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DISPERSIVE CRAB CROSSING: AN ALTERNATIVE CROSSING ANGLE SCHEME

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ABSTRACT

In order to maximize luminosity, many colliding beam storage ring designs are employing two rings, each carrying many bunches. Since the distance between bunches is small, beam-beam interactions other than the desired one at the interaction point occur unless the beams are quickly separated once outside of the high energy physics detector. One possible interaction region geometry is to have the beams collide at an angle. For luminosity degradation and beam stability reasons, this scheme has been augmented with transverse deflecting RF cavities which generate a correlation between horizontal position and time within the bunch, causing the beams to collide in a head-on fashion in their center of momentum inertial frame. The required transverse impedance, in addition to the development costs, of the transverse deflection cavities make this idea unattractive. In this paper an alternate scheme is proposed which makes use of the already existing accelerating RF cavities in the rings.

INTRODUCTION

The beam-beam interaction has the effect of limiting the product of the single bunch current and the number of interactions per bunch per turn. Therefore, to attain the high luminosities required by colliding beam storage rings such as B-meson factories, multiple bunches per beam with only one beam-beam crossing per turn is desirable. Due to limitations in magnet strengths and physical space near an interaction region, a useful optics configuration is to bring the beams into collision with a relative crossing angle.

There are two disadvantages with this scheme. First, when the crossing angle times the bunch length is comparable to the beam width a decrease in luminosity is suffered as compared to head-on collision geometries.[1] Second, when beams cross at an angle synchrobetatron resonances are excited.[2] Since the synchrotron tune of electron-positron storage rings is on the order of 0.05, excitation of synchrotron sidebands modulating major coupling resonances fills the tune plane with "forbidden" regions, reducing the beam-beam limit and hence the beam current.

In the case of linear colliders the concept of "crab crossing" was invented to circumvent the luminosity loss issue.[3] Later, crab crossing was proposed to eliminate synchrobetatron resonance excitation in storage rings.[4] In the remainder of this

paper the original scheme discussed in refs. 3 and 4 will be referred to as "transverse" crab crossing.

TRANSVERSE CRAB CROSSING

Transverse crab crossing involves correlating the horizontal and longitudinal beam emittance envelopes such that in their center of momentum frame the bunches appear to collide head-on. The correlation is accomplished through the use of a set of horizontal deflecting RF cavities set 90° in betatron phase advance from the interaction point. A diagram of this scheme appears in figure 1.

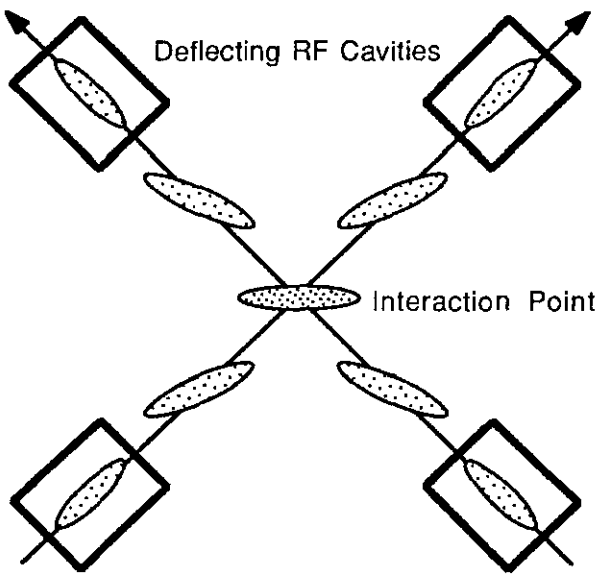

Fig. 1. Sketch of an interaction region in which an exaggerated crossing angle is assumed. The relative positions of the four horizontal deflection cavities and their effect on the beam envelopes are included.

Assume that a horizontal deflecting RF cavity of wavelength λ has an equivalent voltage amplitude V_x. The cavity is placed in a storage ring of energy E_o at a lattice location 90° upstream of the interaction point. Figure 2 contains a sketch of this interaction region, indicating the deflection applied to a bunch and the resultant beam envelope trajectory. If β^* and β_c are the values of the β-function[10] at the interaction point and the RF cavity respectively, then the crossing angle exhibited by the beam at the interaction point is given by

$$\theta = \frac{2\,\pi}{\lambda}\,\frac{e\,V_x}{E_o}\,\sqrt{\beta^*\,\beta_c} \quad . \tag{1}$$

The assumption in this equation is that the bunch length is small compared to λ, such that the RF wave is approximated by a straight line. Note that the crab angle, which should equal the crossing angle of the beam, is proportional to the square root of β^*. In flat beam e+e- interaction region designs, this functional dependence restricts crab crossing to the horizontal plane.

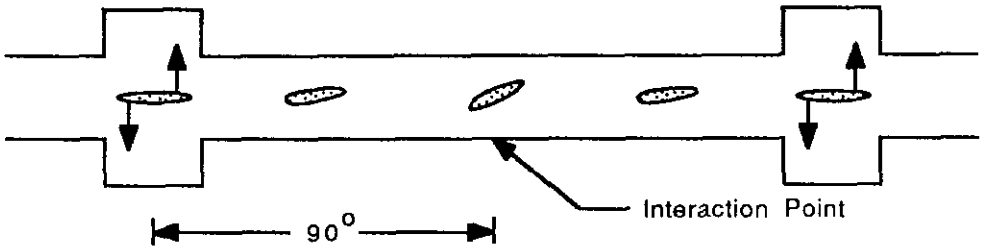

Fig. 2. Sketch of the evolution of one bunch in a single ring undergoing crab crossing. At the interaction point the deflection in the upstream deflecting RF cavity is converted into an azimuth dependent transverse offset.

Looking at figure 2, it is straightforward to visualize crab crossing by treating the head and tail of the bunch as individual particles. These particles receive opposing kicks in the upstream RF cavity, which generate free betatron oscillations. At the interaction point these oscillations reach their maximum amplitudes, and then proceed to advance another 90° at the crossing of the downstream cavity. At this point the betatron oscillation is removed.

Table I. Values of a PEP based asymmetric B-meson factory transverse crab crossing design.

Parameter	Units	Ring #1	Ring #2
E_o	GeV	3	9
eV_x	MeV	1.4	2.5
λ	m	0.42	0.42
β^*	m	0.5	0.5
β_c	m	10	10
θ	mrad	15.6	9.3

As an instructive example, take the parameters (see table I) of a recent asymmetric B-meson factory design.[5] A total of almost 8 MV of transverse voltage must be produced to produce the required crossing angle of 25 mrad.

A number of problems exist with the concept of transverse crab crossing when it is applied to storage rings. These include tolerance issues such as RF voltage and phase regulation necessary to insure beam collisions.[6,7] In addition, the implications of transverse transient beam loading[8], especially in the presence of an azimuthal bunch distribution which contains a gap to prevent ion trapping[9], has not been seriously studied yet. The addition of a transverse deflecting cavity will partial negate the great care taken to minimize the transverse impedance of the storage ring, making transverse instabilities more likely to occur. Finally, the cost and time required to research and development such deflecting cavities is substantial.

DISPERSIVE CRAB CROSSING

With the above arguments in mind, it is desirable to ponder whether any alternatives exist. Clearly, the fundamental concept of crab crossing the beams to attain functional, high luminosity beam collisions is worth preserving. In fact, an alternative method for generating the crab crossing angle is possible. The basic idea is to replace the transverse deflecting RF cavities with the acceleration RF cavities in which the horizontal dispersion is nonzero. Since these cavities already exist, many of the above objections are eliminated. This alternative scheme is called "dispersive" crab crossing.

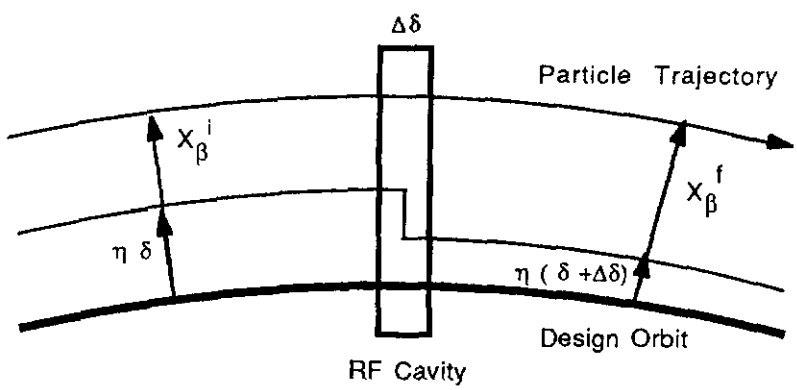

Fig. 3. Change in particle coordinates caused by an instantaneous fractional energy change $\Delta\delta$.

Relative to the design orbit, the horizontal position of a particle is the sum of its betatron position[10] (x_β) and its equilibrium orbit position[1], which is equal to the local horizontal dispersion (η_x) times the particle's fractional energy

offset ($\delta = (E-E_o)/E_o$). Upon passage through a RF cavity, assume that the particle is decelerated relative to a synchronous particle. Since the particle cannot change its physical location instantaneously, the change in equilibrium orbit position is compensated by a matching change in betatron amplitude. Figure 3 contains a sketch of this situation.

The change in betatron position, and for generality sake betatron angle, produced by a fractional energy deflection relative to a synchronous particle, are given by

$$\Delta x_\beta = - \eta_x \, \Delta\delta \quad , \quad \Delta x'_\beta = - \eta'_x \, \Delta\delta \quad . \tag{2}$$

Assuming for the time being that η_x' is zero, the change in betatron position for a small deviation away from the RF synchronous angle (ϕ_s) is

$$\frac{\Delta x_\beta}{s} = \frac{2\,\pi}{\lambda} \frac{e\,V_o\,\eta_c}{E_o} \cos\phi_s \quad , \tag{3}$$

where V_o is the amplitude of the RF voltage and η_c is the value of the dispersion at the cavity. Note that the expression in equation 3 is also the instantaneous betatron angle the beam envelope develops at the RF cavity. Propagating the betatron oscillation of the beam envelope 180° downstream to the interaction point, the equation for the crab angle of the beam is

$$\theta = \frac{2\,\pi}{\lambda} \frac{e\,V_o\,\eta_c}{E_o} \sqrt{\frac{\beta^*}{\beta_c}} \cos\phi_s \quad . \tag{4}$$

Just as in the case of transverse crab crossing, the crab angle scales with the square root of β^*. On the other hand, the angle now depends on the ratio of η_c and the square root of β_c. For the same example B-meson factory used in the discussion of transverse crab crossing, table II contains parameter values for a dispersive crab design.

In the design scenario outlined in Table II, the available RF voltages are 34.2 and 21.3 MV for rings #1 and #2, respectively. Since the synchronous phase angles in the two rings are on the order of a few degrees, they have been neglected. Note that the RF voltages required to generate the same crab angles are small compared to the total amount of RF available. Therefore, only 5 to 7 cavities need to be placed on either side of the interaction region in both rings. The majority of the cavities can be placed at any other convenient azimuth around the rings.

Table II. Values of a PEP based asymmetric B-meson factory dispersive crab crossing design.

Parameter	Units	Ring #1	Ring #2
E_o	GeV	3	9
eV_o	MeV	4.7	8.3
λ	m	0.42	0.42
β^*	m	0.5	0.5
β_c	m	10	10
η_c	m	3	3
θ	mrad	15.6	9.3

The next issue to contemplate is the placement of the cavities near the interaction region. Since the already existing cavities are being used, phase stability must be a consideration. The RF cavities should be placed such that the beam is always at the stable, synchronous phase angle. This requires that if the upstream cavity is $180^o + n \cdot 360^o$ away from the interaction point, the downstream cavity must be $m \cdot 360^o$ away in order to close the crab induced free betatron oscillations (where m and n are arbitrary integers). Figure 4 contains a sketch of such an interaction region geometry.

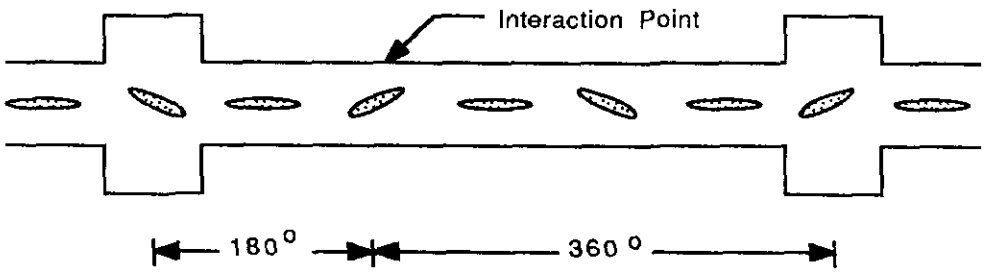

Fig. 4. Example of a dispersive crab crossing interaction region design for a single beam line. Note the asymmetric placement of the RF cavities to insure that the beam always crosses at the stable synchronous phase angle.

SYNCHROBETATRON COUPLING

Potentially the most serious drawback to dispersive crab crossing is the excitation of synchrobetatron coupling due to the existence of horizontal dispersion in the RF cavities.[11] As discussed in detail above, passage through an RF cavity in which dispersion is nonzero causes a horizontal betatron kick. The resultant free betatron oscillation around the ring generally has a different path length as compared to a particle with the same energy but not undergoing betatron oscillations. This causes the particle to arrive at the cavity the next turn with a time delay, determining the next RF/betatron kick to be received. The betatron oscillation dependence of the path length is due to the fact that particles to the outside of the design orbit have a longer path length inside dipole magnets. If the synchrotron (Q_s) and betatron (Q_x) tunes have the relationship

$$i\,Q_x + j\,Q_y = k \quad , \qquad (5)$$

where i, j, and k are integers, the particle motion is unstable.

The widths of these synchrobetatron resonances are beam current independent, and scale as η_c divided by the square root of β_c (exactly the same dependence as the crab angle). But in the dispersive crab crossing scheme the betatron oscillations induced by the RF cavities are purposely cancelled for the majority of the accelerator. Therefore, if the interaction region dipoles between the crab cavities are placed such that the path length through the region is independent of betatron amplitude (a task made significantly easier by the asymmetric placement of the cavities on either side of the interaction region), synchrobetatron resonances will not be excited.

Even if the path length modulation is not compensated, one usually finds that other mechanisms[11], usually beam current dependent, are the dominant sources of synchrobetatron coupling.[12] For example, at CESR the ratio of η_c to the square root of β_c is roughly 3/6 $m^{1/2}$. And yet, attempts to find the source of synchrobetatron resonance excitation have always led away from the RF cavities.[13]

COHERENT INSTABILITIES

Because existing RF cavities are used in the dispersive crab crossing scheme, the introduction of longitudinal impedance by the cavities must already be dealt with. As a result, dispersive crab crossing does not induce a cost with respect to longitudinal instabilities.

In fact, the same can be said with respect to transverse instabilities. In order to minimize the impact of RF cavity generated transverse wake fields, the value of β_c should be as small as possible. But this is exactly the direction required to maximize the crab crossing angle.

TRANSIENT BEAM LOADING

Transient beam loading occurs when beams with high currents are accelerated through RF cavities with a very large shunt impedance, and is a potential problem for at least two reasons. First, potential well distortion[8] disrupts the linear dependence of RF voltage on arrival time at the cavity, destroying the center of momentum frame head-on collision geometry which crab crossing is intended to generated. Second, the gap in the azimuthal bunch distribution around the ring designed to prevent ion trapping will produce a bunch dependent beam loading voltage which is out of phase with the RF drive. The result is that without active compensation, each bunch will see a unique RF phase. Each bunch would then receive a different average energy kick, leading to bunch dependent interaction point offsets in the horizontal plane.

For higher order mode reasons, the RF cavities considered for B-meson factories have recently been designed with very small shunt impedances.[15] As a result, the above two concerns should be greatly diminished. In addition, feed-forward transient beam loading compensation systems are quite common and are well understood.

CONCLUSIONS

Though much work still needs to be done, it is already clear that dispersive crab crossing has many advantages, and no substantial disadvantages, when compared to transverse crab crossing. As in the case of transverse crab crossing, systematic simulation studies of tolerances and impedance effects (such as transient beam loading) need to be done. Such simulation studies are presently underway by the author (dispersive) and A.Piwinski[14] (transverse). Comparisons of these results with experimental results and with one another should be a future priority.

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