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Averaging of the inelastic cross sections measured by the CDF and the E811 experiments.

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1. Introduction

To facilitate the comparison of CDF and D0 cross section measurements in Run II, the collaborations have agreed to use a common $p\bar{p}$ inelastic cross section for luminosity normalization. For this purpose, we present here an average of the CDF and E811 measurements of this quantity.

2. Total and inelastic cross-sections

Both CDF and E811[1] measured the total cross-section for $p\bar{p}$ collisions at 1.8 TeV using the luminosity independent method [1,2]

$$\sigma_{tot} = 16\pi(\hbar c)^2 \frac{b}{1+\rho^2} \frac{N_{el}}{N_{el}+N_{in}},$$

where N_{el} is the rate of elastic and N_{in} is the rate of inelastic $p\bar{p}$ scattering events. The slope b is defined from the measurement of the elastic scattering differential cross section at low four-momentum transfer (t)

$$b = \frac{1}{N_{el}} \frac{dN_{el}}{dt} \Big|_{t\to 0}$$

and it is exactly the same for both experiments (see Table 1). For the slope the E811 experiment used the average of the CDF and the E710 [3] measurements with the error dominated by the CDF measurement. Therefore when comparing the CDF and E811 measurements, which has been discussed also in [4], the b uncertainty should be excluded.

The inelastic cross-section is

$$\sigma_{in} = 16\pi(\hbar c)^2 \frac{b}{1+\rho^2} \frac{N_{el}N_{in}}{(N_{el}+N_{in})^2}.$$

Introducing the ratio of the inelastic and elastic rates $R=N_{in}/N_{el}$, we can re-write the inelastic cross-section as

$$\sigma_{in} = 16\pi(\hbar c)^2 \frac{b}{1+\rho^2} \frac{R}{(1+R)^2}.$$

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[1] The E710 experiment was a predecessor to E811 and also measured the $p\bar{p}$ cross-sections. The E811 measurement is more precise and in many ways supercedes the E710 result. Therefore only E811 is included in the average presented here.

3. Measured values

In both experiments the measured quantities are the number of elastic and inelastic events. Table 1 shows the slope *b*, the number of elastic and inelastic events and their ratio R.

	CDF	E811
N_{el}	78691 ± 1463	508.1K ± 3.5K
N_{in}	240982 ± 2967	1799.5K ± 57.2K
R	3.062 ± 0.068	3.542 ± 0.113
b	16.98 ± 0.25	16.98 ± 0.22

Table 1. Input numbers for the inelastic cross-section.

In comparing the two experiments we should ignore the uncertainty of the slope *b* and compare the measured values of R only. The values of R and therefore all derived cross-sections disagree at a level of 3.6 standard deviations.

To localize the source of disagreement we look at how the inelastic rates were measured. The inelastic rate was measured as a sum of the double-arm rate N_2 (coincidence of two detectors measuring inelastic rates in the p and $\bar{p}$ directions) and the single-arm rate N_1. We could normalize all rates by N_{el} :

$$x = \frac{N_2}{N_{el}}, \quad y = \frac{N_1}{N_{el}}, \quad R = x + y \, .$$

The measurement of the N_{el} and N_2 rates was similar in both experiments and the measurement of N_2 was the most straightforward one. However the measurement of the rate N_1 was quite different.

CDF estimated the rate of the single diffractive (SD) process by measuring the coincidence rate of the $\bar{p}$ elastic detector with the opposite inelastic detector (single diffractive rate). After applying the selection cuts this rate had a small background, however considerable acceptance and detection efficiency corrections were needed to obtain the single diffractive rate. To avoid double counting, the double-arm single diffractive events were subtracted from the total number of the single diffractive events, which gives the estimation of the N_1 rate (32092±1503 events). The small contribution of the single-arm events from the non-diffractive processes (0.6%) was added to the N_2 rate as a simulation-calculated correction. Therefore the corrected number of the single-arm events is 33403±1520.

The E811 experiment measured the exclusive single-arm rate using the inelastic detectors, which should be quite efficient for the single diffractive events. But the background from losses was large (~93%). To obtain the 13% error quoted on the number of single-arm inelastic events, it required the measurement of the background with uncertainty better than 1%, which is a non-trivial task. The measurement of the single-arm rate was done during a special run with missing bunches. Therefore in order to use it in the analysis, in fact, the ratio of the single-arm and double-arm rates was measured: $r = 0.3220 \pm 0.0415$. A small correction to this number due to the final acceptance is δ=0.0107±0.006 [2]. The total number of the single-arm events was estimated as $N_1 = N_2(r+\delta)$.

Table 2 shows the *x* and *y* values measured by the CDF and E811 experiments

	CDF	E811
x	2.638 ± 0.058	2.657 ± 0.023
y	0.424 ± 0.021	0.885 ± 0.115

Table 2. The *x* and *y* ratios measured by the CDF and the E811.

The *x* values are in a very good agreement, but the *y* values disagree. This may lead to the conclusion that E811 has a factor of two more single diffractive events, possibly as a result of the large background subtraction.

However, it is not valid to make direct comparison of the *x* and *y* values because they have different expectation values. The CDF and E811 inelastic detectors had very different acceptances for the two-side events: $\varepsilon_2(CDF) \approx 98.7\%$, $\varepsilon_2(E811) = 88.85 \pm 2.0\%$. The E811 single-arm rate had a lot of non-diffractive events missed by the two-side inelastic trigger and the CDF N_1 rate was due to the single diffractive process only. Therefore in order to check if the E811 inelastic rates are consistent with the CDF rates the acceptance corrections should be taken into account. For more accurate comparison, the non-diffractive and diffractive (e.g. SD) rates for both experiments should be estimated. It's straightforward for CDF. For the E811 experiment the rates are

$$N_{nd} = N_2/\varepsilon_2, \quad N_{sd} = N_2(r+\delta-\frac{1-\varepsilon_2}{\varepsilon_2}).$$

There should be a few percent correction for the N_{sd}(E811) rate to account for the double-arm SD events, which we ignore at this moment. Table 3 shows the rates and their ratios to the elastic rate.

	CDF	E811
N_{nd}	203200 ± 2558	1519.7K ± 34.9K
N_{sd}	37782 ± 1770	279.8K ± 36.3K
N_{nd}/N_{el}	2.582±0.058	2.991±0.069
N_{sd}/N_{el}	0.480±0.029	0.551±0.072
N_{sd}/N_{nd}	0.186±0.009	0.184±0.024

Table 3. The *x* and *y* ratios measured by the CDF and the E811.

At this time we have a remarkable agreement between the CDF and the E811 for the ratio of single diffractive and non-diffractive events N_{sd}/N_{nd}. So both experiments see the same fraction of the single diffractive events. At the same time there is the discrepancy of 4.4 standard deviations between the ratios of the non-diffractive inelastic and elastic events. Similarly, the ratio for the single diffractive rate is also greater for the E811, but the errors are large and the SD ratios are compatible. So it is possible that the source of the CDF/E811 discrepancy is in the measurement of the elastic rates.

Regardless on what is the source of the CDF and E811 disagreement, incorrect measurement of elastic and/or inelastic rates will lead to a discrepancy in measured values of R. Therefore to combine the CDF and E811 measurements we average the ratio of inelastic and elastic rates.

4. Averaging of the CDF and E811 measurements.

To find the mean value of the inelastic cross-section we should average the R measurements, which are not compatible. To average these non-compatible measurements we follow the PDG prescription [5]:

- Find the average of two experiments using the standard approach: $\overline{R} = 3.19$.
- Find the average error using the standard approach: $\sigma_{\overline{R}} = 0.06$
- Calculate χ^2: 13.2
- Scale the error to get $\chi^2 = 1$: $\sigma_{\overline{R}} \to 0.058\sqrt{13.2} = 0.21$.

At the first approximation, ignoring the correlation between the slope b and $\overline{R}$, the inelastic cross-section relative error is

$$\delta^2 = \frac{\sigma_b^2}{b^2} + \frac{\sigma_{\overline{R}}^2}{\overline{R}^2} \cdot \left(\frac{1-\overline{R}}{1+\overline{R}} \right)^2 = (3.8\%)^2 .$$

Finally the average inelastic cross-section (at $\sqrt{s} = 1.8 TeV$) is

$$\overline{\sigma}_{in} \cdot (1+\rho^2) = 60.4 \pm 2.3 \text{ mb},$$

which is 2.2% below the CDF measurement and 5.8% above the E811 measurement.

Now let's take into account the correlation between the slope b and the ratios R. The elastic rate is the raw n_{el} rate measured in each experiment divided by its "acceptance"

$$N_{el} = n_{el} / \left(\exp(-bt_{\min}) - \exp(-bt_{\max})\right),$$

where $(t_{\min}, t_{\max})$ is a range of t used by CDF ($0.04 < t < 0.29$) and E811 ($0.0045 < t < 0.036$). Both measurements depend on the slope b and, in fact, they are anti-correlated. Namely, if we increase b by one standard deviation (1.5%), the CDF value of R increases by ~1% and the E811 value decreases by ~1%. The covariance matrix $cov(R_i,R_j)$ is

$$C = \begin{pmatrix} \sigma_1^2 & \sigma_1\sigma_2\alpha \\ \sigma_1\sigma_2\alpha & \sigma_2^2 \end{pmatrix},$$

where $\sigma_1(\sigma_2)$ is the standard deviation of the ratio $R_1(R_2)$ for the CDF (E811) measurement and the coefficient α is estimated to be –0.09. The average value of the ratio R is

$$\overline{R} = fR_1 + (1-f)R_2,$$

where the weight f

$$f = \frac{\sigma_2^2 - \alpha\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\alpha\sigma_1\sigma_2}$$

can be found by minimization of the variance of $\overline{R}$

$$\text{var}(\overline{R}) = FCF^T, \quad F = (f, 1-f).$$

The same result can be obtained by minimization of the likelihood

$$L = -\log(p(R_1, R_2, \overline{R}, \sigma_1, \sigma_2)),$$

where p is the probability distribution function of two correlated Gaussian variables

$$p = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\alpha^2}} \cdot \exp\left\{ -\frac{1}{2(1-\alpha^2)} \left[\frac{(R_1-\overline{R})^2}{\sigma_1^2} - 2\alpha \frac{(R_1-\overline{R})}{\sigma_1}\frac{(R_2-\overline{R})}{\sigma_2} + \frac{(R_2-\overline{R})^2}{\sigma_2^2} \right] \right\}.$$

Assuming that the standard deviations σ_1, σ_2 are given by the errors listed in Table 1[2], the average value of R is

$$\overline{R} = 3.20 \pm 0.06,$$

which is very close to the number obtained above. Calculating the χ^2

$$\chi^2 = \sum_{i,j} \left(R_i - \overline{R}\right) \cdot C_{ij}^{-1} \cdot \left(R_j - \overline{R}\right) = 12.2; \quad (i, j = 1,2)$$

and applying the same procedure for the scaling of the error of $\overline{R}$, the average R value and the inelastic cross-section are

$$\overline{R} = 3.20 \pm 0.20,$$

$$\overline{\sigma}_{in} \cdot (1 + \rho^2) = 60.3 \pm 2.3 \,\text{mb}.$$

Finally, using the ρ value of 0.135 the average inelastic cross-section at $\sqrt{s} = 1.8$ TeV is

$$\overline{\sigma}_{in} = 59.3 \pm 2.3 \text{ mb}.$$

5. Inelastic cross-section at $\sqrt{s} = 1960$ GeV

In Run-II the Tevatron center of mass energy is 1.96 TeV. There is no measurement of the $p\overline{p}$ scattering cross-sections at this energy. Therefore the extrapolated value of the inelastic cross-section is used for the luminosity measurements. Theory predicts [6,7] that the inelastic cross-section increases with energy as $\ln^2 s$ and the diffraction cross-section increases as $\ln s$. The total cross-section is a mixture of the inelastic and elastic processes, where the elastic cross-section increases more rapidly with energy. The energy dependence of σ_{tot} obtained from the best fit of the experimental data (including cosmic rays data) is $\ln^{2.2} s$ [8], which is consistent with the expectations. However the E710 and E811 measurements favor the $\ln s$ dependence. Table 6 shows the value of the inelastic cross-section at 1.96 TeV and its relative variation for all three options.

	$\overline{\sigma}_{in}$ (1.96TeV)	$\delta\overline{\sigma}_{in} / \overline{\sigma}_{in}$, %
ln(s)	60.0 ± 2.3	1.1
$ln^2(s)$	60.7 ± 2.3	2.3
$ln^{2.2}(s)$	60.8 ± 2.3	2.6

Table 6. Extrapolated inelastic cross-section at 1.96 GeV. The last column is the fractional change relative to the cross section at 1.8 TeV.

[2] Actually the E811 error is underestimated, because the error of the slope *b* is ignored in the *R* ratio.

Assuming the $\ln^2 s$ energy dependence of the inelastic cross-section and assigning an additional 1% systematic error due to uncertainty in the σ_{in} energy dependence the inelastic cross-section at 1.96 TeV is

$$\bar{\sigma}_{in}(1.96TeV) = 60.7 \pm 2.4 \, .$$

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7. References

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