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A Combination of CDF and DØ Limits on the Branching Ratio of $B^0_{s(d)} \rightarrow \mu^+\mu^-$ Decays

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Abstract

We combine the results of CDF and DØ searches for the rare decays $B^0_s \rightarrow \mu^+\mu^-$ and $B^0_d \rightarrow \mu^+\mu^-$. The experiments use $364\,\mathrm{pb}^{-1}$ and $300\,\mathrm{pb}^{-1}$ of data respectively. The limits on the branching ratios are obtained by normalizing the estimated sensitivity to the decay $B^+ \rightarrow J/\psi K^+$ taking into account the fragmentation ratios $f_u/f_{s(d)}$. The combined results exclude branching ratios of $\mathrm{BR}(B^0_s \rightarrow \mu^+\mu^-) > 1.5 \times 10^{-7}$ and $\mathrm{BR}(B^0_d \rightarrow \mu^+\mu^-) > 4.0 \times 10^{-8}$ at 95% confidence level. These are the most stringent limits on these decays at the present time.

1 Introduction

The CDF and DØ experiments have previously reported on searches for the rare decay $B_s^0 \rightarrow \mu^+\mu^-$ [1, 2]. CDF also directly searched for the decay $B_d^0 \rightarrow \mu^+\mu^-$. These decays are highly suppressed in the Standard Model of particle physics with branching ratios of $\mathrm{BR}(B_s^0 \rightarrow \mu^+\mu^-) = 3.5 \times 10^{-9}$ and $\mathrm{BR}(B_d^0 \rightarrow \mu^+\mu^-) = 1.0 \times 10^{-10}$ [3]. However, decays of this type can be significantly enhanced in many scenarios beyond the Standard Model [4]. A combination of results leads to more stringent limits and is of considerable interest in exploring the phase space of the models where strong enhancements for $B_s^0 \rightarrow \mu^+\mu^-$ or $B_d^0 \rightarrow \mu^+\mu^-$ are predicted. In this note we report on a combination of limits in the $B_s^0 \rightarrow \mu^+\mu^-$ and $B_d^0 \rightarrow \mu^+\mu^-$ decay channels.

CDF and DØ use similar methodologies to search for the $B_s^0 \rightarrow \mu^+\mu^-$ decay. CDF applied the same methods to directly search for the decay $B_d^0 \rightarrow \mu^+\mu^-$. Each experiment looks for two oppositely charged muons in the B_s^0 and B_d^0 mass range using dedicated triggers. CDF divides their dataset into two channels: the "Central" channel consists of muon pairs reconstructed in the pseudorapidity region, $|\eta| < 0.6$, and the "Central-Extended" channel consists of dimuon events where one muon is reconstructed in the central region and the second muon in the extended muon system, $0.6 < |\eta| < 1.0$. The two channels have different sensitivities, therefore the optimization is performed separately for each channel. The branching ratio is computed by normalizing the number of signal events to the number of reconstructed $B^+ \rightarrow J/\psi K^+$ [5] events. The branching ratio or limit is then calculated from the equation:

$$\mathrm{BR}(B_s^0 \rightarrow \mu^+\mu^-) = \frac{N_{B_s}^{obs}}{\alpha_{B_s}\epsilon_{B_s}^{\mathrm{total}}} \cdot \frac{\alpha_{B^+}\epsilon_{B^+}^{\mathrm{total}}}{N_{B^+}^{obs}} \cdot \frac{f_u}{f_s} \cdot \mathrm{BR}(B^+ \rightarrow J/\psi K^+) \cdot \mathrm{BR}(J/\psi \rightarrow \mu^+\mu^-), \quad (1)$$

where:

- $N_{B_s}^{obs}$ is the number of observed $B_s^0 \rightarrow \mu^+\mu^-$ candidates;
- α_{B_s} is the geometric and kinematic acceptance of the di-muon trigger for $B_s^0 \rightarrow \mu^+\mu^-$ decays;
- $\epsilon_{B_s}^{\mathrm{total}}$ is the total efficiency (including trigger, reconstruction and analysis requirements) for $B_s^0 \rightarrow \mu^+\mu^-$ events in the acceptance;
- $N_{B^+}^{obs}$, α_{B^+}, and $\epsilon_{B^+}^{\mathrm{total}}$ are similarly defined for $B^+ \rightarrow J/\psi K^+$ decays;
- f_u/f_s accounts for the different b-quark fragmentation probabilities and is: $(0.397 \pm 0.010)/(0.107 \pm 0.011) = 3.71 \pm 0.41$, where the anti-correlation between the uncertainties has been accounted for [6];
- $\mathrm{BR}(B^+ \rightarrow J/\psi K^+) \cdot \mathrm{BR}(J/\psi \rightarrow \mu^+\mu^-) = (1.00 \pm 0.04) \times 10^{-3} \cdot (5.88 \pm 0.10) \times 10^{-2} = (5.88 \pm 0.26) \times 10^{-5}$ are used [6] .

The experiments normalize to the decay mode $B^+ \rightarrow J/\psi K^+$ rather than to the B_s decay $B_s^0 \rightarrow J/\psi\phi$. Normalizing to the decay $B^+ \rightarrow J/\psi K^+$ is preferable since the mode has higher statistics and the branching ratio and lifetime are well known from the measurements at CLEO and the asymmetric B factories. In addition, understanding the efficiency to detect $B_s^0 \to J/\psi\phi$ events is complicated by the presence of CP even and odd decay components which have different lifetimes. Finally normalizing to the mode $B_s^0 \to J/\psi\phi$ does not eliminate the systematic uncertainty from the ratio f_u/f_s since current measurements of the branching ratio of $B_s^0 \to J/\psi\phi$ are calculated using the fragmentation ratio. The expression for $B_d^0 \to \mu^+\mu^-$ is obtained by replacing B_s^0 with B_d^0 and the fragmentation ratio with f_u/f_d which is taken as unity. The limits on the branching ratio BR($B_s^0 \to \mu^+\mu^-$) are also calculated using the ratio f_u/f_s [7] determined from Tevatron data based on the CDF Run 1 analysis [8].

To increase the sensitivity both experiments perform an optimization over the primary discriminating variables, which are based on lifetime, compatibility between the momentum vector of the candidate B meson and the vector between the production and decay vertices, and isolation in a cone around the candidate meson, where isolation is defined as the ratio of sum of momenta of the candidate tracks divided by the sum of momenta the candidate tracks and other tracks in the cone. CDF optimizes for the best limit, and DØ for the best sensitivity, where sensitivity is defined as $\epsilon_{B_s}/(1+\sqrt{N_{BG}})$, and N_{BG} is the number of expected background events.

We use a Bayesian integration method to calculate the combined limits [9]. The method takes into account correlated and uncorrelated systematic uncertainties between the two experiments, and between the two search channels of the CDF analysis.

The key elements in calculating the limit are estimating central values and uncertainties on the efficiency and acceptance for the trigger and reconstruction, and estimating the background. These elements are described in the next section, followed by a section on the limit calculation and a final section where the results are summarized.

2 Acceptance, Efficiency, Backgrounds and Associated Uncertainties

Both experiments evaluate the acceptance and efficiency of their respective triggers, reconstruction code and discriminating variables. By normalizing the limit to the measured decay $B^+ \to J/\psi K^+$ most of the systematic uncertainties of these estimates cancel in the ratio. The sources of systematic uncertainties in the CDF and DØ analyses are discussed in detail in references [1, 2]. Systematic uncertainties on the estimates of the acceptance and efficiency are uncorrelated since they are based on estimates of trigger and reconstruction efficiencies for different detectors. Systematic uncertainties associated with the production of B mesons are possibly correlated since similar event generation programs are used in both analysis. These systematic uncertainties affect the estimates of the efficiencies of the primary discriminating variables. However, the variables that are used are qualitatively different. For instance, the strongest discriminator in each analysis is the lifetime variable. The CDF analysis uses the proper decay time in 3D while DØ selects events based on the 2D transverse decay length significance distribution. The discriminating variable used by DØ is correlated with momentum where the CDF variable is uncorrelated. Therefore the uncertainties on f_u/f_s and BR($B^+ \to J/\psi K^+$) · BR($J/\psi \to \mu^+\mu^-$) are 100% correlated between the two experiments and all other uncertainties are treated as uncorrelated.

The experiments evaluate the background using sideband events. The uncertainties on the backgrounds are dominated by the statistical uncertainties.

The acceptance, efficiency, and background numbers are summarized in Table 1.

	CDF: Central	CDF: Central-Extended	DØ
Luminosity	364pb^{-1}	336pb^{-1}	300pb^{-1}
$(\frac{\alpha_{B^+} \cdot \epsilon^{total}_{B^+}}{\alpha_{B^0_s} \cdot \epsilon^{total}_{B^0_s}})$	0.852 ± 0.084 (±9.9%)	0.485 ± 0.048 (± 9.9%)	0.247 ± 0.019 (± 7.7%)
$N^{obs}_{B^+}$	1785 ± 60 (± 3.4%)	696 ± 39 (± 5.6%)	906 ± 41 (± 5.0%)
Uncor. Uncer.	(±10.4%)	(±11.3%)	(± 9.2%)
f_u/f_s	3.71 ± 0.41 (±11.0%)		
BR($B^+ \to J/\psi K^+$ $\to \mu^+\mu^- K^+$)	$(5.88 \pm 0.26) \times 10^{-5}$ (± 4.0%)		
Cor. Uncer.	(±11.9%)		
$N^{expected}_{back}$	0.81 ± 0.12 (±14.8%)	0.66 ± 0.13 (±19.7%)	4.3 ± 1.2 (±27.9%)
$N^{obs}_{B^0_s}$	0	0	4
ses ($\times 10^7$)	1.04 ± 0.16 (±15.8%)	1.52 ± 0.25 (±16.4%)	0.59 ± 0.09 (±15.0%)
ses ($\times 10^7$)	0.617 (CDF combined)		
Exp Limit 90% CL	3.5×10^{-7}	5.6×10^{-7}	3.5×10^{-7}
Exp Limit 90% CL	2.0×10^{-7} (CDF combined)		

Table 1: A summary of the inputs used in equation 1 to estimate BR($B^0_s \to \mu^+\mu^-$). The relative uncertainties are given parenthetically. The single-event-sensitivity, ses, to a given branching ratio, corresponding to $N^{obs}_{B^0_s} = 1$, and the expected limit at 90% Confidence Level (CL), under a hypothetical repetition of the experiments, are calculated using the inputs. The combined ses and expected limit for the CDF "Central" and "Central-Extended" search channels are also given.

3 Limits

Using the Bayesian integration method we calculate the combined limits. In the case of the DØ search the dimuon mass signal region covers both the B^0_s and B^0_d. The limit on the branching ratio that is extracted in one mode assumes that the branching ratio in the other mode is zero, which results in a conservative limit. This is the case in the framework of Minimal Flavor Violating (MFV) SUSY models, where the CKM matrix is the only source of flavor violation. In MFV SUSY the branching ratio for $B^0_d \to \mu^+\mu^-$ is expected to be suppressed relative to $B^0_s \to \mu^+\mu^-$ by a factor of $|V_{td}/V_{ts}|^2$, making the contribution from $B^0_d \to \mu^+\mu^-$ negligible. The 95% confidence level (CL) limits on the branching ratio BR($B^0_s \to \mu^+\mu^-$) are reported in Table 2. The combined limits are also calculated at 90% CL.

	CDF: Central	CDF: Central-Extended	DØ
Luminosity	364 pb^{-1}	336 pb^{-1}	300 pb^{-1}
Limits at 95%CL	$\mathrm{BR}(B_s^0 \to \mu^+\mu^-) < 2.0 \times 10^{-7}$		$\mathrm{BR}(B_s^0 \to \mu^+\mu^-) < 3.9 \times 10^{-7}$
Combined Limits	$\mathrm{BR}(B_s^0 \to \mu^+\mu^-) < 1.5 \times 10^{-7}$ at 95% CL $\mathrm{BR}(B_s^0 \to \mu^+\mu^-) < 1.2 \times 10^{-7}$ at 90% CL		

Table 2: A summary of the limits on the branching ratio $\mathrm{BR}(B_s^0 \to \mu^+\mu^-)$.

For the branching ratio $\mathrm{BR}(B_d^0 \to \mu^+\mu^-)$ the factor f_u/f_d is taken to be unity. The efficiency for the B_d^0 channel is estimated to be 8% lower than in the B_s^0 channel for the DØ search, and the uncorrelated uncertainty is 10.2%. The CDF efficiency and errors are the same as for the $B_s^0 \to \mu^+\mu^-$ channel. The limits on the branching ratio $\mathrm{BR}(B_d^0 \to \mu^+\mu^-)$ are reported in Table 3.

	CDF: Central	CDF: Central-Extended	DØ
Luminosity	364 pb^{-1}	336 pb^{-1}	300 pb^{-1}
Limits at 95%CL	$\mathrm{BR}(B_d^0 \to \mu^+\mu^-) < 5.1 \times 10^{-8}$		$\mathrm{BR}(B_d^0 \to \mu^+\mu^-) < 11.1 \times 10^{-8}$
Combined Limits	$\mathrm{BR}(B_d^0 \to \mu^+\mu^-) < 4.0 \times 10^{-8}$ at 95% CL $\mathrm{BR}(B_d^0 \to \mu^+\mu^-) < 3.2 \times 10^{-8}$ at 90% CL		

Table 3: A summary of the limits on the branching ratio $\mathrm{BR}(B_d^0 \to \mu^+\mu^-)$.

The limits on the branching ratios $\mathrm{BR}(B_s^0 \to \mu^+\mu^-)$ and $\mathrm{BR}(B_d^0 \to \mu^+\mu^-)$ can be compared to the values expected in the SM. Currently the 95% CL limit on $\mathrm{BR}(B_s^0 \to \mu^+\mu^-)$ is approximately a factor of 50 larger than the SM branching ratio while the limit on $\mathrm{BR}(B_d^0 \to \mu^+\mu^-)$ is approximately 400 times larger.

The uncertainty on the ratio f_u/f_s is the largest systematic uncertainty in limit calculation. This error dominates the external sources of systematic uncertainties. In order to facilitate recalculation of the limit as the measured value of f_u/f_s is improved we give the limits factoring out this contribution in Table 4.

The limit on branching ratio $\mathrm{BR}(B_s^0 \to \mu^+\mu^-)$ can also be calculated using an evaluation of the fragmentation fractions based only on Tevatron data. This average, $f_u/f_s = (0.398 \pm 0.010)/(0.120 \pm 0.021) = 3.32 \pm 0.59$, differs slightly from the world average value and is dominated by the comparison of the mixing probabilities between B_s^0 and B_d^0 mesons, χ_s and χ_d. Limits calculated using this value of f_u/f_s are reported in Table 5.

The limits were checked using independent programs for Bayesian limit integration developed separately by the CDF and DØ collaborations. In addition, the cutoff of the assumed prior distribution was varied and the distributions used for the uncertainties in the calculation were taken as Gaussian distributions with a cutoff and Gamma functions. The limits were found to vary by less than 5% under these tests.

	CDF: Central	CDF: Central-Extended	DØ
Luminosity	364 pb^{-1}	336 pb^{-1}	300 pb^{-1}
Limits at 95%CL	$\text{BR}(B_s^0 \to \mu^+\mu^-)/(f_u/f_s) < 5.1 \times 10^{-8}$		$\text{BR}(B_s^0 \to \mu^+\mu^-)/(f_u/f_s)$ $< 10.1 \times 10^{-8}$
Combined Limits	$\text{BR}(B_s^0 \to \mu^+\mu^-)/(f_u/f_s) < 3.9 \times 10^{-8}$ at 95% CL $\text{BR}(B_s^0 \to \mu^+\mu^-)/(f_u/f_s) < 3.1 \times 10^{-8}$ at 90% CL		

Table 4: A summary of the limits on the branching ratio $\text{BR}(B_s^0 \to \mu^+\mu^-)/(f_u/f_s)$.

	CDF: Central	CDF: Central-Extended	DØ
Luminosity	364 pb^{-1}	336 pb^{-1}	300 pb^{-1}
Limits at 95%CL	$\text{BR}(B_s^0 \to \mu^+\mu^-) < 1.9 \times 10^{-7}$		$\text{BR}(B_s^0 \to \mu^+\mu^-) < 3.7 \times 10^{-7}$
Combined Limits	$\text{BR}(B_s^0 \to \mu^+\mu^-) < 1.4 \times 10^{-7}$ at 95% CL $\text{BR}(B_s^0 \to \mu^+\mu^-) < 1.1 \times 10^{-7}$ at 90% CL		

Table 5: A summary of the limits on the branching ratio $\text{BR}(B_s^0 \to \mu^+\mu^-)$ using the Tevatron determination of f_u/f_s.

4 Conclusion

We report on a combination of CDF and DØ limits on the branching ratios $\text{BR}(B_s^0 \to \mu^+\mu^-)$ and $\text{BR}(B_d^0 \to \mu^+\mu^-)$. The limits are obtained using a relative normalization to the decay $B^+ \to J/\psi K^+$. The individual limits are combined using a Bayesian integration technique that takes into account correlated and uncorrelated systematic uncertainties between the two experiments. The limits are found to be robust under several tests. The combined limits are:

- $\text{BR}(B_s^0 \to \mu^+\mu^-) < 1.5 \times 10^{-7}$ at 95% CL
- $\text{BR}(B_s^0 \to \mu^+\mu^-) < 1.2 \times 10^{-7}$ at 90% CL
- $\text{BR}(B_d^0 \to \mu^+\mu^-) < 4.0 \times 10^{-8}$ at 95% CL
- $\text{BR}(B_d^0 \to \mu^+\mu^-) < 3.2 \times 10^{-8}$ at 90% CL

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