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Effects of Chromaticity Sextupoles on the INS Lattice

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A first medium size imaginary-γ_t proton synchrotron is to be built at KEK in Japan. The chromaticity-correction sextupoles placed inside the modules are not designed for nonlinearity confinement or cancellation. The effects of these sextupoles as contributing to non-linearity and beat factors are studied.

I INTRODUCTION

In order to reduce beam loss, the 50-GeV proton synchrotron designed by the Institute for Nuclear Study of Japan (INS) will operate with an imaginary-γ_t [1]. The flexible momentum-compaction (FMC) modules [2] in the lattice are roughly 3 FODO-cell long. To cancel chromaticities, there will be in general less locations to place the sextupoles, and these sextupoles are repeated every module instead of every FODO cell as in the conventional FODO-cell lattice. As a result, these sextupoles will be stronger and the cancellation of their nonlinear effects will not be as easy. In this note, we are going to analyze the effects of these chromaticity sextupoles, hoping that they would not be too intolerable.

II SMEAR

For truly linear motion, the particle trajectory in a transverse phase space at a certain location along the ring maps out in a Poincaré section a perfect ellipse which is an invariant. In the presence of nonlinearities, however, the trajectory fluctuates about the ellipse from turn to turn. The rms fractional value of this fluctuation is called the *single-particle smear.* Based on past accelerator experience [3], the linear aperture of the Superconducting Super Collider (SSC) is defined [4] quantitatively as the region within which the smear is less than 7% and the on-momentum tune shift with amplitude is less than 0.005. For example, in the experiment E778 performed at the Fermilab Tevatron, the multiparticle smears were measured [5] for various sextupole excitation currents. The results appeared to agree excellently with multi-particle simulations. Single-particle trackings were then performed with exactly the same machine inputs as the multiparticle simulations to convert the observed smear of the beam to that of a single particle. In fact, the horizontal smear has been computed using first-order perturbation theory and the results agree with tracking [6]. Here, we are going to extend the formulas to include both transverse planes. The derivation follows closely those in Ref 5.

The distortion of the horizontal amplitude $\mathcal{A}_x$ at horizontal phase advance ψ_x and that of the vertical particle amplitude $\mathcal{A}_y$ at vertical phase advance ψ_y of a particle are given by [7, 8]

$$
\begin{aligned}
\delta\mathcal{A}_x(\psi_x) \;=\;& \mathcal{A}_x^2[(A_1\sin\varphi_x - B_1\cos\varphi_x) + (A_3\sin 3\varphi_x - B_3\cos 3\varphi_x) \\
-\;& \mathcal{A}_y^2[(A_+\sin\varphi_+ - B_+\cos\varphi_+) - (A_-\sin\varphi_- - B_-\cos\varphi_-) \\
&\qquad\qquad\qquad\qquad\qquad + 2(\bar{A}\sin\varphi_x - \bar{B}\cos\varphi_x)]\ ,
\end{aligned}
$$

$$
\delta\mathcal{A}_y(\psi_x) = -2\mathcal{A}_x\mathcal{A}_y[(A_+\sin\varphi_+ - B_+\cos\varphi_+) + (A_-\sin\varphi_- - B_-\cos\varphi_-)]\ . \tag{2.1}
$$

Here, B_1, B_3, $\bar{B}$, B_+, and B_- are the 5 distortion functions introduced by Collins [7, 9, 10]. The A's are the derivatives of the B's, similar to the situation that the Courant-Synder α's are the derivatives of the betatron functions β's. They are evaluated at ψ_x and the corresponding ψ_y. Note that only ψ_x is included in the argument of $\mathcal{A}_x$ and $\mathcal{A}_y$ because ψ_x and ψ_y are related. The distortion functions depend linearly on the strength of the kth sextupole placed along the ring through two parameters:

$$
s_k = \lim_{\ell\to 0}\left[\left(\frac{\beta_x^3}{\beta_0}\right)^{1/2}\frac{B_y''\ell}{2(B\rho)}\right]_k\ , \qquad \bar{s}_k = \lim_{\ell\to 0}\left[\left(\frac{\beta_x\beta_y^2}{\beta_0}\right)^{1/2}\frac{B_y''\ell}{2(B\rho)}\right]_k\ , \tag{2.2}
$$

where β_0 is just some reference betatron function for dimensional purpose and is set to 1 m for convenience. The phases φ_x, φ_y ($\varphi_\pm = 2\varphi_y \pm \varphi_x$) are the instantaneous phases of the particle describing its instantaneous positions along the ellipses of the horizontal and vertical Poincaré sections. The amplitudes $\delta\mathcal{A}_x$ and $\delta\mathcal{A}_y$ are related to the invariant emittances by

$$
\epsilon_x = \frac{\pi\mathcal{A}_x^2}{\beta_0}\,, \qquad \epsilon_y = \frac{\pi\mathcal{A}_y^2}{\beta_0}\,. \tag{2.3}
$$

The single-particle smears at phase advance ψ_x in the horizontal and ψ_y in vertical phase spaces are defined as the fractional rms distortions:

$$
S_x(\psi_x) = \left(\frac{\langle(\delta\mathcal{A}_x)^2\rangle}{\mathcal{A}_x^2}\right)^{1/2}\ , \qquad S_y(\psi_x) = \left(\frac{\langle(\delta\mathcal{A}_y)^2\rangle}{\mathcal{A}_y^2}\right)^{1/2}\ . \tag{2.4}
$$

This implies the averaging over the instantaneous phases φ_x and φ_y. The results are

$$
\begin{aligned}
S_x^2 \;=\;& \tfrac{1}{2}\mathcal{A}_x^2(A_3^2 + B_3^2 + A_1^2 + B_1^2) - 2\mathcal{A}_y^2(A_1\bar{A} + B_1\bar{B}) \\
&+\frac{\mathcal{A}_y^4}{2\mathcal{A}_x^2}(A_+^2 + B_+^2 + A_-^2 + B_-^2 + 4\bar{A}^2 + 4\bar{B}^2)\ , \qquad\qquad (2.5) \\
S_y^2 \;=\;& 2\mathcal{A}_y^2(A_+^2 + B_+^2 + A_-^2 + B_-^2)\ .
\end{aligned}
$$

A family of thin sextupole of integrated strength -0.10 m^{-2} each is placed close to the first D-quadrupole of the FMC module. Two other families of sextupoles are placed near the inner D-quadrupoles and center F-quadrupoles in each FMC module. These latter families are then adjusted to null out the chromaticities of the whole ring. The rms smears are computed for the reference 4-6-3 lattice [1] for the 50 GeV JHP ring designed by the INS and are plotted in Figs. 1 and 2 for a quarter of the ring, assuming emittances of $\epsilon_x = \epsilon_y = 50\pi \times 10^{-6}$ m. Figure 2 is for the first family of sextupoles set to -0.10 m^{-2} while Fig. 1 is for this family of sextupoles not to be excited. It is clear that with this extra family of sextupoles, the smears become smaller. We see that the rms vertical smear reaches only 0.15%, which is very small, and the rms horizontal smear are still smaller. The full smears will be roughly $\sqrt{2}$ times the rms values, which are much less than the SSC criterion. This lattice has a 4-fold symmetry. The smears computed for one quarter would be the same as the smears for the whole ring. We also see that the vertical smear is constant and has a jump only when it encounters a sextupole. This reflects exactly the properties of the distortion functions. The horizontal smears behave similarly, although there is an interference term between (A_1, B_1) and $(\bar{A}, \bar{B})$.

For a regular FODO-cell lattice, we usually prefer 60° or 90° phase advances in both transverse planes so that the distortions will be cancelled in every 6 or 4 cells, respectively. Otherwise, the distortion waves will flow along the whole ring and can accumulate to big values. For flexible momentum-compaction modules, one would prefer 270° horizontal phase advance and 180° vertical phase advance for the same cancellation. The FMC module in the present lattice has horizontal and vertical tune advances of 0.74051 and 0.53038, respectively. However, the smears are small and therefore the requirement of special phase advances will not be necessary. The largest contribution of the smears comes from B_+. This is because including the long straight sections, the betatron tunes of the whole ring is $\nu_x = 21.7723$, $\nu_y = 15.1462$, and $2\nu_y + \nu_x = 52.0648$, which is too close to a third-order sum resonance.

III AMPLITUDE DEPENDENCE OF TUNES

Another measure of nonlinearity is the amplitude dependence of betatron tunes. If the dependence is large, the tune spreads may overlap a major resonance, thus decreasing the dynamical aperture of the ring.

In terms of the distortion function, the lowest-order tune spreads due to sextupoles are:

$$\Delta\nu_x = -\frac{\mathcal{A}_x^2}{4\pi}\sum_k (B_3 s + 3B_1 s)_k - \frac{\mathcal{A}_y^2}{2\pi}\sum_k (B_+\bar{s} + B_-\bar{s} - 2B_1\bar{s})_k \ ,$$

$$\Delta\nu_y = -\frac{\mathcal{A}_x^2}{2\pi}\sum_k (B_+\bar{s} + B_-\bar{s} - 2B_1\bar{s})_k - \frac{\mathcal{A}_y^2}{4\pi}\sum_k (B_+\bar{s} - B_-\bar{s} + 4\bar{B}\bar{s})_k \ , \tag{3.1}$$

where the summation goes over all the sextupoles which are assumed to be thin. For a ring with N-fold symmetry, it can be shown that the tune spreads are just N times the tune spread of one superperiod. The total tunes of the whole ring are

$$\nu_x = 21.7723 + 126\frac{\epsilon_x}{\pi} - 146\frac{\epsilon_y}{\pi} \ ,$$

$$\nu_y = 15.1462 - 146\frac{\epsilon_x}{\pi} + 148\frac{\epsilon_y}{\pi} \ , \tag{3.2}$$

where the emittances are measured in m. The magnitude of the family of sextupoles near the entrance D-quadrupole of each FMC module has been carefully adjusted to -0.10 m^{-2}, so that 3 detunings are of roughly the same magnitude and have the desirable signs. We see that with $\epsilon_x = \epsilon_y = 50\pi \times 10^{-6}$ m, the largest tune spread is only 0.0074. The SSC has been designed as a storage ring and therefore the more stringent linearity criterion of $\delta\nu < 0.005$ is desired. Here, the JHP is not designed for storage and $\delta\nu < 0.0074$ is therefore quite acceptable.

In general, there are two ways to reduce the detunings of a ring consisting of FMC modules. The first way is to arrange the sextupoles inside one FMC module in such a way that the detunings for one module are small, for example, spacing two sextupoles of the same family 180° apart. The second way is to construct different types of FMC modules with the same matching Twiss parameters, so that the detunings for different types will have different signs. An example of the latter is given in a design of the Fermilab Main Injector lattice [11]. Thus, the total detunings of the whole ring will be cancelled to a certain extent. However, such attempts are not necessary for the INS lattice, because the detunings are already small.

IV BETATRON BEATINGS

Particles with a momentum offset δ will see a different set of betatron functions.

The fractional changes in betatron functions are called "beat factors". At a phase advance ψ, the beat factor per unit momentum offset is given by [12]

$$\left.\frac{\Delta\beta}{\beta}\right|_{\psi} = -\frac{1}{2\sin 2\pi\nu}\int_{\psi}^{\psi+2\pi\nu} k(\psi')\beta^2(\psi')\cos 2(\pi\nu+\psi-\psi')d\psi' \ . \tag{4.1}$$

In the above, the phase advance ψ, field gradient k, and tune ν assume their horizontal or vertical values for the horizontal or vertical beat factor. Note that the field gradient $k(\psi)$ along the ring receives contribution from the quadrupoles, the sextupoles, the centripetal force of the dipoles, and also the dipole edges. In an ideal FODO-cell lattice, chromaticity-correction sextupoles are placed just next to the quadrupoles. The beat factors will be very small since they receive contributions from the dipoles only. Here, in the INS lattice with FMC modules, sextupoles are not placed beside every quadrupole. As a result, a sizable beat wave will propagate in the lattice. Figure 3 shows the beat factors per unit momentum spread before any sextupole is fired, and Fig. 4 shows the beat factors when 3 families of sextupoles are excited to null the chromaticities. We see that the the beat factors actually increase when the sextupoles are excited. This is because the phase advances between the modules do not allow the effects of the sextupoles to cancel. However, for a momentum spread of $\delta = 0.5\%$, the maximum beat factor is only 8%, which is not too much.

The beat factor can be made complex by introducing the imaginary part

$$\Delta\alpha - \alpha\left.\frac{\Delta\beta}{\beta}\right|_{\psi} = -\left.\frac{d}{d\psi}\frac{\Delta\beta}{2\beta}\right|_{\psi} = -\frac{1}{2\sin 2\pi\nu}\int_{\psi}^{\psi+2\pi\nu} k(\psi')\beta^2(\psi')\sin 2(\pi\nu+\psi-\psi')d\psi' \ . \tag{4.2}$$

If we denote the real part by B and the imaginary part by A, the vector (B, A) rotates at a tune of 2ν when there is no field gradient. Whenever it passes through a field gradient k of infinitesimal length ℓ, A increases by

$$\Delta A = -\frac{\beta k\ell}{2} \tag{4.3}$$

while B remains unchanged. Thus the magnitude of the beat vector is an invariant unless it passes through a field gradient. These magnitudes are plotted in Fig. 5 and 6 for the situation before and after the correction sextupoles are excited.

The harmonic analysis of the beat factors are also important, because it gives us a clue to reduce the beat factors. Each beat factor can be written as

$$\left.\frac{\Delta\beta}{\beta}\right|_{\psi} = -\frac{\nu}{\pi}\sum_{p}\frac{J_p e^{ip\psi/\nu}}{4\nu^2 - p^2} , \tag{4.4}$$

where the summation extends over all positive and negative integers and the Fourier components are given by

$$J_p = \int_0^{2\pi\nu} k(\psi')\beta^2(\psi')e^{-ip\psi'/\nu}d\psi' . \tag{4.5}$$

For a left-right symmetric lattice, choosing the point of symmetry as the point having zero phase advances, the J_p's become real. Each beat factor can now be rewritten as

$$\left.\frac{\Delta\beta}{\beta}\right|_{\psi} = -\frac{J_0}{2\pi\nu} - \frac{2\nu}{\pi}\sum_{p>0}\frac{J_p \cos\frac{p\psi'}{\nu}}{4\nu^2 - p^2} , \tag{4.6}$$

where

$$J_p = \int_{-\pi\nu}^{\pi\nu} k(\psi')\beta^2(\psi')\cos\frac{p\psi'}{\nu}d\psi' . \tag{4.7}$$

The INS lattice has a left-right symmetry except for the third family of sextupoles which is placed only at one side of the entrance D-quadrupole of each module. However, the asymmetry is small and so are the $\mathcal{Im}\ J_p$'s. Therefore, we can assume the J_p's to be real.

Since the lattice is 4-fold symmetric, J_p vanishes unless p is a multiple of 4. By definition, $J_0 = 0$ for both horizontal and vertical because the chromaticities are zero. Some of the lower-order J_p's have been computed and are listed in Table I. The 2nd and 6th columns show the contributions of the quadrupoles, while the 3rd and 7th columns the contributions of the sextupoles. The total contributions including those from the dipoles are listed in the 4th and 8th columns. In the 5th and 9th columns, we list the contributions of J_p's to their respective beat factor per unit momentum offset as is indicated in each term of the summation of Eq. (4.6) but not including the cosine term.

We first notice that the J_0's are not exactly zero. This is because Eq. (4.7) is only first order; for example, the betatron function used inside the integral is only the unperturbed one. Nevertheless, this gives us a measurement of the error involved. The contributions of the sextupoles are exactly -4π times the chromaticities.

We see that the sextupoles do produce beat waves in the harmonic space. This is because they have not been placed at the proper phase advances for confinement or cancellation. The tunes of the lattice are $\nu_x = 21.7723$ and $\nu_y = 15.1462$, so that the important Fourier components are $p = 44$ for the horizontal and $p = 28$ and 32 for the vertical. The horizontal $J_{44} = 12.72$ and is small compared to, for example, the sextupole contribution of J_0. For the vertical, $J_{32} = -30.65$ may also be considered as small. But $J_{28} = -161.53$ is large and contributes 46.62 to $\Delta\beta_y/\beta_y$. This comes about because the straight section has a vertical phase advance of 0.60 which is not too far from the vertical phase advance of 0.53 for each FMC module. Thus, the contributions of the sextupoles add up. In fact, we can see from Fig. 4 that the vertical beat factor does have roughly a 7-fold symmetry in a superperiod. To reduce this contribution, the vertical phase advance of the straight section must be increased.

The beat factors are closely related to the integer and half-integer stop bands. There are also other stop bands originated from the sextupoles. For example, if the horizontal tune is tuned to near $\nu_x = 21.667$ instead, there will be a resonance at $3\nu_x = 65$. However, this is not important because of the 4-fold symmetry of the lattice. On the other hand, it will be extremely bad to tune the lattice to $\nu_x = 21.333$, because the resonance $3\nu_x = 64$ is a systematic one, since 64 is divisible by 4. As was discussed in Sec. II, the third-integer resonance $2\nu_y + \nu_x = 52$ will also be excited. Therefore, it will be nice to redesign the straight section, so that the vertical phase advance will become, say, 0.8 instead. In this way, both the third-integer sum resonance and the large vertical beat factor at harmonic 28 can be avoided.

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Table I: The horizontal and vertical J_p's of the INS lattice, showing their contributions from the quadrupoles and sextupoles.

	Horizontal J_p				Vertical J_p			
p	quads	sext.	total	$\Delta\beta/\beta$	quads	sext.	total	$\Delta\beta/\beta$
0	−323.79	325.20	0.42	−0.00	−263.90	267.80	2.69	−0.06
4	−2.08	71.27	68.99	−2.03	3.72	50.33	53.83	−2.30
8	1.59	−65.43	−63.68	1.92	−2.94	−48.57	−51.35	2.32
12	−0.94	55.39	54.35	−1.72	1.65	45.18	46.70	−2.32
16	0.48	−40.11	−39.60	1.34	0.12	−39.32	−39.13	2.28
20	−1.00	16.13	15.13	−0.57	−2.13	29.48	27.32	−2.04
24	5.18	31.41	36.63	−1.54	3.48	−10.06	−6.58	0.75
28	−43.97	−289.12	−333.66	16.63	19.51	−180.79	−161.53	46.62
32	−40.82	−205.18	−246.51	15.67	18.05	−48.58	−30.65	−11.12
36	23.42	79.73	103.36	−9.55	−17.53	19.60	2.15	0.22
40	−18.97	−32.73	−51.79	9.70	17.31	−4.37	12.88	0.73
44	16.87	−4.14	12.72	17.69	−15.12	−4.75	−19.85	−0.75
48	−16.67	43.11	26.50	3.60	9.52	5.72	15.23	0.42
52	21.03	−96.68	−75.73	−5.20	2.99	12.91	15.90	0.34
56	−48.29	234.18	185.92	8.31	−70.92	−221.64	−292.68	−5.09
60	−124.29	398.05	273.56	8.90	−26.87	−152.10	−179.04	−2.57
64	32.74	−26.86	5.90	0.15	−0.71	84.75	84.07	1.02

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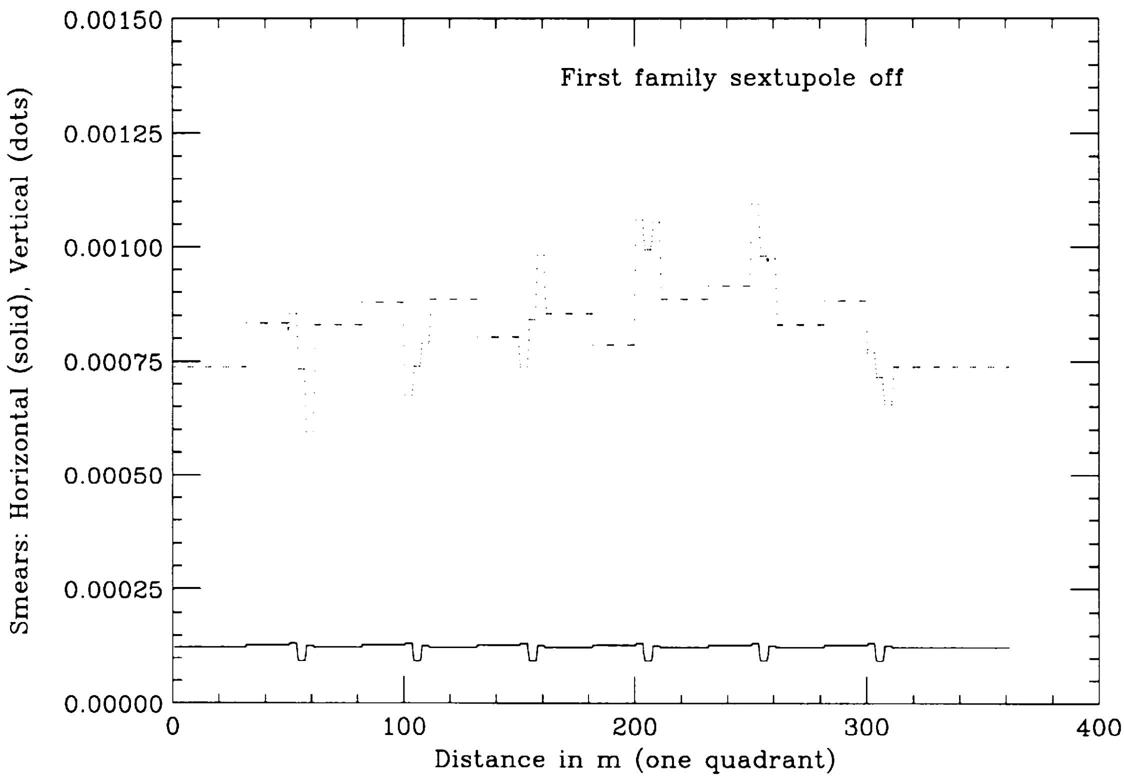

Figure 1: Horizontal (solid) and vertical (dashes) smears of the INS lattice when only two families of sextupoles are excited. Only one superperiod of the lattice is shown starting from the center of a straight section.

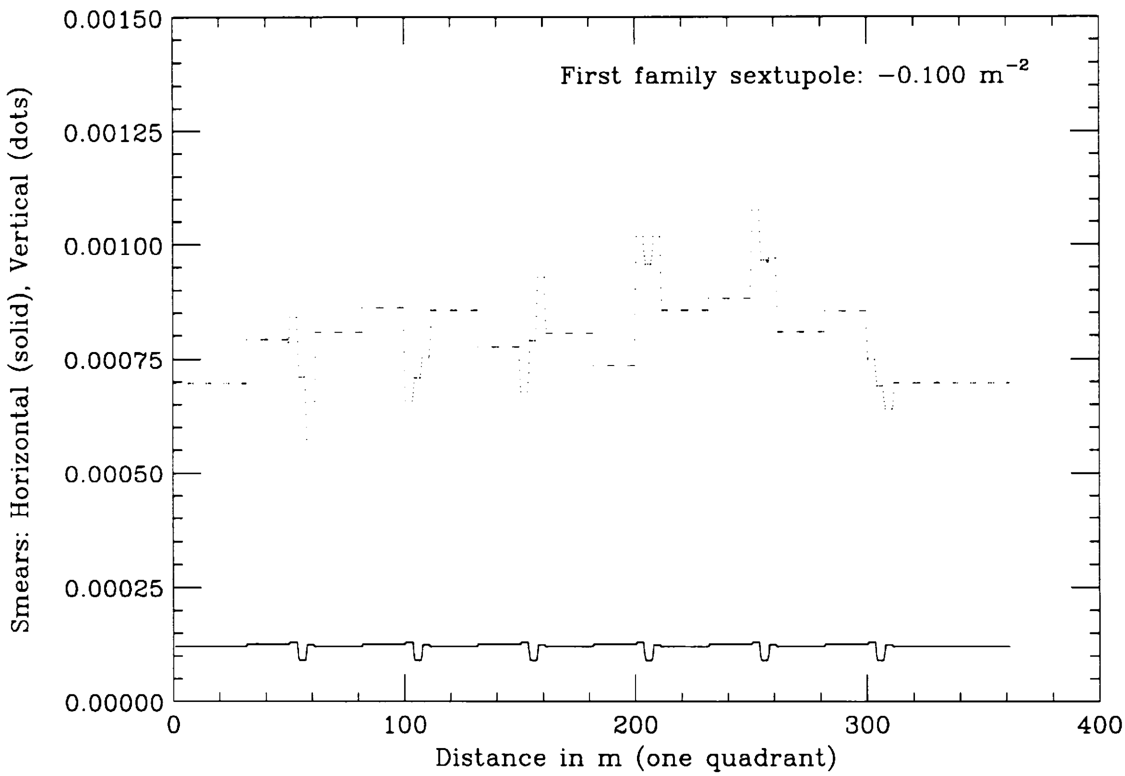

Figure 2: Horizontal (solid) and vertical (dashes) smears of the INS lattice when three families of sextupoles are excited. Only one superperiod of the lattice is shown starting from the center of a straight section.

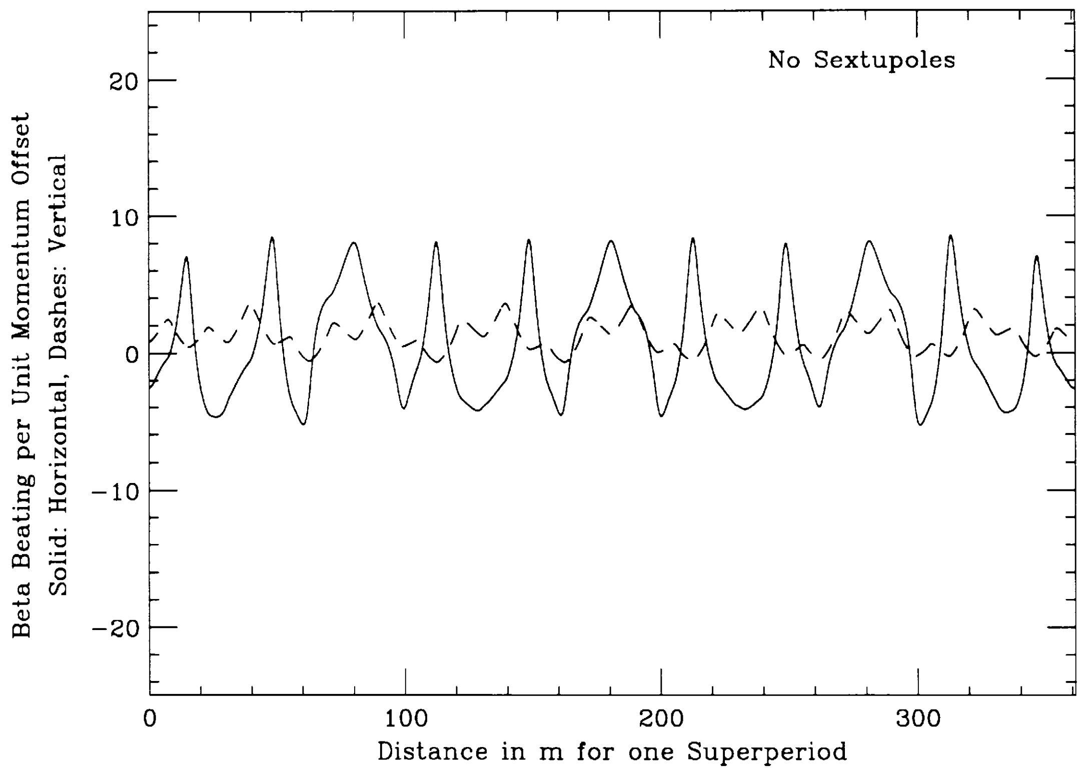

Figure 3: Horizontal (solid) and vertical (dashes) beat factors of the INS lattice when all the sextupoles are not excited. Only one superperiod of the lattice is shown starting from the center of a straight section.

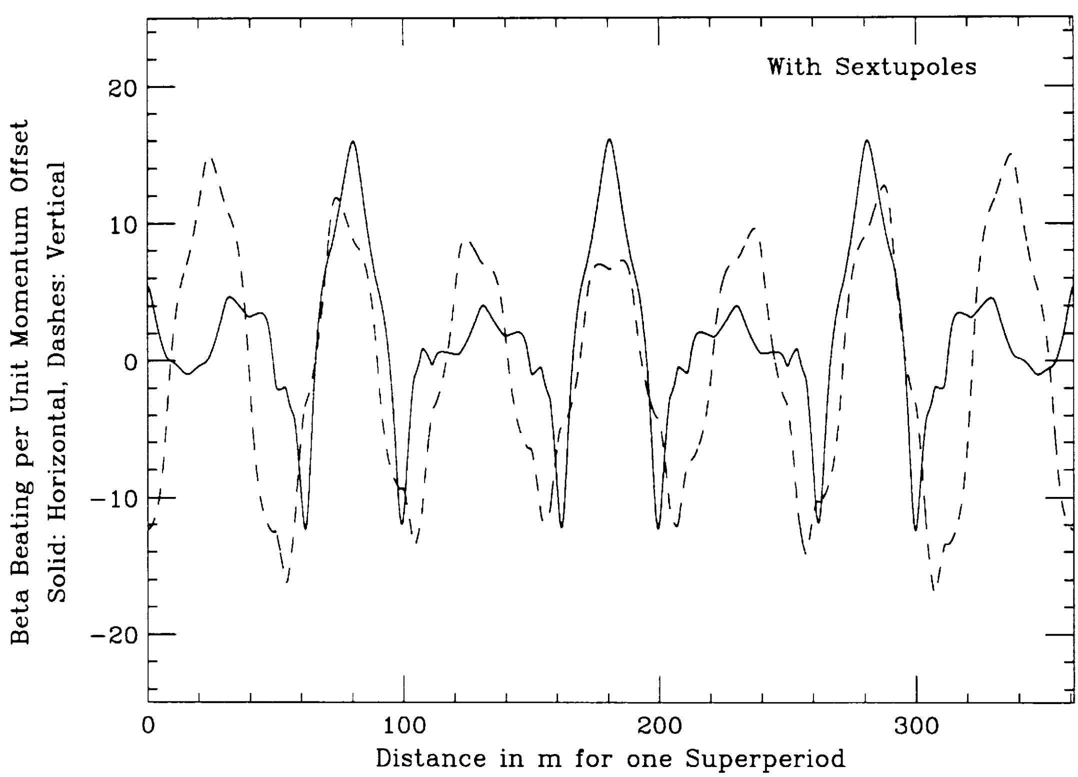

Figure 4: Horizontal (solid) and vertical (dashes) beat factors of the INS lattice when three families of sextupoles are excited. Only one superperiod of the lattice is shown starting from the center of a straight section.

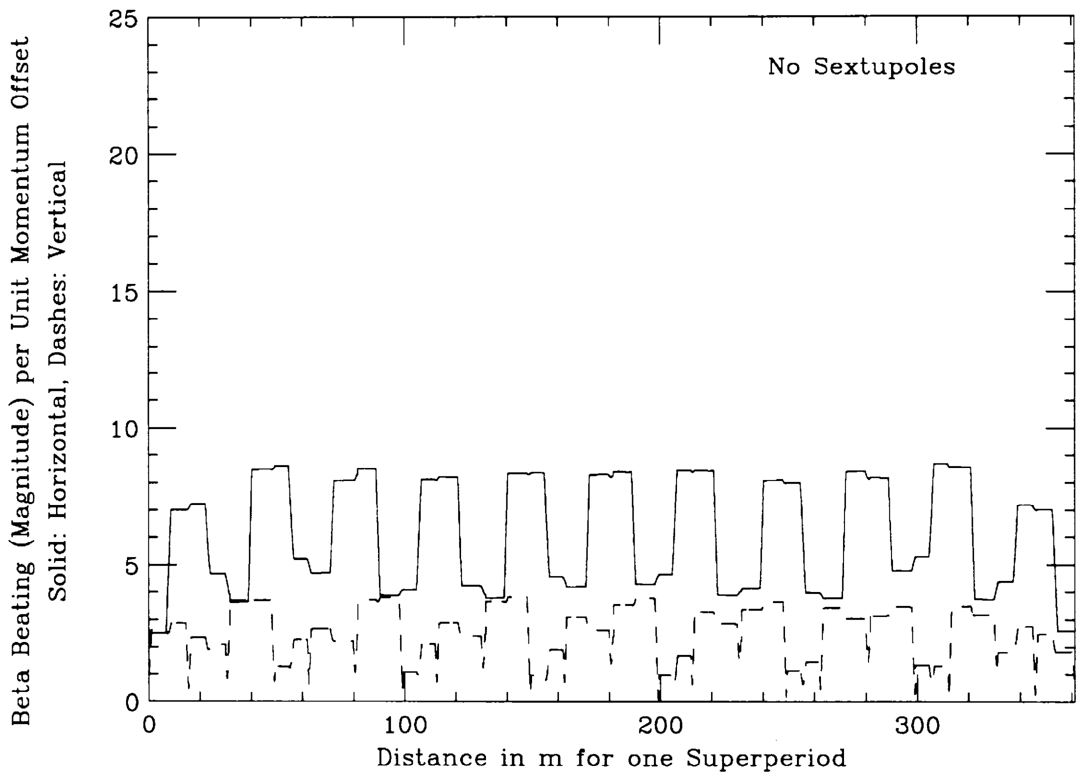

Figure 5: Magnitudes of the complex horizontal (solid) and vertical (dashes) beat factors of the INS lattice when all the sextupoles are not excited. Only one superperiod of the lattice is shown starting from the center of a straight section.

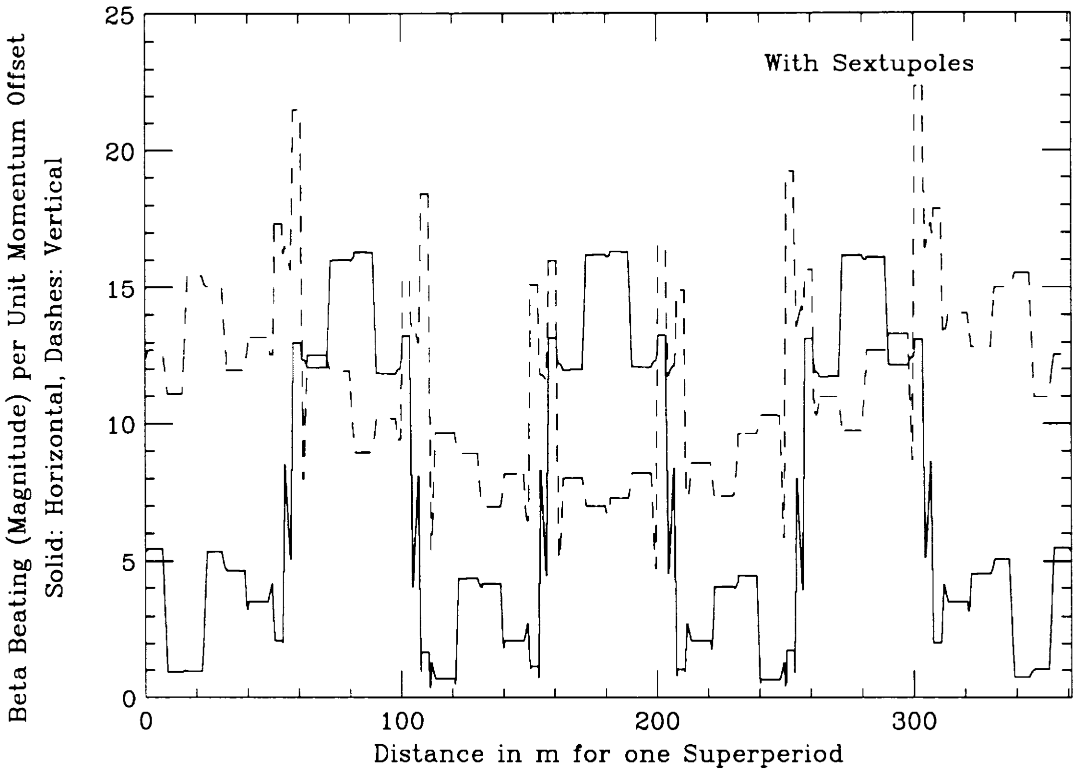

Figure 6: Magnitudes of the complex horizontal (solid) and vertical (dashes) beat factors of the INS lattice when three families of sextupoles are excited. Only one superperiod of the lattice is shown starting from the center of a straight section.