1.1 Beam-Beam Effects in Muon Colliders

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1.1.1 Introduction

Muon Collider (MC) - proposed by G.I. Budker and A.N. Skrinsky more than 40 years ago – is now considered as the most exciting option for the energy frontier machine in the post-LHC era. It has a number of important advantages over e+e– colliders: better energy resolution, larger cross-section of scalar particles production etc. [1]. However, taking into account relatively high transverse emittance which can be obtained with ionization cooling, the bunch population should be as high as $\sim 2\cdot 10^{12}$ in order to achieve competitive luminosities. This brings to the forefront the beam-beam effects, coherent instabilities and their interplay.

Table 1: Baseline muon collider parameters [2].

Parameter	***Unit***	***Value***
Beam energy	TeV	0.75
Repetition rate	Hz	15
Average luminosity / IP	10^{34}/cm^2/s	1.1
Number of IPs, N_{IP}	-	2
Circumference, C	km	2.73
β^*	cm	1 (0.5-2)
Momentum compaction, α_p	10^{-5}	-1.3
Normalized emittance, $\varepsilon_{\perp N}$	π·mm·mrad	25
Momentum spread	%	0.1
Bunch length, σ_s	cm	1
Number of muons / bunch	10^{12}	2
Beam-beam parameter / IP, ξ	-	0.09
RF voltage at 800 MHz	MV	16
Betatron tunes	-	20.56 / 16.58
Synchrotron tune	-	0.00057

1.1.2 Incoherent beam-beam effect

An important feature of a muon collider necessary for achieving high luminosity is small beta-function at IP, $\beta^* \leq 1$cm, which is more typical for e+e– factories than for TeV-range circular machines. As a result the final focus quadrupoles excite very strong chromatic beta-wave which should be suppressed with sextupoles as close to the origin

as possible. The beam-beam interaction changes phase advances between the sextupoles across the IP making the problem more complicated: it must be taken into account already at the stage of lattice design.

A successful IR lattice design providing sufficiently large momentum acceptance and dynamic aperture of the whole ring with little sensitivity to the beam-beam effect was presented in [2]. Its basic parameters are cited in Table 1. The key to success was to arrange the optics so that the sextupoles correcting chromaticity in one plane were located at minima of beta-function in the other plane with phase advances from IP being multiples of π. In the result the beam-beam interaction reduces the beta-function values at the minima further suppressing spherical aberrations produced by these sextupoles.

1.1.2.1 Dynamic aperure

The problem with dynamic aperture in a muon collider is somewhat alleviated by limited number of turns the muons spend in the machine: their lifetime at 0.75 TeV is just 1700 turns. But they may be dumped even earlier – after about 1000 turns – to reduce heat deposition in magnets without significant impact on the integrated luminosity. This means that high order resonances will have little chance to show up.

A preliminary study of incoherent beam-beam effect was performed in the weak-strong approximation with the help of MAD8. The strong bunch was represented by 23 slices according to Zholents-Shatilov algorithm [3]. Figure 1 shows 1000 turns "diagonal" dynamic aperture obtained along the line A_x=A_y with fixed values of the beam-beam parameter, the test particle momentum deviation and longitudinal position at the center of the bunch. More comprehensive studies using LIFETRAC code [3] are underway.

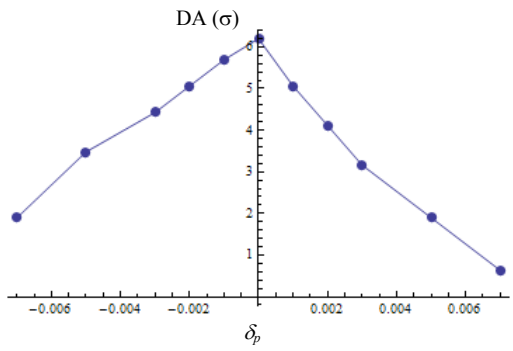

Figure 1: Dynamic aperture vs. constant momentum deviation in the presence of beam-beam interaction (ξ = 0.1/IP)

1.1.2.2 Dynamic beta effect

Conventional wisdom suggests choosing phase advances between IPs to be just above multiples of π. Then the beam-beam interaction reduces the beta-function values at IPs enhancing the luminosity – the phenomenon known in circular colliders as the "dynamic beta effect".

However, this effect increases beta-functions at the IR quadrupole locations and may be detrimental in a TeV-range muon collider where the quadrupole aperture is restricted by high gradient requirements and the necessity of protection from the muon

decay products [4]. From this point of view "neutral" phase advances – odd multiples of $\pi/2$ – are preferable. Then with 2 IPs we get half-integer tunes which are also beneficial for orbit stability and low detuning with amplitude in a bare lattice.

Still - as R. Palmer pointed out - there will be some luminosity enhancement by the beam-beam interaction due to a large length of the bunches ($\sigma_s \sim \beta^*$) which is akin to the "disruption" effect in linear colliders. Strong-strong simulation of this effect in linear approximation for the beam-beam force showed that at given parameters it almost completely compensates for the luminosity reduction due to the "hour-glass" effect [5].

With these considerations in mind, the current version of the MC lattice was chosen to have two identical superperiods with phase advances between the IPs being $41\times\pi/2$, $33\times\pi/2$ in the two planes. But the requirements of coherent oscillations stability may necessitate a different choice of phase advances.

1.1.3 Coherent beam-beam oscillations

The synchrotron tune in the muon collider is very low (see Table 1) as a consequence of a very small (by absolute value) momentum compaction factor needed to obtain sufficiently short bunches with moderate RF voltage. As the longitudinal motion is virtually frozen the beams are subject to a transverse BBU-like instability known in linear accelerators. Its rise-time in a single bunch may be as short as a few hundred turns [6]. A possibility was discussed of using RF quadrupoles to taper the tunes along the bunch and provide BNS damping [6].

The beam-beam interaction may render such a complication unnecessary: the instability – which is in essence a single-particle response to the wakefield generated by the head of the bunch – should be strongly suppressed by the beam-beam tunespread. For parameters of Table 1 the decoherence time of the initial (driving) perturbation is just $1/(N_{IP}\,\xi) = 5$ turns.

However, under conditions typical to a muon collider it may be more difficult to stabilize coherent beam-beam modes: in absence of a tunesplit between the two beams of approximately equal intensities there will be discrete spectral lines of π- and Σ-modes well separated from the incoherent tunespread [7]. The natural suppression mechanism by the synchrotron sidebands will not work due to a very small ratio of the synchrotron tune to the beam-beam parameter [7]. Numerical 3D simulations with realistic impedances are necessary to determine if the instability is strong enough to develop during limited lifespan of the muon beams and if it can be suppressed by chromaticity.

If there will be found a potential for instability, a remedy can be applied which was once considered for LHC: redistribution of phase advances between the superperiods [7]. In particular, a 180° phase advance difference between sectors IP1→IP2 and IP2→IP1 would completely suppress the coherent beam-beam modes. In our case the phase advances can be chosen as 20π, 17π in one half of the ring and 21π, 16π in the other half without changing the total tunes. However, such redistribution will create difficulties with dynamic beta increase in quadrupoles as discussed in the previous section and should be considered as the last resort.

1.1.4 References

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