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FN-432
0401.000

MICROWAVE INSTABILITY CRITERION FOR OVERLAPPED BUNCHES*

K.-Y. Ng
Fermi National Accelerator Laboratory, Batavia, Illinois 60510

May 1986

*Presented at the Second Conference on the Intersections Between Particle and Nuclear Physics, May 26-31, 1986, Lake Louise, Canada.

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

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King-Yuen Ng
Fermi National Accelerator Laboratory[1], Batavia, IL 60510

ABSTRACT

Debunching can be a method to measure Z/n of a storage ring by timing the start of microwave instability. However, if this instability begins to show up when two or more bunches overlap each other, the situation becomes more complex, because one is confused of which local current and energy spread should be used. An analysis shows that exactly the same microwave instability criterion should be used as if there is only one bunch.

INTRODUCTION

During debunching, the energy spread of a bunch becomes smaller and smaller. Eventually, Landau damping fails and microwave instability starts. By measuring the time when instability starts, the Z/n of the storage ring can be inferred. However, this instability may start when two or more bunches overlap each other. One may wonder whether one should take the total energy spread of the bunches or the RMS energy spread of one bunch in the Keil-Schnell criterion. Also, one is not sure whether the total local current of the overlapped bunches or the local current of a single bunch should be used in the criterion. This problem is solved in this paper[2].

THE DISPERSION RELATION

Consider two overlapped Gaussian bunches as shown in Fig. 1. At any azimuthal point in the overlap, the dispersion relation is

$$1 = -(\Delta\Omega_o/n)^2 \int F'(\omega)/(\Delta\Omega/n-\omega)\,d\omega, \qquad (1)$$

where $\Delta\Omega_o/n = [ie\eta\omega_o^2 I_t(Z/n)/(2\pi\beta^2 E)]^{1/2}$ is the growth without Landau damping, η the frequency dispersion coefficient, ω_o the revolution frequency, β the velocity of a bunch particle of energy E in unit of c, and $\Delta\Omega/n$ is the coherent frequency per revolution harmonic of the perturbing wave in excess of ω_o. Note that we have used the total local current I_t which is equal to the sum of the local currents I_1 and I_2 of the two individual bunches. The normalized frequency distribution function is

$$F(\omega) = (\sqrt{2\pi}\sigma)^{-1}\{a_1 \exp[-(\omega-\omega_1)^2/2\sigma^2] + a_2 \exp[-(\omega-\omega_2)^2/2\sigma^2]\}, \qquad (2)$$

where σ is the RMS revolution frequency spread of each bunch which is considered to be Gaussian, ω_1, ω_2 are respectively the mean deviations of revolution frequencies of the bunches from that of a synchronized particle (we take $\omega_1<0$ and $\omega_2>0$). The fraction of each bunch in the overlap is represented by $a_i = I_i/I_t$, i = 1,2.

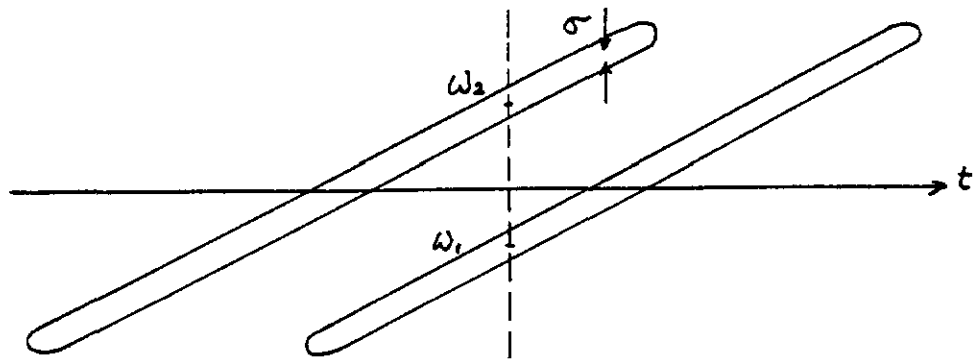

Figure 1

Let us consider the case when Z/n is imaginary; i.e., $(\Delta\Omega_o/n)^2$ is real. Then the thresholds are given by

$$1 = -(\Delta\Omega_o/n)^2 \int F'(\omega)/[\mathrm{Re}(\Delta\Omega/n)-\omega]d\omega, \tag{3}$$

where $\mathrm{Re}(\Delta\Omega/n)$ is any of the 3 zeros of $F'(\omega)$ which are ω_1, ω_2 and another one in between. Equation (3) can be solved exactly:

$$(\Delta\Omega_o/n)^{-2} = -\sigma^{-2}[1-i\sqrt{\pi}a_1u_1w^*(u_1)-i\sqrt{\pi}a_2u_2w^*(u_2)], \tag{4}$$

where $u_i = (\Delta\Omega/n-\omega_i)/\sqrt{2}\sigma$ and $w(u)$ is the complex error function. Then, at one zero, for example, $u_1=0$, Eq. (4) becomes

$$(\Delta\Omega_o/n)^{-2} = -\sigma^{-2}[1-2a_2K\exp(-K^2)\int_0^K\exp(t^2)dt], \tag{5}$$

where $K=\Delta\omega/\sqrt{2}\sigma$ and $\Delta\omega=|\omega_1-\omega_2|$. During debunching, we always have $2K>>1$; Eq. (5) can therefore be simplified to

$$(\Delta\Omega_o/n)^{-2} = -\sigma^{-2}\{1-a_2[1+(\sigma/\Delta\omega)^2]\} = -a_1/\sigma^2+a_2/\Delta\omega^2, \tag{6}$$

Neglecting the last term and putting in the relation between σ and the RMS energy spread σ_E of a bunch, we get

$$ie\eta\omega_o^2I_t(Z/n)/(2\pi\beta^2E) = -(\eta\omega_o^2\sigma_E/E)^2/a_1. \tag{7}$$

Recalling that $I_1 = a_1I_t$, this is just the same stability criterion of a single Gaussian bunch with RMS energy spread σ_E and local current I_1. Similarly, with $u_2 = 0$, we obtain the same stability criterion with σ_E and I_2 for the second bunch.

This result can also be visualized as follows. Consider two coasting beam with frequencies $\omega_o+\omega_{1,2}$ and each has a RMS spread of σ. Imagine a small perturbing current wave of the form $\exp(in\theta-i\Omega t)$ where θ is the azimuthal angle around the accelerator ring. If the coherent frequency $\Omega\sim n(\omega_o+\omega_1)$, it will set the particles in the first beam to oscillate with harmonic n and eventually lead to a growth if σ is not large enough to destroy the coherency. If $\sigma<<|\omega_1-\omega_2|$, the particles in the second beam will not be affected. On the other hand,

if $\Omega \sim n(\omega_0+\omega_2)$, it can only drive a growth of harmonic n in the second beam while the first one will not affected. Thus the stability criterion applies to each beam individually. In debunching, the bunches are long and resemble coasting beams so we expect the same reasoning applies to overlapped bunches as well.

THE STABILITY CURVE

The stability curve in the $(\Delta\Omega_0/n)^2$-plane is shown in the Fig. 2 with $(\omega_1/\sqrt{2}\sigma)^2=10$, $|\omega_1|=\omega_2$ and $\alpha_1=\alpha_2$. It wraps around the origin twice in two Riemannn sheets as the real part of the coherent frequency shift $\Delta\Omega/n$ increases, the cut being the positive imaginary axis. The real coherent frequency shift $Re(\Delta\Omega/\sqrt{2}n\sigma)$ is marked along the curve. For the shake of clarity, only one half of the curve is plotted. The other half is just an mirror image about the cut. The two identical intercepts it makes with the negative imaginary axis in two the different sheets correspond to $Re(\Delta\Omega/n) = \omega_{1,2}$ for the two bunches if Z/n is capacitive. The intercept it makes with the positive imaginary axis corresponds to the threshold criterion of Eq. (4) using the third zero of $F'(\omega)$ and corresponds to substituting $\sim\Delta\omega$ in the stability criterion and is therefore $(\Delta\omega/2\sqrt{2}\sigma)^2$ farther away from the origin than the two other intercepts. Any Z/n corresponding to a point inside the center region of the curve is completely stable. Thus, different from the situation of a single bunch, a big enough inductive Z/n above transition can also lead to instability.

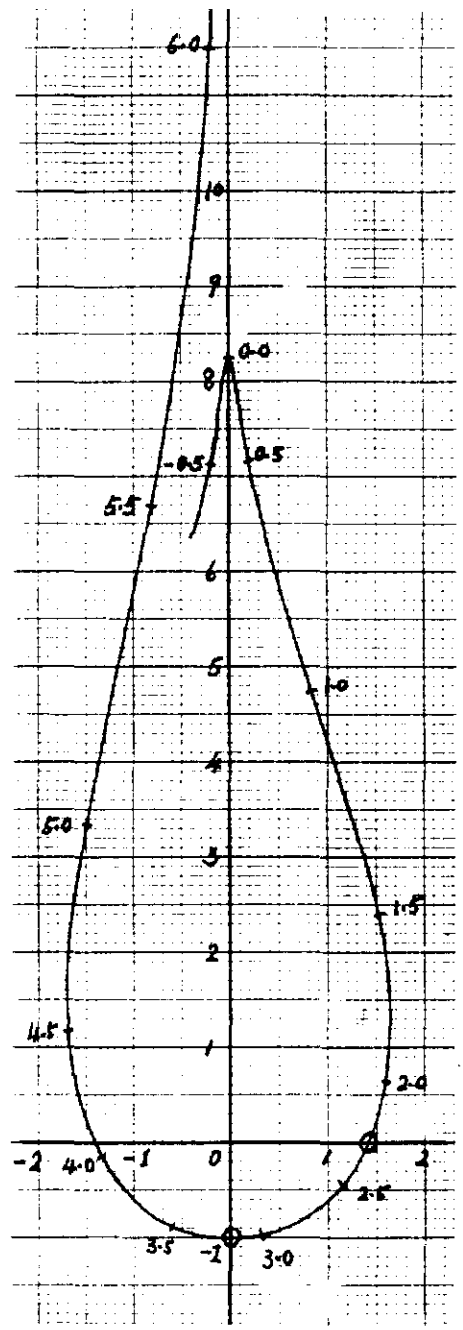

Figure 2

1. Operated by the Universities Research Association, Inc., under a contract with the U.S. Department of Energy.
2. For the debunching experiment and other analysis, see K. Y. Ng Fermilab Report TM-1389.