Fermi National Accelerator Laboratory

FN-517

Longitudinal Emittance Blow-Up Due to Coherent Motion of Coupled Bunches

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June 1989

LONGITUDINAL EMITTANCE BLOW-UP DUE TO COHERENT MOTION OF COUPLED BUNCHES

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Abstract

This note describes phase-space dilution effects due to the presence of a single coupled bunch mode driven by a sharp parasitic resonance of rf cavities. A longitudinal phase-space simulation of the net emittance blow-up is carried out for various beam intensities. The resulting plot (final emittance vs bunch intensity) reveals exponential emittance growth.

1. Introduction

The physical presence of vacuum structures can be expressed in terms of wake fields experienced by the beam. The beam environment considered here consists of parasitic higher order modes of the r.f. cavities. These resonances may have high enough Q's to allow consecutive bunches to interact through mutually induced wake fields. The cumulative effect of such fields as the particles pass through the cavity may be to induce a coherent buildup in synchrotron motion of the bunches, i. e. a longitudinal coupled-bunch instability[1].

The colliding mode operation of the present generation of high energy synchrotrons and the accompanying r.f. manipulations, make considerations of individual bunch area of paramount importance. Thus, a longitudinal instability in one of a chain of accelerators, while not leading to any immediate reduction in the intensity of the beam in that accelerator, may cause such a reduction of beam quality that later operations are inhibited (resulting in a degradation in performance).

In this study we employ a longitudinal phase-space tracking code (ESME)[2] to simulate the longitudinal emittance blow-up caused by specific coupled bunch modes arising in the Fermilab Booster. One of the obvious advantages of the simulation compared to existing analytic formalisms, e.g. based on the Vlasov equation[3], is that it allows consideration of the instability in a self-consistent manner with respect to the changing accelerating conditions. Furthermore this scheme allows to model nonlinearities of the longitudinal beam dynamics, which are usually not tractable analytically.

Included in the simulation is the investigation of some universal scaling of the resulting beam emittance with increasing bunch intensity.

2. Longitudinal Phase-Space Tracking with High-Q Resonances

The tracking procedure used in ESME consists of turn-by-turn iteration of a pair of Hamilton-like difference equations describing synchrotron oscillation in $\theta-\varepsilon$ phase-space ($0 \leq \theta \leq 2\pi$ for the whole ring and $\varepsilon = E - E_o$, where E_o is the synchronous particle energy). Knowing the particle distribution in the azimuthal direction, $\rho(\theta)$, and the revolution frequency, ω_o, after each turn, one can construct a wake field induced voltage generated by a single parasitic resonance.

Based on the analytic model of coupled bunch modes proposed by Sacherer[1] one can formulate a simple resonance condition for the m-th dipole mode driven by the longitudinal impedance $Z(\omega)$ sharply peaked at ω_c. This condition is given by:

$$\omega_c = (nM + m)\, \omega_o \pm \omega_s \ ,$$

where n is an integer. Since ω_o is time dependent (acceleration) and ω_c is fixed (geometry), and knowing that the width of the impedance peak is governed by ω_c/Q one can clearly see that the above resonance condition is maintained over a finite time interval. This serves as a guide in the simulation since it allows us to select an appropriate time domain where the mode of interest crosses the resonance and will more likely become unstable.

In the early stages of this study[4] we tentatively identified the parasitic resonance at $f_c = 87.7$ MHz with $Q = 3311$ and the shunt impedance $Z_s = 9.14\times10^6$ Ohm as the offending part of the impedance giving rise to a coupled bunch instability with harmonic number $m = 56$. This mode crosses the resonance later in the booster cycle; at time 22.42×10^{-3} sec and it takes 0.59×10^{-3} sec to sweep through the full width at half maximum of the resonance. Therefore the appropriate time interval to study the $m = 56$ mode is chosen as $20 - 25\times10^{-3}$ sec. The r.f. system of the Fermilab Booster provides 84 accelerating buckets. As a starting point for our simulation each bucket in $\theta-\varepsilon$ phase-space is populated with 100 macro-particles according to a bi-Gaussian distribution matched to the bucket so that 95% of the beam is confined within the contour of the longitudinal emittance of 2×10^{-2} eV–sec. Each macro-particle is assigned an appropriate effective charge to simulate the desired beam intensity.

3. Summary – Emittance Blow-up

The simulation results (in terms of histories of selected phase-space characteristics) are collected in Figs.1–4. They correspond to increasing bunch intensities of: 6×10^9 ppb, 6×10^{10} ppb, 4×10^{11} ppb and 6×10^{11} ppb respectively. The top part of each figure ,a), represents motion of a single bunch centroid which is a signature of a specific coupled bunch mode developing in the system. The lower plot, b), illustrates the resulting longitudinal emittance blow-up triggered by the instability. In the low intensity region (See Figs.1 and 2) the collective motion of all 84 bunches is perfectly coherent. Although its amplitude is reaching macroscopic values (comparable to the bucket size) the beam is still confined to the stable phase-space regions at the end of the cycle. The last two intensities (See Figs.3 and 4) correspond to the incoherent region; initially coherent synchrotron motion of individual bunches builds up very quickly to the threshold value and substantial fraction of the beam is kicked out the buckets by its own wake field. The trapped beam is still undergoing synchrotron motion but the coupling between bunches is highly diminished. The remaining particles float in the unstable region resembling a coasting beam at the end of the cycle.

The above results are summarized in Fig.5. One can see an exponential behavior of the final emittance with increasing intensity, which agrees with earlier experimental predictions[5].

References

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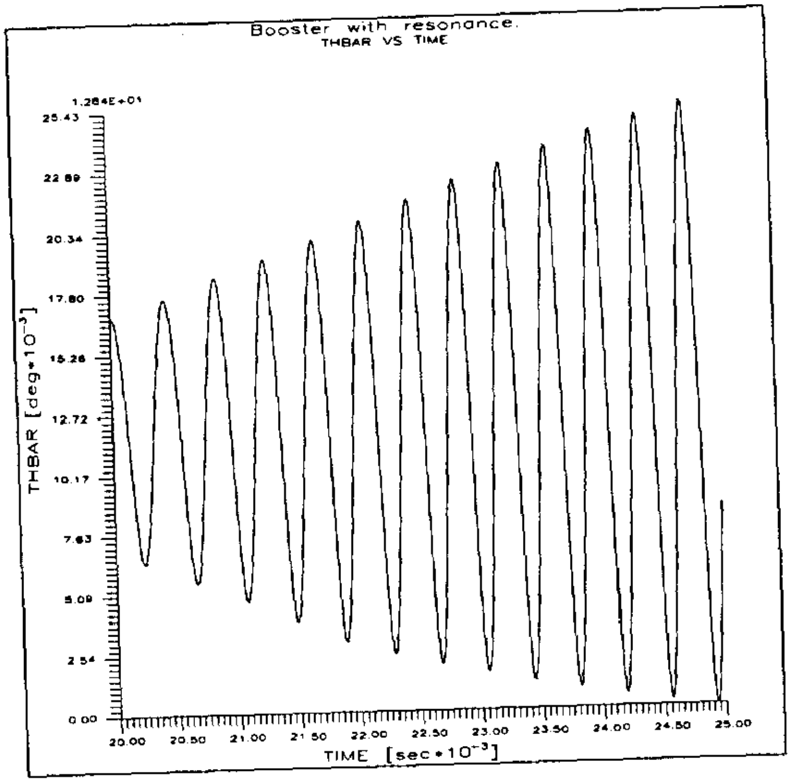
[3] S. Krinsky and J.M. Wang, Particle Accelerators, **17**, 109 (1985)

[4] S. Stahl and S.A. Bogacz, Phys. Rev. D, **37**,1300 (1988)

[5] M. Harrison, Private Communication

$6{\times}10^9$ ppb

a)



b)

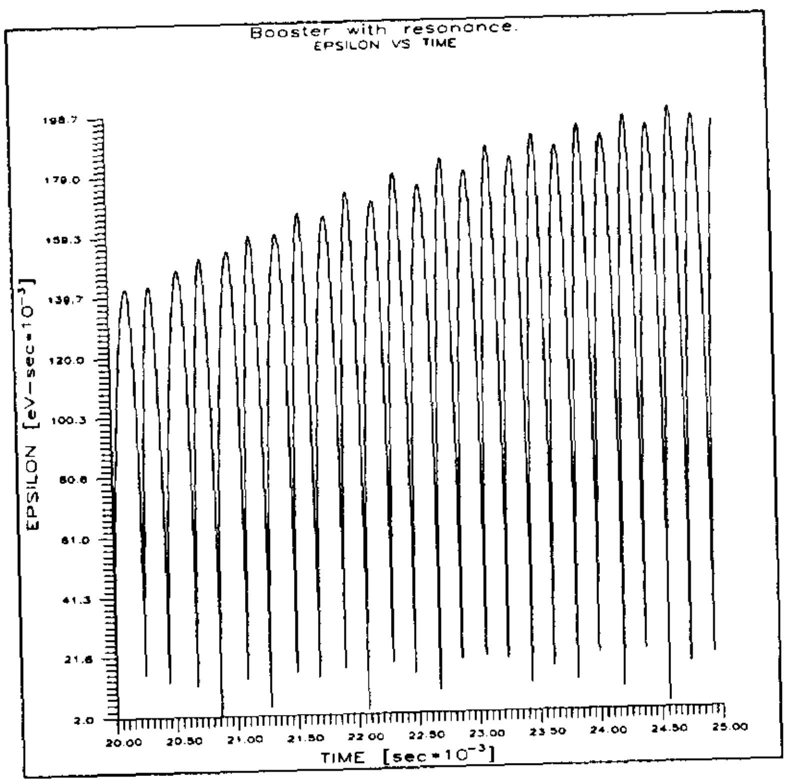


Fig.1

a)

6×10^{10} ppb

b)

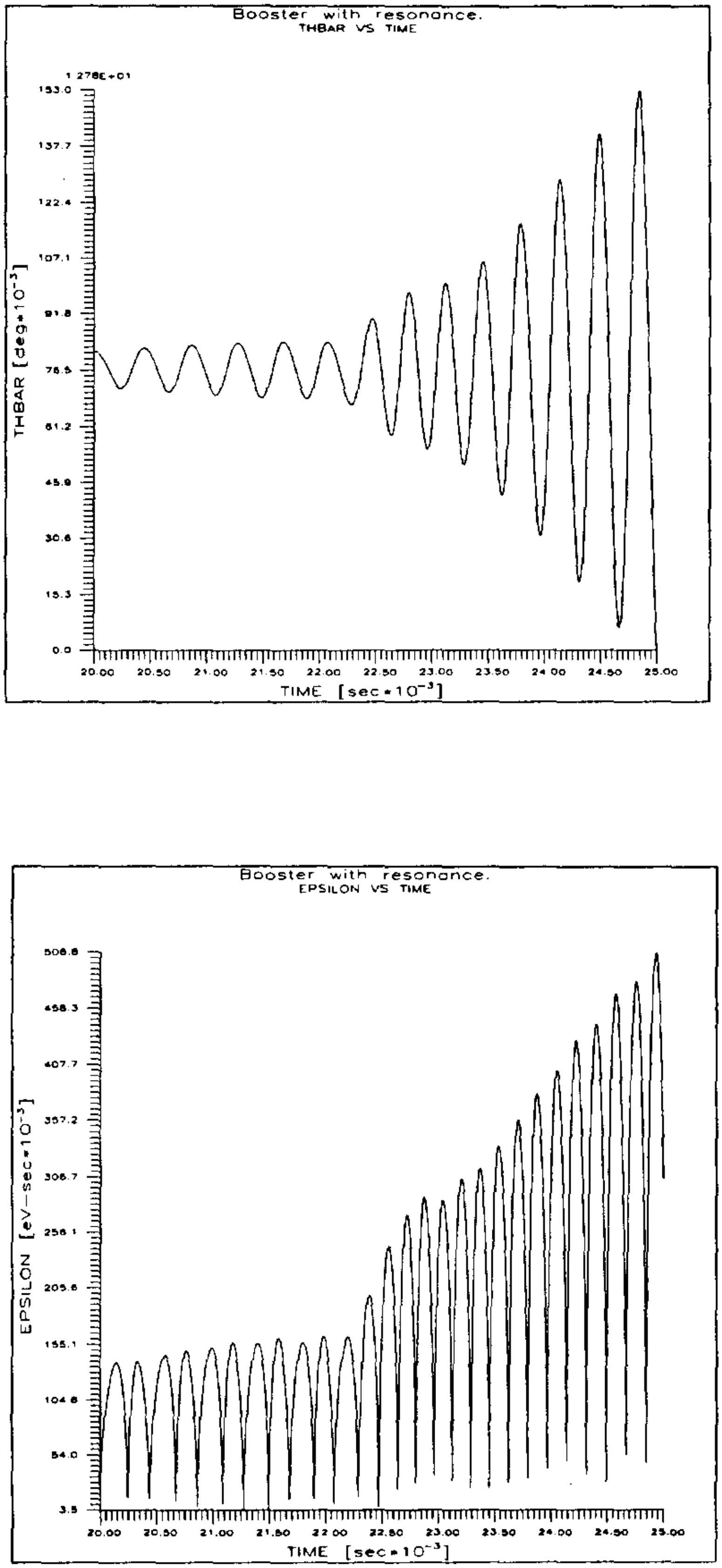
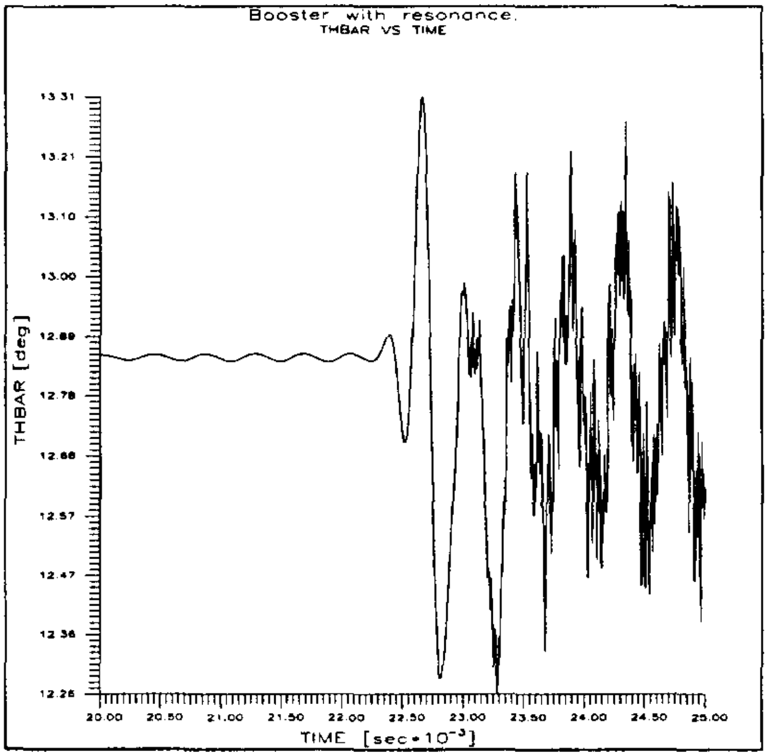


Fig.2

4×10^{11} ppb

a)



b)

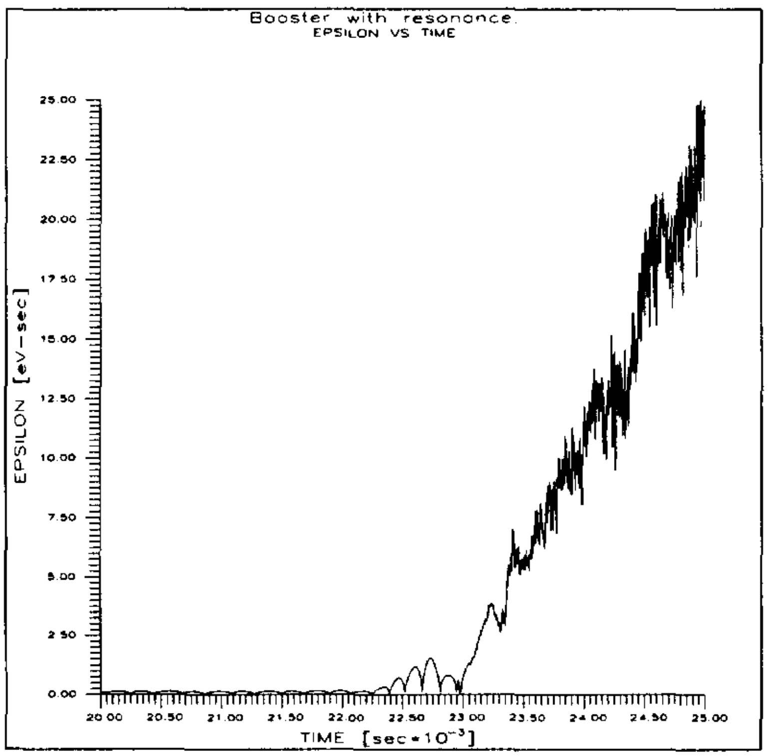
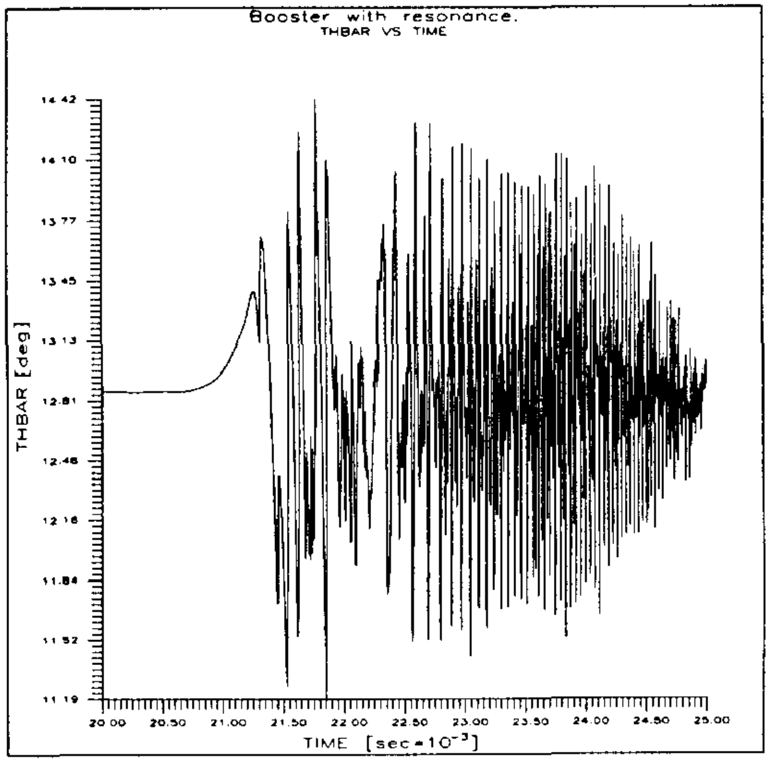


Fig.3

a)

6×10^{11} ppb



b)

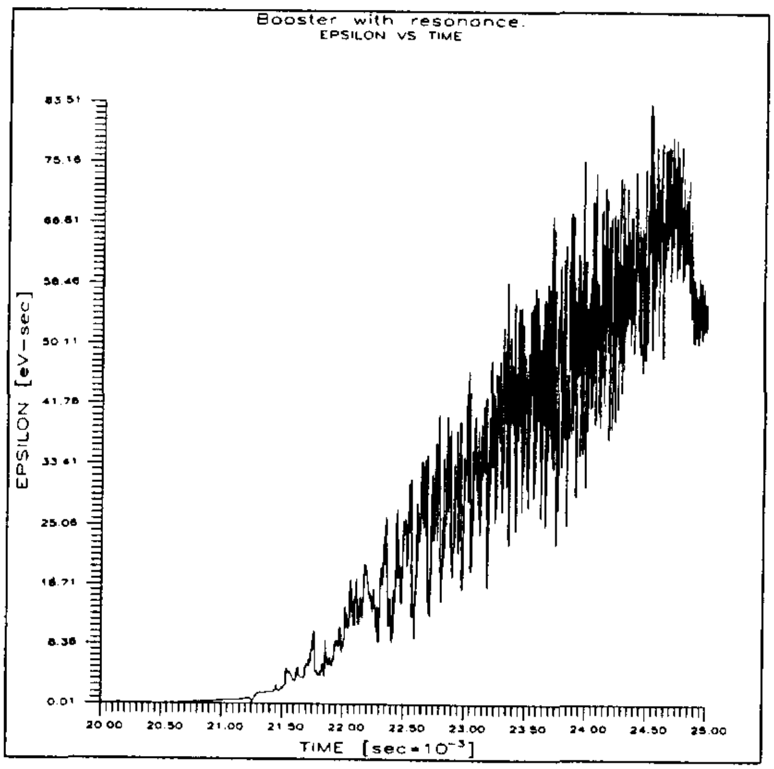


Fig.4

Emittance Blow-up - Coupled Bunch Instability

(Booster with one parasitic resonance)

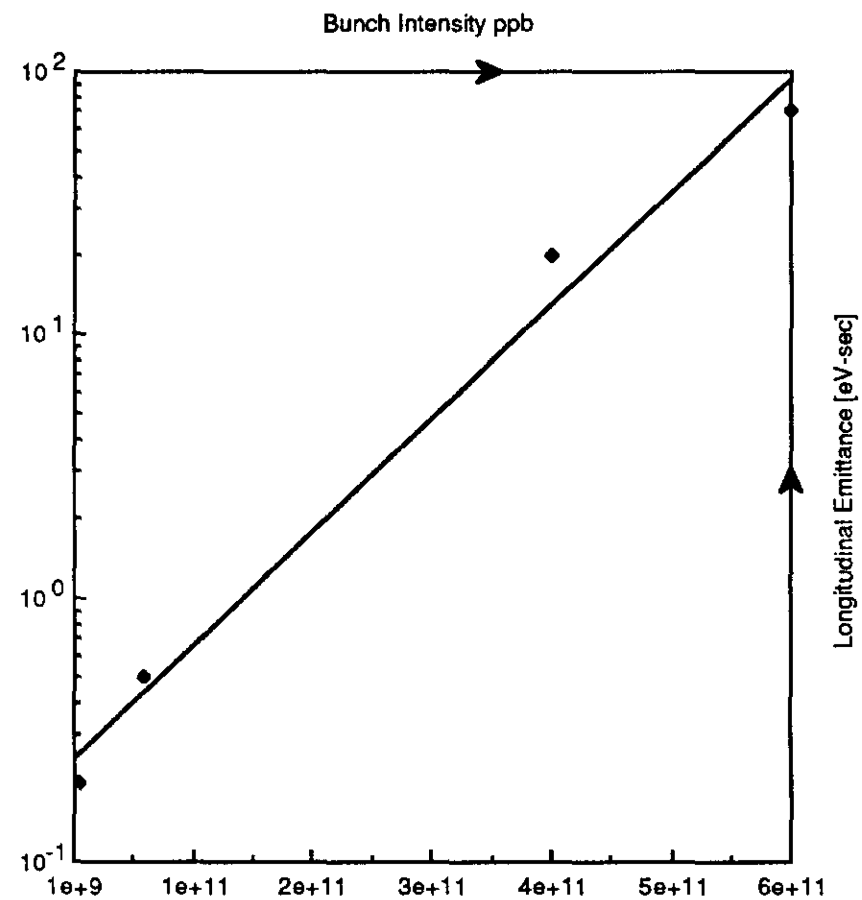


Fig.5