\left[\gamma_\mu \frac{(1-\gamma_5)}{2}\right]\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} + (\bar{u}\ \bar{c}\ \bar{t})\left[\gamma_\mu \frac{(1-\gamma_5)}{2}\right](U_{QUARK})\begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$= (\bar{\nu}_e \bar{\nu}_\mu \bar{\nu}_\tau)\left[\gamma_\mu \frac{(1-\gamma_5)}{2}\right]\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} + (\bar{u}\ \bar{c}\ \bar{t})\left[\gamma_\mu \frac{(1-\gamma_5)}{2}\right]\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}$$

Where U_{LEPTON} (U_{QUARK}) is the Kobayashi-Maskawa mixing matrix for the ν^n leptons (d^n quarks). Here, the matrix elements of U_{LEPTON} and U_{QUARK} are interpreted as the relative coupling strengths of charged leptons to neutral leptons (charged +2/3 quarks to charged -1/3 quarks) within their own generations ($\Delta n = 0$) and to other generations ($\Delta n \neq 0$). From the quark sector, transitions to other generations are observed to be supressed relative to transitions within the same generation by factors of ~ 20. Note further that if there were no mixing between d^n quarks (ν^n), then the d^n (ν^n) would be absolutely stable, u^n quarks (ℓ^n charged leptons) would be confined to decay only to the d^n quark (ν^n) within their own generation. (i.e. a "$\Delta n_{gen} = 0$" rule; violated by the observed amount of mixing!)

For fermions in general, (for 3 generations)

$$U_f = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix}$$

$$U_{ij} / U_{ij}\delta^{ij} \simeq 20; \quad i,j = 1,\ 2,\ 3; \quad \delta^{ij} = 1,\ i=j.$$

Thus, the U_{LEPTON} and U_{QUARK} mixing matrices are:

$$U_{LEPTON} = \begin{pmatrix} U\nu_1 e & U\nu_1\mu & U\nu_1\tau \\ U\nu_2 e & U\nu_2\mu & U\nu_2\tau \\ U\nu_3 e & U\nu_3\mu & U\nu_3\tau \end{pmatrix} \qquad U_{QUARK} = \begin{pmatrix} Uud & Uus & Uub \\ Ucd & Ucs & Ucb \\ Utd & Uts & Utb \end{pmatrix}$$

It may be that $U_{LEPTON} = R\ U_{QUARK}$, where R is (another) unitary matrix which relates the mixing of leptons to that for the quarks.

Note that, for example, the existence of non-vanishing matrix elements such as $U_{\nu_2\tau}$ has experimental consequences for neutrino production of τ leptons in ν_μ interactions (see Ref.[233,120]), for the τ lifetime [38,39], and the $F^+ \to \tau^+\nu_\tau$ branching ratio (see below).

The weak neutral current is of the form:

$$\begin{aligned} J_\mu^N = {} & (\bar\nu_e\ \ \bar\nu_\mu\ \ \bar\nu_\tau)\,[\tfrac{1}{2}\gamma_\mu \tfrac{(1-\gamma_5)}{2}] \begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} \\ & + (\bar e\ \ \bar\mu\ \ \bar\tau)\,[-\tfrac{1}{2}\gamma_\mu \tfrac{(1-\gamma_5)}{2} + \sin^2\theta_W\gamma_\mu] \begin{pmatrix}e\\ \mu\\ \tau\end{pmatrix} \\ & + (\bar u\ \ \bar c\ \ \bar t)\,[+\tfrac{1}{2}\gamma_\mu \tfrac{(1-\gamma_5)}{2} - \tfrac{2}{3}\sin^2\theta_W\gamma_\mu] \begin{pmatrix}u\\ c\\ t\end{pmatrix} \\ & + (\bar d'\ \ \bar s'\ \ \bar b')\,[-\tfrac{1}{2}\gamma_\mu \tfrac{(1-\gamma_5)}{2} + \tfrac{1}{3}\sin^2\theta_W\gamma_\mu] \begin{pmatrix}d'\\ s'\\ b'\end{pmatrix} \end{aligned}$$

Because of the unitary nature of the mixing matrices ($U\ U^\dagger = 1$)

$$\begin{aligned} J_\mu^N = {} & (\bar\nu_1\ \bar\nu_2\ \bar\nu_3)\,[\tfrac{1}{2}\gamma_\mu \tfrac{(1-\gamma_5)}{2}] \begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix} \\ & + (\bar e\ \ \bar\mu\ \ \bar\tau)\,[-\tfrac{1}{2}\gamma_\mu \tfrac{(1-\gamma_5)}{2} + \sin^2\theta_W\gamma_\mu] \begin{pmatrix}e\\ \mu\\ \tau\end{pmatrix} \\ & + (\bar u\ \ \bar c\ \ \bar t)\,[+\tfrac{1}{2}\gamma_\mu \tfrac{(1-\gamma_5)}{2} - \sin^2\theta_W\gamma_\mu] \begin{pmatrix}u\\ c\\ t\end{pmatrix} \\ & + (\bar d\ \ \bar s\ \ \bar b)\,[-\tfrac{1}{2}\gamma_\mu \tfrac{(1-\gamma_5)}{2} + \sin^2\theta_W\gamma_\mu] \begin{pmatrix}d\\ s\\ b\end{pmatrix} \end{aligned}$$

For the fermion f the coupling $[\Gamma_\mu^f]$ has a V-A term depending on T_3^f and a vector term depending on the charge Q_f;

$$[\Gamma_\mu^f] = [T_3^f\gamma_\mu\frac{(1-\gamma_5)}{2} - Q_f\sin^2\theta_W\gamma_\mu]$$

'For (most) low energy phenomena ("low" = $Q^2 < M_W^2$) the weak interactions of quarks and leptons may be described by an effective current-current "point" or four-fermion interaction, the effective interaction Lagrangian is of the form:

$$\mathcal{L}_{WEAK}^{EFF} = \frac{G}{\sqrt{2}}\, 4\,(J^{c\mu} J_\mu^{c\dagger} + 2\rho J^{N\mu} J_\mu^N) \qquad \frac{G}{\sqrt{2}} = \frac{\pi\alpha}{2M_W^2 \sin^2\theta_W}$$

$$\rho = M_W^2 / M_Z^2 \cos^2\theta_W$$

For $\sin^2\theta w = 0.22 \pm 0.01$, the calculated masses for the W^+ and Z° (including the effects of radiative corrections) are [232]:

$$M_{W^+} = 82.6 \pm 1.7 \text{ GeV}/c^2$$

$$M_{Z^\circ} = 93.4 \pm 1.4 \text{ GeV}/c^2$$

II. WEAK DECAYS OF QUARKS AND LEPTONS

The decays of quarks and leptons are observed to proceed only by the weak charged V-A current J_W^c, to first order in G_F. Due to the fundamental nature of quark-lepton universality, (i.e. that quarks and leptons couple with the same strength $G_F/\sqrt{2}$ to the weak charged current), the decays of leptons and quarks are identical, aside from mass factors and radiative (QED and QCD) corrections.
Since the decays of quarks and leptons have with masses $m \ll M_W^2$ we may use the effective weak Hamiltonian

$$H_W^{eff} = \frac{G_F}{\sqrt{2}} J_\mu^+(x) \; J^{\mu -}(x) \; + \text{h.c.}$$

to calculate the (inclusive) decay rate for $N \to n\alpha\bar{\beta}$ via "W-radiation" (Fig.1):

$$\Gamma = \frac{1}{2M_N} \sum_{spin} \sum_{n,\alpha,\beta} (2\pi)^4 \; \delta^4(P_N - P_n) <N|H_W^{eff}(0)|n><n|H_W^{eff}(0)|N>$$

$$\Gamma = \left(\frac{G_F}{\sqrt{2}}\right)^2 \frac{1}{(2\pi)^2} \frac{1}{2S+1} \int d^4k \, \delta(k^2)\, \theta(k_0) \int d^4k' \, \delta(k'^2)\, \theta(k_0') \, \delta^4(P_N - (P_n + k + k'))$$

$$\times \sum_{spin\, \alpha,\beta} <0|J^{\nu\dagger}(0)|\alpha\bar{\beta}><\alpha\bar{\beta}|J^\mu(0)|0>$$

$$\times \frac{1}{2M_n} \sum_n <N|J_\nu^\dagger(0)|n><n|J_\mu(0)|N>$$

The decay rate for leptons and (free) quarks with $N \to n\alpha\bar{\beta}$,

$$N = L, Q \qquad \alpha = \ell, q \qquad n = \ell, q \qquad \bar{\beta} = \bar{\nu}, \bar{q}$$

summed over all admissible quark and lepton flavors, and spins, is given by:

$$\Gamma^\circ = \frac{G_F^2}{192\pi^3} M_N^5 \sum_{n,\alpha,\bar{\beta}} |U_{Nn}|^2 |U_{\alpha\beta}|^2 \phi(m_n, m_\alpha, m_\beta; m_N)$$

Where U_{ij} are the ij-th matrix elements for the K-M mixing matrices for the quarks (and/or leptons), and $\phi(m_n, m_\alpha, m_\beta; m_N)$ is a mass factor (which is a complicated expression, and is given in refs.217,218,230.) For the special case where one of the daughter masses n is heavy and the others zero (or negligibly small), as for $c \to s\, \ell^+ \nu_\ell$:

$$\phi(x,0,0;1) \equiv f(x) = (1 - x^4)(1 - 8x^2 + x^4) - 12x^4 \ln(x^2)$$

Where $x \equiv m_n/m_N$. For two daughter particles with equal masses:

$$\phi(x,x,0;1) \equiv g(x) = (1 - \tfrac{7}{2}x^2 - \tfrac{1}{8}x^4 - \tfrac{3}{16}x^6)(1 - x^2)^{1/2}$$

$$+ \; 3x^4(1 - \tfrac{1}{16}x^4) \ln((1 + \sqrt{1-x^2})/\sqrt{x})$$

Where $x \equiv (2m_n/m_N)$

II-1. SEMI-LEPTONIC DECAYS

From the above formulae, the decay rate for leptons and free quarks $N \rightarrow n\, \ell\, \nu_\ell$ is given by:

$$\Gamma^{o}_{SL} = \frac{G_F^2}{192\pi^3} M_N^5 \sum_n |U_{Nn}|^2 |U_{\ell\nu_\ell}|^2 \phi(m_n, m_\ell, m_{\nu_\ell}; m_N)$$

Where $N = L$, Q and $n = \nu_L$, q.

The semi-leptonic decay rate is altered by radiative effects from the renormalization of the N-n vertex by virtual and real photon emission (inner bremmstrahlung) for quarks and leptons, and similarly, by virtual and real gluon emission (for free quarks only). The Feynman diagrams associated with first-order radiative corrections are shown in Fig.2.

Calculation of the radiative QED corrections for (free) quark decay requires the use of the complete (rather than effective) weak Hamiltonian due to the fractional quark (rather than integer) charges [246]. The same is true for radiative QED corrections for "semi-hadronic" decays of leptons, e.g. $L^+ \rightarrow \nu_L\, u\, \bar{d}$.

Thus, for lepton decay (only) the radiative QED corrections are [245]:

$$\Gamma^{r}_{SL} = \Gamma^{o}_{SL}\left(1 - \frac{\alpha}{2\pi}\left(\pi^2 - \frac{25}{4}\right)\right) = \Gamma^{o}_{SL}(1 - 0.0042)$$

$$\alpha = e^2/\hbar c = 1/137$$

Where the first term in the inner parenthesis represents the contribution from virtual photon emission (loop effects) and acts to damp the decay. The second contribution from real photon emission (i.e. inner bremmstrahlung) acts to enhance the decay. It can be seen that one-loop virtual photon emission dominates the higher order QED effects, interfering destructively with the "bare" decay diagram. Note that the magnitude of the first-order QED corrections are rather small, on the order of 0.4%.

In exact analogy to the QED corrections, the QCD corrections to the semi-leptonic decay rate for (free) quarks may be calculated by replacing α by $\alpha_s(m_Q^2) \sum \lambda^i \lambda_i = \frac{4}{3}\alpha_s(m_Q^2)$, as there is a one-to-one correspondence between QED and QCD diagrams for radiative corrections [193-196].

$$\text{Thus, } \Gamma_{SL}^{q} = \Gamma_{SL}^{o}\left(1 - \frac{4}{3}\left(\frac{\alpha_s(M_Q^2)}{2\pi}\right)\left(\pi^2 - \frac{25}{4}\right)\right)$$

Where the strong coupling $\alpha_s(M_Q^2)$ is the running quark-gluon coupling "constant" evaluated at the mass M of the quark Q.

$$\alpha_s(m_Q^2) = \frac{12\pi}{b_n}\left[\ln(m_Q^2/\Lambda^2)\right]^{-1}$$

Where $b_n = 11N - 2n$, $N = 3$ for SU(3)

n = number of relevant quark flavors at mass scale m_Q.

Λ = scale parameter reflecting the onset of scaling, determined from deep-inelastic lepton-hadron scattering processes.

For charm decays, the "radiative corrections" due to real and virtual gluon emission are quite large, reducing the 0-th order calculation by approximately 35% (for Mc = 1.65 GeV, Λ = 0.5 GeV, and α_s = 0.6)

The calculated semi-leptonic decay rate for a free charmed quark c to first order in α_s is therefore:

$$\Gamma_{SL}^{ch} = \frac{G_F^2}{192\pi^3}M_c^5\left(1 - \frac{4}{3}\left(\frac{\alpha_s(M_c^2)}{2\pi}\right)\left(\pi^2 - \frac{25}{4}\right)f\left(\frac{m_s}{m_c}\right)\right.$$

$$= 1.6 \pm 0.3 \times 10^{11}\ \sec^{-1}$$

The largest source of uncertainty in the above calculation originates from the uncertainty in the charmed quark mass (on which the decay rate varys as the fifth power).

II-2. NON-LEPTONIC DECAYS

The non-leptonic decay rate for leptons and free quarks with $N - n\alpha\bar{\beta}$; $\alpha, \bar{\beta}$ quarks, summed over all admissible quark and lepton flavors is given by:

$$\Gamma^{\circ}_{NL} = \frac{G_F^2}{192\pi^3} M_N^5 \sum_{n,\alpha,\bar{\beta}} |U_{Nn}|^2 |U_{\alpha\bar{\beta}}|^2 \phi(m_n, m_\alpha, m_{\bar{\beta}}; m_N)$$

Again, the effects of gluon emission modify this result:

$$\Gamma^{g\ell}_{NL}(L) = \Gamma^{\circ}_{NL}\left(1 + \frac{4}{3}\left(\frac{\alpha_s(M_L^2)}{2\pi}\right)\left(\frac{3}{2}\right)\right)$$

for lepton "semi-hadronic" decay [192]

$$\Gamma^{g\ell}_{NL}(Q) = \Gamma^{\circ}_{NL}\left(1 - \frac{4}{3}\left(\frac{\alpha_s(M_Q^2)}{2\pi}\right)\left(\pi^2 - \frac{25}{4}\right) + \frac{4}{3}\left(\frac{\alpha_s(M_Q^2)}{2\pi}\right)\left(\frac{3}{2}\right)\right)$$

$$= \Gamma^{\circ}_{NL}\left(1 - \frac{4}{3}\left(\frac{\alpha_s(M_Q^2)}{2\pi}\right)\left(\pi^2 - \frac{31}{4}\right)\right)$$

for (free) quark decay [192]

Where for free quark decay, the first correction term is the same as in the semi-leptonic decay of the free quark, from the N,n vertex, the second correction term is from gluon exchange renormalization effects at the $\alpha, \bar{\beta}$ vertex. The effect of real gluon emission from final state $\alpha, \bar{\beta}$ quarks in lepton or free quark decay is insignificant [192]. The corrections due to virtual gluon exchange between the N-n and $\alpha - \bar{\beta}$ quarks are also negligible for free quark decay [192]. The non-leptonic decay rate for free charmed quarks, to first order in α_s is therefore:

$$\Gamma^{g\ell_1}_{NL}(c) = 2.1 \pm 0.6 \times 10^{11}\ \mathrm{sec}^{-1}$$

The total decay rate for free charmed quark decay, to first order in α_s is thus:

$$\Gamma^{TOT}_{CH}(q\ell) = 2\Gamma^{g\ell_1}_{SL}(c) + 3\Gamma^{g\ell_1}_{NL}(c)$$

(For 2 leptons e, μ, and 3 colors of quarks $\alpha, \bar{\beta}$.)

$$\Gamma^{TOT}_{CH}(WL) = 9.6 \pm 2.0 \times 10^{11}\ \mathrm{sec}^{-1}$$

$$\tau^{g\ell_1}_{CH} = 10.3 + 2.7 - 1.8 \times 10^{-13}\ \mathrm{sec}$$

The semi-leptonic branching ratio, to first order in α_s is

$$Br_{g\ell}(c \rightarrow s\, \ell^+ \nu_\ell) = 16 \pm 2\ \%$$

Note that for the free-field decay of a charmed quark (i.e. no strong interaction effects):

$$\Gamma^{0}_{sl} \approx \Gamma^{0}_{nl} = 2.9\pm0.8 \times 10^{11} \ sec^{-1}$$

$$\Gamma^{0}(c\to s l^{+}\nu_{l}) = 14.6 \pm 4.1 \times 10^{11} \ sec^{-1}$$

$$\tau^{free}_{CH} = 6.8{+}2.7{-}1.5 \times 10^{-13} \ sec$$

$$Br_{free}(c\to s l^{+}\nu_{l}) = 20\%$$

Second-order QCD corrections to the non-leptonic decays of free quarks have recently been calculated [198], and have been found to increase the non-leptonic rate over the first-order calculation by approximately 22% (55%) for Λ = 0.25 (0.50), such that:

$$\Gamma^{q/2}_{NL} = 1.22\,\Gamma^{q/1}_{NL} \quad (\Lambda = 0.25)$$

$$\Gamma^{q/2}_{NL} = 1.55\,\Gamma^{q/1}_{NL} \quad (\Lambda = 0.50)$$

The total decay rate for free charmed quarks is increased accordingly:

$$\Gamma^{TOT}_{CH}(q/2) = 1.1 \quad (1.3) \times 10^{12} \ sec^{-1}$$

$$\tau^{q/2}_{CH} = 9.1 \quad (7.6) \times 10^{-13} \ sec$$

for Λ = 0.25 (0.50)

The semi-leptonic branching ratio is correspondingly reduced:

$$Br_{q/2}(c \to s\, l^{+} \nu_{l}) = 14\% \ (12\%)$$

In another, independent approach, the renormalization effects of the strong interactions (in the framework of QCD) upon the weak interactions for non-leptonic decays from hard gluon exchange between quarks at the four fermion vertex have been calculated to first order in α_s [185,186,199] with the use of the operator product (short distance) expansion and renormalization group techniques, allowing the effective weak Hamiltonian to be expressed in terms of local operators involving quark fields. The effective weak non-leptonic Hamiltonian for quark decay may thus be written [185,186] as:

$$H^{eff}_{w} = \frac{G_F}{\sqrt{2}} U_{\alpha f} U^{*}_{q Q} \left[\tfrac{1}{2}(f_{+} + f_{-})(\alpha\bar{\beta})(\bar{q}Q) + \tfrac{1}{2}(f_{+} - f_{-})(\bar{q}\beta)(\bar{\alpha}Q)\right]$$

Where $\bar{q}_1 q_2$ is a left-handed color-symmetric V-A current

$$\bar{q}_1 q_2 = \tfrac{1}{2} q^{a}_{1}(1-\gamma_5) q_{2a} \qquad a = \text{color index}$$

The coefficients f_+ and f_- embody short distance renormalization effects due to hard gluon exchange between quarks.

$$f_- = \prod_{k=n}^{6} \left[\frac{\alpha_s(M^2_{k+1})}{\alpha_s(M^2_k)} \right]^{12/b_k} \qquad k=1,2\ldots 7=d,u,s,c,b,t,W^+$$

$$f_+ = 1/\sqrt{f_-} \qquad b_k = 33-2k$$

$f_- = f_+ = 1$ in the free quark limit.

$$\alpha_S(Q^2) = \frac{12\pi}{b_k \ln(Q^2/\Lambda^2)} \text{ for } m^2_k < Q^2 < m^2_{k+1}$$

$$\text{for } m^2_k < Q^2 < m^2_{k+1} \quad : \quad \Lambda^2_k = (m^2_{k+1})^{b_{k+1}-b_k/b_{k+1}} (\Lambda^2_k)^{b_k/b_{k+1}}$$

The calculated values for f_-, f_+ and $\alpha_S(M^2_Q)$ for s,c,b,t are [199,216]:

	f_-	f_+	$\alpha_S(M^2_Q)$
t	1.26	0.89	0.20
b	1.56	0.80	0.34
c	2.15	0.68	0.60
s	2.80	0.60	0.80

It can be seen that the ratio f_-/f_+ grows as the mass scale decreases. The strong coupling constant $\alpha_S(M^2_Q)$ also grows as the mass scale decreases. To understand the physical meaning of the coefficients f_- and f_+ we (Fierz) rearrange the terms in the weak Hamiltonian into the form:

$$H^{eff}_w = \frac{G_f}{\sqrt{2}} U_{\alpha\bar{\beta}} U^*_{qQ} [f_- O^- + f_+ O^+]$$

Where the Fierz transformation of the quark operators is of the form [203,232]:

$$(\bar{q}_1 q_2)(\bar{q}_3 q_4) = \tfrac{1}{3}(\bar{q}_1 q_4)(\bar{q}_3 q_2) + \tfrac{1}{2}(\bar{q}_1 \lambda^a q_4)(\bar{q}_3 \lambda_a q_2)$$

The operators O^- (O^+) consist of color anti-symmetric (color symmetric) (V-A) X (V-A) ((V-A) X (V+A)) four quark operators, e.g. $O^\pm \sim \frac{1}{2}[(\bar{s}u)_L(\bar{u}d)_L \pm (\bar{s}d)_L(\bar{u}u)_L]$ (color indices supressed). The operator O^- (O^+) is pure $\Delta I = 1/2$ (a mixture of $\Delta I = 3/2, 1/2$) and to lowest order in α_s, O^- and O^+ are renormalized multiplicatively. The terms in the effective weak Hamiltonian with the coefficients f_- and f_+ therefore represent transitions which are pure $\Delta I = 1/2$ or a mixture of $\Delta I = 1/2$, 3/2 and transform as members of the SU(4) $[20_s]$ and $[84_s]$, respectively, of the non-leptonic H^{eff}_W. (This aspect is discussed in greater detail, below.)

The net contribution to the non-leptonic decay rate for $Q \rightarrow q\,\alpha\,\bar{\beta}$ is:

$$\Gamma_{NL}^{q\ell H_W} = a_3 \Gamma_{NL}^{\circ}\ ,\ a_3 = \frac{1}{3}(f_-^2 + 2f_+^2)$$

Where the factor 1/3 is from the color average over the intial state and the factor of 3 is the color sum over final color states. The effects of soft gluon exchange are not incorporated in this (first order) calculation. Additional factors may enter if the decay products are correlated (in flavor, color, momentum or spin) with the spectator quarks [204].

The total decay rate for a free charmed quark within this context is

$$\Gamma_{CH}^{TOT}(H_W) = 2\Gamma_{SL}^{q\ell} + 3\Gamma_{NL}^{q\ell H_W}$$

$$\Gamma_{CH}^{TOT}(H_W) = 1.9 \pm 0.6 \times 10^{12}\ \mathrm{sec}^{-1}$$

$$\tau_{CH}^{H_W} = 5.1 \pm 1.6 \times 10^{-13}\ \mathrm{sec}$$

The semi-leptonic branching ratio is $Br_{H_W}(c\text{-}sl\nu) = 8 \pm 2\%$ Note that this is close to the average of the semi-leptonic branching ratio for D^+ and D° decays [160].

THE STRUCTURE OF THE CHARGED WEAK CURRENT Jw

A Perspective on the Origin of The Lifetime Differences Between Charmed Mesons

The D lifetime was expected to be longer than the D on the basis of the structure of the non-leptonic part of the effective weak Hamiltonian Hw, within the framework of SU(4):

$$H_W^{eff} = \frac{G_F}{\sqrt{2}} J_\mu^+ J^{\mu-} + H.C.$$

This can be seen from the following. The weak charged current J_μ^+ transforms as a [15] in SU(4), thus:

$$H_W \sim [15] \otimes [15^*] + H.C.$$

$$[15] \otimes [15^*] = [84_s] \oplus [45_a] \oplus [45_a^*] \oplus [20_s] \oplus [15_a] \oplus [15_s] \oplus [1_s]$$

As Hw involves symmetric transitions the relevant SU(4) multiplets are the

$$H_W^{eff} \sim [84_s] \oplus [20_s] \oplus [15_s] \oplus [1_s]$$

For charm changing transitions, only the $[84_s]$ and the $[20_s]$ contribute [37,41-44]. The $[84_s]$ and the $[20_s]$ can be further decomposed into SU(3) charm-changing subgroups:

$$[84_s] = \underbrace{\{6\} \oplus \{6^*\}}_{\Delta C=\pm 2} \oplus \underbrace{\{3\} \oplus \{15\} \oplus \{3^*\} \oplus \{15^*\}}_{\Delta C=\pm 1} \oplus \underbrace{\{1\} \oplus \{8\} \oplus \{27\}}_{\Delta C=0}$$

$$[20_s] = \underbrace{\{6\} \oplus \{6^*\}}_{\Delta C=\pm 1} \oplus \underbrace{\{8\}}_{\Delta C=0}$$

The $\Delta C = \pm 1$ terms in Hw which transform as a {6} and a {15} under SU(3) can be expressed most clearly (after a Fierz rearrangement [185,186] of terms in Hw as:

$$H_W^{eff} = \frac{G_F}{\sqrt{2}} [f_- O^- + f_+ O^+]$$

where the operators O^-, (O^+) consist of color anti-symmetric (color symmetric) (V-A) x (V-A) (V-A) x (V+A) four-quark operators, e.g.

$$O^\pm = 1/2[(\bar{c}s)_L(\bar{d}u)_L \pm (\bar{c}u)_L(\bar{d}s)_L]$$
(color indices supressed)

The operator O^- is pure $\Delta I = 1/2$ in nature (i.e. connects initial and final states with $\Delta I = 1/2$) and concerns that part of H_W^{eff} which transforms as a $\{\dot{6}\}$ under SU(3) for $\Delta C = \pm 1$ transitions.

The operator O^+ is a mixture of $\Delta I = 1/2, 3/2$ in nature and concerns that part of H_W^{eff} which transforms as a $\{15\}$ under SU(3) for $\Delta C = \pm 1$ transitions.

In the absence of strong interactions between quarks (i.e. the "naive" free quark model) the coefficients $f_- = f_+ = 1$.

In the presence of strong interactions between quarks, these coefficients may differ from unity. In the context of first order QCD, the effects of hard gluon exchange at the four-fermion vertex may be computed using the operator product expansion and renormalization group techniques [185,186,199]. The effects of hard gluon exchange are embodied in the coefficients f_- and f_+ for the $\Delta I = 1/2$ and $\Delta I = 1/2, 3/2$ transitions of the $\{6\}$ and $\{15\}$. The values of f_-, f_+ depend primarily upon the value of the strong coupling constant $\alpha_S(M_Q)$ at the mass scale M_Q associated with the problem. For charm:

$$f_- = 2.15, \quad f_+ = f_-^{-1/2} = 0.68$$

$$\text{for} \quad \alpha_S(M_c^2) = 0.6$$

Thus, transitions occurring within the $\{6\}$ will be enhanced by a factor of ~ 3.2 in amplitude (~ 10.0 in rate) over transitions occurring within the $\{15\}$ for $\Delta C = \pm 1$ transitions. Thus $\{6\}$ enhancement (or $\{6\}$ dominance) for charm in SU(4) is the analog of octet enhancement for strange particle decays in SU(3).

Now, it has been shown [21,42] that the Cabibbo-favored two-body decays of the D^+ to two pseudoscalar (or vector) mesons may proceed only via the SU(3) $\{15\}$ of the SU(4) $[84_S]$ of H_W^{eff}, while similar decays for the D° (and F^+) meson proceed via the SU(3) $\{6\}$ of the SU(4) $[20_S]$. For the multi-body (e.g. three-body) decays, the decays of the D^+ were expected to be supressed relative to the D° and F^+, as the Cabibbo-favored decays of the D^+ are "exotic" in flavor, in the usual quark model sense (having strangeness $S = -1$ and net positive charge), while the decays of the D° and F^+ mesons are "non-exotic". Hence, from the above, the expectations were that the non-leptonic rate for the D^+ was believed to be supressed relative to the corresponding non-leptonic rates for the D° and F^+ mesons. Consequently, the D^+ meson was expected to be longer lived than the D°.

Another effect, due to the occurrence of strong "color clustering" for the $c\bar{d} \rightarrow (s\bar{d}) + (u\bar{d})$ decay process, the $\bar{d}\ \bar{d}$ anti-quarks interfering destructively in the final state [204,219], is expected to further supress the D^+ decay rate. This can be seen more clearly from the following. The operators O^- and O^+ of the effective weak Hamiltonian H_W^{eff} are linear combinations of the operator O_1 and O_2, i.e.

$$O^{\pm} = 1/2(O_1 \pm O_2)$$

Where $O_1 = (\bar{c}s)_L (\bar{d}u)_L$ and $O_2 = (\bar{c}u)_L (\bar{d}s)_L$

The decay amplitudes for the D^+ and D° in terms of the operators O_1 and O_2 are given by [243]:

$$D^+ : \quad \begin{aligned} O_1 |c\bar{d}> &= (\bar{d}s)_L + (\bar{d}u)_L \\ O_2 |c\bar{d}> &= (\bar{d}s)_L + (\bar{d}u)_L \end{aligned}$$

$$D^\circ : \quad \begin{aligned} O_1 |c\bar{u}> &= (\bar{u}s)_L + (\bar{d}u)_L \\ O_2 |c\bar{u}> &= (\bar{u}u)_L + (\bar{d}s)_L \end{aligned}$$

Thus, it can be seen that cancellation between the two amplitudes for O_1 and O_2 can occur for the D^+ but not for the D°, thus acting to suppress the decay rate for the D^+ (and therefore increasing its lifetime). See also Fig.4.

The quantitative impact of this phenomena on the non-leptonic decay rates for charmed particles is summarized below for Cabibbo-favored decays, assuming exact cancellation for the color-connected W-radiation diagrams for the D^+. (Note that for Cabibbo-unfavored c-decays, the reverse situation occurs for the D^+ and F^+.)

$$\Gamma_{NL}^{\gamma\ell} (D^+)_{H_W} = \frac{4}{3} f_+^2 \Gamma_{NL}^{0}$$

$$\Gamma_{NL}^{\gamma\ell} (D^\circ)_{H_W} = \frac{1}{3}(f_-^2 + 2f_+^2)\Gamma_{NL}^{0}$$

$$\Gamma_{NL}^{\gamma\ell} (F^+)_{H_W} = \frac{1}{3}(f_-^2 + 2f_+^2)\Gamma_{NL}^{0}$$

$$\Gamma_{NL}^{\gamma\ell} (\Lambda_c^+)_{H_W} \simeq \frac{1}{3}(f_-^2 + 2f_+^2)\Gamma_{NL}^{0}$$

II-3. WEAK DECAYS of PSEUDOSCALAR MESONS

A. Annihilation and $W^{\pm}$-Exchange Diagrams

Additional decay mechanisms exist for pseudoscalar mesons which are not available to leptons or free quarks, via interactions between the quark-antiquark pair. Pseudoscalar mesons couple to the spin-0 component of the virtual W through the divergence of the axial vector current [204]. Thus, for charged mesons, the $Q\bar{q}$ pair can annihilate into a virtual W^{+} which can then "decay" to a $\ell\nu_{\ell}$ lepton pair or a $\alpha\bar{\beta}$ quark pair (Figs.3,4). Since the W^{+} boson carries no color, the annihilating $Q\bar{q}$ pair must be in a color singlet state. For neutral mesons, the $Q\bar{q}$ quarks can exchange a W^{+} transforming themselves to $\alpha\bar{\beta}$ quarks (Figs.3,4). As the Lorentz structures of the annihilation (s-channel) and exchange (t-channel) diagrams are identical for momenta << M_W, being related in this limit by a Fierz reordering of the effective four-fermion vertices, the decay rates for annihilation and $W^{\pm}$-exchange will have the same form [203,204,205].

For annihilation decays to leptons or quarks [209,210]:

$$\Gamma^{0}_{ANN} = \frac{G_F^2}{8\pi} M_P^5 \left(\frac{f_P^2}{M_P^2}\right) |U_{qQ}|^2 \, |U_{\alpha\bar{\beta}}|^2 \rho\left(\frac{M_\alpha^2}{M_P^2}, \frac{M_\beta^2}{M_P^2}\right)$$

For exchange decays to quarks:

$$\Gamma^{0}_{EXCH} = \frac{G_F^2}{8\pi} M_P^5 \left(\frac{f_P^2}{M_P^2}\right) |U_{\alpha Q}|^2 \, |U_{\bar{q}\bar{\beta}}|^2 \rho\left(\frac{M_\alpha^2}{M_P^2}, \frac{M_\beta^2}{M_P^2}\right)$$

Where [211]: $\rho(x,y) = g_p(x,y)\,\lambda^{1/2}(1,x,y)$

$$g_p(x,y) = x + y - (x - y)^2$$

$$\lambda(x,y,z) = x^2 + y^2 + z^2 - 2(xy + yz + zx)$$

For leptonic decays, and annihilation to quarks or W-exchange where one of the final state quarks or leptons has negligible mass,

$$\rho(x, 0) = (1 - (x)^2)(x) \qquad n = \ell, \alpha .$$

The factor $x = (m_n^2/m_P^2)$ represents the effect of helicity supression in the decay, which arises because the coupling of the virtual W^{+} to the left-handed $n = \ell, (\alpha)$ lepton ℓ (quark α) is constrained to be in a total angular momentum J = 0 state, since the spin of the pseudoscalar meson is zero. Therefore the helicity of $n = \ell, \alpha$ must be opposite to its spin, which introduces a factor of m_n/E_n into the decay amplitude. The annihilation (and also the W-exchange) rate vanishes in the limit $m_n \to 0$.

f_P is the axial current coupling constant, or "decay constant" of the pseudoscalar meson, defined as [209,210]:

$$<0 \mid J^A_\mu(0) \mid P(q)> = i\, f_P\, q_\mu\, U_{\bar{q}Q}$$

Where $J^A_\mu(0)$ is the axial vector current, q_μ is the four-momentum of the pseudoscalar meson P.

Numerically, $f_P \sim \sqrt{\frac{6}{M_P}}\,|\psi_P(0)|$, where $\psi_P(0)$ is the wave function of the meson at the origin. The ratio (f_P^2/M_P^2) represents the probability of annihilation of the $Q\bar{q}$ pair. As M_Q increases, the annihilation rate is supressed, unless f_P also increases. For exact SU(6) symmetry (all quark masses equal) all decay constants f_P are equal. If SU(6) is not spontaneously broken the decay constants are expected to be approximately equal [21,186]. The decay constants for some of the pseudoscalar mesons have been calculated [205,206] (the values for f_π and f_K are experimentally measured quantities [207,208]) and are summarized below. The theoretical calculations for the decay constants f_P have uncertainties on the order of factors of ~ 2, due to the assumptions used in the calculations.

<u>PSEUDOSCALAR MESON DECAY CONSTANTS</u>

Pseudoscalar Meson	Decay Constants f_P (MeV)
π	93
K	114
D	~150
F	~170
$B_{u,d}$	~500
B_s	~520

B. Gluon Bremmstrahlung

The helicity supression effects in pseudoscalar decay may be partially overcome by emission of hard photons and/or gluons from the initial state; the photon or gluon carries away one unit of angular momentum, leaving the $Q\bar{q}$ pair in a spin-one state. (Soft gluons may play an important role here too.) As gluon emission is expected to have a much more pronounced effect on the annihilation and W-exchange decay rate, we neglect the radiative QED effects.

It would seem at first glance that to first order in α_s that there can be no gluon contribution to the annihilation of $Q\bar{q}$ since the initial $Q\bar{q}$ pair must occur in a color singlet state. Thus, gluon effects should only appear in second order QCD since at least two gluons must be emitted to form a color singlet hadronic state. However, from consideration of the Fock state of the pseudoscalar meson, soft gluons are expected to be present in the initial meson wave function, thus all possible $Q\bar{q}$ color states will be populated according to their statistical weights. Hence first order QCD effects may be present in $Q\bar{q}$ annihilation and W-exchange.

The effects of gluon radiation and exchange have been calculated in first-order QCD for the annihilation and W-exchange diagrams using a non-relativistic model for the Qq system [212-215]. This calculation is expected to be valid for heavy quark $Q\bar{q}$ systems, where the non-relativistic approximation is expected to hold. It will not be so with the light qq systems (e.g. π, K) as the assumption of non-relativistic behavior is not valid for light quark systems. Thus, for heavy quarks:

For $Q\bar{q}$ annihilation:

$$\Gamma^{g}_{ANN} = \frac{G_F^2}{108\pi^2} M_P^5 |U_{\bar{q}Q}|^2 |U_{\bar{\beta}\alpha}|^2 \, a_8^- \left\{ \alpha_s(m_Q^2) \left[|F_V|^2 + |F_A|^2 \right] \right\} g_P\left(\frac{M_\alpha^2}{M_P^2}, \frac{M_{\bar{\beta}}^2}{M_P^2} \right)$$

For $Q\bar{q}$ W-exchange:

$$\Gamma^{g}_{EXCH} = \frac{G_F^2}{108\pi^2} M_P^5 |U_{\alpha Q}|^2 |U_{\bar{\beta}\bar{q}}|^2 \, a_8^+ \left\{ \alpha_s(m_Q^2) \left[|F_V|^2 + |F_A|^2 \right] \right\} g_P\left(\frac{M_\alpha^2}{M_P^2}, \frac{M_{\bar{\beta}}^2}{M_P^2} \right)$$

Where $a_8^- = \frac{1}{4}(f_- - f_+)^2 \qquad a_8^+ = \frac{1}{4}(f_- - f_+)^2$

$$F_V \simeq \frac{|\Psi_P(0)|}{M_{\bar{q}} M_Q}(m_P)^{1/2} = \frac{f_P}{\sqrt{6}}\left(\frac{M_P}{M_{\bar{q}} M_Q}\right)$$

$$F_A \simeq |\Psi_P(0)| \frac{M_{\bar{q}} - M_Q}{M_{\bar{q}} M_Q M_P}(m_P)^{1/2} = \frac{f_P}{\sqrt{6}}\left(\frac{M_{\bar{q}} - M_Q}{M_{\bar{q}} M_Q}\right)$$

Thus:

$$\Gamma^{g\ell}_{ANN} = \frac{G_F^2}{108\pi^2} M^5{}_P \left(\frac{f_P^2}{M_{\bar q}^2}\right) |U_{\bar q Q}|^2 \, |U_{\beta\alpha}|^2 \; a_g^- \;\; \alpha_s(m^2_Q) \;\; g_P\left(\frac{M_\alpha^2}{M_P^2}, \frac{M_\beta^2}{M_P^2}\right)$$

$$\Gamma^{g\ell}_{EXCH} = \frac{G_F^2}{108\pi^2} M^5_P \left(\frac{f_P^2}{M_{\bar q}^2}\right) |U_{\alpha Q}|^2 \, |U_{\bar\beta \bar q}|^2 \; a_g^+ \;\; \alpha_s(m^2_Q) \;\; g_P\left(\frac{M_\alpha^2}{M_P^2}, \frac{M_\beta^2}{M_P^2}\right)$$

Contributions for gluon emission from final state quarks will be supressed by helicity factors (powers of m_α^2 , m_β^2) and are neglected. Note that the helicity supression factor (m_α^2/M_P^2) is not present for these diagrams, and that the annihilation supression factor ($f_P^2/M_{\bar q}^2$) contains the light $\bar q$ quark mass $m_{\bar q}$ rather than the heavy Q quark mass m_Q . Since the W-boson is colorless, QCD renormalization effects via first-order gluon exchange and second-order gluon emission at the weak $Q\bar q$ vertex are expected to be important with regard to first-order gluon emission. Note further that the gluon radiation contribution to annihilation vanishes in the free quark limit, $f_- = f_+ = 1$. Also note that the semi-leptonic decays of $Q\bar q$ charged pseudoscalar mesons to light charged leptons (e and μ) are also enhanced by gluon emission. The possibility of the remnant gluons forming "glueballs" from such leptonic decays depends upon whether glueball states exist in nature, and whether or not their masses are "allowed" by the phase space of pseudoscalar meson leptonic decay.

Note that for the decays of the charmed D^+ , D° and F^+ pseudoscalar mesons, the contribution of the W-annihilation diagram (with or without gluons) for the D+ meson is Cabibbo-supressed. Hence, the contribution of this diagram to the total decay rate for the D^+ is small; The lifetime for the D+ will be correspondingly longer than that for the D° or F^+ mesons.

C. Penguin Diagrams

The last weak decay mechanism we consider is the class of diagrams known as "penguin" diagrams, or "color-radius" interactions. The lowest order penguin diagrams contribute a term to the effective weak Lagrangian of the form:

$$\mathcal{L}_W^{eff}(\text{Penguin}) = G_W^{eff}(k^2)\; U_{\bar{\alpha}Q}\; U^*_{\beta\bar{\alpha}}\; (\bar{Q}\;(\gamma_\mu(1-\gamma_5)\lambda^a\,\beta)$$
$$\times \sum_\alpha (\bar{q}\gamma^\mu\lambda^a q + \bar{\alpha}\gamma^\mu\lambda^a\alpha) + \text{h.c.}$$

Where the effective weak coupling constant is given by:

$$G_W^{eff}(k^2) = 32\frac{G_F}{\sqrt{2}}F(k^2)$$

The form factor $F(k^2)$ is given in first-order QCD by the Feynman integral:

$$F(k^2) = \frac{\alpha_s}{4\pi}\int_0^1 dz\; z\;(1-z)\;\ln\left\{\frac{M^2_{\alpha_1} - k^2 z(1-z)}{M^2_{\alpha_2} - k^2 z(1-z)}\right\} = \frac{\alpha_s}{4\pi} f(k^2)$$

$$f(k^2) = \ln\left(\frac{M_W^2}{M^2_{\alpha_1}}\right) - \ln\left(\frac{M_W^2}{M^2_{\alpha_2}}\right)$$
$$+\;\left(1 - \frac{4M^2_{\alpha_2}}{k^2}\right)\ln\left[1 - \frac{k^2}{2M^2_{\alpha_2}}\left(1 - \left(1 - \frac{4M^2_{\alpha_2}}{k^2}\right)^{1/2}\right)\right.$$
$$-\;\left(1 - \frac{4M^2_{\alpha_1}}{k^2}\right)\ln\left[1 - \frac{k^2}{2M^2_{\alpha_1}}\left(1 - \left(1 - \frac{4M^2_{\alpha_1}}{k^2}\right)^{1/2}\right)\right.$$

Where k is the four-momentum transfer carried by the gluon $k^2 = m^2_{\bar{q}} - m_Q m_{\bar{q}}$, the invariant mass of the t-channel exchanged virtual gluon, assuming the initial $Q\bar{q}$ to be at rest. λ^a is a color SU(3) matrix.

The integral over the four-momentum k of the gluon is separated into two regions, $\mu^2 < k^2 < m_Q^2$ with the number of operative quark flavors n such that $n_{loop} = n_Q$, the generalized GIM mechanism being inoperative in this region. The second region has $m_Q^2 < k^2 < m_W^2$, with $n_{loop} = n_Q + 1$; the GIM mechanism is operative in this region, and (partial) cancellation occurs. In the SU(6) limit, total cancellation occurs as the αt quarks in the loop have the same masses, the total amplitude vanishes.

For $k^2 << m_Q^2, m_{\bar{q}}^2$ (appropriate for B meson decay $b\bar{q}\to s\bar{q}$)

$$k^2 \simeq m_b m_{\bar{q}} << m_b^2, m_{\bar{q}}^2 \quad (\bar{q}=\bar{u},\bar{d})$$

$$F(k^2) \simeq \frac{\alpha_s}{4\pi}\left(\frac{1}{6}\right)\left[\ln\left(\frac{M_W^2}{M^2_{\alpha_1}}\right) - \ln\left(\frac{M_W^2}{M^2_{\alpha_2}}\right) + O\left(\frac{k^2}{M_b^2}\right)\right]$$

For $m_\beta^2 << k^2 < m_Q^2$ (appropriate for K decay $s\bar{q}-d\bar{q}$)

$$k^2 \simeq m_s m_{\bar{q}}, \qquad (\bar{q}=\bar{u},\bar{d})$$

$$F(k^2) \simeq \frac{\alpha_s}{4\pi}\left(k^2/\,30m_\beta^2\right)$$

For $k^2 >> m_Q^2, m_\beta^2$ (appropriate for $D^{+,0}, F^+$ meson decay $c\bar{q}\to d\bar{q}$)

$$k^2 \simeq m_c m_{\bar{q}}, \qquad (\bar{q}=\bar{u},\bar{d},\bar{s}) \text{ for } D^{+,0}, F^+ \text{ mesons}$$

$$F(k^2) \simeq \frac{\alpha_s}{4\pi}\left(\frac{M_Q^2 - M_\beta^2}{k^2}\right)$$

The mass parameters m_u, m_d, m_s, m_c.. cannot be identified with the constituent masses of the quarks, as they appear in the evaluation of the divergences of currents, and thus characterize the violation of chiral symmetry [204]. The appropriate masses are the current masses of the quarks, $m_u = m_d = 5.4$ MeV, $m_s = 150$ MeV, $m_c = 1500$ MeV [228-230].

A naive "free quark" estimate for the decay rate via the penguin mechanism is given by [204]:

$$\Gamma_{penguin}(Q\bar{q}\to\beta\bar{q}) = \frac{G_F^2}{8\pi} M_P^5 \left[\frac{f_P^{o\,2}}{M_P^2}\right][F(k^2)] \,|U_{\bar{q}Q} \; U^*_{\bar{\beta}\alpha}|^2 \; g_P\left(\frac{M_\beta^2}{M_P^2}, \frac{M_{\bar{q}}}{M_P^2}\right)$$

Where f_P^o is the effective decay constant, $f_P^o > f_P$ since no helicity supression factor is involved in the decay. A crude estimate for f_P^o is [204]

$$f_P^o \simeq \frac{M_P}{(M_Q+M_{\bar{q}})} f_P = \frac{|\psi_P(0)|}{M_m}$$

The helicity supression is absent for penguins, as both the initial and final state quarks may be either left- or right-handed, as can be seen by the structure of the effective weak Lagrangian for penguins. The effective weak Lagrangian is also seen to be pure $\Delta I = 1/2$ in nature for the $Q-\beta$ transition, as gluons cannot transmit isospin. Thus, this diagram, in conjunction with the previously mentioned "octet enhancement" mechanism of $\Delta S = 1$ transitions is believed to fully explain the $\Delta I = 1/2$ rule for hyperon and K-decay. The penguin diagram does not contribute to the dominant $\Delta C = -\Delta S = +1$ transitions since the basic four-fermion coupling is exotic in flavor, containing no identical quarks. Thus, the penguin diagram is not expected to contribute significantly in the decays of charmed particles [186,205,228-230]. The coupling constant renormalization becomes weaker as the mass scale increases, as the average momentum transfer is characterized by M_Q^2. Thus, for bottom decay, the penguin diagram may still contribute to the total rate, but is expected to be supressed by a factor of order $\alpha_s(m_b^2)/\alpha_s(\mu^2)$ relative to strange particle decay [186,205]. Higher order QCD corrections to the penguin diagram were found not to substantially change the lowest order result [228-230].

D. Non-Perturbative Effects

Several phenomena have been proposed as (partial) explanation of the differences in charmed particle lifetimes, which are non-perturbative in origin. One is that soft gluons play an important role in charmed meson (and charmed baryon) decay [216], although the quantitative effects are difficult to determine, the qualitative effects can be more easily understood, as the presence of (primordial) gluons in the initial state allows the W-exchange or W-annihilation diagram to occur in a unsupressed manner; the constraints due to the color iso-singlet nature of the charmed particles being taken care of "naturally" (and with a factor of 3 increase in rate over that of simple (hard) gluon bremmstrahlung, when one considers the true Fock state of the particle).

Final state interactions between the daughter hadrons in the decay of the charmed particle have been considered [220] and found to have non-negligible effect on particular decay modes. Likewise, the existence of a narrow S-channel $K-\pi$ resonance near the D mass has been proposed [220] as an expanation for the observed differences in D^+, D° exclusive, non-leptonic decay modes, (i.e. their branching ratios) as measured by experiments at SLAC [139-144]. However, while such a resonance may (dramatically) affect partial rates, it is not expected to have a large effect (if any) on the total rate, and hence on charmed particle lifetimes.

Phenomenological arguments have also been made to account for the difference in charmed particle lifetimes, on the basis of "quark number conservation" (i.e. the $\Delta n_t = 0$ Rule) [41,239-241]. Decays of charmed particles with a different total number of quarks in the final state than in the initial state are supressed (e.g. compare D^+ decays with those for the D°; $q\bar{q}$ pairs such as $u\bar{u}$, $d\bar{d}$, etc. are not counted in the final state).

There exists one non-pertubative phenomenon which can have a non-negligible impact on the lifetimes, from the Pauli exclusion principle. For the D^+, the spectator $\bar{d}$-quark and the $\bar{d}$-quark from the virtual W^+ decay cannot simultaneously be in the same quantum spatial, spin and color state. For the non-leptonic Cabibbo-favored (Cabibbo-unfavored) decays of D^+ (F^+) mesons, a supression factor of 5/6 must be included to take into account the identical fermion (Pauli exclusion principle) effects in the color-connected non-leptonic W-radiation diagrams for the D^+ (F^+) mesons (see Fig.4). The factor of 5/6 is obtained by assumption of equal probability of population of J = 0, 1 states for the initial $Q\bar{q}$ meson, and equal probability of flip/non-flip in the decay of the heavy quark, application of angular momentum conservation rules and Clebsch-Gordanry, and color counting of particle states.

This effect is actually taken into account in a different manner when the effective weak non-leptonic Hamiltonian is used, for these same diagrams (see above).

II-4 WEAK DECAYS of BARYONS

The weak decays of baryons will occur via the same mechanisms as for the mesons, with exception of the annihilation diagram. The "naive" free-quark semi-leptonic and non-leptonic decay rates for Qqq baryons should be similar to that of corresponding Qq mesons. The W-exchange diagram does not suffer from helicity supression effects as the initial "Qq" pair may be either in a total angular momentum state J = 0 or 1, with approximately equal probability. The occurrence of W-exchange may only take place within baryons when at least two of the three constituent quarks have net non-zero charge. The penguin diagram is known to be important for hyperon decay; from the arguments presented above, its role in heavier baryons is expected to have less impact, although it remains to be seen experimentally whether or not this is so. If the dominant contribution to the decay of heavy baryons comes from W-radiation, then naive expectations are such that the lifetimes of of mesons and baryons containing a heavy quark Q will be approximately equal.

For the W-exchange diagram for heavy quarks Q, the partial rate due to this process has been calculated to be [176]:

$$\Gamma^{\gamma\delta}_{EXCH}(B) = \frac{G_F^2}{2\pi} f_-^2 \left|\psi_B(0)\right|^2 \left|U_{\alpha Q}\right|^2 \left|U_{\beta q_\gamma}\right|^2 \left[\frac{\Delta^2}{(M_Q^2 + M_{q_\gamma}^2)}\right] (\Delta^4 - 4M_\alpha^2 M_\beta^2)^{1/2}$$

$$\text{Where} \quad \left|\psi_B(0)\right| = \frac{9}{16\pi} \frac{1}{\alpha_s(M_Q^2)} \frac{M_\beta^2 M_Q^2}{(M_Q - M_\beta)} \Big(\underset{I=1}{M^B_{1/2^+}} - \underset{I=0}{M^B_{1/2^+}} \Big)_{Q\gamma\gamma}$$

$$\Delta^2 = (M_Q + M_{q_\gamma})^2 - M_\alpha^2 - M_\beta^2$$

and $U_{\alpha Q}$, $U_{\beta q_\gamma}$ are the K-M matrix elements for the $Q\alpha$, $q\beta$ vertices involved in the W-exchange process.

The SU(4) weight diagrams for the lowest-lying spin-0, spin-1 mesons and spin-1/2, spin-3/2 baryons containing u, d, s and c quarks are shown in Fig.3.

The weak decays of the D^{+}, D° and F^{+} mesons are shown in Fig.4 for decays via the W^{+}-radiation, W^{+}-exchange/annihilation and penguin diagrams. Similarly, the decay diagrams for some of the ground state (S-wave) baryons are shown in Fig.5.

The "interested reader" is referred to the following papers which consist of seminal articles, review papers on the weak decays of charm or other articles which were of particlar use in writing this document. (This is also only a small subset of many excellent articles):

[21,37,38,39,186,191,197,199,202,204,205,206,210-217,219,

223-232,243].

III. PARTICLE LIFETIMES

The decay mechanisms discussed above are believed to account for the most dominant modes of the (weak) decays of quarks and leptons. Other mechanisms not discussed (e.g. pseudoscalar meson decay via higher order weak interactions, radiative (QED and QCD) effects) are expected to have only small contributions to the total rate.

To determine particle lifetimes, a necessary link must be made between the weak decays of quarks (and heavy leptons such as the τ, which have appreciable non-leptonic decay modes) and the decays of the physical hadrons (or leptons). The assumption is made that the final state quarks and/or gluons "hadronize" with unit probability into final state hadrons (mesons and baryons) and that the "micro-details" of the hadronization process have little or no effect on the total decay rate (and thus the particle lifetime). The last assumption may be incorrect, as it has been (emphatically) pointed out [220] that final state interactions may significantly change exclusive decay rates, particularly if there exist s-channel resonances with the same final state particles as those for weak (non-exotic) decays in the region of the mass of the weakly decaying meson or baryon. Other non-perturbative effects (such as the Pauli exclusion principle) may alter the decay rate (see discussion above).

The total rate Γ^{TOT} is just the sum of all the partial rates:

$$\Gamma^{TOT} = \sum_i \Gamma_i$$

for leptons:

$$\Gamma_L^{TOT} = \sum_{e\mu\tau} \Gamma_{NH} + 3\Gamma_{HAD}$$

for free quarks:

$$\Gamma_Q^{TOT} = \sum_{e\mu\tau} \Gamma_{SL} + 3\Gamma_{NL}$$

for mesons:

$$\Gamma_M^{TOT} = \sum_{e\mu\tau} \Big(\Gamma_{SL}^{gl} + \Gamma^{0}_{\substack{ANN\\LEPTONIC}} + \Gamma^{gl}_{\substack{ANN\\LEPTONIC\\(SL)}} \Big) + 3\Gamma_{NL}^{gl} + 3\left\{ \begin{matrix} \Gamma^{0}_{\substack{ANN\\NL}} + \Gamma^{gl}_{\substack{ANN\\NL}} \\ \Gamma^{0}_{\substack{EXCH\\NL}} \quad \Gamma^{gl}_{\substack{EXCH\\NL}} \end{matrix} \right\} + 3\Gamma_{PENGUIN}$$

for baryons (crudely)

$$\Gamma_B^{TOT} = \sum_{e\mu\tau} \Gamma_{SL}^{gl} + 3\Gamma^{gl}_{NL_B} + 3\Gamma^{0}_{\substack{EXCH\\NL_B}} + 3\Gamma^{gl}_{\substack{EXCH\\NL_B}} + 3\Gamma_{PENGUIN_B}$$

The lifetime of a particle is defined as $\tau \equiv 1/\Gamma^{TOT}$.

Coupling Constants

$$G_F = (1.16632 \pm 0.00004) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

$$\alpha = e^2/4\pi = 1/137.0038$$

$$\alpha_s(m_c) = 0.60 \quad N_c = 3\ ,\ n = 4,\ \Lambda = 0.5$$

$$f_- = 2.15 \quad f_+ = 0.68$$

$$\frac{G_F^2}{192\pi^3} = 3.474 \times 10^{10}\ \mathrm{sec}^{-1}\ /\mathrm{GeV}^5$$

The values used for the matrix elements of the Kobayashi-Maskawa mixing matrix are those given in Refs.[188-192].

$$V \doteq \begin{array}{c} \begin{array}{ccc} u & c & t \end{array} \\ \begin{pmatrix} 0.9739 & -0.20 & -0.11 \\ 0.220 & 0.95 - 0.75 \times 10\ i & 0.20 + 1.3 \times 10\ i \\ 0.068 & -0.22 + 2.40 \times 10\ i & 0.97 - 4.1 \times 10\ i \end{pmatrix} \end{array} \begin{array}{c} \\ d \\ s \\ b \end{array}$$

Decay Constants

$$f_D \doteq 150\ \mathrm{MeV} \qquad f_F \doteq 170\ \mathrm{MeV}$$

Note that for the following calculations, ALL rates are in units of 10^{10} sec^{-1} and ALL lifetimes are in units of 10^{-13} sec.* Semi-leptonic rates are for individual leptons, except where explicitly notesd, and all non-leptonic rates are without the color factor of 3 (included in the final rate calculation for each particle type).

* EXCEPT WHERE EXPLICITLY INDICATED

A. Free Charmed Quark Decays

$$\Gamma^{0}_{CH} = 5.70 \times 10^{11}\ \mathrm{sec}^{-1}$$

Semi-leptonic Decays	Non-leptonic Decays
No gluons ("bare" diagram):	
29.1	29.3
Radiative gluon corrections: 1-st order QCD Corrections:	
15.7	21.4
2-nd order QCD Non-Leptonic Corrections:	
	33.1
H_W^{eff} (N.L.) Hard Gluon Exchange Corrections:	
	54.1

Total rate for free charmed quark decay:

$$\Gamma^{TOT}_{CH} = 2\Gamma^{CH}_{SL} + 3\Gamma^{CH}_{NL}$$

The total rate, lifetime and semi-leptonic branching ratio for free charmed quark decay (from the above four methods) are:

Γ^{TOT}_{CH} Total Rate	τ_{CH} ($\times 10^{-13}$ sec) Charmed Quark Lifetime	$B_r(c \to s \ell^+ \nu_\ell)$ S.L. Branching Ratio
No Gluons (Bare Diagram):		
146.0	6.8	20%
1-st Order QCD N.L. Corrections:		
96.0	10.4	16%
2-nd Order QCD N.L. Corrections:		
131.0	7.6	12%
H_W^{eff} (N.L.) Hard Gluon Exchange Corrections:		
194.0	5.1	8%

B. D^+ D° F^+ and Λ_c^+ Decays

Semi-Leptonic Decays

1-st Order QCD Radiative Corrections:

D^+ Meson	D° Meson	F^+ Meson	Λ_c^+ Baryon
15.7	15.7	15.7	15.7

Non-Leptonic Decays

	D^+ Meson	D° Meson	F^+ Meson	Λ_c^+ Baryon
1-st Order QCD Radiative Corrections:	17.9	21.4	21.2	21.3
2-nd Order QCD Radiative Corrections:	27.7	33.1	32.8	32.9
H_W^{eff} (N.L.) Hard Gluon Exchange Corrections:	18.0	54.1	53.7	53.9

W^+-Annihilation/Exchange Diagrams

Leptonic Annihilation (No Gluons)

	D^+ Meson	D° Meson	F^+ Meson	Λ_c^+ Baryon
e	3.6×10^3	0	1.1×10^5	0
μ	1.5×10^6	0	0.48	0
τ	3.4×10^8	0	7.20	0

Non-leptonic W+ Annihilation/Exchange (No Gluons):

D^+ Meson	D^0 Meson	F^+ Meson	Λ_c^+ Baryon
0.08	2.79	2.17	see *

Gluon Bremmstrahlung (1-st Order QCD):
Leptonic W^+-Annihilation:

	D^+ Meson	D^0 Meson	F^+ Meson	Λ_c^+ Baryon
e	0.41	0	7.19	0
μ	0.41	0	7.19	0
τ	0	0	0	0

Non-Leptonic W^+-Annihilation/Exchange:

D^+ Meson	D^0 Meson	F^+ Meson	Λ_c^+ Baryon
0.41	32.7	7.19	104.89 *

* includes W^+-exchange with and without gluons.

The mesonic rates differ by a factor of 3 from those of Refs.[212-215] due to differences in color counting; the inclusion of the presence of gluons in the initial meson state results in a factor of 3 increase in rate for the gluon bremmstrahlung diagram over that for single gluon emission from a meson consisting of a (bare) $Q\bar{q}$-pair.

<u>Penguin Diagram</u>

D^+ Meson	D^0 Meson	F^+ Meson	Λ_c^+ Baryon
0.13	0.26	0.28	0.30

Total Decay Rates

$$\Gamma_{CH}^{TOT} = \sum \Gamma_{SL}^{q\ell} + 3\Gamma_{NL} + \sum \Gamma^{0}_{LEPTONIC} + 3\Gamma^{0}_{\substack{ANN\\EXCH}} + \sum \Gamma^{\gamma\ell}_{LEPTONIC} + 3\Gamma^{\gamma\ell}_{\substack{ANN\\EXCH}} + 3\Gamma_{PENGUIN}$$

D^+ Meson	D^0 Meson	F^+ Meson	Λ_c^+ Baryon
1-st Order QCD Corrections:			
87.9	202.9	146.0	410.9
2-nd Order N.L. QCD Corrections:			
117.3	238.0	180.8	445.8
H_W^{eff} (N.L.) Hard Gluon Exchange Corrections:			
88.2	301.0	243.5	508.7

Charmed Particle Lifetimes - Theory

Thus, the theoretical charmed particle lifetimes are (in units of 10^{-13} sec)

D^+ Meson	D^0 Meson	F^+ Meson	Λ_c^+ Baryon
1-st Order QCD Corrections:			
11.4	4.9	6.8	2.4
2-nd Order N.L. QCD Corrections:			
8.5	4.2	5.5	2.2
H_W^{eff} (N.L.) Hard Gluon Exchange Corrections:			
11.3	3.3	4.1	2.0

Measured Lifetimes of Charmed Particles (E-531 and LEBC)

The lifetimes of the D^+, D^0, F^+ and Λ_c^+ as measured by this experiment and LEBC are:

D^+ Meson	D^0 Meson	F^+ Meson	Λ_c^+ Baryon
E-531:			
$10.3^{+10.3}_{-4.2}$	$3.2^{+1.0}_{-0.6}$	$2.0^{+1.8}_{-0.8}$	$2.3^{+1.0}_{-0.6}$
LEBC:			
$9.3^{+4.6}_{-2.3}$	$3.0^{+1.8}_{-0.8}$	($\sim$2.3)	

Thus, it can be seen that there appears to be reasonable agreement between theory and experiment, which come into closest agreement when the non-leptonic rate as obtained with the effective weak non-leptonic Hamiltonian Hw is used. The overall agreement is the poorest for the F^+, which could be due to the limited experimental statistics of

3 (or 4) events, (see thesis for details), or perhaps may be an indication of the presence of additional decay processes which have not been included in the above analysis.

One such (as yet, un-accounted for) decay process for the F^+ meson which could occur, were there to exist in Nature a charged, light Higgs scalar, i.e. H^+ or P^+, (predicted to occur in some theories of extended technicolor (the P^+) and also in some theories of electro-weak interactions with non-minimal Higgs structure (the H^+), see thesis in Chapter 3 for references.). Such a particle, were it to exist, would couple semi-weakly to quarks (and leptons), with a coupling proportional to the quark (or lepton) mass(es). Hence the effects of such a coupling would be expected to show up most naturally in the $F^+(c\bar{s})$ charm system, before their subsequent detection of its effects on the lifetimes of the $D^+(c\bar{d})$ or $D^\circ(c\bar{u})$ mesons. Such a particle would also have observable effects in the charmed-strange baryons, for the same reasons, but with the opposite effect, of increasing the lifetime (!).

The existence of a charged light Higgs scalar has obvious ramifications for heavier quark systems, e.g. truth and beauty mesons and baryons. Further discussion of this topic can be found in the thesis, Chapter 3, Event 635-4949, the (long-lived) neutral charmed baryon candidate.

Whether or not such a particle truely exists, and has the above described behavior for heavy particle systems remains to be seen, experimentally.

C. D^+, D°, F^+ and Λ_c^+ Semi-Leptonic Branching Ratios

Theoretical calculations of the semi-leptonic branching ratios are

D^+ Meson	D° Meson	F^+ Meson	Λ_c^+ Baryon
	1-st Order QCD Corrections:		
18.3%	7.7%	15.7%	3.8%
	2-nd Order N.L. QCD Corrections:		
13.7%	6.6%	12.7%	3.5%
	H_W^{eff} (N.L.) Hard Gluon Exchange Corrections:		
18.3%	5.2%	9.4%	3.1%

"Hybrid" calculations of the D , D , F and semi-leptonic branching ratios may be obtained from the product of theoretical semi-leptonic partial widths for free charmed quark decay,

$$\Gamma_{THY}(c \rightarrow s\, \ell^+ \nu_\ell) = 1.6 \pm 0.3 \times 10^{11}\ sec^{-1}$$

$$\Gamma_{THY}(D \rightarrow X\, e^+ \nu_e) = 2.2 \pm 0.5 \times 10^{11}\ sec^{-1}$$

(see Refs.[21,193-196,201], thesis, and above sections) and the measured D^+, D°, F^+ and Λ_c^+ lifetimes. The branching ratios are denoted as Br(1) and Br(2), respectively. For comparison, we obtain the semi-leptonic branching ratios for the D°, F^+ and Λ_c^+ using the experimentally determined D^+ semi-electronic partial width as obtained from the product of the (averaged) D^+ semi-electronic branching ratio as measured by Mk.II and DELCO [139-144] experiments at SLAC:

$$Br_{exp}(D^+ \rightarrow X^\circ e^+ \nu_e) = 20.5^{+3.6}_{-2.1}\,\% \quad \text{with } \tau_{D^+} = 10.3^{+10.3}_{-4.2} \times 10^{-13}\ sec$$

$$\Gamma_{exp}(D^+ \rightarrow X^\circ e^+ \nu_e) = 2.1^{+2.0}_{-0.9} \times 10^{11}\ sec^{-1}$$

and the measured D^+ lifetime, to determine the branching ratios for the other charmed particle states, (denoted by Br(3)). For the F^+ meson, the semi-leptonic width has been incremented by ~1.5% to account for non-negligible contributions from the W^+-annihilation diagram.

<u>Hybrid Determination of Semi-Leptonic Branching Ratios</u>

D^+ Meson	D^0 Meson	F^+ Meson	Λ_c^+ Baryon
	Br(1) = $\Gamma_{THY}(c \to s \ell^+ \nu_\ell) \cdot \tau_{D^+}$:		
$16.5^{+16.8}_{-7.4}$ %	$5.1^{+1.9}_{-1.5}$ %	$4.7^{+4.3}_{-2.1}$ %	$3.6^{+2.1}_{-1.3}$ %
	Br(2) = $\Gamma_{THY}(D^+ \to X^0 e^+ \nu_e) \cdot \tau_{D^+}$:		
$22.7^{+23.0}_{-10.1}$ %	$7.0^{+2.5}_{-2.0}$ %	$5.9^{+5.0}_{-2.4}$ %	$5.1^{+3.0}_{-1.6}$ %
	Br(3) = $\Gamma_{EXP}(D^+ \to X e \nu) \cdot \tau_{D^+}$:		
$20.5^{+3.6}_{-2.1}$ %*	$6.5^{+5.6}_{-4.4}$ %	$5.5^{+6.0}_{-4.2}$ %	$4.6^{+4.6}_{-3.5}$ %

* Input (Experimental Branching Ratio for D^+)

III-2. $F^+ \rightarrow \tau \nu_\tau$ Leptonic Branching Ratio

An estimate of the leptonic $F^+ \rightarrow \tau^+ \nu_\tau$ branching ratio may be obtained from

$$Br(F^+ \rightarrow \tau^+ \nu_\tau) = \Gamma(F^+ \rightarrow \tau^+ \nu_\tau)\ \tau_{F^+}$$

Where the theoretical partial leptonic width for $F^+ \rightarrow \tau^+ \nu_\tau$ is [209-211]:

$$\Gamma(F^+ \rightarrow \tau^+ \nu_\tau) = \frac{G_F^2}{8\pi} M_F^5 \left(\frac{f_F^2}{M_F^2}\right) |U_{c\bar{s}}|^2 |U_{\tau\nu_\tau}|^2 \left(\frac{M_\tau^2}{M_F^2}\left(1 - \left(\frac{M_\tau^2}{M_F^2}\right)\right)^2\right.$$

$$\Gamma(F^+ - \tau^+ \nu_\tau) = 7.2 + 21.6 - 5.4 \text{ X } 10^{10} \text{ sec}$$

Where $G_F = 1.16632 \times 10^{-5}\ \text{GeV}^{-2}$

$M_F = 2030\ \text{MeV}/c^2$ $\qquad$ $M_\tau = 1784\ \text{MeV}/c^2$

$f_F = 170$ MeV

$M_{\nu_\tau} = 0\ \text{MeV}/c^2$ $\qquad$ $U_{\tau\nu_\tau} = 1.00$

Using the F^+ lifetime as measured in this experiment, the $F^+ \rightarrow \tau^+ \nu_\tau$ branching ratio is

$$Br(F^+ \rightarrow \tau^+ \nu_\tau) = 1.4\% + (^{+1.3}_{-0.6}\ \%)\text{exp} + (^{+4.3}_{-1.1}\ \%)\text{thy}$$

$$Br(F^+ \rightarrow \tau^+ \nu_\tau) = 1.4^{+4.4}_{-1.1}\ \%$$

NOTE:

$$Br(D^+ \rightarrow \tau^+ \nu_\tau) = 0.035^{+0.110}_{-0.028}\ \%$$

$$\text{IF } m_{\nu_\tau} \leq 84.3\ \text{MeV}/c^2$$

III-3. A Determination of the Matrix Element Ucs in the Kobayashi-Maskawa Mixing Matrix

From Ref.[192] the matrix element Ucs of the Kobayashi-Maskawa mixing matrix is given by:

$$|Ucs|^2 = \frac{1}{1.5 \times 10^{11}\ sec^{-1}} \quad \frac{\Gamma(D^+ \to K^\circ e^+ \nu_e)}{|f_+^{D\to K}(0)|^2}$$

$$\text{Where} \quad \Gamma(D^+ \to K^\circ e^+ \nu_e) \simeq 1/2\ \Gamma(D^+ \to X^\circ e^+ \nu_e)$$

$$\Gamma_{Exp}(D^+ \to X^\circ e^+ \nu_e) = 2.1^{+1.5}_{-0.9} \times 10^{11}\ sec^{-1}$$

The value assumed for the form factor at $Q^2 = 0$ is $f_+^{D\to K}(0) = 1$. Thus:

$$|Ucs|^2 = 0.70^{+0.50}_{-0.30}$$

IV. CALCULATION OF THE TAU LEPTON LIFETIME

The theoretical calculation of the tau lepton lifetime has much less uncertainty associated with it than for charmed particles, solely due to the nature of the charged leptons: Simplicity, i.e. the tau lepton is (presumably) a point particle, not a composite object (at least at the energy scale associated with its own weak decays). Hence there are fewer factors and parameters to obscure theoretical predictions. The strong interactions and their 1-st order QCD corrections affect only the hadronic decay modes of the tau. The QED corrections have been tested on the muon lifetime, and have been found to be valid. In calculating the tau lifetime, the full W-S Hamiltonian was used, complete with first order QED and QCD corrections, with the effects of the W boson included in the calculation. (see Ref.[246].)

The tau lepton is an interesting "proving" ground for theory, in that QCD predictions may be tested more cleanly than elsewhere, and also mixing effects in the lepton sector may likewise be tested; for if there are measureable e-u-t mixings either in the neutral leptons (or charged leptons) then the tau lifetime will be increased accordingly, and the non-hadronic branching ratios will differ from those as predicted with no lepton mixing. The tau lifetime is calculated assuming no lepton mixing to be present. The partial widths for each of the four possible decay diagrams are summarized below.

<u>Non-Hadronic Decays</u>

$$\Gamma(\tau^+ \rightarrow e^+ \nu_e \bar{\nu}_\tau) = 62.49 \times 10^{10} \text{ sec}^{-1}$$

$$\Gamma(\tau^+ \rightarrow \mu^+ \nu_\mu \bar{\nu}_\tau) = 60.79 \times 10^{10} \text{ sec}^{-1}$$

<u>Hadronic Decays</u>

$$\Gamma(\tau^+ \rightarrow u\ \bar{d}\ \bar{\nu}_\tau) = 55.64 \times 10^{10} \text{ sec}^{-1}$$

$$\Gamma(\tau^+ \rightarrow u\ \bar{s}\ \bar{\nu}_\tau) = 1.98 \times 10^{10} \text{ sec}^{-1}$$

The total rate for tau lepton decay is therefore

$$\Gamma(\tau^+ \rightarrow \text{all}) = 296.11 \times 10^{10} \text{ sec}^{-1}$$

The tau lifetime is therefore

$$\tau_{\tau^+} = 3.38 \times 10^{-13} \text{ sec}$$

IV-1. Hadronic and Non-Hadronic Branching Ratios

The branching ratios of $\tau^+ \to e^+ \nu_e \bar{\nu}_\tau$ and $\tau^+ \to \mu^+ \nu_\mu \bar{\nu}_\tau$ are:

$Br(\tau^+ \to e^+ \nu_e \bar{\nu}_\tau) = 21.1\%$ (c.f. 17.0±1.1% exp)

$Br(\tau^+ \to \mu^+ \nu_\mu \bar{\nu}_\tau) = 20.5\%$ (c.f. 17.9±1.5% exp)

The branching ratios of $\tau^+ \to u\, \bar{d}\, \nu_\tau$ and $\tau^+ \to u^+ \bar{s}\, \bar{\nu}_\tau$ are:

$Br(\tau^+ \to u\, \bar{d}\, \bar{\nu}_\tau) = 56.3\%$ (c.f. 59.4±10.0% exp)

$Br(\tau^+ \to u\, \bar{s}\, \bar{\nu}_\tau) = 2.0\%$ (c.f. 3.4±1.0% exp)

The ratio of $\tau^+ \to e^+ \nu_e \bar{\nu}_\tau$ to $\tau^+ \to \mu^+ \nu_\mu \bar{\nu}_\tau$ decays is:

$R(\tau^+ \to e^+\nu_e \bar{\nu}_\tau / \tau^+ - \mu^+\nu_\mu \bar{\nu}_\tau) = 1.03\%$ (c.f. 0.95±0.10% exp)

The ratio of $\tau^+ \to u\, \bar{s}\, \bar{\nu}_\tau$ to $\tau^+ \to u\, \bar{d}\, \bar{\nu}_\tau$ decays is:

$R(\tau^+ \to u\bar{s}\bar{\nu}_\tau / \tau^+ \to u\bar{d}\bar{\nu}_\tau) = 3.6\%$ (c.f. 5.7±1.9% exp)

The experimental measurements for the electronic and muonic branching ratios differ from the theoretical predictions by approximately 3 S.D. ($\sim$3% lower). This could be an indication of a larger-than-expected hadronic rate for the tau, as a shift of 4% in the τ^+ lifetime to $\tau_{\tau^+} = 3.24 \times 10^{-13}$ sec brings experiment and theory into agreement. This corresponds to a shift of α_S from 0.6 to slightly over 0.8. (This may also be taken as an indication of the level of uncertainty in the calculation of the tau lifetime.) Another possibility is that the discrepancy between experiment and theory could be construed as possible evidence for mixing phenomena in the lepton sector.

V. INCLUSIVE Λ_c^+ POLARIZATION IN NEUTRINO INTERACTIONS

An investigation of polarization effects in the production and decay of the Λ_c^+ was made, using the eight events observed in this experiment and the WA-17 Λ_c^+ event. While it is fully understood that the statistical significance of such an investigation is minimal at best, such an investigation was nevertheless pursued, if not to overcome a strong curiousity and desire to learn; then to "lay down" a framework such that future endeavors of this nature (with statistically meaningful results) will be induced to occur. Such investigations of this type are also an independent consistency check on the claim as to whether or not the observed decays of charmed baryons produced by the weak interactions of neutrinos with target nuclei are indeed charmed baryon decays, and not background nuclear interactions, as will be shown below.

The angular distribution of the daughter baryon, in the charmed baryon rest system is of the form [160,247,248]:

$$I = 1 + \alpha \mathbb{P}\cdot\hat{q}$$

Where α = Lee-Yang asymmetry parameter
$\mathbb{P}$ = charmed baryon polarization vector
$\hat{q}$ = unit vector along the direction of the daughter baryon, in the charmed baryon rest frame.

The quantity $\alpha\mathbb{P}\cdot\hat{q} = \alpha\mathbb{P}\cos\theta_x + \alpha\mathbb{P}\cos\theta_y + \alpha\mathbb{P}\cos\theta_z$

where the $\cos\theta_x$, $\cos\theta_y$, $\cos\theta_z$ are the direction cosines in the Λ_c^+ rest frame; $\hat{x}$ is a unit vector lying in the Λ_c^+ production plane, $\hat{y}$ is a unit vector normal to the Λ_c^+ production plane, and $\hat{z}$ is a unit vector lying in the Λ_c^+ production plane, along the Λ_c^+ direction (in the lab) and perpendicular to $\hat{x}$. (See Fig.6.)

The $\cos\theta_k$ distributions are shown in Fig.7 for decays of $\Lambda_c^+ \rightarrow \Lambda^0\pi^+\pi^-\pi^+$ and $\Lambda_c^+ \rightarrow p\,K^-\pi^+\pi^0$ etc., and also for the "resonant", quasi-two-body decay hypotheses $\Lambda_c^+ \rightarrow \Sigma^-(1385)\pi^+\pi^+$, $\Lambda_c^+ \rightarrow \Delta^{++}K^{*-}$, etc.

As the Λ_c^+ is a $J^p = 1/2^+$ iso-singlet state (as is the Λ^0), the asymmetry parameter α is expected to be the similar for differing decay modes, as is the case for $\Lambda^0 \rightarrow p\pi^-$, $\Lambda^0 \rightarrow n\,\pi^0$. Hence all events are used in this study. (Not to mention the fact that there would be no statistics with which to do such a polarization analysis.)

It can be seen from Fig.7 that even with these few events that the product of $\alpha\mathbb{P}$ is non-zero in the X and Z directions, and consistent with zero polarization in the Y-direction. The components of $\alpha\mathbb{P}$ in the X, Y, and Z directions were determined via the maximum likelihood method, where the likelihood function $\mathcal{L}(\alpha\mathbb{P}_k)$ for the k-th component (x-y-x) of $\alpha\mathbb{P}$ is defined as:

$$\mathcal{L}(\alpha\mathbb{P}_k) = \prod_{i=1}^{N} (1 + \alpha\mathbb{P}_k \cos\theta_i)^N$$

The results are shown below. The 1 S.D. limits are defined as $\ln(\mathcal{L}) - 0.5$.

INCLUSIVE Λ_c^+ POLARIZATION

(Components of $\alpha\mathbb{P}$ in the Λ_c^+ Rest Frame)

$\alpha\mathbb{P}x$ (Daughters)	0.44 + (0.58,-0.70)
$\alpha\mathbb{P}x$ (Grand-Daughters)	0.12 + (0.66,-0.72)
$\alpha\mathbb{P}y$ (Daughters)	-0.54 + (0.43,-0.42)
$\alpha\mathbb{P}y$ (Grand-Daughters)	-0.73 + (0.41,-0.27)
$\alpha\mathbb{P}z$ (Daughters)	-1.00 + (0.30,-oo)
$\alpha\mathbb{P}z$ (Grand-Daughters)	-0.98 + (0.24,-0.18)

The product $\alpha\mathbb{P} = \left\{(\alpha\mathbb{P}x)^2 + (\alpha\mathbb{P}y)^2 + (\alpha\mathbb{P}z)^2\right\}^{1/2}$

$\alpha\mathbb{P}$ (Daughters)	1.46 + (1.30,-oo)
$\alpha\mathbb{P}$ (Grand-Daughters)	1.30 + (1.05,-0.97)

The "daughter" baryons are the $\Sigma^-(1385)$, Δ^{++} and include "direct" protons. The "grand-daughter" baryons are the Λ°, protons (from "daughter $\Sigma^-(1385)$ and Δ^{++} (strong) decay) and "direct" protons.

The -1 S.D. limit for the $\alpha\mathbb{P}z$(Daughter) component is infinite as the likelihood function diverges for $\alpha\mathbb{P} < -1.0$.

For the strong interactions, the production of baryons with non-zero asymmetry parameter and net polarization $\mathbb{P}$ must have the polarization vector normal to the production plane (as required by parity conservation in the strong interactions). For production of charmed baryons by neutrinos (an intrinsically (and maximally) parity-violating process, due to the "handedness" of the incident neutrino, and also the weak charged current interaction between quarks and leptons) the polarization is not required to lie along the y-axis.

The observed distributions in $\cos\theta_h$ for $\hat{x}$, $\hat{y}$, and $\hat{z}$ components tend to indicate that the Λ_c^+ production process is indeed parity-violating and also that the product of $\alpha\mathbb{P}$ (asymmetry parameter) x (polarization) is non-zero and also large (most likely both α and $\mathbb{P}$ are close to unity, however the independent values for α and $\mathbb{P}$ cannot be determined separately, from these data; if the polarization $\mathbb{P}$ is assumed to be nearly 100% negative polarization, then the asymmetry parameter α is close to +1.0.)

The statistical nature of the data is such that the probability of observing 7 decays in the negative $\cos\theta z$ region and 2 decays in the positive $\cos\theta z$ region is 20%, as the probability of such an occurrence depends on the difference in populations between the forward and backward hemispheres.

The presence of a net, non-zero (and large) polarization is expected for heavy quark production in neutrino interactions [150] (see Fig.8).

If the Λ_c^+ decays are assumed to proceed through resonant Δ^{++} and $\Sigma^*(1385)$ states, then the polarization of these baryon states (in the Λ_c^+ center of mass), as shown in Fig.7 should be more closely corellated to the initial polarization. The results, although limited by the statistics indicate this to be so.

Because of the iso-singlet nature of the Λ_c^+ charmed baryon, the spin of the Λ_c^+ is carried entirely by the c quark, the spin part of the Λ_c^+ wave function being

$$\chi_{\Lambda_c^+} = \frac{1}{\sqrt{2}}\, c\uparrow\, (u\uparrow d\downarrow - u\downarrow d\uparrow)$$

in exact analogy to the Λ°. The mechanism for generation of a net polarization $\mathbb{P}$ for the Λ_c^+ can be thus be understood as a consequence of the nature of the weak interactions, and the nature of the incoming neutrino (having negative helicity). It should be noted that this study is with a sample of inclusive, rather than exclusively produced Λ_c^+'s; Evidence for the resonant charmed state Δ^{++} is present in several of the events within the data sample. This in no way invalidates the above results, as the presence of such resonant states merely serves to "wash out" the initial state polarization. (Furthermore, many experiments have been done to investigate the nature of hyperon polarization, almost all of which dealt with inclusive production mechanisms [250-258].)

The systematic effects due to the acceptance of the experiment and scanning biases can be seen to be small, as the angular acceptance has been shown to be greater than the observed production spectrum of charmed particles produced in neutrino interactions. The scanning biases are negligibly small for the Λ_c^+, as the flight lengths are short, (~100um.), due to the short Λ_c^+ lifetime and a relatively low parent momentum spectrum (with large opening angles in the decay of the Λ_c^+ as a consequence). Hence a high scanning efficiency (> 95%) for finding Λ_c^+ decays in the emulsion. However, the proper method for determination of such biases can only be accomplished with the aid of a complete Monte Carlo program. This was not done with the limited statistics of the present sample, but MUST be done for any serious investigation of polarization phenomena in ν-interactions.

These results should be compared with lambda polarization in neutrino and anti-neutrino interactions [133], where it was found that the lambda polarization is consistent with strong production, rather than weak production mechanisms, and also the magnitude of the lambda polarization is less than for Λ_c^+'s.

WORLD SUMMARY OF VISIBLE DECAYS OF CHARMED PARTICLES

The following table summarizes the charm decay candidates observed in experiments other than E-531, both prior to, during and since this experiment. No attempt has been made to determine mean lifetimes from these data, due to the large variations and uncertainties in scanning efficiencies from one experiment to another, and also because of the uncertainty in parent identity in many cases.

The notation used below for the decay modes is as follows:

1. Particles which have been unambiguously identified (with > 90% C.L.) are indicated in the same manner as for our experiment, i.e. underlining with — (~) for counter (emulsion) identification.

2. (A/B) denotes ambiguity as to particle type for a given daughter particle.

3. (A/B) (C/B) denotes a correlated ambiguity between daughter particles, i.e. {A with C} OR {B with D}.

4. For the case of neutral particles, "( )" also denotes "not observed".

WORLD SUMMARY OF VISIBLE DECAYS OF CHARMED PARTICLES

EXPERIMENT INSTUTION	BEAM -TGT	OBSERVED DECAY MODE	CHARGED PRONGS	D.L. (μM)	MOMENTUM (GEV/C)	MASS (MEV/C^2)	LIFETIME ($X10^{-13}$SEC)
Nagoya	C.R.	$X^{\pm} \to \pi^{\pm}\pi^{0}$	1	13800		1.78	0.22
1971	emul	$X^{\pm} \to \mu^{\pm}\pi^{0}$	1	48800		2.95	0.36
[45]							
Nagoya	C.R.	$D^{\pm} \to K^{0}\pi^{+}\pi^{-}(\pi^{0})$	3	12900	217-430	0-C	1.9-3.7
1975	emul	$D^{\pm} \to K^{0}\pi^{+}\pi^{-}(\pi^{0})$	3	12800	176-352	0-C	2.3-4.6
[47,48]							
E-247	ν-emul	Nor D^{+}, Λ_c^{+} (F^{+}?)	3	182			6.0
1976							
[59]							
SKAT	ν-B.C.	$\Sigma^{0}, D^{0} \to (\mu^{-}/\pi^{-})e^{+}\nu_e$	2	4800	5.0	1.4-2.5	60
1977							
[97]							
Russian	p-nucl	$(\pi^{0}/K^{0})\pi^{\mp}(\Sigma^{\pm}/p)$	3	63		2.2±.3	0.07
1978	400GeV	$K^{\mp}\pi^{\pm}p$	3	90		2.1±.3	0.13
[60]		$\pi^{\mp}\Sigma^{\pm}$	2	25		2.1±.3	0.04
		$\pi^{\mp}\Sigma^{\pm}$	2	29		2.0±.2	0.44
		$K^{-}p$	2	35		2.3±.2	0.15
		$\pi^{0}p$	2	22		2.5±.4	0.23
		$(K^{0}/\Sigma^{0})(3\pi)^{\pm}$	4	28		2.1±.3	0.04
		$\Sigma^{\pm}(2\pi)^{\mp}e^{\pm}\nu_e$	4	77		2.2±.3	0.29
		$\Sigma^{\mp}(2\pi)^{\pm}e^{\pm}\bar{\nu}_e$	4	12		2.5±.5	0.01
WA-17	ν-emul	$(\pi^{0}/K^{0}/\Lambda^{0})\pi^{+}\pi^{-}\pi^{+}$	3	906	12-38	0-C	2-5
1979	B.C.	$\Lambda_c^{+} \to pK^{-}\pi^{+}$	3	354	3.7	2295±15	7.3
[64-66]		$D^{+}, F^{+}, \Lambda_c^{+}$	3	96	5-15	0-C	0.5-1.2
		$(\pi^{-}/K^{-})\pi^{+}(K^{0}/\pi^{0})$	2	182		0-C	0.9-0.6
		$(\pi^{-}/K^{-})\pi^{+}(K^{0}/\pi^{0})$	2	54		0-C	5.0-0.2
		$(\pi^{-}/K^{-})\pi^{+}(K^{0}/\pi^{0})$	2	115		0-C	2.4-0.4
Nagoya	p-emul	$(K^{0}/\Lambda^{0})\pi^{0}$	0	2390			11.7
1978	400GeV	$(K^{\pm}/\Sigma^{\pm})e^{\mp}\nu_e$	2	320			0.38
[61]							
Nagoya	p-emul	$(\pi^{+}/K^{+}/\mu^{+})(\pi^{-}/\mu^{-}/K^{-})\pi^{0}(K^{0}/\nu_\mu)$	2	4700	69-105	0-C	2.8-4.2
1979	400GeV	$(\pi^{-}/K^{-}/\mu^{-})(\pi^{+}/\mu^{+}/K^{+})\pi^{0}(K^{0}/\nu_\mu)$	2	4250	60-92	0-C	2.9-4.2
[62]							
Nagoya	p-emul	$\pi^{\pm}K^{\mp}(\pi^{0}/K^{0})$	2	980	13-107	0-C	0.6-4.9
1980	340GeV	$K^{\mp}\pi^{\pm}\pi^{\mp}(\pi^{0})$	4	1630	24-30	0-C	3.4-4.0
[63]							
		$\pi^{\pm}(K^{\mp}/\pi^{\mp})(\pi^{0}/K^{0})$	2	445	4-14	0-C	1.9-7.1
		$(K^{\mp}/\pi^{\mp})\pi^{+}\pi^{-}(\pi^{0}/K^{0}/\Lambda^{0})$	2	435	5-28	0-C	1.0-6.8
		$\pi^{\pm}(K^{\mp}/\pi^{\mp})(\pi^{0}/K^{0})$	2	2290	29-117	0-C	1.2-4.8
		$\pi^{\pm}(K^{\mp}/\pi^{\mp})(\pi^{0}/K^{0})$	2	2150	14-99	0-C	1.3-9.6
		$\pi^{\pm}\pi^{-}\pi^{+}\pi^{-}\pi^{+}\pi^{0}$	5	2350	4-5	0-C	32-36
		$(K^{\pm}/\pi^{\pm})\pi^{+}\pi^{-}(\pi^{0}/K^{0}/\Lambda^{0})$	3	5150	37-62	0-C	5-10
BFHSW	ν-B.C.	$C1 \to h^{+}h^{-}e^{+}(\nu_e)$	3	6200	111-237	0-C	1.6-3.5
1979		$N1 \to h^{-}e^{+}(\nu_e)$	2	6700	8-40	0-C	10-52
[98-100]		$C_0 \to h^{+}h^{-}e^{+}(\nu_e)$	3	11000	35-63	0-C	9-20
		$N2 \to h^{-}e^{+}(\nu_e)$	2	8700	38-135	0-C	4-14
		$U1 \to h^{-}(K^{0}?)e^{+}(\nu_e)$	2,3	3-6000		0-C	
E-564	ν-emul	$F^{+} \to \pi^{+}\pi^{-}\pi^{+}\pi^{0}$	3	50	2.4+.3	2017±25	1.5
1980	B.C.						
[114]							
Ω-SPEC	γ-emul	$\bar{D}^{0} \to \pi^{+}\pi^{-}\pi^{-}K^{+}$	4	123		1866±8	0.23
1979-80		$\Lambda_c^{+} \to \Lambda^{0}\pi^{+}$	1	50		2330±50	0.57
[67-70]		$\bar{D}^{0} \to K^{+}\pi^{+}\pi^{-}\pi^{-}$	4	124		1847±30	0.86
		$D^{-} \to \pi^{-}\pi^{+}\pi^{-}(K^{0})$	3	94		0-C	0.6-0.9
		$D^{0} \to$ >2 body	2	267		0-C	0.5-0.9
		$C^{0} \to$ >3 body	3	685			7.1-14.2
		$C^{0} \to$ >3 body	3	980			3.8-7.6
		$D^{0} \to$ >2 body	2	44			.04-.13
		$C^{-} \to$ >3 body	3	260			1.0-5.4
		$C^{0} \to$ >3 body	3	32			0.23-0.66
		$C^{\mp} \to$ >2 body	1	1900			1.7-40.0
		$C^{0} \to$ >3 body	3	3236			
		$C^{0} \to$ >2 body	1	341			
•LEBC	π^{-}-B.C.	$D^{0} \to K^{-}\pi^{+}\pi^{0}\pi^{0}$	2	4100	119.+.6	1857±22	2.1±0.1
1981	360GeV	$\bar{D}^{0} \to K^{+}\pi^{+}\pi^{-}\pi^{-}$	4	7500	78.5+.3	1862±9	5.9±0.1
	[244]						

KINEMATIC FITTING PROGRAMS

To obtain improved resolution and reduce systematic error on the parent mass, momentum and therefore the decay time, each charm candiate was kinematically fit with as high a constrained fit as possible. The problem of energy and momentum conservation in particle decays is an inherently non-linear one, which can be solved analytically for the cases in which there are no constraints (i.e. < 0-C fits). This situation occurs for events with a missing neutral (e.g. semi-leptonic decays) where only -1C and 0C fits are possible. -1C fits test kinematic consistency for a given decay hypothesis, whereas 0C fits yield two solutions from the quartic -1C mass vs. missing neutral momentum curve, for a assumed parent mass.

For over-determined situations, 1C, 2C, 3C fits were obtained using a kinematic fitting program which utilized the method of linearized least squares along with Lagrange multipliers to fit the parent slopes (X' and Y') and momentum for a 1C fit; the parent mass and momentum for a 2C fit; the parent momentum for a 3C fit, subject to the constraints of energy and momentum conservation. The fitting program was developed along the concepts outlined in ref.[221].

A χ^2 was defined as:

$$\chi^2(m, x, \alpha) = (m-m^0)^T Gm (m-m^0) + 2\alpha^T f(x, m)$$

Where

m^0 = initial value of measured quantity

m = fitted value of measured quantity

x = unknown quantities (e.g. Pc, Mc)

Gm = error matrix (Gm = weighting matrix)

α = Lagrange multipliers

f_k = Energy and momentum constraint equations.

$$f1 = Ec - \sum_i Ei = 0$$

$$f2 = Pcx - \sum_i Pxi = 0$$

$$f3 = Pcy - \sum_i Pyi = 0$$

$$f4 = Pcz - \sum_i Pzi = 0$$

The χ^2 is then minimized with respect to the measured quantities m, the unknown quantities x, and the Lagrange multipliers α. For χ^2 to be a minimum, the first derivatives of χ^2 with respect to these three parameters must be zero:

$$d\chi^2/dm = 2[(m-m^\circ)^T Gm + \alpha^T fm] = 0$$

$$d\chi^2/dx = 2\alpha^T fx = 0$$

$$d\chi^2/d\alpha = 2f(x,m) = 0$$

(these are just the constraint eqns.)

The constraint equations are expanded to first order in x, m around the minimum in the 3N(m) +(4-N(c)) dimensional space, N(m) = number of measured quantities, N(c) = number of constraints.

$$f^\nu + f_x^\nu (x^{\nu+1} - x^\nu) + f_m^\nu (m^{\nu+1} - m^\nu) = 0$$

Where ν = iteration step number.

from $d\chi^2/dm$, $m^{\nu+1} = m^\circ - G_m^{-1} f_m^{\nu T} \alpha^{\nu+1}$

Insertion of this equation into the above equation, gives

$$f^\nu + f_x^\nu (x^{\nu+1} - x^\nu) + f_m^\nu (m^{\nu+1} - m^\nu) = 0$$

Defining $R \equiv f^\nu + f_m^\nu (m^\circ - m^\nu)$; $S \equiv f_m^\nu G_m^{-1} f_m^{\nu T}$, The linearized constraint equations become:

$$R - S\alpha^{\nu+1} + f_x^\nu (x^{\nu+1} - x^\nu) = 0$$

Solving for $\alpha^{\nu+1}$:

$$\alpha^{\nu+1} = S^{-1} [R + f_x^\nu (x^{\nu+1} - x^\nu)]$$

But $d\chi^2/dx = 2\alpha^T f_x = 0$, hence $x^{\nu+1} = x^\nu - (f_x^{\nu T} S^{-1} f_x^\nu)^{-1} f_x^{\nu T} S^{-1} R$

Thus, the problem is solved; $x^{\nu+1}$ is calculated, input to $\alpha^{\nu+1}$, which is in turn used to calculate $m^{\nu+1}$, after which χ^2 itself can be obtained.

This procedure is repeated until the derivatives of χ^2 are less than predetermined values. For example, the requirement that the derivatives of χ^2 with respect to the Lagrange multipliers be less than 0.0001 physically means that energy and each of the three components of momentum be conserved to within 100KeV. The requirements on the other derivatives were the same, $d\chi^2/dm$ and $d\chi^2/dx$ both $< 1E-4$, although the physical meaning of these derivatives is not as obvious as $d\chi^2/d\alpha$.

The physically measured quantities associated with the decay were parameterized in term of the slopes, $X'=dX/dZ$, $Y'=dY/dZ$, and the inverse momentum $Q=1/P$, as the measured errors associated with these quantites were gaussian in their distributions for charged tracks. This is not the case for parametrization with θ, ϕ, and P.

The errors associated with the measured and fitted quantities and the correlations between the measured and fitted quantities were obtained from the respective error matrices:

$$G^{-1}_{m^{\nu+1}} = \frac{dg}{dm^\circ} G^{-1}_m \left(\frac{dg}{dm^\circ}\right)^T$$

(error matrix of measured quantities)

$$m^{\nu+1} \equiv g(m^\circ) \qquad\qquad x^{\nu+1} \equiv h(m^\circ)$$

$$G^{-1}_{x^{\nu+1}} = \frac{dh}{dm^\circ} G^{-1}_m \left(\frac{dh}{dm^\circ}\right)^T$$

(error matrix of unknown quantities)

$$dg/dm^\circ = 1 - G^{-1}_m f^T_m \frac{d\alpha}{dm^\circ} \quad \text{substitute in } \alpha^{\nu+1}:$$

$$dg/dm^\circ = 1 - G^{-1}_m f^T_m S^{-1}\left[\frac{dR}{dm^\circ} + f_x \frac{d(x^{\nu+1}-x^\nu)}{dm^\circ}\right]$$

$$dg/dm = 1 - G^{-1}_m f^T_m S^{-1}[f_m - f_x (f^T_x S^{-1} f_x)^{-1} f^{-1}_x S^{-1} f_m]$$

$$dh/dm = (f^T_x S^{-1} f_x)\ f^{-1} S^{-1}_x f_m$$

Therefore, $G^{-1\,\nu+1}_m = G^{-1}_m - G^{-1}_m f^T_m S^{-1} f^{-1}_m G^{-1}_m + G^{-1}_m f^T_m S^{-1} f_x (f^T_x S^{-1} f_x)^{-1} f^T_x S^{-1} f_m G^{-1}_m$

$$G^{-1\,\nu+1}_x = (f^T_x S^{-1} f_x)^{-1} \quad \text{and} \quad C^{\nu+1}_{mx} = -G^{-1}_m f^T_m S^{-1} f_x (f^T_x S_x f_x)^{-1}$$

The errors associated with the initial measured quantities, m are assumed to be uncorrelated. Note that the errors associated with the measured quantities after the ν-th iteration are "reduced" due to correlations in the measured quantities arising from couplings in the constraint equations. Note also that correlations between the fitted variables exist, even when the measured quantities are uncorrelated.

A confidence level for the fit was obtained after "convergence" upon a solution, of the form:

$$\text{C.L.} = \int_{\chi^2}^{\infty} d\chi^2 P_{n_D}(\chi^2)$$

where $P_{n_D}(\chi^2)d\chi^2 = \frac{1}{2^n\Gamma(n)}(\chi^2)^{h-1}e^{-\chi^2/2}d\chi^2$, for $\chi^2 > 0$

n_D = number of degrees of freedom (4 - N(c))

$$h = n_D/2$$

For n_D degrees of freedom the χ^2 has a mean of n_D and variance of $2n_D$.

After the charm decay candidate had acheived an acceptable kinematic fit, the decay time τ_D^{cH} was then obtained using the fitted (or assumed) values for the parent mass and momentum, along with the measured flight length For 2-C (3-C) fits, respectively. i.e.:

$$ct_d^{cH} = \frac{m_{cH}}{P_{cH}} L$$

The errors on the fitted decay time were obtained using the matrix elements from the unknown quantities error matrix, G , and the measured error for the flight length, i.e.:

$$\frac{\sigma_{t_d}^2}{t_d^2} = \frac{\sigma_{m_{cH}}^2}{M_{cH}^2} - \frac{\sigma_{P_{cH}}^2}{P_{cH}^2} + 2\frac{\sigma_{m_{cH}P_{cH}}^2}{M_{cH}P_{cH}} + \frac{\sigma_L^2}{L^2}$$

A Monte Carlo program tested the kinematic fitting program, in which 10,000 fake charm decay events were fit with (normally distributed) shifts in the measured quantities, m . The correct χ^2 distributions and confidence levels were obtained, with the expected behavior for the derivatives of χ^2 . The errors obtained were also in accordance with expected behavior, as the width of the distribution for a particular quantity was found to be equal to the mean of the error distribution for that quantity. The Monte Carlo results are in good agreement with the results for the charm decay candidates. The flow chart of the basic kinematic fitting program is shown in Fig.9.

HINTS AT THE FUTURE

This Ph.D candidate wishes to share some of the wonder, joy, and excitement he has experienced at various times during the past few years of working on this experiment, as embodied in a picture of a 25 TeV event, observed in a Nagoya University cosmic-ray emulsion chamber experiment [51,52] as shown in Fig.10. There are eight observed decay candidates in this (single) event, some of which are sequential in nature. Note that the incident particle is neutral. Several of these types of high-energy cosmic ray events have been observed.

LIST OF REFERENCES

1. C.H.Llewellyn-Smith, "Topics in Quantum Chromodynamics", in Quantum Flavordynamics, Quantum Chromodynamics and Unified Theories, edited by K.T.Mahanthappa and James Randa, Nato Advanced Study Institutes Series, (Series B Physics) p.59,60,76-86, 1980.
and
O.W.Greenberg, C.A.Nelson, Physics Reports, 32C , (1977), 93-94.
and
J.S.Bell, R.Jackiw, Nuovo Cimento 61 , (1969), 47.
and
S.l.Adler, Phys.Rev. 177 , (1969), 2426.
and
S.M.Bilenky, "Introduction to Feynman Diagrams", (Oxford, Pergammon Press, 1974), P.163-166.

2. R.L.Glasser, N.Seeman, and B.Stiller, Phys.Rev. 123 (1961), 1014.

3. H.Schwe, F.M.Smith, and W.H.Barkas, Phys.Rev. 136 (1964), 1839.

4. M.Gell-Mann and A.Pais, Phys.Rev. 97 (1955), 1387.

5. K.Lande, E.T.Booth, J.Impeduglia, L.M.Lederman and W.Chinowsky, Phys.Rev. 103 (1956), 1901.
ibid., Phys.Rev. 105 (1957), 1925.

6. T.D.Lee and C.N.Yang, Phys.Rev. 104 , (1956), 254.

7. C.S.Wu, E.Ambler, R.W.Hayward, D.D.Hoppes and R.P.Hudson, Phys.Rev. 105 , (1957), 1413.

8. J.H.Christenson, J.W.Cronin, V.L.Fitch, and R.Turlay, Phys.Rev.Lett. 13 , (1964) 138.

9. A.Abashian, R.J.Abrams, D.W.Carpenter, G.P.Fisher, B.M.K.Nefkens, J.H.Smith, Phys.Rev.Lett. 13 , (1964), 243.

10. A.Pais and O.Piccioni, Phys.Rev. 100 , (1955) 1487.

11. N.Cabibbo, Phys.Rev.Lett. 10 , (1963), 531.

12. B.Pontecorvo, JETP 33 , (1957), 549.

13. Z.Maki, M.Nakagawa and S.Sakata, Prog.Theor.Phys. 28 , (1962), 247.

14. B.Pontecorvo, JETP 53 , (1967), 1717.

15. B.Pontecorvo, JETP 26 , (1968), 984.

16. V.Gribov and B.Pontecorvo, Phys.Lett. 28B , (1969), 493.

17. G.Feinberg, M.Goldhaber and B.Steigman, Phys.Rev. D18 , (1979), 1602.

18. R.E.Marshak and R.N.Mohapatra, Phys.Lett. 91B , (1980), 222.

19. R.E.Marshak and R.N.Mohapatra, Phys.Rev.Lett. 44 , (1980), 1316.

20. B.J.Bjorken and S.L.Glashow, Phys.Lett. 11 , (1964), 255.

21. M.K.Gaillard, B.W.Lee and J.L.Rosner, Rev.Mod.Phys. 47 , (1975), 277.

22. S.L.Glashow, Nucl.Phys. 22 , (1964), 579.

23. S.Weinberg, Phys.Rev.Lett. 19 (1967), 1264.

24. A.Salam, in "Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium no.8), edited by N.Svartholm, (Almquist and Wiksell, Stockholm, 1968), 367.

25. S.L.Glashow, J.Iliopoulos, L.Maiani, Phys.Rev. D2 (1970), 1285.

26. S.H.Aronson et al., Phys.Rev.Lett. 25 (1970), 1057.

27. W.C.Carithers et al., Phys.Rev.Lett. 30 , (1973), 1336.
ibid., 31 , (1973), 1025.

28. Y.Fukushima et al., Phys.Rev.Lett. 36 , (1976), 348.

29. G.D.Cable, R.H.Hildebrand, C.Y.Pang and R.Steining, Phys.Rev. D8 , (1973), 3807.

30. H.I.Vainhstein, I.B.Kriplovich, JETP 18 (1973), 141.

31. E.Ma, Phys.Rev. D9 , (1974), 1303.

32. A.Gavrielides, Ph.D.Thesis, University of Minnesota, 1974, (unpublished).

33. S.Adler, 1974 (unpublished).

34. S.Joglekar, 1974.

35. M.K.Gaillard, B.W.Lee, Phys.Rev. D10 , (1974), 897.

36. G.Altarelli, N.Cabibbo and L.Maiani, Phys.Rev.Lett. 35 , (1975), 635.

37. M.Einhorn, Academic Lecture Series, FNAL 75/1, (1975).

38. J.L.Rosner, Institute For Advanced Study, Princeton Preprint C00-2220-102, (1977), 26.
and
Orbis Scientiae, Deeper Pathways in High Energy Physics, edited by A.Perlmutter and L.F.Scott, (Plenum, New York, 1977), P.489.

39. J.L.Rosner, Bartol Conference, University of Delaware, (1978), 297.

40. N.Cabibbo and L.Maiani, Phys.Rev.Lett. 35 , (1978), 109.

41. T.Hayashi, M.Nakagawa, H.Nitto and S.Ogawa, Prog.Theor.Phys. 49 , (1973), 350.
ibid., 52 , (1974), 636.

T.Hayashi et al., Prog.Theor.Phys. 55 , (1976), 968.

42. M.Einhorn, C.Quigg, Phys.Rev. D12 , (1975), 2015.

43. G.Altarelli, N.Cabibbo and L.Maiani, Nucl.Phys. B88 , (1975), 285

44. R.L.Kingsley, S.B.Tremain, F.Wilczek and A.Zee, Phys.Rev. D11 , (1975), 1919.

45. K.Niu, E.Mikumo, Y.Maeda, Prog.Theor.Phys. 46 , (1971), 1644.

46. T.Hayashi et al., Prog.Theor.Phys. 47 , (1972), 280.

47. K.Hoshino et al., in "Proceedings of the Fourteenth International Conference on Cosmic Rays, Munich, W.Germany, 1975. (Max Planck Institute fur Extraterestrische Physik,Garching, W.Germany, 7 (1975), p.2442.

48. K.Niu, in "Proceedings of the Nineteenth International Conference on High Energy Physics", Tokyo, Japan, 1978. (Physical Society of Japan, Tokyo, (1979), 447.)

49. M.Kaplon, B.Peters and D.Ritson, Phys.Rev. 85 , (1952), 900.

50. K.Nishikawa, Jour.Phys.Soc.Japan, 14 , (1959), 880.

51. Kuramata et al., in "Proceedings of Thirteenth International Cosmic Ray Conference", Denver, 3 , (1973), 2239.

52. S.Kuramata, in "Compilation of X-Particles (Short-Lived Particles Observed by Emulsion Techniques)", Summer Study on Weak Interactions, Aichi University of Education, Japan, Sept. 1977, (unpublished).

53. J.J.Aubert et al., Phys.Rev.Lett. 33 , (1974), 1404.

54. J-E.Augustin et al., Phys.Rev.Lett. 33 , (1974), 1046.

55. G.Goldhaber et al., Phys.Rev.Lett. 37 , (1976), 225.

56. I.Peruzzi et al., Phys.Rev.Lett., 37 , (1976), 569.

57. J.E.Wiss et al., Phys.Rev.Lett., 37 , (1976), 1531.

58. G.Feldman et al., Phys.Rev.Lett., 38 , (1977), 1313.

59. E.H.S.Burhop et al., Phys.Lett. 65B , (1976), 299.

60. A.A.Komer et al., JETP Lett. 28 , (1978), 453.

61. N.Ushida et al., Lett.Nuovo Cimento, 23 , (1978), L 577.

62. H.Fuchi et al., Phys.Lett. 85B , (1979), 135.

63. H.Fuchi et al., Nagoya University Preprint DPNU-42-80, (1980), (Submitted to Lett.Nuovo Cimento).

64. C.Angelini et al., Phys.Lett. 80B , (1979), 428.

65. C Angelini et al., Phys.Lett. 84B , (1979), 159.

66. D Allasia et al., Phys.Lett. 87B , (1979), 287.

67. M.I.Adamovich et al., Phys.Lett. 89B , (1980), 427.

68. M.I.Adamovich et al., Phys.Lett. 99B , (1981), 271.

69. A.Conti et al., Contributed paper to "International Symposium on Lepton-Photon Interactions at High Energies, (Fermilab, Illinois, 1979.)

70. M.I.Adamovich et al., in "Proceedings of the Twentieth International Conference on High Energy Physics", (Madison, Wisconsin, 1980.)

71. H.Deden et al., Phys.Lett., 58B , (1975), 361.

72. J.Blietschau et al., Phys.Lett., 60B , (1976), 207.

73. E.G.Cazzoli et al., Phys.Rev.Lett., 34 , (1975), 1125.

74. W.Braunschweig et al., Phys.Lett. 63B , (1976), 471.

75. J.von Krogh et al., Phys.Rev.Lett. 36 , (1976), 710.

76. H.Deden et al., Phys.Lett., 67B , (1977), 474.

77. P.Bosetti et al., Phys.Rev.Lett., 38 , (1977), 1248.

78. P.Bosetti et al., Phys.Lett., 73B , (1978), 380.

79. C.Baltay et al., Phys.Rev.Lett., 38 , (1977), 1248.

80. C.Baltay et al., Phys.Rev.Lett., 39 , (1977), 62.

81. C.Baltay et al., Phys.Rev.Lett., 41 , (1978), 73.

82. C.Baltay et al., Phys.Rev.Lett., 42 , (1979), 1721.

83. A.M.Cnops et al., Phys.Rev.Lett., 42 , (1979), 197.

84. J.Blietschau et al., Phys.Lett., 86B , (1979), 108.
ibid., 99B , (1981), 159.

85. G.S.Abrams et al., Phys.Rev.Lett., 44 , (1980), 10.

86. B.Knapp et al., Phys.Rev.Lett. 37 , (1976), 882.

87. M.S.Atiya et al., Phys.Rev.Lett. 43 , (1979), 414.

88. M.C.Goodman et al., Phys.Rev. D22 , (1980), 537.

89. P.Avery et al., Phys.Rev.Lett., 44 , (1980), 1309.
and
P.Avery, Ph.D. thesis, University of Illinois, 1980, (unpublished).

J.J.Russell et al., Phys.Rev.Lett., 46 , (1981), 799.
and
J.J.Russell, Ph.D. thesis, University of Illinois, 1981, (unpublished).

90. D.Aston et al., Phys.Lett. 94B , (1980), 113.

ibid., 100B , (1981), 91.
and
D.Aston et al., CERN preprint EP/81-41, (1981), submitted to Nucl.Phys.B.

91. R.Brandelik et al., Phys.Lett. 70B , (1977), 132.
ibid., 80B , (1979), 412.

92. K.Giboni et al., Phys.Lett. 85B , (1979), 437.

93. W.Lockman et al., Phys.Lett. 85B , (1979), 443.

94. D.Drijard et al., Phys.Lett., 85B , (1979), 452.

95. A.Kernan, in the "1979 International Symposium on Lepton Photon Interactions at High Energies", (Fermilab, Illinois, 1979), p.535.

96. J.Sandweiss et al., Phys.Rev.Lett., 44 , (1980), 1104.

97. D.S.Baranov et al., Phys.Lett., 70B , (1977), 269.

98. H.C.Ballagh et al., LBL-Fermilab-Hawaii-Washington-Wisconsin Collaboration Preprint UH-511-351-79 (1979), (unpublished).

99. D.D.Reeder, in the "Proceedings of the |979 International Symposium On Lepton and Photon Interactions at High Energies" (Fermilab, Illinois, 1979), p.553.

100. A.M.Cnops et al., Brookhaven-Columbia-Collaboration preprint BNL-26309 (1979), (unpublished).

101. A.Benvenuti et al., Phys.Rev.Lett. 34 , (1975), 419.

102. A.Benvenuti et al., Phys.Rev.Lett. 35 , (1975), 1199.

103. A.Benvenuti et al., Phys.Rev.Lett. 35 , (1975), 1203.
ibid., 35 , (1975), 1249.

104. B.C.Barish et al., Phys.Rev.Lett. 36 , (1976), 939.

105. B.C.Barish et al., Phys.Rev.Lett. 39 , (1977), 981.

106. A.Benvenuti et al., Phys.Rev.Lett. 41 , (1978), 1204.

107. P.C.Bosetti et al., Phys.Lett. 73B , (1978), 380.

108. O.Erriquez et al., Phys.Lett. 77B , (1979), 227.

109. L.Voyvodic, in "Proceedings of the 1979 International Symposium on Lepton-Photon Interactions at High Energies", Fermilab, Illinois, 1979.

110. N.W.Reay, in "Proceedings of the Twentieth International Cosmic Ray Conference", Kyoto, Japan, 1979, (unpublished).
and
N.W.Reay, in "Particles and Fields - 1979", AIP Conference Proceedings 59 , (Am.Inst.Phys., New York, 1980), p.77.

111. J.D.Prentice, in the "Proceedings of the International Symposium on Lepton and Photon Interactions", (Fermilab, Illinois, 1979), p.563.

112. J.D.Prentice, in the "Proceedings of Baryon Spectroscopy Conference", (Toronto, Ontario, Canada, 1980).

113. K.Niu, in the "Proceedings of the Twentieth International Conference on High Energy Physics, (Madison, Wisconsin, 1980).

114. R.Ammar et al., Phys.Lett. 94B , (1980), 118.

115. N.Ushida et al., Phys.Rev.Lett. 45 , (1980), 1049.

116. N.Ushida et al., Phys.Rev.Lett. 45 , (1980), 1053.

117. F.Nezrick, FNAL TM-555, (1975).

118. J.Grimson and S.Mori, FNAL TM-824, (1978).

119. S.Mori, FNAL TM-888, (1979).

120. D.Bailey, Ph.D. Thesis, McGill University, 1982, (unpublished).

121. D.Pitman, Ph.D. Thesis, University of Toronto, 1982, (unpublished).
and
D.Pitman, M.Sc. Thesis, University of Toronto, 1979, (unpublished).

122. T.Hara, Ph.D Thesis, Kobe University, 1982, (unpublished).

123. M.Gutzwiller, Ph.D.Thesis, The Ohio State University, 1981, (unpublished).

124.R.M.Egloff, Ph.D.Thesis, University of Toronto, 1979, (unpublished).

125.M.Peshkin, J.L.Rosner, Nucl.Phys. B122 , (1977), 144.

126.A.Bodek, in the "Proceedings of the Calorimeter Workshop, (Fermilab, May, 1975, edited by M.Atac, 1975) p.229.

127.A.Grant, NIM 131 , (1975), 167.

128.C.Baltay, in "Particles and Fields - 1979", (APS/DPF Montreal, edited by B.Margolis and D.G.stairs, AIP Conference Proceedings 59 (American Institute of Physics, New York, 1980), p.25.

129.W.Sippach, "Nevis Drift Chamber TDC System", 1977, (unpublished),
and
W.Sippach, "8-Channel Time Recorder Module for Drift Chamber Readout System", 1977, (unpublished).

130.L.Pape et al., Contributed Paper to the Topical Conference on Neutrino Physics, Oxford, July, 1978
and
Cern preprint EP Phys.78-25.

131.J.W.Chapman et al., Phys.Rev. D14 , (1976), 5.

132.J.P.Berge et al., Phys.Rev. D18 , (1978), 1359.
and
J.P.Berge et al., FNAL.PUB.79/51 (1979).
and
P.H.Stuntebeck et al., Phys.Rev. D9 , (1974), 608.

133.J.P.Berge et al., FNAL.PUB.80/44 (1980).

134.C.F.Powell, P.H.Fowler, D.H.Perkins, The study of Elementary Particles by the Photographic Method" Pergammon Press, 1959.

135.W.H.Barkas, "Nuclear Research Emulsions", Academic Press, New York, London, 1963, p.263-463.

136.W.T.Eadie et al., "Statistical Methods in Experimental Physics", North Holland Publishng Company, New York, p.155.

137.H.Winzeler, Nucl.Phys. 69 , (1965), 661.
and
H.Winzeler et al., Nuovo Cimento 17 , (1960), 8.

138.P.L.Jain et al., Nucl.Phys. 67 , (1965), 641.

139.V.Luth, in the "Proceedings of the 1979 International Symposium on Lepton and Photon Interactions at High Energies", (Fermilab, Illinois, 1979), p.78.

140.J.Kirkby, in the "Proceedings of the 1979 International Symposium on Lepton and Photon Interactions at High Energies", (Fermilab, Illinois, 1979), p.107.

141.J.Dorfan, in "Particles and Fields - 1979", edited by B.Margolis and D.G.Stairs, (AIP, New York, 1980), p.159.

142.R.H.Schindler, Ph.D.Thesis, Stanford University, 1979.

143.R.H.Schindler et al., SLAC-pub-2507, LBL-10905, (1981), submitted to Phys.Rev.D.

144.W.Bacino et al., Phys.Rev.Lett. 45 , (1980), 329.

145.G.Donaldson, private communication. We are grateful to him for kindly providing us with the likelyhood function for the D , D lifetime ratio as obtained from the DELCO data.

146.X.Y.Pham, Phys.Rev.Lett. 45 , (1980), 1663.

147.I.I.Bigi, Institute fur Theoretische Phyisk der RWTH, Aachen, Frankfurt,Germany preprint, 1981.

148.C.R.C. Handbook of Chemistry and Physics, 52 Ed. C.R.C., (1972), B-245, and references therein.

149.M.Abramowitz, I.A.Stegun, Handbook of Mathematical Functions, NBS Applied Mathematics Series 55, (1970), 295.

150.S.J.Barish et al., Phys.Lett. 66B , (1977), 291.
and
W.Lerche et al., Nucl.Phys. B142 , (1978), 65.
and
J.Hanlon et al., Phys.Rev.Lett. 45 , (1980), 1817.
and
N.Armenise et al., Phys.Lett. 102B , (1981), 374.

151.D.P.Stanley and D.Robson, Phys.Rev.Lett. 45 , (1980), 235.

152.E.Golowich and E.Hagg, NSF-ITP Santa Barbara preprint NSF-ITP-81-39, (1981).

153.H.J.Lipkin

154.X.Yicheng, Institute four Theoretische Physic, University of Heidelberg, W.Germany preprint HD-THEP-81-12, (1981).

155.J.Franklin, D.B.Lichtenberg, W.Namgung and D.Carydas, Indiana University preprint IUHET-64, (1981).

156.J.Ellis, M.K.Gaillard, D.V.Nanopoulos and P.Sikivie, CERN preprint TH-2938, (1980).

157.E.Farhi and L.Susskind, CERN preprint TH-2975, (1980).

158.S.Dimopoulos, S.Raby, F.Wilczek, Santa Barbara preprint NSF-ITP-81-31, (1981).

159.P.Fayet, LPTENS preprint 81/9, (1981).

160.Particle Data Group, Rev.Mod.Physics 52 , (1980).

161.E.J.Moniz et al., Phys.Rev.Lett. 26 , (1971), 445.

162.A.Firestone et al., Phys.Rev.Lett. 16 , (1966), 556.

163.A.D.Brody et al., Phys.Rev.Lett. 26 , (1971), 1050.

164.G.W.Brandenburg et al., Phys.Rev. D9 , (1974), 1939.

165.G.W.Brandenburg et al., Nucl.Phys. B45 , (1972), 397.

166.G.W.Brandenburg et al., Phys.Rev.Lett. 28 , (1972), 932.

167.J.N.Carney et al., Nucl.Phys. B107 , (1976), 381.

168.N.A.McCubbin and L.Lyons, Nucl.Phys. B86 , (1975), 13.

169.S.L.Baker et al., Nucl.Phys. B99 , (1975), 211.

170.B.W.Lee, C.Quigg, J.L.Rosner, Phys.Rev. D15 , (1977), 157.

171.A.DeRujula, H.Georgi, S.L.Glashow, Phys.Rev. D12 , (1975), 147.

172.L.A.Copley, N.Isgur and G.Karl, Phys.Rev. D20 , (1979), 768.

173.K.Maltman and N.Isgur, Phys.Rev. D22 , (1980), 1701.

174. C.Avilez, T.Kobayashi, J.G.Korner, Phys.Rev. D17, (1978), 709.

175. J.G.Korner, G.Kramer, J.Willrodt, Z.Phys.C, Part. and Fields, (1979), 117.

176. V.Barger, J.P.Leveille, P.M.Stevenson, Phys.Rev.Lett. 44, (1980), 226.

177. J.G.Korner, G.Kramer, J.Willrodt, Phys.Lett. 78B, (1978), 492.
ibid., (erratum), 81B, (1979), 419.

178. R.L.Kingsley, S.B.Treiman, F.A.Wilczek and A.Zee, Phys.Rev. D12, (1975), 106.

179. H.J.Lipkin, in the "Proceedings of Baryon Spectroscopy Conference" (Toronto, Ontario, 1980), p.10.

180. H.J.Lipkin, Phys.Lett. 70B, (1977), 113.

181. H.J.Lipkin, ANL preprint CP-80-71, (1980),

182. N.Isgur and H.J.Lipkin, Phys.Lett. 99B, (1981), 151.

183. Berthold Stech, Phys.Rev.Lett. 38, (1977), 304.

184. C.Quigg and J.L.Rosner, Phys.Rev. D17, (1978), 239.

185. M.K.Gaillard, and B.W.Lee, Phys.Rev.Lett. 33, (1974), 108.

186. M.K.Gaillard, in the "Proceedings of the 1979 International Symposium on Lepton and Photon Interactions at High Energies", (Fermilab, Illinois, 1979), p.397.
and
M.K.Gaillard, "Weak Decays of Heavy Quarks" FNAL pub. 78/64 Thy, (1978) (See also 1978 SLAC Summer School Notes).

187. M.Kobayashi, K.Maskawa, Prog.Theor.Phys. 49, (1973), 652.

188. R.E.Schrock and L.L.Wang, Phys.Rev.Lett. 41, (1978), 1692.

189. R.E.Schrock, S.B.Treiman and L.L.Wang, Phys.Rev. D19, (1979), 2148.

and

R.E.Schrock, M.B.Voloshin, Phys.Lett. 87B , (1979), 375.
and
V.Barger, W.F.Long and S.Pakvasa, Phys.Rev.Lett. 42 , (1979), 1585.
and
R.E.Schrock, S.B.Treiman and L.L.Wang, Phys.Rev.Lett. 42 , (1979), 1589.

190.V.Barger and S.Pakvasa, Phys.Rev.Lett. 43 , (1979), 812.
and
M.Suzuki, Phys.Rev.Lett. 43 , (1979), 818.
and
M.Suzuki, Phys.Lett. 85B , (1979), 91.

191.L.L.Wang, BNL preprint 28280 (1980).

192.S.Pakvasa, S.F.Tuan, J.J.Sakurai, Phys.Rev. D23 , (1981), 2799.

193.N.Cabibbo and L.Maiani, Phys.Lett. 79B , (1978), 109.

194.N.Cabibbo, G.Corbo and L.Maiani, Nucl.Phys. B155 , (1979), 93.

195.M.Suzuki, Nucl.Phys. B145 , (1978), 420.

196.G.Altarelli, N.Cabibbo and L.Maiani, Nucl.Phys. B88 , (1975), 285.

197.B.Guberina, R.D.Peccei, R.Ruckl, Max Planck Institute fur Physik und Astrophysik, Munich Preprint MPI-PAE/PTH 56/79 (1979),
and
ibid., Preprint MPI-PAE/PTH 56/79 (1979).

198.G.Altarelli, G.Curci, G.Martinelli and S.Petrarcha, Phys.Lett. 99B , (1981), 141.

199.J.Ellis, M.K.Gaillard and D.V.Nanopoulos, Nucl.Phys. B100 , (1975), 313.

200.R.L.Kingsley, S.B.Treiman, F.Wilczek and A.Zee, Phys.Rev. D11 , (1975), 1919.

201.D.Fakirov and B.Stech, Nucl Phys. B133 , (1978), 315.

202.C.Quigg, Z.Phys.C, Particles and Fields, 4 , (1980), 55.

203.J.C.Pati and C.H.Woo, Phys.Rev. D3 , (1971), 2920.

204. S.Wolfram, Caltech preprint CALT-68-790, (1980).

205. J.Ellis, M.K.Gaillard, D.V.Nanopoulos and S.Rudaz, Nucl.Phys. B131 , (1977), 285.

206. M.Suzuki, Univ. of Calif., Berkeley, preprint UCB-PTH-80/4, (1980).

207. G.Ebel Nucl.Phys. B33 , (1971), 317.

208. R.E.Marshak, Riazuddin and C.P.Ryan, "Theory of Weak Interactions in Particle Physics", First ed. Wiley Interscience, New York, (1969), p.446.

209. H.Pietschmann, "Formulae and Results in Weak Interactions", Springer-Verlag, Wien, (1974), p.36.

210. C.Quigg, "Lectures on Charmed Particles", FNAL-78/37, p.47.

211. R.E.Schrock, Institute for Theoretical Physics, State Univ. of New York at Stony Brook preprint ITP-SB-80-56.

212. M.Bander, D.Silverman and A.Soni, Phys.Rev.Lett. 44 , (1980), 7.
ibid. (erratum), 44 , (1980), 962.

213. H.Fritzsch and P.Minkowski, Univ. of Bern preprint (1979).

214. V.Barger, J.P.Leveille and P.M.Stevenson, Phys.Rev. D22 , (1980), 693.

215. V.Barger, R.N.Cahn, Y.Kang, J.P.Leveille S.Pakvasa and M.Suzuki, (1979), unpublished.

216. N.Deshpande, M.Gronau, D.Sutherland, FNAL 79/70, (1979).

217. J.L.Cortes, X.Y.Pham, and A.Tounsi, Laboratoire de Physique Theoretique et Hautes Energies, Paris preprint PAR-LPTHE 80/31, (1980).

218. J.D.Bjorken, C.H.Llewellyn-Smith, Phys.Rev. D7 , (1973), 887.

219. B.Guberina, S.Nussinov, R.D.Peccei and R.Ruckl, Phys.Lett. 89B , (1979), 111.

220. H.J.Lipkin, FNAL preprint 79/84 (1979).

221.Cern Summer School (Yellow Report), Herceg Novi, (1964), p.92-99.

222.F.James and M.Roos, Nucl.Phys. B172 , (1980), 475.

223.I.I.Bigi, CERN preprint Ref.Th. 2798-CERN (1979).

224.S.P.Rosen, Phys.Rev.Lett. 44 , (1980), 4.

225.C.Quigg, "The Status of Charm", Wine and Cheese talk given at Fermilab, Sept.28,1979.

226.R.J.Cashmore, "The Current Status of Charm", Oxford Ref. 48/80, (1980).
(to be published in Progress in Particle and Nuclear Physics.)

227.M.S.Chanowitz, in "The Proceedings of the International Symposium on High Energy e+e- Interactions", Vanderbilt University, May, 1980.

228.A.I.Vainstein, V.I.Zakharov, M.A.Shifman, JETP Lett. 22 , (1975), 55.

229.M.B.Wise, E.Witten, Phys.Rev. D20 , (1980), 1216.

230.L.F.Abbot, P.Sikivie and M.B.Wise, Phys.Rev. D21 , (1980), 768.

231.T.G.Rizzo, Phys.Rev. D18 , (1978), 1569.

232.J.E.Kim, P.Langacker, M.Levine, H.H.Williams, Rev.Mod.Phys. 53 , (1981), 211.
and
S.Dawson, J.S.Hagelin, L.Hall, Phys.Rev. D23 , (1981), 2666.

233.N.Ushida et al., "Upper Limits to $\nu_\mu - \nu_\tau$ Oscillation and $\nu_\mu - \tau$ Coupling", submitted to Phys.Rev.Lett., June, 1981.

234.C.Quigg, private communication.

235.H.J.Lipkin, private communication.

236.J.L.Rosner, private communication.

237.S.Ogawa, private communication.

238.S.Sawada, private communication.

239.S.Sawada, Prog.Theor.Phys. 63 , (1980), 572-579.

ibid., 55 , (1976), 818-831.

240.M.Matsuda, M.Nakagawa, S.Ogawa, Prog.Theor.Phys. 63 , (1980), 351.
and
M.Matsuda, M.Nakagawa, S.Ogawa, "Quark Number Conservation and New Particle Decays", Meijo University Preprint MCU-DP-042, (1981).

241.I.I.Bigi, L.Stodolsky, SLAC Pub 2410, (1979).

242.E.Ma, S.Pakvasa and W.Simmons, University of Hawaii Preprint UH-511-369-79, (1979).

243.D.G.Hitlin, "Weak Decays of Strange and Heavy Quarks", 1980 SLAC Summer School, P.67-140.

244.B.Adeva et al., CERN Preprint CERN/EP 81-28, (1981), submitted to Phys.Lett.B.

245.R.E.Behrends, R.J.Finkelstein and A.Sirlin, Phys.Rev. 101 , (1956), 866.
and
S.M.Berman, Phys.Rev. 112 , (1958), 267.
and
T.Kinoshita and A.Sirlin, Phys.Rev. 113 , (1959), 1652.

246.A.Sirlin, Nucl.Phys. B100 , (1975), 291.
and
A.Sirlin, Rev.Mod.Phys. 50 , (1978), 573.

247.G.Kallen, "Elementary Particle Physics", Addison-Wesley, Reading, Ma. (1964), p.471.

248.T.D.Lee and C.N.Yang, Phys.Rev. 108 , (1957), 1615.

249.A.Bartl, H.Fraas and W.Majerotto, University of Wien, Austria preprint UW-TH Phy-80-41, (1980).
and
A.Bartl, private communication.

250.K.Heller et al., Phys.Rev.Lett. 41 , (1978), 607.

251.P.Skubic et al., Phys.Rev. D18 , (1978), 3115.

252.L.Schachinger et al., Phys.Rev.Lett. 41 , (1978), 1348.

253.C.Wilkinson et al., Phys.Rev.Lett. 46 , (1981), 803.

254.P.T.Cox et al., Phys.Rev.Lett. 46 , (1981), 877.

255. L.C.Schachinger, Ph.D.Thesis, Rutgers University, 1978 (unpublished).

256. R.Grobel, Ph.D.Thesis, University of Wisconsin, 1980 (unpublished).

257. P.T.Cox, Ph.D.Thesis, University of Michigan 1980 (unpublished)

258. L.Pondrom, Fermilab Report, 1979.

Figure 1.

WEAK DECAYS OF QUARKS AND LEPTONS

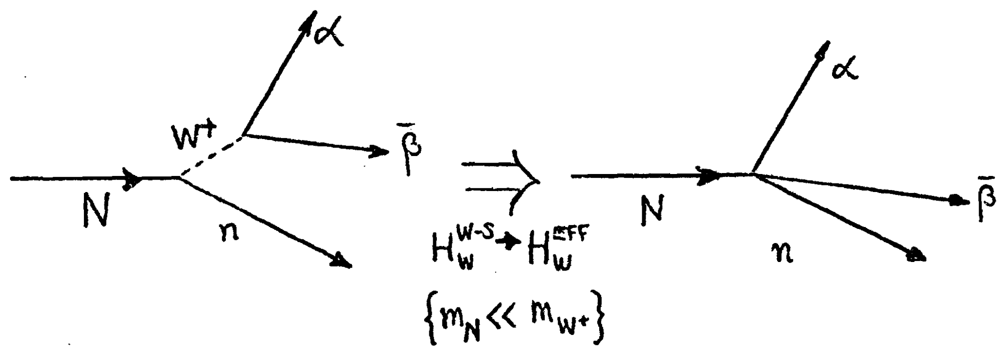
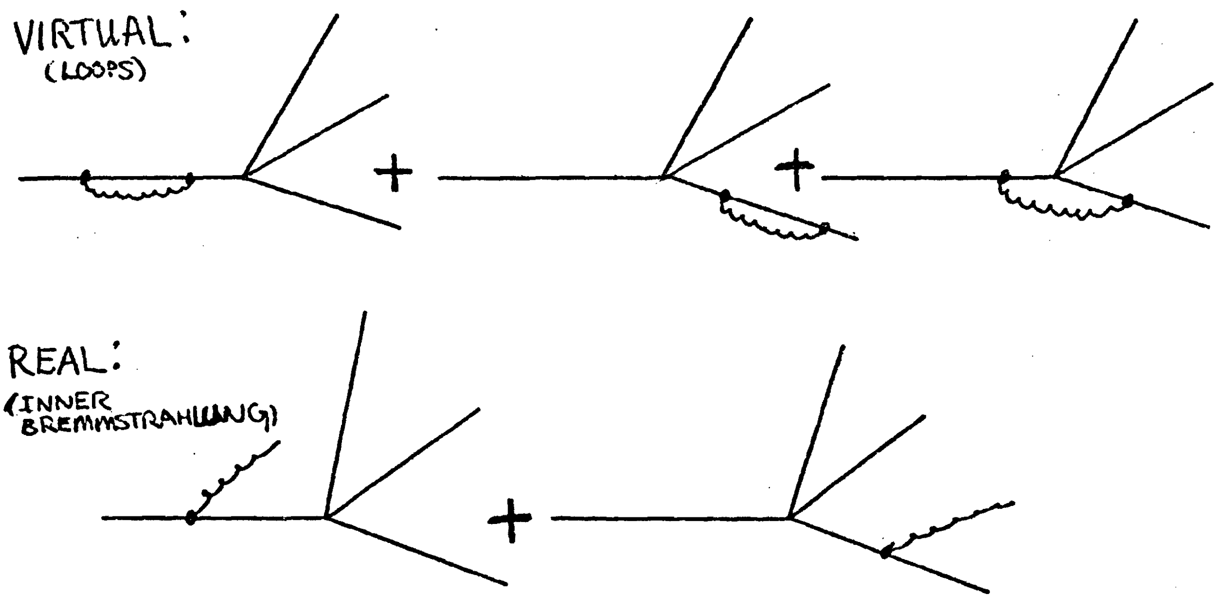


Figure 2.

FIRST-ORDER RADIATIVE CORRECTIONS TO WEAK DECAYS

(EM AND STRONG)

{QED} + {QCD}



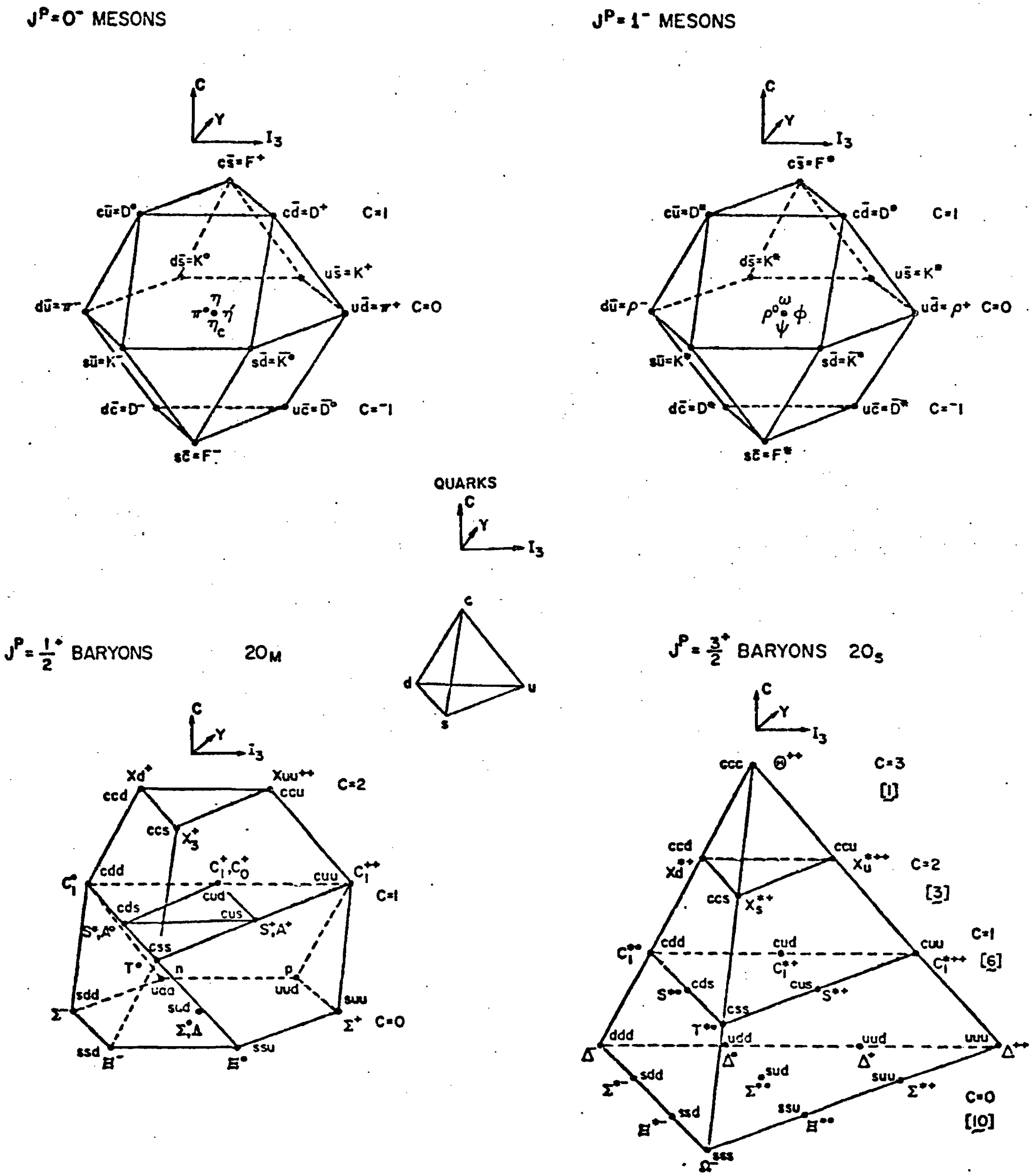


Figure 3

Meson and Baryon SU(4) Multiplets

Figure 4.

CHARMED PSEUDOSCALAR MESON DECAY

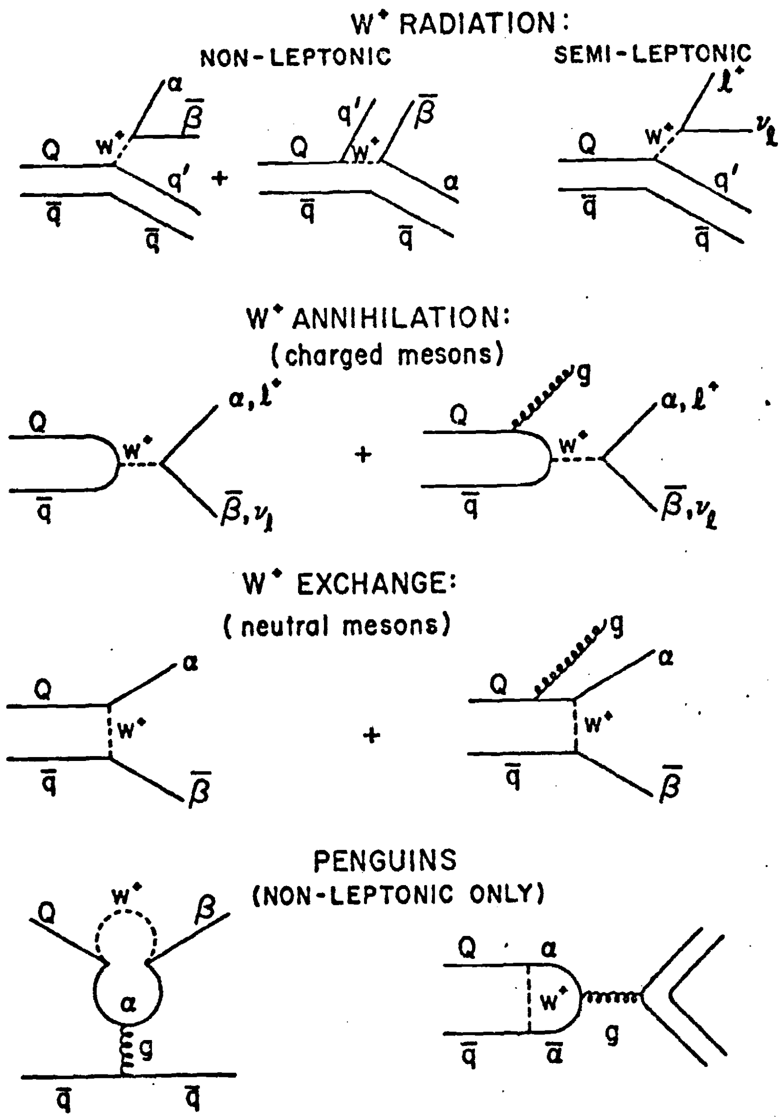
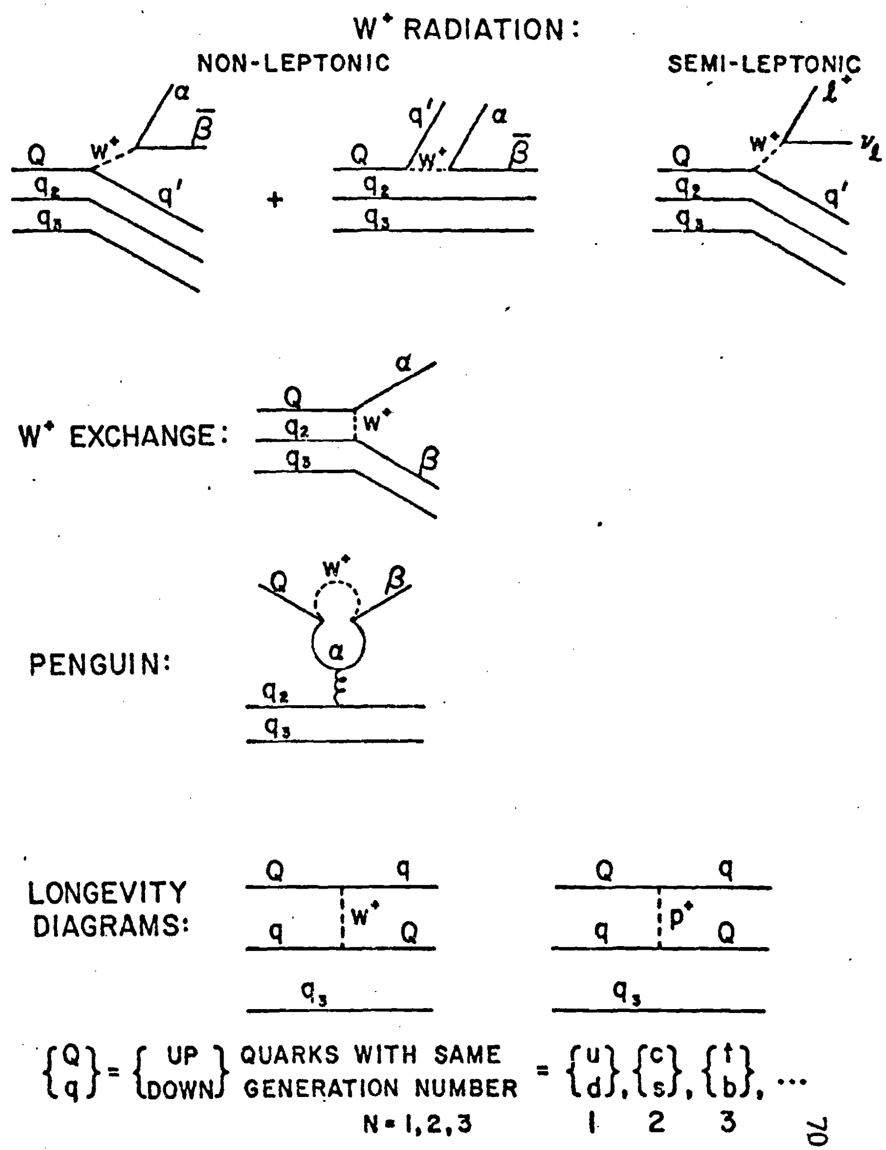


Figure 5.

CHARMED BARYON DECAY



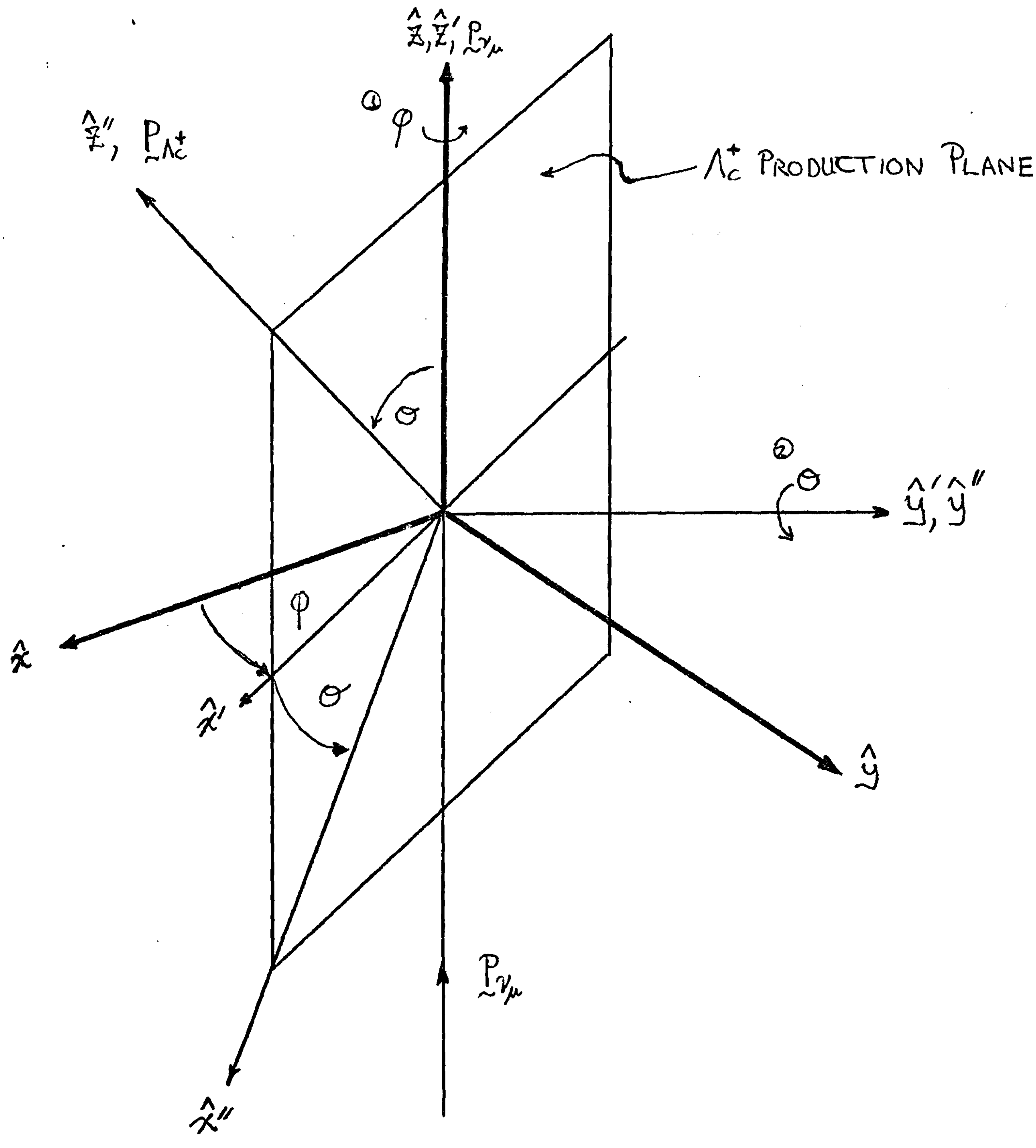
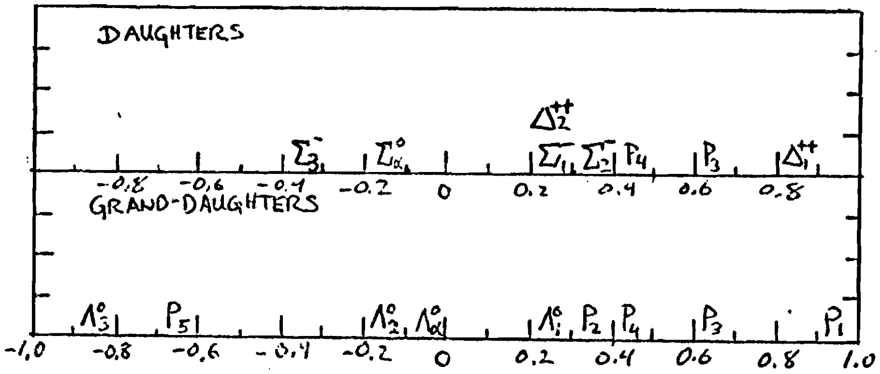


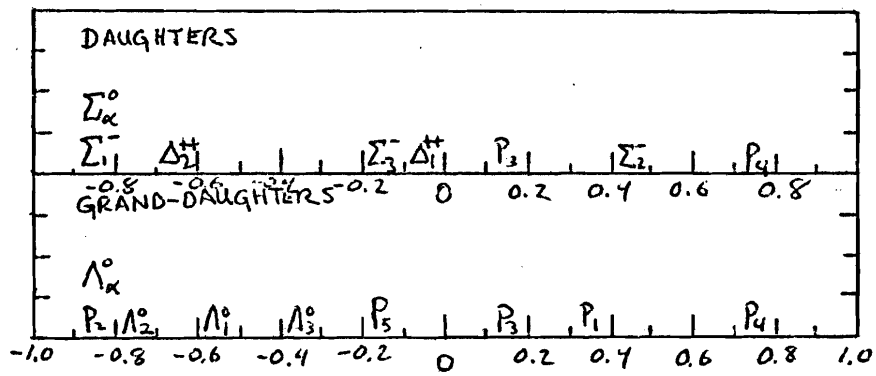
Figure 6. Λ_c^+ Production Plane

COMPONENTS OF α_{P} IN Λ_c^+ REST FRAME

$\cos\Theta_X$



$\cos\Theta_Y$



$\cos\Theta_Z$

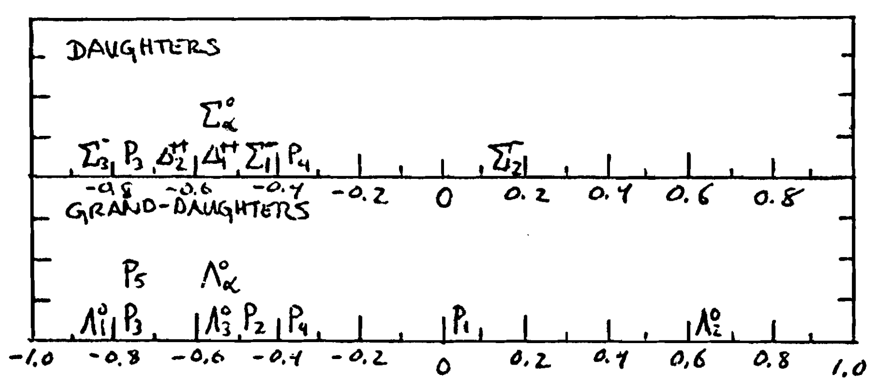
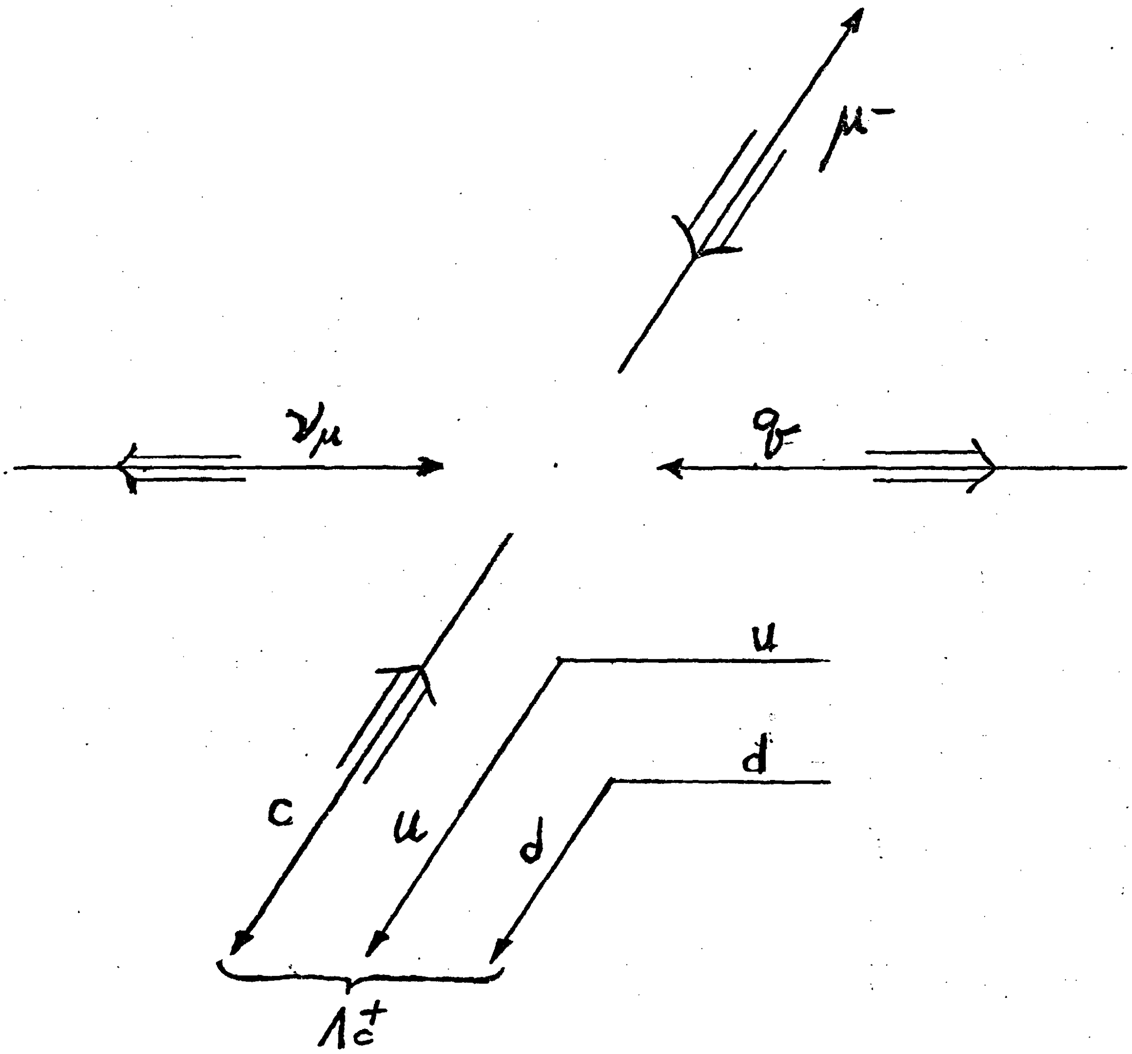


Figure 8.

CHARMED BARYON POLARIZATION IN NEUTRINO INTERACTIONS



$$\chi_{\Lambda_c^+} = \frac{1}{\sqrt{2}}\, c\uparrow (u\uparrow d\downarrow - u\downarrow d\uparrow)$$

PREDICTED
BARYON POLARIZATION $\mathbb{P}$ vs. $Z = P_B / P_{HAD}$
IN ν_μ- INTERACTIONS

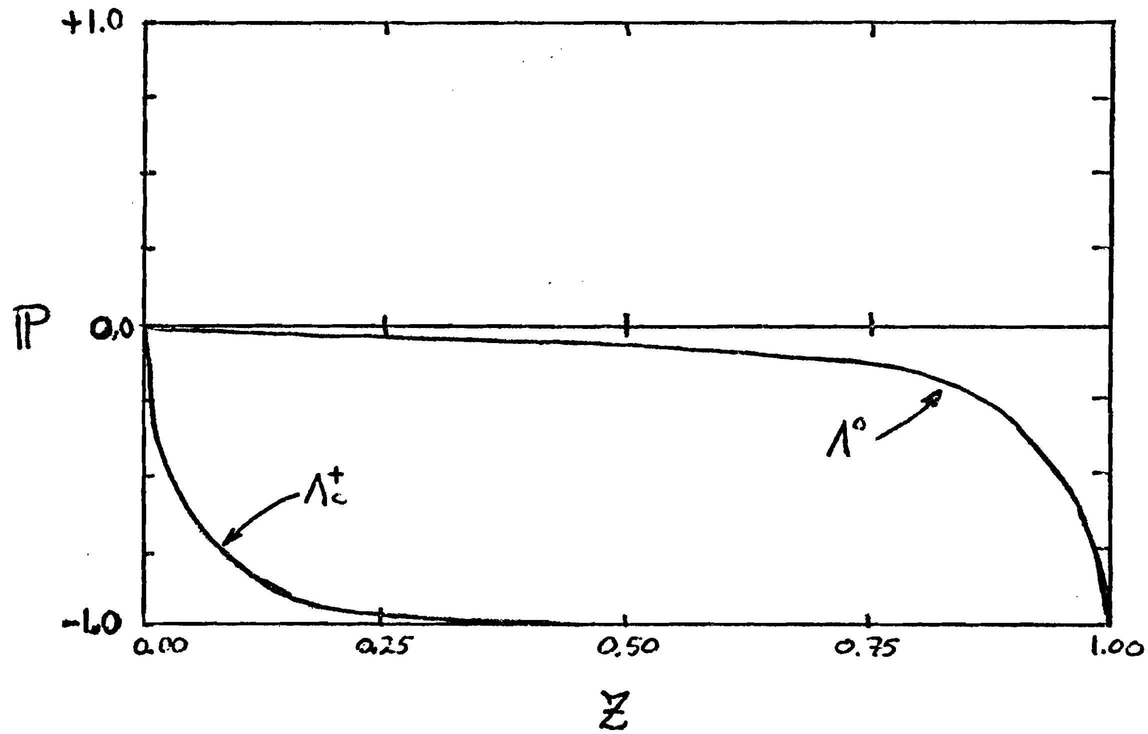
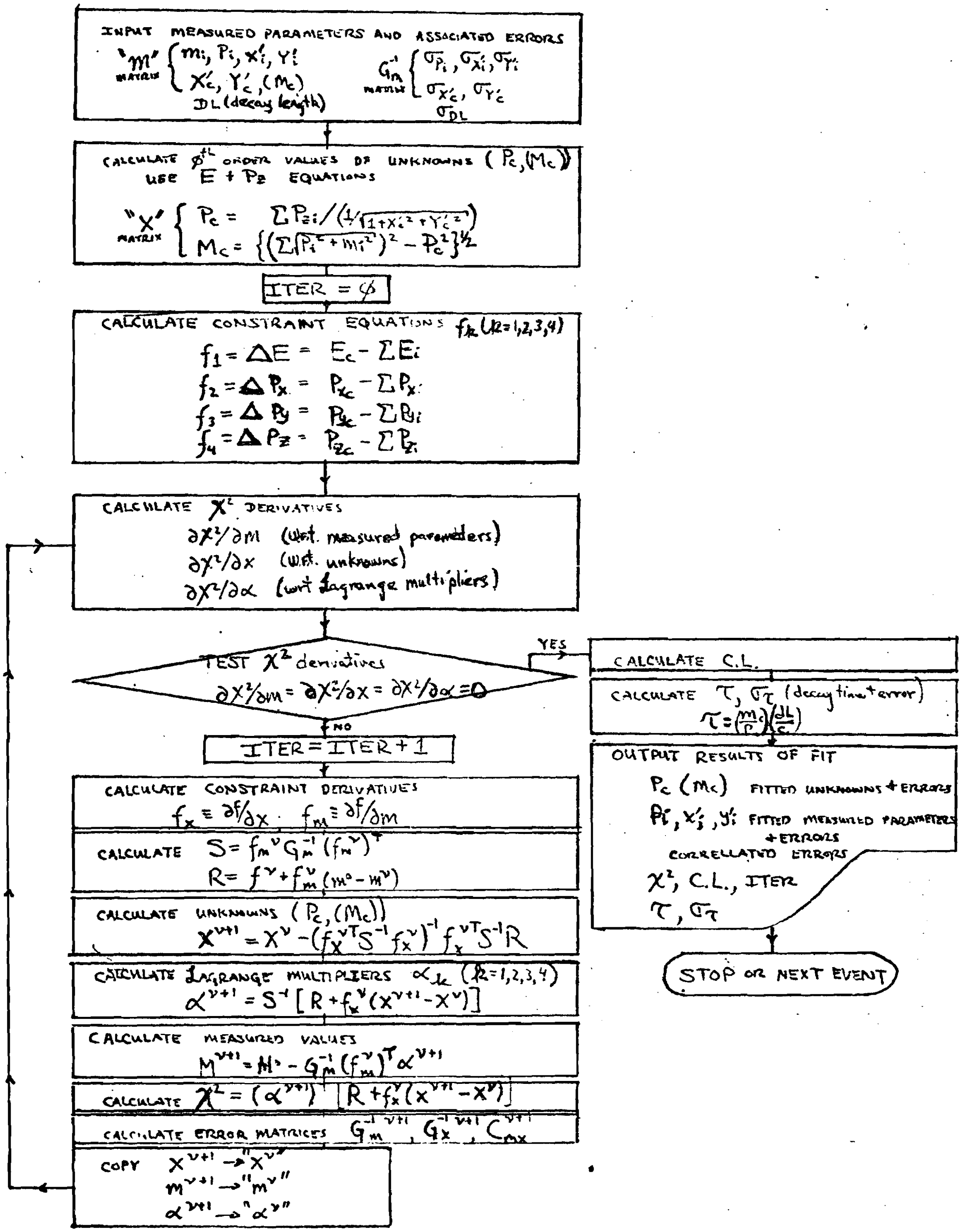


Figure 9. KINEMATIC FITTING PROGRAM



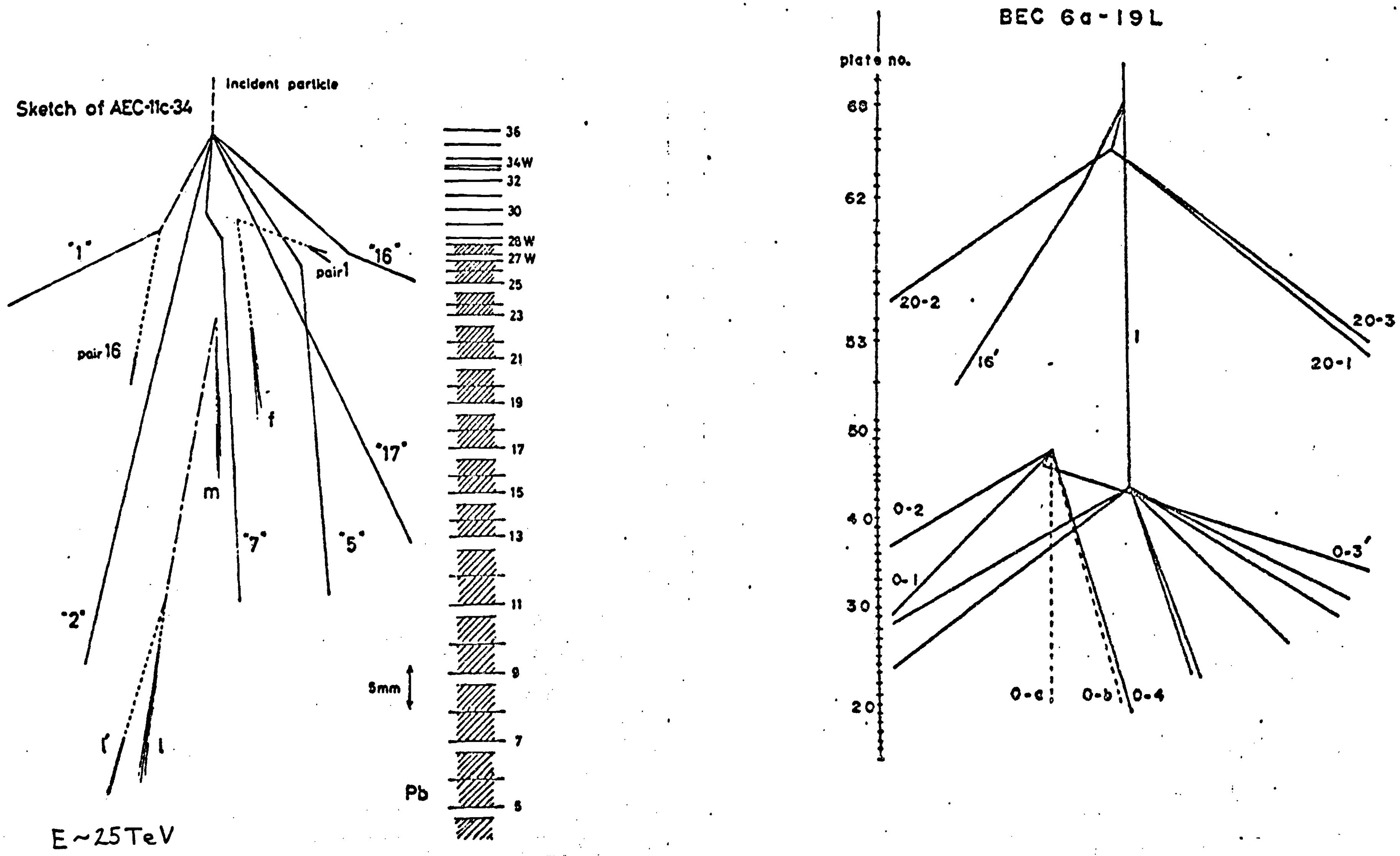


Figure 10. High Energy Cosmic Ray Events

(Nagoya University)