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A Study of the Autin-Wildner IR Scheme

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Abstract

The Autin-Wildner scheme [1] of implementing an interaction region (IR) with $\beta_x^* = \beta_y^*$ using two quadrupoles of equal but opposite strength is investigated. The impacts on chromaticities from quadrupole strength, low beta, and clearance from IP to first quadrupole are studied.

I. INTRODUCTION

Specifications for a 2 TeV muon collider call for a round beam at the point of collision [2]. Recently, Autin and Wildner [1] suggested a doublet scheme of quadrupoles to achieve the low-betatron functions for a round beam. The method is sketched in Fig. 1, where both the focusing and defocusing quadrupoles have the same focal length f, and are considered to be *thin*. The

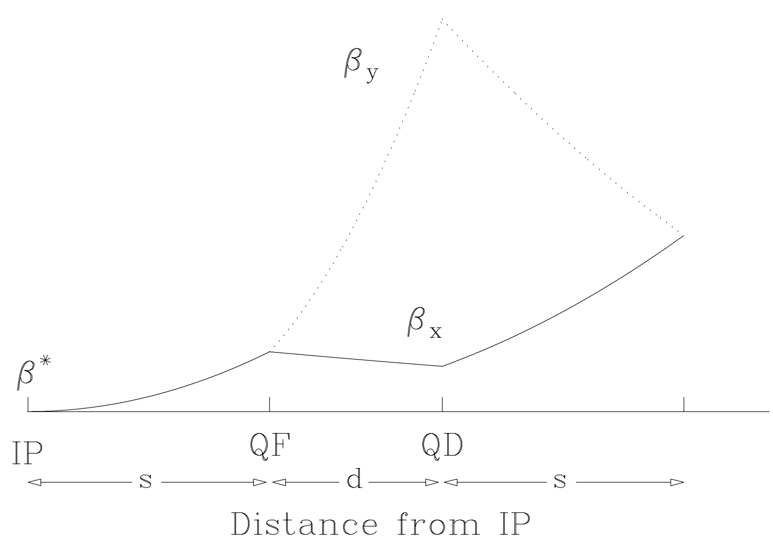

Figure 1: The Autin-Wildner doublet scheme at the IR.

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first quadrupole, say the focusing one, QF, is at a distance s from the IP. It focuses the horizontal betatron function β_x, while it increases the rate of divergence of the vertical betatron function β_y. The second quadrupole QD is defocusing and positioned at a distance d from QF. It reverses the rise of β_y. The Autin-Wildner theorem says that, if the distance d between the two quadrupoles satisfies

$$f = \sqrt{sd}\,, \tag{1.1}$$

then, at a distance s downstream from QD, the two betatron functions are equal and their slopes obey the relation,

$$\alpha_x + \alpha_y = 0\,. \tag{1.2}$$

The proof of this theorem is straightforward but rather tedious. In this paper, we will study some general properties of the Autin-Wildner scheme: its feasibility and how the chromaticities are affected by the low-beta value at the IP, the quadrupole strength, and the clearance between the IP and first quadrupole. Although thin lenses are used in this model, the overall conclusions of our study should not be altered much when using a thick-lens calculation; therefore our results should serve as some guide lines in the actual design.

II. MINIMUM QUADRUPOLE LENGTH

The defocusing quadrupole will experience the largest betatron value and therefore have the largest aperture. If we require a 5-sigma aperture for the beam, then the half aperture of the quadrupole is given by

$$y = 5\sqrt{\frac{\beta_{yD}\epsilon}{\pi}}\,, \tag{2.1}$$

where the unnormalized rms emittance ϵ of the beam is related to the normalized emittance by

$$\epsilon = \frac{\epsilon_N}{\beta\gamma}\,, \tag{2.2}$$

with β and γ being the relativistic parameters of the beam. The vertical betatron function β_{yD} at QD can be derived using the relation of Eq. (1.1):

$$\beta_{yD} = \beta^*\left(1+\frac{f}{s}\right)^2 + \frac{s^2}{\beta^*}\left(1+\frac{f}{s}+\frac{f^2}{s^2}\right)^2\,. \tag{2.3}$$

With the pole-tip field B, the maximum field gradient of the quadrupole is therefore $G = B/y$, and the maximum quadrupole strength is

$$\frac{1}{f} = \frac{GL}{(B\rho)} = \frac{BL}{5(B\rho)}\sqrt{\frac{\pi}{\beta_{yD}\epsilon}}\,, \tag{2.4}$$

where $(B\rho)$ is rigidity of the beam and L the length of the *thin* quadrupoles.

For simplicity, let us define a parameter

$$\lambda = \frac{BL}{5(B\rho)}\sqrt{\frac{\pi}{\epsilon}} = \frac{\sqrt{\beta_{yD}}}{f} \tag{2.5}$$

to denote the quadrupole length. Using Eqs. (2.3) and (2.4), one arrives at

$$\lambda = \frac{\sqrt{\beta^{*2}(f+s)^2 + (f^2+fs+s^2)^2}}{\sqrt{\beta^*}fs} . \tag{2.6}$$

In the case where the quadrupoles are extremely strong; i.e., $f \ll s$, the separation between the quadrupoles $d = f^2/s \to 0$, and the vertical betatron function at QD is just given by the simple quadratic expression

$$\beta_{yD} = \beta^* + \frac{s^2}{\beta^*} . \tag{2.7}$$

Then we have

$$\lambda = \frac{\sqrt{\beta_{yD}}}{f} \to \frac{s}{f\sqrt{\beta^*}} . \tag{2.8}$$

On the other extreme, when $f \gg s$, the focusing quadrupole is so weak that it does not significantly affect the vertical betatron function β_y, the defocusing quadrupole, being equal in strength to the focusing one, is also weak and it can only reverse the growth of β_y when the latter becomes very large. This is because the bending power of a quadrupole is given by $\Delta\alpha = \beta/f$. Here, the distance of QD from the IP is

$$s + d \approx d = \frac{f^2}{s} , \tag{2.9}$$

so that β_{yD} can be computed according to Eq. (2.7), and we obtain

$$\lambda = \frac{\sqrt{\beta_{yD}}}{f} \to \frac{f}{s\sqrt{\beta^*}} . \tag{2.10}$$

We learn from Eqs. (2.8) and (2.10) that $\lambda \to \infty$ as $f \to \infty$ or $f \to 0$. Thus, not all quadrupole lengths are possible and there is a minimum length.

In general, we do not want to keep s constant. Instead, we like to keep the distance s_0 from the IP to the *front end* of the first quadrupole constant. In other words,

$$s = s_0 + \frac{L}{2} , \tag{2.11}$$

and Eq. (2.10) is an implicit function of the quadrupole length. To find the minimum quadrupole length, we equate $d\lambda/df$ to zero to obtain

$$f = (\beta^{*2}s + s^3)^{1/3} \approx s\,. \tag{2.12}$$

This implies

$$\lambda_{\rm min} \approx \frac{3}{\sqrt{\beta^*}}\,, \tag{2.13}$$

or

$$L_{\rm min} \approx \frac{15(B\rho)}{B}\sqrt{\frac{\epsilon}{\beta^*\pi}}\,. \tag{2.14}$$

It is important to note that the minimum allowable quadrupole length is independent of s_0.

Let us study the situation of a low-beta of $\beta^* = 3$ mm and a 2 TeV muon beam with normalized emittance $\epsilon_N = 50\times 10^{-6}\pi$ m. If permanent magnets with a maximum pole-tip field of $B = 1$ T are used, we find immediately that the minimum quadrupole length is $L_{\rm min} = 93.9$ m, which implies that such magnets are too weak to be practical in building the IR. Even with conventional magnets and boosting the maximum field to $B = 2$ T, $L_{\rm min} = 47.0$ m is still too long to be practical.

Now, let us consider the strongest superconducting magnets which have a pole-tip field of 9.5 T. We obtain $L_{\rm min} = 9.88$ m. The distance d between the centers of the quadrupoles must be larger than L, or obviously the physical placement of the quadrupoles will not be possible. For a clearance of $s_0 = 6.5$ m between the IP and the first quadrupole, the quadrupole length L and the quadrupole separation d are plotted as functions of quadrupole focal length f in Fig. 2. We see that the physical allowable region starts at $f > 10.07$ m and the quadrupole length changes very slowly about its minimum value.

III. CHROMATICITY

Since the vertical betatron function will be much larger than the horizontal, the vertical chromaticity will be larger also. In this simple model, the vertical chromaticity receives its contribution from the defocusing quadrupole only and is given by

$$\xi_y = -\frac{\beta_{yD}}{4\pi f} = -\frac{\beta^{*2}(f+s)^2 + (f^2+fs+s^2)^2}{4\pi\beta^* s^2 f}\,. \tag{3.1}$$

We can see with the help of Eq. (2.3) that $\xi_y \to -\infty$ regardless of whether $f \to 0$ or ∞. Thus, the chromaticity has a minimum also. In Fig. 2, the chromaticity is plotted as dots. It is clear that to minimize chromaticity, we must choose a quadrupole focal length of ~ 10 m and a quadrupole focal length as small as possible. In fact, it is true in general that, to minimize

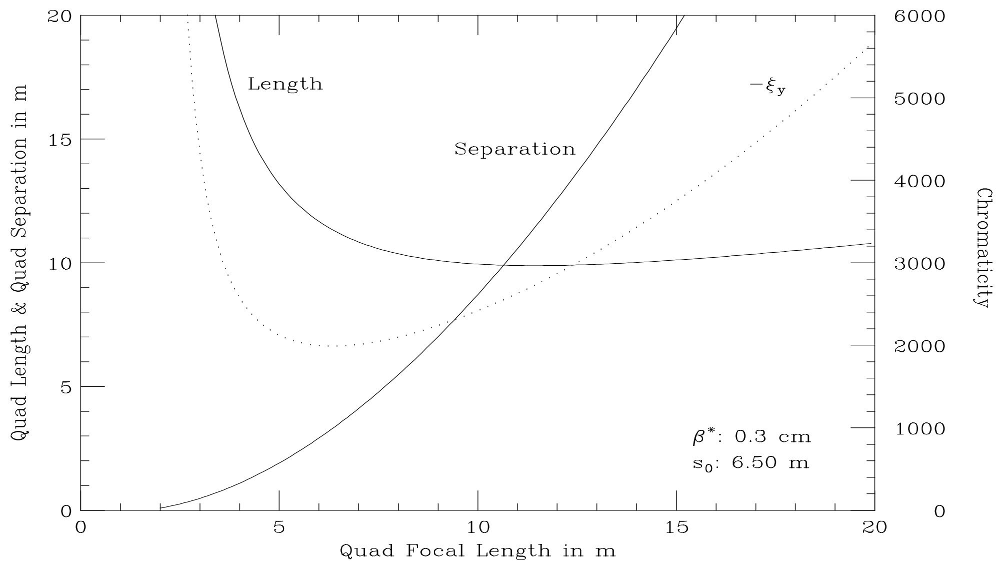

Figure 2: Quadrupole length and chromaticity versus quadrupole focal length when $\beta^* = 3$ mm and IP clearance $s_0 = 6.5$ m.

chromaticity, we should choose f at the position in the plot when L crosses d. In other words, the distance between the two quadrupoles must be made as short as possible.

IV. CLEARANCE BETWEEN IP AND QUADRUPOLE

For the detector acceptance, an open angle of $\theta \lesssim \pm 150$ mrad at the IP is required. Therefore, the clearance between the IP and the first quadrupole can actually be smaller. If the IR quadrupoles can be placed closer to the IP, the betatron functions at the quadrupoles will be smaller. However, for smaller β_x, the bending power of a quadrupole will not be so efficient. This can be seen as follows. At the center of the first IR quadrupole,

$$\beta_x \approx \frac{s^2}{\beta^*} \,. \tag{4.1}$$

The maximum strength of QF is

$$\frac{1}{f} = \frac{BL}{5(B\rho)}\sqrt{\frac{\epsilon}{\pi\beta_x}} \,. \tag{4.2}$$

The bending efficiency of the quadrupole can be defined as the change of the Twiss parameter α. Therefore, at the first IR quadrupole,

$$\Delta\alpha_x = \frac{\beta_x}{f} \approx \frac{BL}{5(B\rho)}\sqrt{\frac{\epsilon}{\pi\beta^*}}\, s \,. \tag{4.3}$$

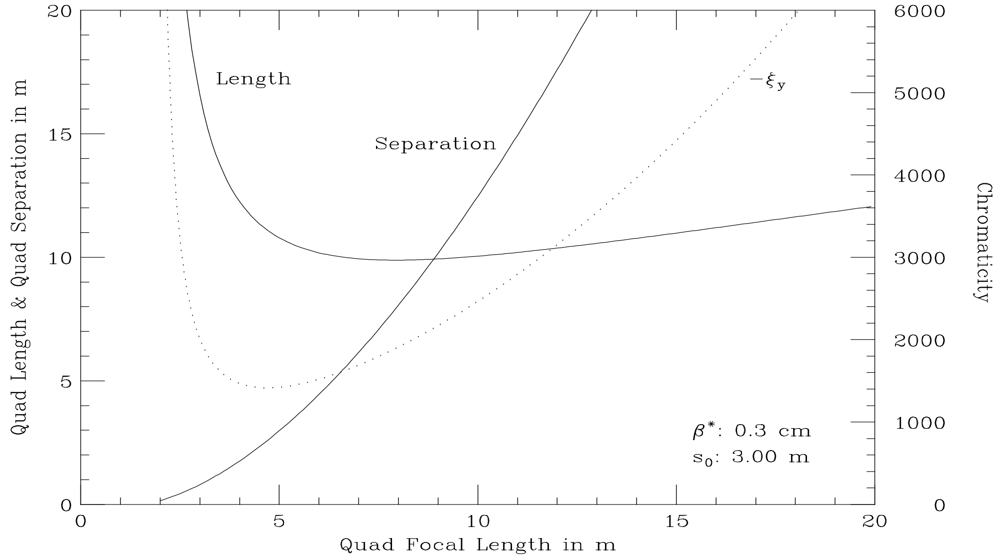

Figure 3: Quadrupole length and chromaticity versus quadrupole focal length when $\beta^* = 3$ mm and IP clearance $s_0 = 3.0$ m.

Nevertheless, the vertical chromaticity which is given by β_y at QD is in general somewhat smaller when s_0 is shortened. In Fig. 3, we make a similar plot with s_0 shortened from 6.5 m to 3 m. We see that the maximum possible strength of the quadrupoles is increased so that the focal length becomes 8.9 m. The length of the quadrupoles is still around 10 m. The chromaticity is reduced from $\xi_y = -2600$ to -2200. The reduction is only about 15%, although s_0 has been reduced from 6.5 m to 3 m. However, if we use the distance from the IP to the center of QF, this distance is actually reduced from $s = s_0 + L/2 = 11.5$ m to 8.5 m only.

In passing, it is worth pointing out that the natural chromaticity of an IR with $\beta^* = 3$ mm and $s_0 = 6.5$ is in practice of order -6000 [3], much larger than the value of -2600 quoted above. This is because due to the limitation of the quadrupole strengths, the doublet scheme is not able to lower the Twiss parameters to reasonable values to match to a normal cell or module of the collider ring. More quadrupoles are required and this raises the chromaticities.

V. LARGER LOW-BETAS

From Eq. (3.1), it is clear that chromaticity will be reduced if β^* is increased, the reason being that β_y which is inversely proportional to β^*, increases less rapidly. We perform the same plot of quadrupole length and vertical chromaticity versus the quadrupole focal length in Fig. 4, for an IR

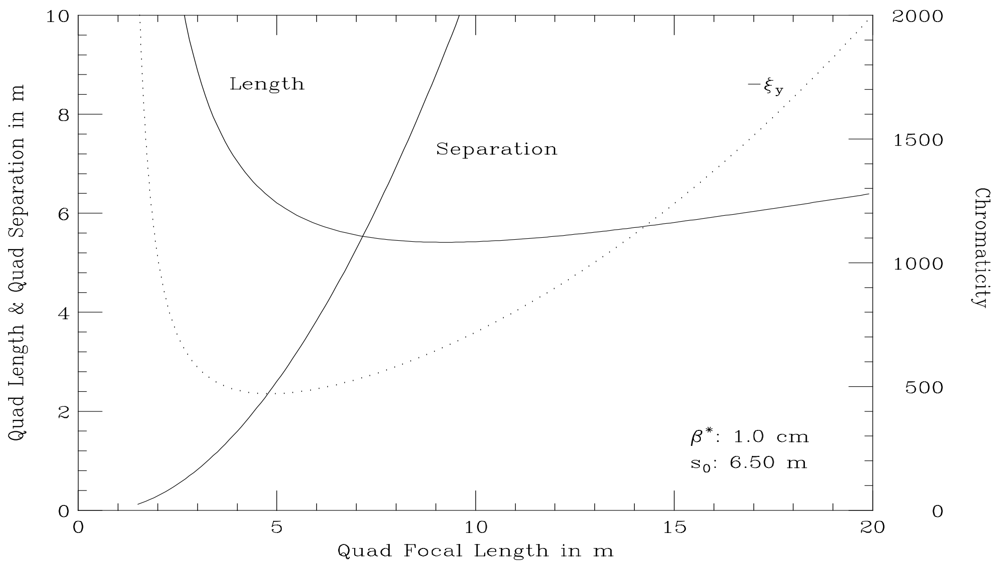

Figure 4: Quadrupole length and chromaticity versus quadrupole focal length when $\beta^* = 1$ cm and IP clearance $s_0 = 6.5$ m.

with $\beta^* = 1$ cm and IP clearance $s_0 = 6.5$ m. We see that the quadrupole length is around 5.5 m long and the focal length can be made as short as $f = 7.2$ m, which is shorter than the distance from the IP to the center of the first quadrupole, $s = s_0 + L/2 = 9.25$ m. This implies that the first quadrupole is capable of bending β_x by so much that α_x after passing through the quadrupole becomes positive, so that β_x starts dropping down after exiting this first quadrupole. In contrast, in the 3-mm low-beta IR described in Sec. II above, the first quadrupole has $f_{\min} = 10.07$ m, while $s = s_0 + L/2 = 11.4$ m, and β_x actually continues to increase after leaving the focusing quadrupole. Later, after passing through the defocusing quadrupole β_x will start to sky-rocket and another focusing quadrupole will be necessary to reverse its slope eventually. In other words, this Autin-Wildner doublet scheme is actually not feasible in the 3-mm low-beta IR.

According to Fig. 4, a vertical chromaticity as low as -500 can be achieved for the 1-cm IR. In fact, in an actual design [4], we did succeed in obtaining such an IR with $\xi_y \sim -500$.

Similarly, we plot in Fig. 5, the quadrupole length and vertical chromaticity as functions of quadrupole focal length when the low-beta is $\beta^* = 3$ cm. Now the quadrupole length can be made about 3.2 m and the quadrupole focal length $f_{\min} = 5.2$ m. Here, the first IR quadrupole is very strong. The horizontal betatron function β_x drops very fast after passing through the focusing quadrupole and does not rise significantly even after passing through the defocusing quadrupole, so that when it crosses the vertical betatron function at a distance $2s+d$ from the IP, where $\alpha_x+\alpha_y=0$, both $|\alpha_x|$ and $|\alpha_y|$ are small and can be matched easily to ordinary cells or modules of the collider. In contrast, in the situation of the 3-mm low-beta IR, although we still have $\alpha_x+\alpha_y=0$ when β_x and β_y cross according to Eq. (1.2), both $|\alpha_x|$ and $|\alpha_y|$ may be of order several thousands. Therefore, many more quadrupoles will be needed to control the betatron functions after this point, and chromaticities increase. Going back to the 3-cm low-beta IR, from Fig. 5 the vertical chromaticity can be as low as ~ -140. This prediction was also realized in an actual design [4]. There, we deviate from the Autin-Wildner model by employing a stronger focusing quadrupole and a weaker defocusing quadrupole, so that the horizontal betatron function continues to drop even after passing through the defocusing quadrupole. In this way, both β_x and β_y can be controlled more easily.

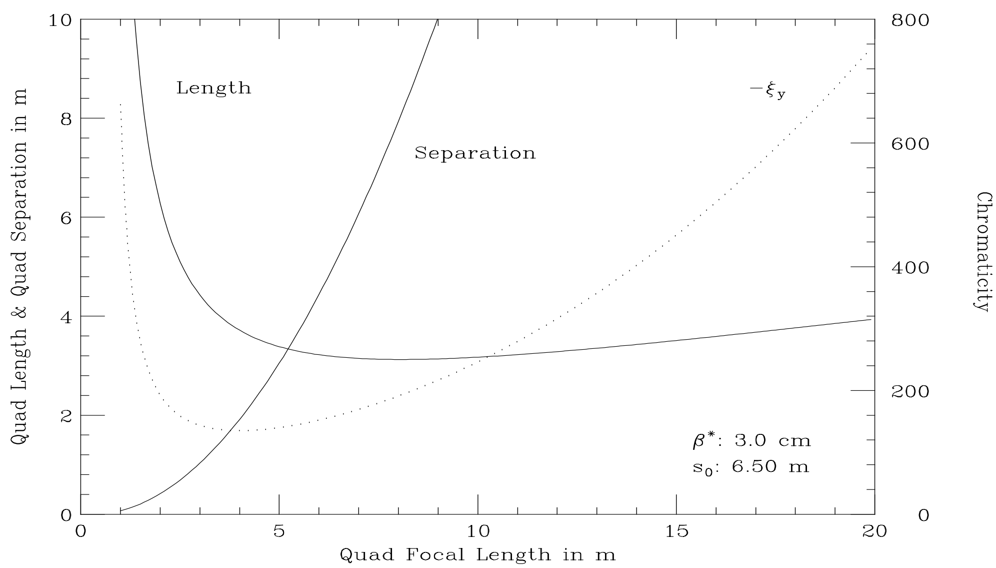

Figure 5: Quadrupole length and chromaticity versus quadrupole focal length when $\beta^*=3$ cm and IP clearance $s_0=6.5$ m.

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