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PASSIVE LANDAU CAVITY FOR THE LNLS LIGHT SOURCE ELECTRON RING

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Abstract

The electron storage ring at the LNLS light source at Brazil proposes the installation of a passive Landau cavity to alleviate its longitudinal coupled-bunch instabilities. Here, we compute the required shunt impedance of the higher-harmonic cavity, discuss the possible Robinson instability for the two-rf system, and estimate the fastest collective instabilities that can be damped. Finally we investigate the adjustment of rf voltage, synchronous phase, rf detuning, and other consequences in the event that the bunch intensity changes.

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1 INTRODUCTION

The synchrotron light source electron ring at LNLS, Brazil suffers from longitudinal coupled-bunch instabilities. A brief list of information of the ring is tabulated in Table I. Landau damping of the instability can come from the spread in the synchrotron frequency. When the synchronous angle $\phi_s \neq 0$, the computation of synchrotron frequency spread is tedious. A numerical computation is shown in Fig. 1 for various $\Gamma = \sin\phi_s$. These curves can be represented approximately by the following expression:

$$\frac{\Delta\omega_s}{\omega_s} = \left(\frac{\pi^2}{16}\right)\left(\frac{1+\sin^2\phi_s}{1-\sin^2\phi_s}\right)(h\tau_{_L} f_0)^2 \ , \tag{1.1}$$

where $\tau_{_L}$ is the total length of the bunch and ϕ_s is the synchronous angle. The expression is valid for small-amplitude oscillation. The mth azimuthal mode will be stable if [1]

$$\frac{1}{\tau} \lesssim \frac{\sqrt{m}}{4}\Delta\omega_s \ , \tag{1.2}$$

where $1/\tau$ is the growth rate ignoring Landau damping. Notice that a large synchronous angle enhances the synchrotron frequency spread by the factor

$$F = \frac{1+\sin^2\phi_s}{1-\sin^2\phi_s} \ . \tag{1.3}$$

Table I: Some information of the synchrotron light source electron ring at LNLS, Brazil, with a single rf system.

Circumference	93.21204	m
Total energy E_0	1.37	GeV
Revolution frequency $f_0 = \omega_0/(2\pi)$	3216	kHz
Rf harmonic h	148	
Rf voltage $V_{\rm rf}$	350	kV
Momentum compaction α	0.0083	
Synchrotron frequency f_s	22.10	kHz
Natural energy spread $\Delta E/E_0$	5.89×10^{-4}	
Natural bunch length σ_τ	30.0	ps
Damping times: τ_x	13.2	ms
τ_y	12.6	ms
τ_s	6.2	ms
Synchronous phase angle $\pi-\phi_s$	19.0	degrees

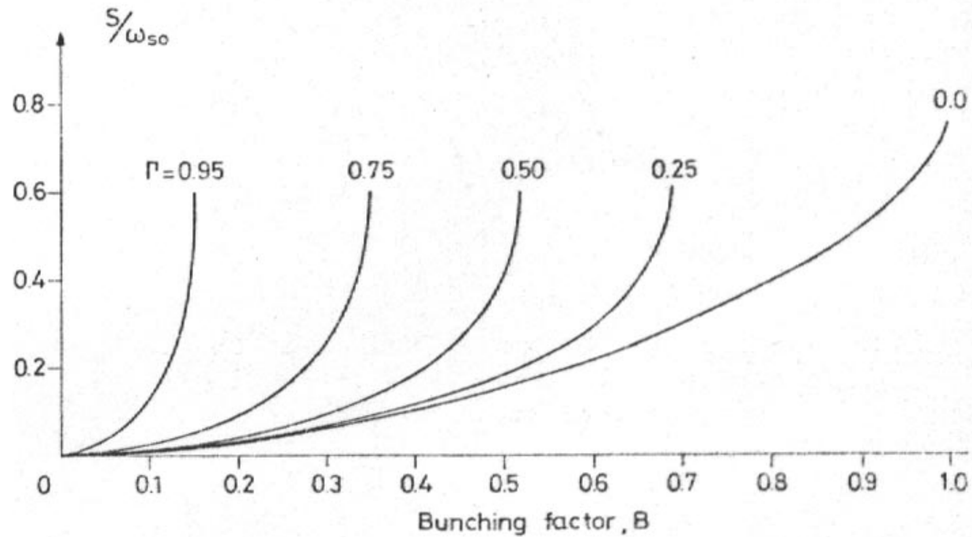

Figure 1: Synchrotron frequency spread S as a function of single-bucket bunching factor $B \approx \tau_L f_0$ for various values of $\Gamma = \sin\phi_s$. τ_L is full bunch length, f_0 is revolution frequency, ϕ_s is synchronous angle, and ω_{s0} is unperturbed angular synchrotron frequency.

The synchronous phase angle of $\phi_s = \pi - 19.0°$ at the LNLS ring contributes an enhancement factor of 1.24. Using the natural full bunch length of $\tau_L = 2\sqrt{6}\sigma_\tau$ where $\sigma_\tau = 30.0$ ps at $V_{\rm rf} = 350$ kV, the angular synchrotron frequency spread is $\Delta\omega_s = 694$ Hz. This spread is able to damp an instability in the dipole azimuthal mode that has a growth rate of $\tau^{-1} < 173\ {\rm s}^{-1}$ or a growth time $\tau > 5.77$ ms, which is roughly the same as the longitudinal damping time. In other words, the damping due to the spread of the synchrotron frequency is no better than the damping from the synchrotron radiation. Presumably, the longitudinal coupled-bunch instability at LNLS is much faster.

2 HIGHER-HARMONIC CAVITY

In order to Landau damp longitudinal coupled-bunch instability, a large spread in synchrotron frequency inside the bunch is required. One way to do this is to install a higher-harmonic cavity, sometime known as *Landau cavity* [2]. For example, the higher-harmonic cavity has resonant frequency $m\omega_{\rm rf}$, where $\omega_{\rm rf}$ is the resonant angular frequency of the fundamental rf cavity. The total rf voltage seen by the beam particles becomes

$$V(\tau) = V_{\rm rf}[\sin(\phi_s - \omega_{\rm rf}\tau) - r\sin(\phi_m - m\omega_{\rm rf}\tau)] - \frac{U_s}{e} \ , \tag{2.4}$$

where the modified synchronous phase angle ϕ_s is to compensate for the U_s, the radiation energy loss or any required acceleration. We would like the bottom of the potential well, which is the integral of $V(\tau)$, to be as flat as possible. The rf voltage seen by the synchronous particle is compensated to zero by the energy lost to synchrotron radiation. In addition, we further require

$$\left.\frac{\partial V}{\partial \tau}\right|_{\tau=0} = 0 \ , \qquad \text{and} \qquad \left.\frac{\partial^2 V}{\partial \tau^2}\right|_{\tau=0} = 0 \ , \tag{2.5}$$

so that the potential will become quartic instead. We therefore have 3 equations in 3 unknowns:

$$\sin\phi_s = r\sin\phi_m + \frac{V_s}{V_{\rm rf}} \ , \tag{2.6}$$

$$\cos\phi_s = rm\cos\phi_m \ , \tag{2.7}$$

$$\sin\phi_s = rm^2\sin\phi_m \ , \tag{2.8}$$

where we have introduced, for convenience, the radiation voltage drop $V_s = U_s/e$. Eliminating ϕ_m from Eqs. (2.6) and (2.8), ϕ_s is obtained:

$$\sin\phi_s = \frac{m^2}{m^2-1}\frac{V_s}{V_{\rm rf}} \ . \tag{2.9}$$

From Eqs. (2.7) and (2.8), ϕ_m is solved:

$$\tan\phi_m = \frac{\dfrac{m}{m^2-1}\dfrac{V_s}{V_{\rm rf}}}{\sqrt{1-\left(\dfrac{m^2}{m^2-1}\dfrac{V_s}{V_{\rm rf}}\right)^2}} \ . \tag{2.10}$$

Finally from Eq. (2.7), we obtain the voltage ratio

$$r = \sqrt{\frac{1}{m^2} - \frac{1}{m^2-1}\frac{V_s^2}{V_{\rm rf}^2}} \ . \tag{2.11}$$

For small amplitude oscillation, the potential becomes

$$-\int V(\tau)d(\omega_{\rm rf}\tau) \longrightarrow \frac{m^2-1}{24}(\omega_{\rm rf}\tau)^4 V_{\rm rf}\cos\phi_s \ , \tag{2.12}$$

which is quartic and the small-amplitude synchrotron frequency is (see Appendix)

$$\frac{\omega_s(\tau)}{\omega_{s0}}=\frac{\pi}{2}\left(\frac{m^2-1}{6}\right)^{1/2}\frac{\omega_{\rm rf}\tau}{K(1/\sqrt{2})}\left[\frac{1-\left(\dfrac{m^2}{m^2-1}\dfrac{V_s}{V_{\rm rf}}\right)^2}{1-\left(\dfrac{V_s}{V_{\rm rf}}\right)^2}\right]^{1/4}\;, \tag{2.13}$$

where the last factor equals $\sqrt{\cos\phi_s/\cos\phi_{s0}}$ and can usually be neglected; its deviates from unity by only $\sim\frac{1}{2}[V_s/(mV_{\rm rf})]^2$ if the synchronous angle is small. In above, ω_{s0} is the synchrotron frequency at zero amplitude when the higher-harmonic cavity voltage is turned off, and $K(1/\sqrt{2})=1.854$ is the complete elliptic integral of the first kind which is defined as

$$K(t)=\int_0^{\pi/2}\frac{d\theta}{\sqrt{1-t^2\sin^2\theta}}\;. \tag{2.14}$$

We see that the synchrotron frequency is zero at zero amplitude and increases linearly with amplitude. This large spread in synchrotron frequency may be able to supply ample Landau damping to cure the longitudinal coupled-bunch instability.

In the situation where there is no radiation loss and no acceleration, $U_s=0$, the solution of Eqs. (2.9) to (2.11) simplifies, giving $\phi_s=\phi_m=0$ and the ratio of the voltages of higher-harmonic cavity to the fundamental $r=1/m$. Of course, it is also possible to have $r\neq 1/m$. Then the synchrotron frequency at the zero amplitude will not be zero, but the spread in synchrotron frequency can still be appreciable. When $m=2$, i.e., having a second harmonic cavity, the mathematics simplifies. The synchrotron frequencies for various values of r are plotted in Fig. 2. Here, $r=0$ implies having only the fundamental rf while $r=\frac{1}{2}$ the situation of having the synchrotron frequency linear in amplitude for small amplitudes. In between, the synchrotron frequency spread decreases as r decreases. Notice that for $0.3\lesssim r<0.5$, the synchrotron frequency has a maximum near the rf phase of $\sim 110^\circ$. Particles near there will have no Landau damping at all and experience instability. Thus the size of the bunch is limited when a double cavity is used. Also the size of the bunch cannot be too small because of two reasons. First, the average synchrotron frequency may have been too low. Second, the central region of the phase space is a sea of chaos, implying that a bunch of small longitudinal area will be blown up to the first well-behaved torus. [3].

A Landau cavity increases the spread in synchrotron frequency, therefore it is ideal in damping mode-mixing instability and coupled-bunch instability. However, it may be not helpful for the Keil-Schnell type longitudinal microwave instability [4]. This method was

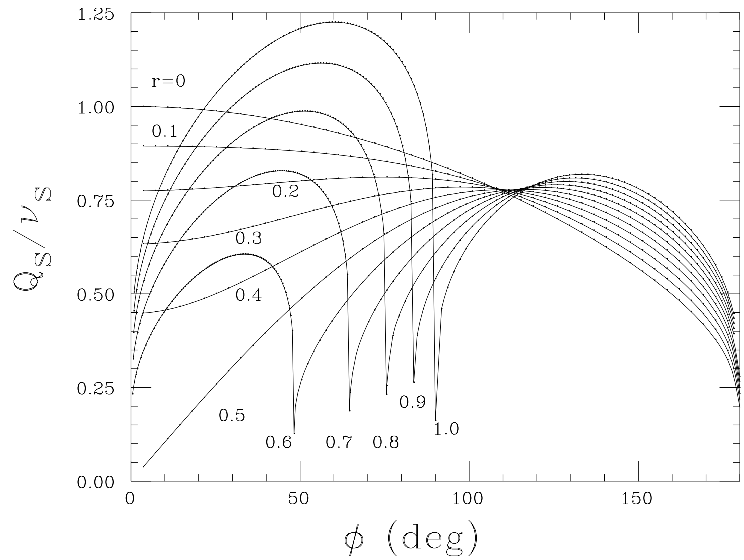

Figure 2: The normalized synchrotron tune of a double rf system as a function of the peak rf phase ϕ for various voltage ratio r. Here, the higher-harmonic cavity has frequency twice that of the fundamental. When $r > \frac{1}{2}$, the center of the bucket becomes an unstable fixed point and two stable fixed points emerge [3].

first applied successfully with a third harmonic cavity to increase Landau damping at the Cambridge Electron Accelerator (CEA) [5]. It was later applied at the ISR with a 6th harmonic cavity to cure mode-mixing instability [6]. Recently, a third-harmonic cavity has been reported in the SOLEIL ring in France to give a relative frequency spread of about 200%. However, since the center frequency has been dramatically decreased (not exactly to zero), the net result is a poor improvement in the stabilization. The gain in the stability threshold has been only 30% [7].

Actually, with a higher-harmonic cavity, the bunch becomes more rectangular-like in the longitudinal phase space, or particles are not so concentrated at the center of the bunch. Assuming the bunch area to be the same, the Boussard-modified Keil-Schnell threshold is proportional to the energy spread [4, 10]. Since the bunch becomes more flattened, the maximum energy spread which is at the center of the bunch is actually reduced, and so will be the instability threshold. However, spreading out the particles longitudinally does help to increase the bunching factor and decrease the incoherent self-field or space-charge tune shift. At the Proton Synchrotron Booster at CERN, a rf system with higher harmonics 5 to 10 has raised the beam intensity by about 25 to 30% [8]. For the Cooler Ring at the Indiana University Cyclotron Facility, a double cavity has been able to quadruple the beam intensity [3].

3 PASSIVE LANDAU CAVITY

Higher-harmonic cavities are useful in producing a large spread in synchrotron frequency so that single-bunch mode-mixing instability and coupled-bunch instability can be damped. However, the power source to drive this higher-harmonic rf system can be rather costly. One way to overcome this is to do away with the power source and let the higher-harmonic cavity be driven by the beam-loading voltage of the circulating beam.

For a cavity with a high quality factor, the beam loading voltage is just the i_b, the current component of a bunch *per bunch separation* at the cavity resonant frequency, multiplied by the impedance of the cavity. Thus, for a Gaussian bunch,

$$i_b = 2I_0 e^{-\frac{1}{2}(mh\omega_0\sigma_\tau)^2} \ , \tag{3.1}$$

where σ_τ is the rms bunch length and $\omega_0/(2\pi)$ is the revolution harmonic. Here, m is the ratio of the resonant frequencies of the higher-harmonic cavity to the fundamental rf cavity and h is the fundamental rf harmonic. For a short bunch, $i_b \approx 2I_0$ with I_0 being the average current of the bunch per bunch separation or the total average current of all the bunches in the ring. Applying to the proposed LNLS higher-harmonic cavity with $m = 3$, this is true for a bunch of $\sigma_\tau = 30$ ps to within 3.5%.

The higher-harmonic cavity must have suitable shunt impedance R_s and quality factor Q, and this can be accomplished by installing necessary resistor across the cavity gap. Thus, R_s and Q can be referred to as the loaded quantities of the cavity. For a particle arriving at time τ ahead of the synchronous particle, it sees the total voltage

$$V(\tau) = V_{\rm rf}\sin(\phi_s - \omega_{\rm rf}\tau) - i_b R_s\, \mathcal{R}e\left[\frac{1}{1+i2Q\delta}e^{im\omega_{\rm rf}\tau}\right] - \frac{U_s}{e} \ , \tag{3.2}$$

where $\omega_{\rm rf} = h\omega_0$ is the angular rf frequency determined by the resonator in the rf klystron that drives the fundamental rf cavity and the negative sign in front of i_b indicates that this beam loading voltage is induced by the image current and opposes the beam current. In above,

$$\delta = \frac{1}{2}\left(\frac{\omega_r}{m\omega_{\rm rf}} - \frac{m\omega_{\rm rf}}{\omega_r}\right) \approx \frac{\omega_r - m\omega_{\rm rf}}{\omega_r} \tag{3.3}$$

represents the deviation of the resonant angular frequency ω_r of the higher-harmonic cavity from the mth multiple of the rf angular frequency. Of course, this is related to the detuning angle ψ of the higher-harmonic cavity, which we introduce in the usual way as

$$\tan\psi = 2Q\delta \ . \tag{3.4}$$

Now, Eq. (3.2) can be rewritten as

$$V(\tau)=V_{\rm rf}\sin(\phi_s-\omega_{\rm rf}\tau)-i_bR_s\cos\psi\cos(\psi-m\omega_{\rm rf}\tau)-V_s\ . \tag{3.5}$$

Again to acquire the largest spread in synchrotron frequency, we require

$$V(0)=0\,,\quad V'(0)=0\,,\quad V''(0)=0\,, \tag{3.6}$$

so that the potential for small amplitudes becomes quartic,

$$U(\tau)=-\int V(\tau)d\tau=-\frac{\tau^4}{4!}V'''(0)\ . \tag{3.7}$$

Since we are having exactly the same quartic potential as in an rf system with an active Landau cavity, we expect the synchrotron frequency to be exactly the same as the expression given by Eq. (2.13) when the oscillation amplitude is small.

The set of requirements, however, are different from that of the active Landau cavity system. Here, the requirements are

$$\begin{aligned}
V_{\rm rf}\sin\phi_s &= i_bR_s\cos^2\psi+V_s\ , && (3.8)\\
V_{\rm rf}\cos\phi_s &= -m\,i_bR_s\cos\psi\sin\psi\ , && (3.9)\\
V_{\rm rf}\sin\phi_s &= m^2i_bR_s\cos^2\psi\ . && (3.10)
\end{aligned}$$

For an electron machine which is mostly above transition, the synchronous angle ϕ_s is between $\frac{1}{2}\pi$ and π. Thus, from Eq. (3.9), we immediately obtain

$$\sin 2\psi>0\Longrightarrow 0<\psi<\frac{\pi}{2}\ , \tag{3.11}$$

and from Eqs. (3.3) and (3.4), $\omega_r>m\omega_{\rm rf}$. This means that the beam in the higher-harmonic cavity is Robinson unstable [9], as is illustrated in Fig. 3. Of course, the fundamental rf cavity should be Robinson stable, and it will be nice if the detuning is so chosen that the beam remains stable after traversing both cavities.

4 APPLICATION TO LNLS

The synchrotron light source electron ring at LNLS, Brazil would like to install a passive Landau cavity with $m=3$ in order to alleviate the longitudinal coupled-bunch instabilities.

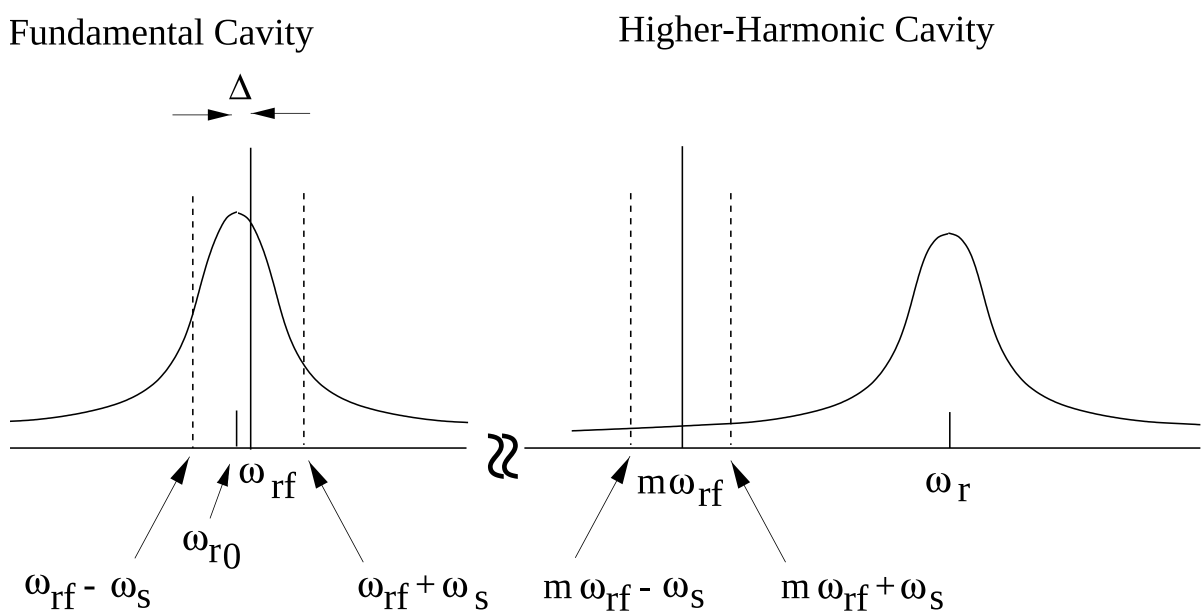

Figure 3: For the higher-harmonic cavity, the resonant frequency ω_r is above the mth multiple of the rf frequency. The beam will be Robinson unstable above transition. For the fundamental cavity, the angular resonant frequency ω_{r0} is below the angular rf frequency $\omega_{\rm rf} = h\omega_0$, and the beam will be Robinson stable. The detuning of the fundamental rf should be so chosen that the beam will be stable after traversing both cavities.

The fundamental rf system has harmonic $h = 148$ or rf frequency $f_{\rm rf} = \omega_{\rm rf}/(2\pi) = 476.0$ MHz with a tuning range of ± 10 kHz, and rf voltage $V_{\rm rf} = 350$ kV. To overcome the radiation loss, the synchronous phase is set at $\phi_{s0} = 180^\circ - 19.0^\circ$. This gives a synchrotron tune at small amplitudes $\nu_s = 6.87 \times 10^{-3}$ or a synchrotron frequency $f_s = 22.1$ kHz.

With the installation of the passive Landau cavity, the synchronous phase must be modified to a new ϕ_s, which is obtained by solving Eqs. (3.8) and (3.10):

$$\sin\phi_s = \left(\frac{m^2}{m^2-1}\right)\left(\frac{V_s}{V_{\rm rf}}\right) = \frac{m^2}{m^2-1}\sin\phi_{s0} \ . \tag{4.1}$$

Thus,

$$\phi_{s0} = 180^\circ - 19.0^\circ \quad \Longrightarrow \quad \phi_s = 180^\circ - 21.49^\circ \ , \tag{4.2}$$

where $m = 3$ has been used. The detuning ψ of the higher-harmonic cavity can be obtained from Eqs. (3.9) and (3.10), or

$$\tan\psi = -m\cot\phi_s \qquad \Longrightarrow \qquad \psi = 82.53^\circ \ . \tag{4.3}$$

Finally from Eq. (3.10),

$$i_b R_s = \frac{V_{\rm rf}\sin\phi_s}{m^2\cos^2\psi} \ . \tag{4.4}$$

With $i_b = 2I_0 = 0.300$ A and $V_{\rm rf} = 350$ kV, we obtain the shunt impedance of the higher-harmonic cavity to be $R_s = 2.81$ MΩ. The power taken out from the beam is

$$P = \frac{1}{2}\frac{i_b^2 R_s}{1+\tan^2\psi} = 2.14 \text{ kW} \ , \tag{4.5}$$

which is not large when compared with the power loss due to radiation

$$P_{\rm rad} = NU_s f_0 = I_0 V_{\rm rf}\sin\phi_{s0} = 17.09 \text{ kW} \ , \tag{4.6}$$

where N is the total number of electrons in the ring. The higher-harmonic cavity has a quality factor of $Q = 45000$ and a resonant frequency $f_r \sim 3f_{r0} = 1428$ MHz. From the detuning, it can easily found that the frequency offset is $f_r - 3f_{\rm rf} = 121$ kHz.

Now let us compute the growth rate for one bunch at the coherent frequency Ω. For one particle of time advance τ, we have from Sacherer integral equation for a short bunch [1, 11],

$$\Omega^2 - \omega_s(\tau)^2 = \frac{i\eta e I_0}{E_0 T_0}\sum_q (q\omega_0+\Omega) Z_0^\parallel(q\omega_0+\Omega) \ , \tag{4.7}$$

where η is the slip factor and we have retained the dependency of the synchrotron frequency ω_s on τ because of its large spread in the presence of the higher-harmonic cavity. From Eq. (2.13), this dependency is

$$\frac{\omega_s(\tau)}{\omega_{s0}} = \frac{\pi}{2}\left(\frac{m^2-1}{6}\right)^{1/2}\frac{\omega_{\rm rf}\tau}{K(1/\sqrt{2})}\sqrt{\frac{\cos\phi_s}{\cos\phi_{s0}}} \, , \tag{4.8}$$

where the last factor amounts to 0.9920 and can therefore be safely abandoned. Thus, the average ω_s^2 over the whole bunch just gives the square of the rms frequency spread,

$$\langle\omega_s^2\rangle = \sigma_{\omega_s}^2 = \left[\frac{\pi\omega_{s0}}{2}\sqrt{\frac{m^2-1}{6}}\frac{\omega_{\rm rf}\sigma_\tau}{K(1/\sqrt{2})}\right]^2 \ . \tag{4.9}$$

The FWHM natural bunch length at $V_{\rm rf} = 350$ kV is 70.6 ps; thus $\sigma_\tau = 30.0$ ps. This gives $\sigma_{\omega_s} = 12.2$ kHz.

Since the synchrotron frequency is now a function of the offset from the stable fixed point of the rf bucket, a dispersion relation can be obtained from Eq. (4.7) by integrating over the synchrotron frequency distribution of the bunch. Here, we are interested in the growth rate without damping, which is given approximately by

$$\frac{1}{\tau} = \mathcal{I}m\,\Omega \approx \frac{\eta e I_0 \omega_{\rm rf}}{2E_0 T_0 \bar{\omega}_s} \Big\{ \Big[\mathcal{R}e\, Z_0^{\parallel}(\omega_{\rm rf} + \bar{\omega}_s) - \mathcal{R}e\, Z_0^{\parallel}(\omega_{\rm rf} - \bar{\omega}_s) \Big] + m \Big[\mathcal{R}e\, Z_0^{\parallel}(m\omega_{\rm rf} + \bar{\omega}_s) - \mathcal{R}e\, Z_0^{\parallel}(m\omega_{\rm rf} - \bar{\omega}_s) \Big] \Big\} \ , \tag{4.10}$$

where the mean angular synchrotron frequency can be computed from Eq. (4.8) to be

$$\bar{\omega}_s = \sqrt{\frac{2}{\pi}} \sigma_{\omega_s} \ . \tag{4.11}$$

This can be computed easily by substituting into the expression for $\mathcal{R}e\, Z$. However, the differences in Eq. (4.10) can also be approximated by derivatives. For the higher-harmonic cavity, both the upper and lower synchrotron side-bands lie on the same side of the higher-harmonic resonance as indicated in Fig. 3. Also their difference, $4\sigma_\omega/(2\pi) = 7.76$ kHz is very much less than the cavity detuning $(\omega_r - m\omega_{\rm rf})/(2\pi) = 121$ kHz. Recalling that

$$\mathcal{R}e\, Z(\omega) = R_s \cos^2 \psi \ , \tag{4.12}$$

where the detuning ψ is given by Eq. (3.4), the second term can be written as a differential,

$$\mathcal{R}e\, Z_0^{\parallel}(m\omega_{\rm rf} + \bar{\omega}_s) - \mathcal{R}e\, Z_0^{\parallel}(m\omega_{\rm rf} - \bar{\omega}_s) \approx \left[R_s \cos^2 \psi \sin 2\psi \, \frac{2Q}{\omega_r} \right] 2\bar{\omega}_s \ . \tag{4.13}$$

For the fundamental cavity, the detuning is usually $\Delta = -10$ kHz at injection and is reduced to $\Delta = -2$ kHz in storage mode when the highest electron energy is reached. Thus, the upper and lower synchrotron side-bands lie on either side of the resonance peak as illustrated in Fig. 3. Since $|\Delta| \ll \sigma_{\omega_s}$ and the resonant peak is symmetric about ω_{r0}, we can also write the first term of Eq. (4.10) as a differential about $\bar{\omega}_s$. Thus,

$$\begin{aligned} &\mathcal{R}e\, Z_0^{\parallel}(\omega_{\rm rf} + \bar{\omega}_s) - \mathcal{R}e\, Z_0^{\parallel}(\omega_{\rm rf} - \bar{\omega}_s) \\ &\quad = \mathcal{R}e\, Z_0^{\parallel}(\omega_{r0} + \Delta + \bar{\omega}_s) - \mathcal{R}e\, Z_0^{\parallel}(\omega_{r0} - \Delta + \bar{\omega}_s) \approx \left[R_s \cos^2 \psi_{\omega_s} \sin 2\psi_{\omega_s} \, \frac{2Q}{\omega_{r0}} \right] 2\Delta \ , \end{aligned} \tag{4.14}$$

where $\omega_{r0}/(2\pi) = 476.00$ MHz is the resonant frequency of the fundamental cavity and ψ_{ω_s}, which is similar to a detuning angle, is defined as

$$\tan \psi_{\omega_s} = 2Q \, \frac{\bar{\omega}_s}{\omega_{r0}} \ . \tag{4.15}$$

We arrive at

$$\frac{1}{\tau} = \frac{2\eta e I_0 Q}{E_0 T_0} \left[\frac{2\Delta}{\bar{\omega}_s} R_s \cos^2 \psi_{\omega_s} \sin 2\psi_{\omega_s} \Big|_{\text{fundamental}} + R_s \cos^2 \psi \sin 2\psi \Big|_{\text{higher harmonic}} \right] . \quad (4.16)$$

The square bracketed factor in Eq. (4.16) becomes

$$\left[\frac{2\Delta}{\bar{\omega}_s} R_s \cos^2 \psi_{\omega_s} \sin 2\psi_{\omega_s} \Big|_{\text{fundamental}} + R_s \cos^2 \psi \sin 2\psi \Big|_{\text{higher harmonic}} \right] = (-0.1513 + 0.0122)\ \mathrm{M}\Omega\,, \quad (4.17)$$

where we have used for the fundamental cavity, the shunt impedance $R_s = 3.84$ MΩ, and quality factor $Q = 45000$. The damping rate is 36600 s^{-1} or a damping time of 0.022 ms, and the two-rf system turns out to be Robinson stable. However, it is important to point out that the growth rate formula given by Eq. (4.10) is valid only if the shift and spread of the synchrotron frequency are much less than some unperturbed synchrotron frequency. Here, the synchrotron frequency is linear with the offset from the stable fixed point of the longitudinal phase space and the spread is therefore very large. Thus, Eq. (4.10) can only be viewed as an estimate. The employment of a mean synchrotron angular frequency $\bar{\omega}_s$ can also be questionable. Note that the assumption of the mean synchrotron angular frequency in Eq. (4.11) is not sensitive to the higher-harmonic-cavity term in Eq. (4.10) but is rather sensitive to the fundamental-cavity term. For example, if we use $\bar{\omega}_s = 1.5\sigma_{\omega_s}$ instead, the damping time increases to 0.751 ms, while $\bar{\omega}_s = 2.0\sigma_{\omega_s}$ makes the system Robinson unstable with a growth time of 0.596 ms. With this uncertainty, it may be better to increase the detuning Δ of the fundamental to at least $\Delta \sim -4$ KHz so that it becomes more certain that the two-rf system will be Robinson stable. Otherwise, the purpose of the higher-harmonic cavity can be defeated, because some or most of the spread of the synchrotron frequency obtained will be used to fight the Robinson instability created instead of other longitudinal collective instabilities of concern.

Now let us estimate how large a Landau damping we obtain from the passive Landau cavity coming from the spread of the synchrotron frequency. Following Eq. (1.2), the stability criterion is roughly

$$\frac{1}{\tau} \lesssim \frac{\omega_s(\sqrt{6}\sigma_\tau)}{4}\ , \quad (4.18)$$

where the synchrotron angular frequency spread is given by Eq. (2.13). The spread in synchrotron angular frequency has been found to be $\omega_s(\sqrt{6}\sigma_\tau) = 39.6$ kHz. In other words, the higher-harmonic cavity is able to damp an instability that has a growth time longer than 0.101 ms, an improvement of 57 fold better than when the higher-harmonic cavity is absent. Thus, theoretically, this Landau damping is large enough to alleviate the Robinson antidamping of higher-harmonic cavity.

5 THE SHUNT IMPEDANCE

We notice that the required shunt impedance of the passive Landau cavity $R_s = 2.81$ MΩ is large, although it is still smaller than the shunt impedance of 3.84 MΩ of the fundamental cavity. It is easy to understand why such large impedance is required. The synchronous angle for a storage ring without the Landau cavity is usually just not too much from 180°, here $\phi_{s0} = 180^\circ - 19.0^\circ$, because of the compensation of a small amount of radiation loss. The rf gap voltage phasor is therefore almost perpendicular to the beam current phasor. In order that the beam-loading voltage contributes significantly to the rf voltage, the detuning angle of the passive higher-harmonic cavity must therefore be large also, here $\psi = 82.53^\circ$. In fact, without radiation loss to compensate, the beam-loading voltage phasor would have been exactly perpendicular to the beam current phasor. Since $\cos\psi = 0.130$ is small, the shunt impedance of the higher-harmonic cavity must therefore be large. In some sense, the employment of the higher-harmonic cavity is not efficient at all, because we are using only the tail of a large resonance impedance, as is depicted in Fig. 3. This is not a waste at all, however, because we can do away with the generating source for this cavity. Also, the large detuning angle implies not much power will be taken out from the beam as it loads the cavity, only 2.14 kW here. On the other hand, the detuning of the fundamental cavity need not be too large. This is because the rf gap voltage is supplied mostly by the generator voltage and only partially by the beam loading in the cavity.

The most important question here is how do we generate a large shunt impedance for the higher-harmonic cavity. Usually it is easy to lower the shunt impedance by adding a resistor across the cavity gap. Some other means will be required to raise the shunt impedance, in case it is not large enough. One way is to coat the interior of the higher-harmonic cavity with a layer of medium that has a higher conductivity. However, it is hard to think of any medium that has a conductivity very much higher than the copper surface of the cavity. For example, the conductivity of silver is only slightly higher. Another way to increase the conductivity significantly is the reduction of temperature to the cryogenic region. Notice that R_s/Q is a geometric property of the cavity. Raising R_s will raise Q also. However, a higher quality factor is of no concern here, because the requirements in Eqs. (3.8), (3.9), and (3.10) depend on the detuning ψ only and are independent of Q. With the same detuning ψ, a higher Q just implies a smaller frequency detuning, or a smaller frequency offset between the resonant angular frequency ω_r of the higher-harmonic cavity and the mth multiple of the rf angular frequency.

Another way to achieve a lower shunt impedance requirement is to reduce the rf voltage. We can rewrite Eq. (4.4) as

$$\frac{i_b R_s}{V_s} = \left(\frac{m^2-1}{m^2}\right)\left(\frac{V_{\rm rf}}{V_s}\right)^2 - 1\ , \tag{5.1}$$

after eliminating ϕ_s and ψ with the aid of Eqs. (4.1) and (4.3). Thus, for a given beam current, lowering $V_{\rm rf}$ will result in a smaller shunt impedance. Notice that the right side is quadratic in $V_{\rm rf}$, a higher $V_{\rm rf}$ will increase the required shunt impedance by very much. For example, with the same radiation loss, increasing $V_{\rm rf}$ from 350 kV to 500 kV will increase the required shunt impedance of the higher-order cavity from 2.81 to 6.12 MΩ. However, lowering $V_{\rm rf}$ by too much is usually not favored because the electron bunches will become too long.

In order to maximize Landau damping, criteria must be met so that the rf potential becomes quartic. As is shown in Fig. 2 for a $m=2$ double rf system, when the rf voltage ratio deviates from $r=1/m=0.5$ by 20% to 0.4, the spread in synchrotron frequency for a small bunch decreases tremendously to almost the same tiny value as in the single rf system. There is a big difference between an active Landau cavity and a passive Landau cavity. In an active Landau cavity, the criteria of Eqs. (2.6), (2.7), and (2.8) are independent of the beam intensity. On the other hand, the criteria for the operation of a passive cavity, Eqs. (3.8), (3.9), and (3.10), depend on the bunch intensity. What will happen when the bunch intensity changes significantly? Let us recall how we arrive at the solution of the 3 equations of the passive two-rf system. The new synchronous phase ϕ_s, as given by Eq. (4.1), is determined solely by the ratio of the radiation loss U_s to the rf voltage $V_{\rm rf}$. while the detuning ψ is just given by $-m\cot\phi_s$. The only parameter that depends on the beam current is the shunt impedance R_s. Thus, the easiest solution is to install a variable resistor across the the gap of the higher-harmonic cavity and adjust the proper shunt impedance by monitoring the intensity of the electron bunches.

In the event that the shunt impedance is not adjustable, one can adjust instead the rf voltage so that Eq. (5.1) remains satisfied with the new current but with the preset R_s. With the new rf voltage, the synchronous phase ϕ_s has to be adjusted so that Eq. (4.1) remains satisfied. This will alter the detuning ψ according to Eq. (4.3). The only way to achieve the new detuning is to vary the rf frequency. This will push the beam radially inward or outward. As the beam current changes by $\Delta I_0/I_0$, to maintain the criteria of the quartic rf potential, the required changes in rf voltage, synchronous angle, and detuning of the higher-harmonic cavity are, respectively,

$$\frac{\Delta V_{\rm rf}}{V_s} = \frac{1}{2}\left[\frac{m^2}{m^2-1}\frac{V_s}{V_{\rm rf}}\right]\left[\frac{m^2-1}{m^2}\frac{V_{\rm rf}^2}{V_s^2}-1\right]\frac{\Delta I_0}{I_0}\ , \tag{5.2}$$

$$\Delta(\pi-\phi_s) = -\left[\left(\frac{m^2-1}{m^2}\frac{V_{\rm rf}}{V_s}\right)^2-1\right]^{-1/2}\frac{\Delta V_{\rm rf}}{V_s}\ , \tag{5.3}$$

$$\Delta\psi = \frac{1}{2m}\left[\left(\frac{m^2-1}{m^2}\frac{V_{\rm rf}}{V_s}\right)^2-1\right]^{-1/2}\frac{\Delta I_0}{I_0}\ , \tag{5.4}$$

where $U_s = eV_s$ is the energy loss per turn due to synchrotron radiation. The change of the detuning angle ψ leads to a fractional change in the rf frequency and therefore a fractional change in orbit radius

$$\frac{\Delta R}{R} = -\frac{m^2-1}{4mQ}\left[\frac{m^2-1}{m^2}\frac{V_{\rm rf}^2}{V_s^2}-1\right]\left[\left(\frac{m^2-1}{m^2}\frac{V_{\rm rf}}{V_s}\right)^2-1\right]^{-1/2}\frac{\Delta I_0}{I_0}\ , \tag{5.5}$$

where R is the radius of the storage ring. These changes are plotted in Fig. 4 for the LNLS double rf system when he beam current varies by $\pm 20\%$. Because of the high quality factors Q of the cavities, the radial offset of the beam turns out to be very small, less than ± 0.14 mm for a $\pm 20\%$ variation of beam current.

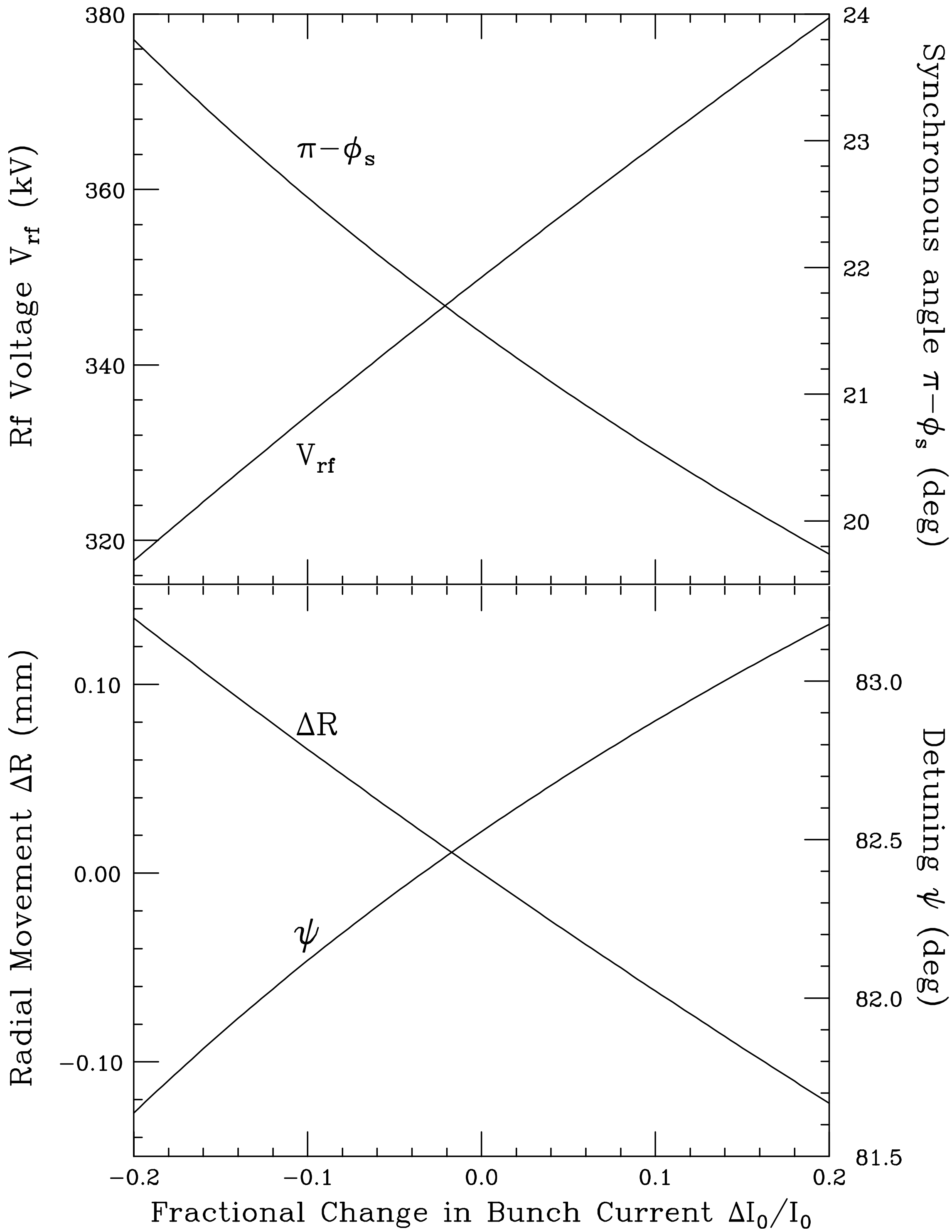

Figure 4: Plots showing the required variations of rf voltage $V_{\rm rf}$, synchronous angle ϕ_s, higher-harmonic-cavity detuning ψ, and beam radial offset Δr to maintain the criteria of the quartic rf potential, when the beam current varies by $\pm 20\%$.

APPENDIX

The easiest way to derive the synchrotron frequency is to use action-angle variables. With the action defined as

$$J = \frac{1}{2\pi} \oint \Delta E \, d\tau \ , \tag{A.1}$$

the synchrotron angular frequency is given by

$$\omega_s = \frac{\partial H}{\partial J} \ , \tag{A.2}$$

where H is the Hamiltonian of the system. Here, we have used as canonical variables τ, the arrival time ahead of the synchronous particle, and ΔE, the energy offset. The use of a Hamiltonian is justified for a time interval much less than the radiation damping time.

However, it is also easy to arrive at the same result without resorting to action-angle variables. The phase equation of a beam particle in the longitudinal phase space is

$$\frac{d\tau}{dt} = -\frac{\eta}{\beta^2 E_0} \Delta E \ , \tag{A.3}$$

where β is the velocity of the particle with respect to the velocity of light. The negative sign on the right side comes about because a particle with $\Delta E > 0$ will arrive late after one turn when it is above transition ($\eta > 0$). The energy equation is

$$\frac{d\Delta E}{dt} = \frac{1}{T_0} \left[eV(\tau) - U_s \right] \ , \tag{A.4}$$

where $V(\tau)$ is the rf voltage seen by the particle of time advance τ. In a two-rf system satisfying the requirements in Eq. (2.5),

$$V(\tau) = \frac{1}{3!} V'''(0) \tau^3 = -\frac{m^2 - 1}{3!} (\omega_{\rm rf}\tau)^3 V_{\rm rf} \cos\phi_s \ , \tag{A.5}$$

where ϕ_s is the modified synchronous angle in the presence of the higher-harmonic cavity. The Hamiltonian is given by

$$H = -\frac{\eta}{2\beta^2 E} (\Delta E)^2 + \frac{m^2 - 1}{4!} \frac{(\omega_{\rm rf}\tau)^4}{2\pi h} eV_{\rm rf} \cos\phi_s \ . \tag{A.6}$$

For a particle having maximum arrival excursion $\hat\tau$, the Hamiltonian assumes the value

$$H = \frac{m^2 - 1}{4!} \frac{(\omega_{\rm rf}\hat\tau)^4}{2\pi h} eV_{\rm rf} \cos\phi_s \ . \tag{A.7}$$

We can therefore solve from Eq, (A.6) the energy offset

$$\Delta E = \pm\sqrt{-\frac{2\beta^2 E_0}{\eta}\,\frac{m^2\!-\!1}{4!}\,\frac{\omega_{\rm rf}^4}{2\pi h}\,eV_{\rm rf}\cos\phi_s}\,\sqrt{\hat\tau^4-\tau^4}\ . \tag{A.8}$$

We now make use of the phase equation of Eq. (A.3) and integrate over one synchrotron period,

$$\frac{2\pi}{\omega_s(\hat\tau)} = \oint dt = \oint -\frac{\beta^2 E_0}{\eta\Delta E}d\tau\ . \tag{A.9}$$

A comment on the sign of the integrand is in order. Above transition ($\eta > 0$) when $\Delta E > 0$, the particle arrival time slips backward, or the integration over τ ranges from $+\hat\tau$ to $-\hat\tau$. If we integrate from $-\hat\tau$ to $+\hat\tau$ instead, the negative sign will be eliminated and the integrand becomes positive. On the other half of the synchrotron oscillation when $\Delta E < 0$ so that the negative sign is cancelled, the particle arrival time slips forward from $-\hat\tau$ to $+\hat\tau$. Similarly, below transition ($\eta < 0$), when $\Delta E > 0$, the particle arrival time slips forward from $-\hat\tau$ to $+\hat\tau$, the integrand is again positive. When $\Delta E < 0$, the arrival time slips backward from $+\hat\tau$ to $-\hat\tau$. Reversing the order of integration, the integrand is again positive. Substituting ΔE from Eq. (A.8) into Eq. (A.9), we obtain

$$\frac{2\pi}{\omega_s(\hat\tau)} = \frac{2}{\hat\tau}\sqrt{-\frac{2\beta^2 E_0}{\eta eV{\rm rf}\cos\phi_s}\,\frac{4!}{m^2\!-\!1}\,\frac{2\pi h}{\omega_{\rm rf}^4}}\,I\ , \tag{A.10}$$

where the integral

$$I = \int_0^1 \frac{du}{\sqrt{1-u^4}} = \frac{K(1/\sqrt2)}{\sqrt2}\ . \tag{A.11}$$

Finally, we make use of the expression for the small-amplitude synchrotron angular frequency ω_{s0} in the absence of the higher-harmonic cavity,

$$\omega_{s0} = \sqrt{-\frac{\eta heV_{\rm rf}\cos\phi_{s0}}{2\pi\beta^2 E_0}}\omega_0\ , \tag{A.12}$$

to arrive at

$$\frac{\omega_s(\hat\tau)}{\omega_{s0}} = \frac{m^2\!-\!1}{4!}\,\frac{\pi\omega_{\rm rf}\hat\tau}{K(1/\sqrt2)}\sqrt{\frac{\cos\phi_s}{\cos\phi_{s0}}}\ . \tag{A.13}$$

Substitution of Eq. (2.9) gives the expression in Eq. (2.13).

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