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On Possibility of Footprint Compression with One Lens in Nonlinear Accelerator Lattice

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Abstract

Electromagnetic interaction of colliding beams along with other nonlinear fields in many cases limits beams lifetime and luminosity. Nonlinearities result in spread of betatron frequencies (footprint) and, thus, may enhance dynamic diffusion of particles due to high order resonances. One of possible ways to eliminate nonlinearities and overcome the difficulties is compensation of nonlinear forces, but, in practice, it's hardly possible to obtain exact linearity of the system. The compensation with a single nonlinear lens can't cope with distributed nonlinearities, nonlinearities due to parasitic crossings, etc. In the article we present a method to compute parameters of nonlinear element (lens) that eliminates both the footprint and resonance strength without achieving full compensation.

1 Introduction

Use of opposite charge particles for compensation of a kick due to counter bunch in colliders is an attractive idea[1], although there are several issues to be solved for its successful realization (see e.g. [2]). One of them is provision of proper non-linear electron charge distribution. For example, Fig.1 demonstrates the effect of the ideal "electron compressor" on the 2-D tune diagram. The largest "leaf" is the pbar footprint due to "head-on" collisions with round Gaussian proton beam with charge distribution $\rho_p(r) = C \cdot exp(-\frac{r^2}{2\sigma^2})$. The smaller one shows the footprint in case when electron beam with charge density profile proportional to $\rho_e(r) = -C \cdot \frac{0.83}{1+(r/\sigma)^8}$ is installed on the pbar orbit. For convenience of presentation we have separated two plots horizontally, in fact the second one would be around zero tune point $\nu_{(x,y)} = 0$. One can see a significant reduction of the tune spread with the electron beam. It was originally thought that the electron beam with Gaussian charge density $\rho_e(r) = -\rho_p(r)$ can lead to complete elimination of the footprint, and thus to compensation of the beam-beam effects.

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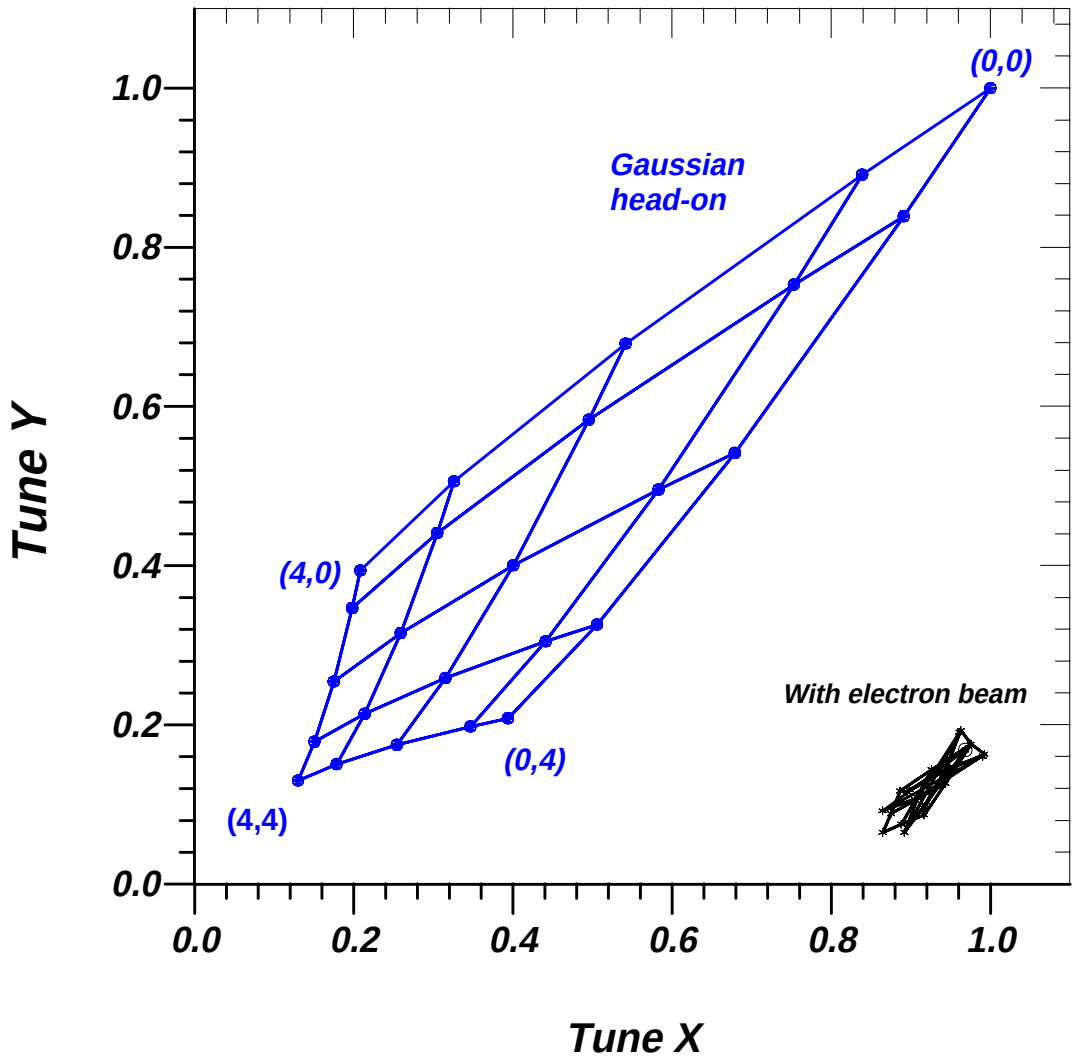

Figure 1: “Electron compression” of head-on footprint of antiprotons.

It was revealed then, that such an idealized picture of the compensation is not adequate to reality. First of all, the beam-beam footprint itself can be significantly distorted by “imperfections” like crossing angle at the interaction point, numerous parasitic interactions in multibunch colliders at the locations where two beams do not actually collide but still interact via long-range electromagnetic forces, etc. The collider focusing lattice itself is not linear and this must be taken into account too. An additional difficulty is that usually nonlinearities are not localized in one element - in contrast, they are distributed over the collider ring - and a single thin lens even with any field distribution can not eliminate all nonlinearities from particle motion. Finally, it does matter what is the ratio of the electron beam length and beta-function at its location. To create some insertion with predetermined two or three dimensional map is an extremely complicated task even from theoretical point of view.

This article is an attempt to investigate the possibility to add a single thin nonlinear lens to some arbitrary nonlinear lattice in such a way that the particle motion in the modified structure would become resonance-free, though nonlinear, and the beam of the particles has zero footprint. After that we apply our results to experimental project of the beam-beam compensation with electron beam (BBCEB) in the Tevatron which is currently under way at FNAL ([3]).

2 Resonance-Free Nonlinear System.

Here we present some of earlier results on the nonlinear systems without resonances [4]. These systems form a subset of integrable systems, i.e., systems with regular motion, and some of them are applicable in colliding beams. As an example, we consider nonlinear (de)focusing due to some specially prepared electron beam.

Let us consider particle motion in the three elements lattice: 1) drift space with a unity length (for simplicity); 2) an axially symmetric thin lens, as a representation of the angular-momentum-preserving linear optics in between interactions; and 3) radial beam-beam kick $k_{bb}(r)$. 2D map for particle trajectory coordinates x, y and angles x', y' is:

$$\begin{aligned} \overline{x} &= x + x' \\ \overline{y} &= y + y' \\ \overline{x}' &= x' + \overline{k}_x \\ \overline{y}' &= y' + \overline{k}_y, \end{aligned} \tag{1}$$

where $k_x = \dfrac{x}{r}k(r)$, $k_y = \dfrac{y}{r}k(r)$, and $r = \sqrt{x^2+y^2}$. Due to conservation of the angular momentum $M = xy' - yx'$, the motion can be reduced to 1D, i.e. to r and $r' = (xx' + yy')/r$.

It can be checked directly, that if the total radial kick function is equal to:

$$k(r) = -2r - k_{bb}(r) = -2r - \frac{br}{1+ar^2}\,, \tag{2}$$

where a and b are free parameters, then, there is an additional invariant of motion:

$$\mathcal{I}_M(r,r') = (1+ar^2)\Big((r'+r)^2 + \frac{M^2}{r^2}\Big) + br(r'+r) + r^2. \tag{3}$$

The variables here are changed to r, r', and we used a simple relation $x'^2 + y'^2 = ((rr')^2 + (xy' - yx')^2)/r^2 = r'^2 + M^2/r^2$. [1]

It is easy to find the corresponding electron charge distribution $\rho_e(r)$ which leads to the necessary kick $k_{bb}(r)$. Indeed, if the electron beam length is much smaller than the beta-function at this point, then

$$k_b b(r) = C\frac{\int_0^r f(r)rdr}{r},$$

[1] the first term in (2) together with the drift forms the linear optics in between the IPs. The optics appears to be a 90° lattice in both X and Y planes, with the matrix of the period:

$$T_x = T_y = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{4}$$

In order to return to the physical units in the result, one should replace 1 to β, and -1 to $-1/\beta$ in the right hand side of the last equation, β is the beta-function at the interaction point.

thus,

$$\rho_e(r) = -\frac{2b}{C}\frac{1}{(1+ar^2)^2}. \tag{5}$$

We do not specify here the value of the constant C, since the constants a, b are arbitrary.

Since we provide two invariants of 2D problem, then the resulting motion is regular, though nonlinear (the frequency of oscillations depends on invariant of oscillations) [4].

3 Numerical Algorithm to Eliminate Footprint.

In previous section we deal with nonlinear integrable system with quite predictable motion. One can use that or similar system in order to eliminate chaotic particle motion. Certain difficulty appears if one takes into account nonlinearities and field errors of the real accelerator which may significantly change the particle motion. It can cause drastic changes if the nonlinear lens brings the system close to higher order resonances. To avoid excitation of these resonances, we need system with weak dependence of betatron frequency on the oscillation amplitude or with no dependence at all.

Such systems are linear in some appropriate variables. Let us consider all 1D classical Hamiltonians [2]

$$H = p^2/2 + U(x)$$

which have the property of constant frequency of oscillations for all initial conditions.

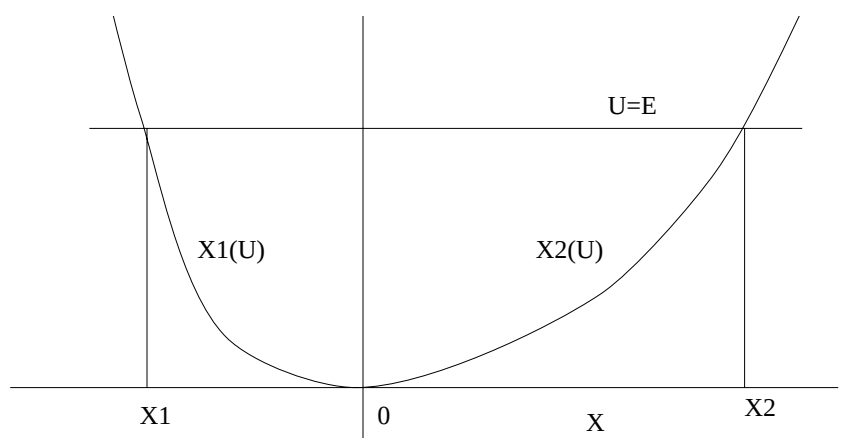

Figure 2: Potential of classical Hamiltonian vs coordinate.

Under conditions that the potentials $U(x)$ have only one minimum and that the oscillation period is known function $T(E)$ of the energy E (a particular value of the above Hamiltonian), the solution of the problem is given by formula (see e.g. [5]):

$$X_2(U) - X_1(U) = \frac{1}{\sqrt{2}\pi}\int_0^U \frac{T(E)dE}{\sqrt{U-E}}, \tag{6}$$

[2] for sake of simplicity, we put the mass of particle is equal to 1

where $X_2(U), X_1(U)$ are left and right boundary for particle's motion for energy $U(X_1) = U(X_2) = E$ - see Fig.2. Reversion of $X_{1,2}(U)$ gives the potential $U(X)$ vs coordinate for both left and right branches of the potential energy graph. Evidently, we are free to choose only one of the functions $X_1(U)$ or $X_2(U)$, while the second one to be determined from Eq.(6). For a constant period $T(E) = T = const$ in the formula yields:

$$X_2(U) - X_1(U) = \frac{\sqrt{2}}{\pi} T \sqrt{U}. \tag{7}$$

This formula has very simple meaning - while one of the branches (e.g. $X_1(U)$) is determined, the second branch must be adjusted to the first one to keep the same period of oscillations for all energies.

In general, symplectic maps have no appropriate "good" invariant (like energy in the example above), and to find general form of the maps with constant frequency is much more difficult problem. We present here a numerical method to construct such maps. It has simple physical meaning but has no reliable mathematical foundation.

The idea is as follows: let us choose our frequency equal to a resonant one, e.g. the 20th order resonance [3]). There is a simple way to know, either the motion for every particular initial condition has that frequency or not: one just has to calculate the squared differences of coordinates and momenta at the beginning and at the end of 20 successive map transformations. E.g. in the normalized variables $x = \sqrt{\epsilon}\cos(\psi)$ and $x' = -\sqrt{\epsilon}\sin(\psi)$ (ψ is a betatron phase), one makes summation over some region of initial conditions and gets a special function F:

$$F = \sum_j (x_f - x_i)^2 + (x'_f - x'_i)^2, \tag{8}$$

where symbols i and f are for initial and final normalized coordinates and angles, respectively, and the index j denotes different phase space elements of initial conditions. When this function is equal to zero $F = 0$, then we have:

1) all the frequencies are equal to the particularly chosen value (1/20 in our case);

2) strength of this resonance is equal to zero.

In fact, for resonant islands we get the same average frequency for all phase space elements of the island, but the motion inside the island has own frequency. It gives the nonzero difference of initial and final conditions after the number of turns equal to number of resonance (20 turns in our case). When the function F is equal to zero, the motion inside the island is degenerate, so the resonance strength is equal to zero.

The numerical code we developed deals with minimization of the function F. The code can find the solution with very small F in all the important cases when there are enough parameters for optimization. It means that the "number" of integrable systems with constant frequency is rather large as for the case of 1D motion with time-independent Hamiltonian as we have shown above. In the next section we present the results of numerical calculations of the nonlinear lens for the Tevatron.

[3] we do so for simplicity. In fact, for an arbitrary frequency, there is always a rational number in its vicinity such as the difference is negligible

4 Application to the Tevatron Electron Beam Lens

Fig.3 shows a schematic view of the Tevatron collider, with three beams: high energy proton and antiproton beams and low-energy high current electron beam.

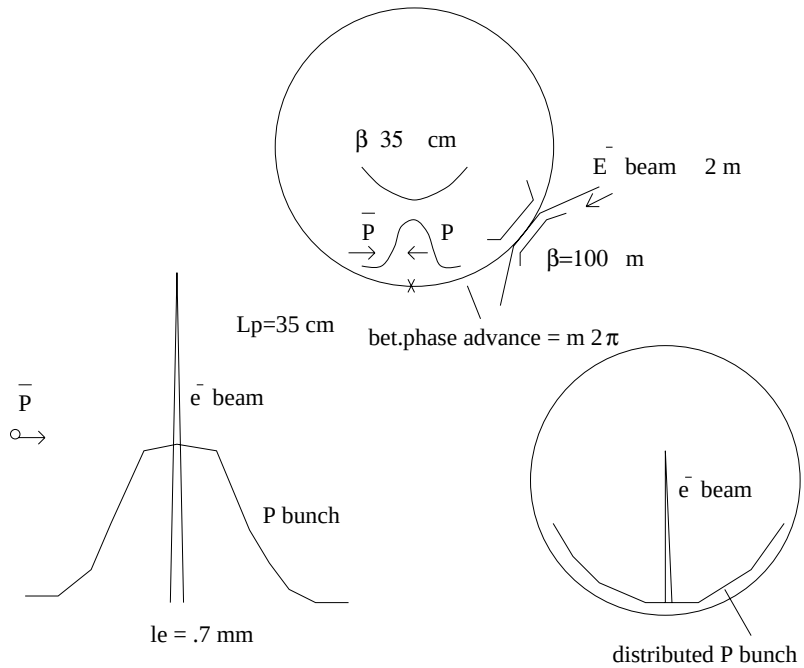

Figure 3: Scheme and some parameters of bunches in the Tevatron.

Evidently, it is beneficial to collide all three beams in one interaction point(IP): if the electron bunch goes along with the proton bunch at interaction region and has the same size, speed, charge, then the electromagnetic force due to protons can be compensated by electrons and test particles, antiprotons, experience no total kick at the IP.

In reality, it is hard to achieve necessary electron current density (equal to the proton one at the IP) with conventional electron sources. Moreover, often there is no space at the interaction region for necessary additional equipment. Thus, the electron beam has to be placed somewhere else at the ring, preferably in the location where transverse beam size is large, e.g. about 1 mm rms in Tevatron if the beta-function is about $\beta^e = 200m$ - compare with about 30 μm rms at the IP $\beta^* = 35$ cm. It is desirable to have betatron phase advance between the IP (marked by cross in top picture in Fig.3) and the electron beam point to be multiple of 2π. It was shown in Ref.[1] that thermionic electron gun can provide necessary beam parameters for compensation of the proton bunch impact on antiprotons.

Nevertheless, there is another difficulty which come from the fact that the proton bunch length of about $\sigma_z \simeq 35cm$ is comparable to beta-function at the interaction point β^*. Therefore, the betatron phase advance for antiprotons at the main IP is large $\phi = \int dz/\beta \simeq \sigma_z/\beta^* \simeq 1$. In contrast, the electron beam length of about 2 m is much less than β^e and corresponding betatron phase advance

of antiprotons passing the electron beam is very small $\phi \simeq 0.01 - 0.02$. In other words, effectively, the electron beam kick looks like delta-function if transformed to the main IP - see low left picture in Fig.3. Consequently, this short impact due to electrons contains a lot of resonance harmonics, although the average actions due to proton and electron beams are the same. One can reduce betatron tune spread by such a lens, but this fact alone does not assure the motion more stable than that with no compensation. For example, if the round proton charge is uniformly distributed along the whole ring - see low right picture in Fig. 3 - than $\bar{p}$ motion is regular in spite of the nonlinear force. Introduction of short electron beam results in time-dependent total force and stochastic dynamics in general. So, the resonance strengths sometimes are more important than the betatron tune spread.

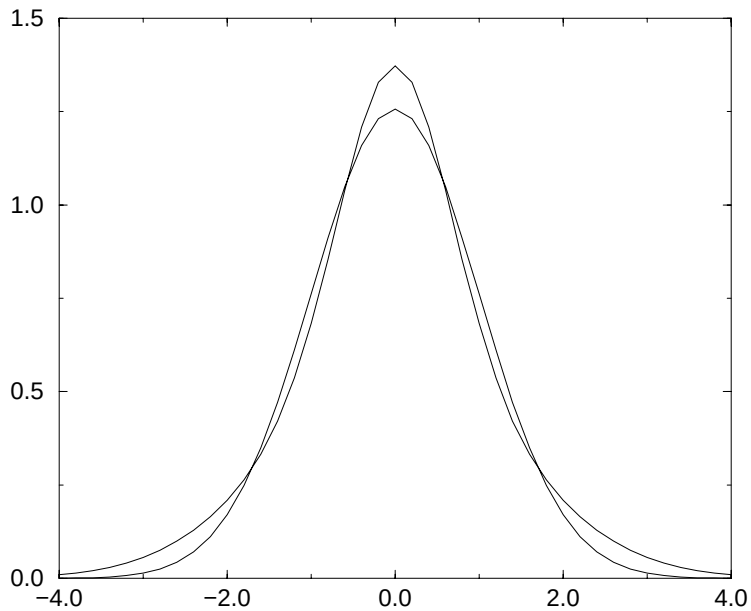

Figure 4: Transverse electron charge distributions necessary for elimination of resonances and tune spread in the antiproton motion: the curve with smaller value at 0 is the Gaussian distribution to compensate very short proton bunch; the curve with higher maximum is optimized electron distribution to compensate 75-cm long proton bunch (rms).

The numerical procedure suggested in the preceding section is used to fix the situation. We operated with transverse electron charge distribution which is sum of the Gaussian distributions with different rms values:

$$\rho_e(r) = \sum_{n=1}^{6} C_n exp(-\frac{n \cdot r^2}{4\sigma^2}),$$

where C_n are variable coefficients for optimization and σ is the rms transverse proton bunch size at the location of the electron beam. In simulations the electron beam produces a delta-function kick because of its short "effective" length, while the proton bunch length is presented as a number of short slices.

We used somewhat different optimization function than F introduced in previous section, namely:

$$\tilde{F} = \sum_j \frac{(x_f - x_i)^2 + (x'_f - x'_i)^2}{x_i^2 + (x'_i)^2}, \tag{9}$$

where j is the index for different initial conditions. The denominator of this expression is added in order to make the same weight of trajectories with small and large amplitudes.

The numerical code finds coefficients C_n depending on the proton bunch length. The synchrotron amplitude of antiprotons is taken to be zero. In the process of optimization the value $\tilde{F}$ usually decreases in factor of 1000. For example, under conditions of zero length proton bunch, the tune shift due to protons is $\xi = 0.05$, with equal horizontal and vertical beta-functions of $\beta_x^* = \beta_y^* = 35cm$ we get $C_1 = 0, C_2 = 1.25, C_3 = 0, C_4 = 0, C_5 = 0, C_6 = 0$ in some units (for normalized variables).

For the proton bunch length of $\sigma_z = 2\beta^*$, we get $C_1 = 0.533, C_2 = .225, C_3 = 0.484, C_4 = 0.325, C_5 = 0.194, C_6 = 0.25$ in the same units. Fig.4 demonstrates the resulting "optimized" distribution and compares it with the Gaussian distribution $C_2 = 1$. Significant difference is clearly seen.

5 Conclusion

We suggest a numerical method to calculate the parameters of single thin lens necessary to compensate effects of accelerator nonlinearities. It can be applied to a problem of non-linear beam-beam effects in collider rings if a specially prepared electron beam can be used for compensation. The primary studies of the method have shown that it works as soon as there are enough parameters to vary non-linear components. Though the method is valid for 1D case of round beam schemes, it can be generalized for two dimensions.

We intend to continue these investigations for the electron beam lens in the Tevatron, and take into account influence of synchrotron motion, the crossing angle effects, etc. on the antiproton beam dynamics.

References

[1] V.Shiltsev, D.Finley,"Electron Compression" of Beam-Beam Tune Spread in the Tevatron", FERMILAB-TM-2008(1997)

[2] *Proceedings of the Mini-Workshop on Beam-Beam Compensation, Fermilab, Feb.1998*, FERMILAB-Conf-98/064 (1998).

[3] V.Shiltsev, "Electron Lens" to Compensate Bunch-to-Bunch tune Spread in TEV33", FNAL-TM-2031 (1997)

[4] V.Danilov and E.Perevedentsev, "Two Examples of Integrable Systems with Round Colliding Beams", *Proc. 1997 PAC*, Vancouver (1997).

[5] L.Landau and E.Lifshits, *Course of Theoretical Physics, vol.1, "Mechanics"*, 4-th edition, 1988.