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AN EXPLORATION OF THE FREQUENCY DEPENDENCE OF MODULATION BLOWUP IN $\bar{p}p$ COLLIDING BEAMS

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An Exploration of the Frequency Dependence of Modulation Blowup in $\bar{p}p$ Colliding Beams

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Abstract

The dependence of beam blow-up in $\bar{p}p$ colliding beams with tune modulation upon the modulation frequency has been explored with a computer simulation. It is determined that a threshold frequency ν_{TH} exists; modulations at frequencies greater than ν_{TH} do not lead to beam blow-up. The relationship between this threshold and colliding beams parameters is discussed.

Introduction

In a previous paper, we had noted that computer simulations of weak-strong $\bar{p}p$ collisions show the possibility of beam blow-up at large modulation amplitudes. In this paper we report results of an investigation of the dependence of this blow-up upon the modulation frequency, with the modulation amplitude fixed.

The basic features of our simulations of the beam-beam interaction have been described previously,[1] and we will only summarize these briefly in the present paper. Particle transport around an accelerator ring is simulated as the product of two transformations: linear transport around the rings represented by a "Courant-Synder"[2] (C-S) matrix with the C-S parameters β_x, β_y and tunes ν_x, ν_y; and a nonlinear beam-beam kick determined by assuming a "zero-length" - "weak-strong" interaction with a round gaussian "strong" beam, with the strength of the interaction determined by the "beam-beam tune shift" $\Delta\nu$. $\Delta\nu = .01$ is used in this paper, a large but possible value for $\bar{p}p$ collisions.

Tune modulation is simulated by changing the tunes ν_x and ν_y from turn to turn following

$$\nu_x = \nu_{xo} + a_x \sin w_x t$$
$$\nu_y = \nu_{yo} + a_y \sin w_y t$$

with $w_x = w_y = 2\pi/Nm$, where Nm is the modulation period in number of turns, and where $t = N-1$, N = turn number $N = 1,2,3...$ In the previous paper,[1] $\nu_{xo} = .3439$, $\nu_{yo} = .1772$ was chosen, a "resonance-free" case with long-time stability.[3] The parameters a_x and a_y were varied and Nm was set at 1000, a

modulation period expected at $\bar{p}p$ colliders. A large modulational blow-up was noted for the cases $a_x = -a_y$, $a_x > .004$, where the modulation carries the beam across low-order resonances (3rd and 6th order).

In the present paper we fix ν_{xo} and ν_{yo} at the same values (.3439, .1772) and fix $a_x = -a_y = .005$, a case known to have modulational blow-up at Nm = 1000, and vary Nm from Nm = 8 to Nm = 100,000. Figure 1 shows the time space swept by the modulation in this case, showing the low order resonances which are crossed.

Simulation Procedure

In our analysis of stability of particle motion two types of simulations have been used: "long-time" simulations and "reversibility tests". These have previously been described in some detail[1,3,4] and are briefly described below.

In a "long-time" simulation 100 initial particle positions are chosen randomly within a 4-D phase space gaussian distribution, and trajectories generated from these initial conditions are tracked for millions of turns of the storage ring. In the present paper 3 million turn long-time simulations were generated in each case, corresponding to 1 minute of beam storage in the Tevatron I. RMS x,y, and r emittances (ϵ_x, ϵ_y and $\epsilon_r = \epsilon_x^2 + \epsilon_y^2$) for the 100 particle ensemble were calculated every 2000 turns after the first thousand turns and changes in these values were monitored. "Doubling times" are calculated from these emittance values from a least squares fit, which obtains a slope or rate of increase (decrease) of these values. The "doubling time" is the average emittance divided by this slope, so a short doubling time indicates beam blow-up.

Individual particle trajectories are also monitored for significant amplitude changes.

In previous simulations of modulational beam blow-up, the blow-up was associated with the appearance of "chaotic" trajectories, where a "chaotic" trajectory is a particle trajectory which diverges exponentially in time from nearby trajectories in phase space. "Reversibility" tests are performed to detect these trajectories. In these tests trajectories are tracked forward 100,000 turns and returned by reversing the transformations, and forward and return positions are compared. "Chaotic" trajectories develop deviations exponentially and, in these cases, can accummulate errors of order unity in 100,000 turns. "Non-chaotic" cases develop errors linearly, and, in our double precision tests, develop errors of order 10^{-20}. A clear empirical separation between these two types is obtained.

Reversibility Test

Reversibility tests for a set of 100 particles were run for Nm = 8, 16, 32, 64, 100, 200, 400, 800, 1000, 2000, 4000, 10,000 and 100,000. For Nm $\leq$ 10,000, the tests included 100,000 forward and return turns. For Nm = 100,000 ten periods were tracked (1,000,000 turns). In each test trajectories were classified as chaotic when the error over the full test length exceeded 10^{-18}. The results of these repeatability tests are summarized in Table I.

A gradual increase in the number of chaotic trajectories from zero for Nm = 16 can be seen with a maximum of $\sim$ 50 (out of 100) for Nm $\geq$ 800. The number appears to remain constant or decrease very slowly for larger Nm

(low modulation frequency). This is illustrated graphically in Figure 2.

A complementary behavior can be seen in the behavior of the Liapunov exponents of the trajectories. The Liapunov exponent is the factor A which appears in the error growth equation

$$\Delta = \Delta_o \, e^{AN}$$

where Δ_o is an initial error and Δ is the exponentially enhanced error at turn N. For Nm 2000, A does not show any significant dependence on Nm for those trajectories that are chaotic with $A \simeq 1.0 \times 10^{-3}$ on average. For Nm > 2000 a noticeable decrease in A occurs, with $A \simeq 10^{-4}$ for Nm = 10,000 **and $A \simeq 10^{-5}$ for Nm = 100,000, apparently approaching a $\sim$ 1/Nm dependence. This behavior is displayed graphically in Figure 3.**

"Long-Time" Simulation Results

We have completed "long-time" simulations for values of Nm = 8 to Nm = 100,000. These simulations included > 3 million turns (1 minute Tevatron time). The results are summarized in Table II, where mean emittance values and "doubling times" are tabulated.

The mean emittance values $\bar{\varepsilon}_x$, $\bar{\varepsilon}_y$ are averaged over the 200,000 turns previous to the indicated turn numbers (N = 0.2, 1., 3. million) where measurements of $\bar{\varepsilon}$ for the 100-particle ensemble are taken every 2000 turns. The "doubling times" T are found from the inverse of the slope of a least squares fit to ε_x, ε_y, $\varepsilon_r = \sqrt{\varepsilon_x^2 + \varepsilon_y^2}$ as a function of time; that is

$$\varepsilon_x = \bar{\varepsilon}_{x_o} + \frac{AN}{N_o} \quad \text{and} \quad T_x = \frac{\bar{\varepsilon}_x}{A} \text{ minutes}$$

where N_o = 3 million turns = 1 minute real time.

All emittance values measured before the indicated turn number are included in the doubling time estimates. Note that a negative doubling time indicates a decreasing emittance.

These doubling times can be compared to "statistically signficant" doubling times T_σ is found from the statistical error σ in the slope A:

$$T_\sigma = \bar{\varepsilon}/\sigma$$

In the tables, S_R, the ratio between the statistically significant doubling time and the radial double time is displayed:

$$S_R = \frac{T_{\sigma_R y}}{T_R} = \frac{A_R}{\sigma_R}$$

When S_R is sufficiently larger than 1, a statistically significant change in beam emittances is indicated. Similar comparisons for x and y emittances have been made but are not explicity included in Table 2. A reasonable estimate can be obtained from symmetry by noting

$$T_{\sigma_x} \sim T_\sigma \sim T_{\sigma_R}$$

from which S_x and S_y can be deduced.

Table II shows that for Nm $\leq$ 100, no significant increases in total beam emittances are observed; extrapolated doubling times are many hours. For Nm = 200 signficant increases appear and for Nm $\gtrsim$ 400 fast beam blow-up occurs with doubling times of several seconds.

For Nm $\gtrsim$ 10,000 a noticeable decrease in the calculated 200,000 turn doubling times can be observed, following the 1/Nm dependence noted in the obeservations of Liapunov exponent, described above.

This behavior is displayed graphically in Figure 4 where the slope parameter A as a function of Nm is plotted. They show a "threshold" dependence with increasing Nm. At Nm = 200, 3 "divergent" trajectories appear. The number increases with Nm to an approximately constant value of 30 for Nm ~ 1000 and decreases again for Nm > 1000.

Discussion of Modulational Beam Blow-Up

This study of the frequency dependence of modulational blow-up provides information which can be used in constructing a theoretical model to describe that blow-up. From this paper and reference 1, we note that beam blow-up can occur when a low order resonance is swept across the beam, provided the "speed of sweeping" (modulation frequency) is not "too fast" and not "too slow". The resonance is mostly seen when the modulation frequency is comparable with the precession frequency of the trajectory in the resonance island region.

In our case, as we can see by inspecting Figure 1, there are two resonances which are actually involved in the sweeping.

Investigation of individual particle trajectories show that they can be classified into three groups:

1. Non-chaotic trajectories; these do not change amplitude significantly.

2. "Non-divergent" "chaotic" trajectories which remain confined to amplitudes near their original values.
3. "Divergent" chaotic trajectories, those which change amplitudes greatly in the 3 million turn simulations. These trajectories participate in the beam blow-up for Nm > 200.

In Figure 2 and Table I we indicate the number of these divergent chaotic trajectories. We can clearly note a strong correlation between the appearance of this last kind of particles and beam growth.

In Figure 5 we show the location of the particles in the tune diagrams. It corresponds to the case Nm = 800. The betatron tune values are those obtained by averaging over the first 800 turns of the simulation. We also show in the same figure the location of the two major resonance when they are the most inside into the beam. Reversibility test and emittance growth measurements were done after 6 million turns. The particles, (A) that are not swept by the two resonances are stable and meet the reversibility test positively. The non-reversible particles that also grow to large emittance values (F) are swept by the two resonance lines. Chaotic particles (C) which do not grow in amplitude are confined in a region between the (A) and (F) particles.

In Figure 6 we show the same distribution 6 million turns later. The (F) particles have been pushed to larger amplitude. These particles are those initially at large amplitude and are those that are chaotic and grow. This is in agreement with some observations of the beam-beam effect in the SPS in CERN with proton and antiproton beams colliding.[5]

References

1. D. Neuffer, A. Riddiford, and A.G. Ruggiero, FN-359, Fermilab, Feb. 1982.
2. E.D. Courant and H.S. Snyder, Annals of Physics 3, 1 (1958).
3. D. Neuffer, A. Riddiford, and A.G. Ruggiero, FN-346, Fermilab, Oct. 1981.
4. D. Neuffer, A. Riddiford, and A.G. Ruggiero, FN-363, Fermilab, March 1982.
5. L. Evans, Private Communication, Summer 1982.

Table I Results of Repeatability Tests

Nm	Number of Chaotic Trajectories (out of 100 total)	Number of Trajectories which diverge to large amplitude
8	0	0
16	0	0
32	10	0
64	7	0
100	5	0
200	13	3
400	28	18
600	29	-
800	40	27
1000	41	-
2000	50	24
4000	46	22
10,000	42	18
100,000	39	7

Table II Long Time Simulation Results

Case I: Nm = 8

Turn Number (millions)	$\bar{\varepsilon}_x$ (mm-mr)	$\bar{\varepsilon}_y$ (mm-mr)	T_x Doubling Times	T_y	T_R (days)	S_R
0.2	.01755	.01878	-.005	.005	.04	0.38
1.0	.01752	.01883	0.071	.085	-4.63	0.036
3.0	.01752	.01882	2.55	1.06	1.49	0.565

Case II: Nm = 32

N_T (M)	$\bar{\varepsilon}_x$ (mm-mr)	$\bar{\varepsilon}_y$ (mm-mr)	T_x (hours)	T_y	T_R	S_R
0.2	.01762	.01894	.083	.278	.133	2.94
1.0	.01777	.01882	.688	-.928	9.30	0.42
3.0	.01791	.01876	1.06	-.928	10.1	2.19
5.0	.01779	.01888	7.4	-17.5	30.0	1.45

Case III: Nm = 64

N_T (M)	$\bar{\varepsilon}_x$ (mm-mr)	$\bar{\varepsilon}_y$ (mm-mr)	T_x (hours)	T_y	T_R	S_R
0.2	.01740	.01906	-.59	.36	1.35	0.26
1.0	.01745	.01891	35.8	-.78	-1.48	2.56
3.0	.01748	.01889	-8.1	22.4	-31.0	0.63

Case IV: Nm = 100

N_T (M)	$\bar{\varepsilon}_x$ (mm-mr)	$\bar{\varepsilon}_y$ (mm-mr)	T_x (hours)	T_y	T_R	S_R
0.2	.01731	.01914	-0.48	0.10	0.19	1.85
1.0	.01734	.01904	1.32	-0.77	2.74	1.35
3.0	.01743	.01906	4.30	-4.39	51.6	0.37

Case V: Nm = 200

N_T (M)	ε_y (mm-mr)	ε_y (mm-mr)	T_x	T_y (minutes)	T_R	S_R
0.2	.01779	.01880	13.2	-1.42	-2.97	6.9
1.0	.01889	.01934	4.6	6.4	5.36	19.4
3.0	.02011	.02091	6.8	9.0	7.71	84.0

Case VI: Nm = 400

N_T (M)	ε_x (mm-mr)	ε_y (mm-mr)	T_x	T_y (seconds)	T_R	S_R
0.2	.02347	.02333	7.8	10.4	8.8	46.
1.0	.05540	.03664	20.2	35.9	24.2	108.
3.0	.07003	.06560	77.6	59.7	69.6	127.

Case VII: Nm = 800

N_T (M)	ε_x (mm-mr)	ε_y (mm-mr)	T_x	T_y (seconds)	T_R	S_R
0.2	.02739	.02535	4.8	10.6	6.4	46.
1.0	.05137	.07967	32.3	15.7	19.5	70.
3.0	.08193	.10930	80.	59.	64.	96.

Case VIII: Nm = 2000

N_T (M)	ε_x (mm-mr)	ε_y (mm-mr)	T_x	T_y (seconds)	T_R	S_R
0.2	.02594	.03051	5.7	4.4	4.8	38.
1.0	.07269	.06938	13.4	14.0	13.7	194.
3.0	.1184	.1553	48.6	40.1	43.3	187.

Case IX: Nm = 4000

N_T (M)	ε_x (mm-mr)	ε_y (mm-mr)	T_x	T_y (seconds)	T_R	S_R
0.2	.02838	.02942	4.6	4.8	4.7	45.
1.0	.08313	.1143	15.4	10.3	11.7	136.
3.0	.1445	.2285	48.2	43.0	44.7	121.

Case X: Nm = 10,000

N_T (M)	$\bar{\varepsilon}_x$ (mm-mr)	$\bar{\varepsilon}_y$ (mm-mr)	T_x (seconds)	T_y (seconds)	T_R	S_R
0.2	0.02762	0.03243	5.7	3.7	4.3	5.1
1.0	0.08577	0.08065	11.1	13.9	12.3	163.
3.0	0.21976	0.25361	38.4	32.7	35.1	361.

Case XI: Nm = 100,000

N_T (M)	$\bar{\varepsilon}_x$ (mm-mr)	$\bar{\varepsilon}_y$ (mm-mr)	T_x (seconds)	T_y (seconds)	T_R	S_R
0.2	0.01875	0.01876	20.8	68.8	30.4	16.
1.0	0.02961	0.02622	17.9	27.4	20.9	70.
3.0	0.06862	0.05353	37.8	43.0	39.8	108.

Figure 1

Figure 1 Area of tune space swept out by

Case C-.005: $\nu_x = 0.3439 + 0.005 \sin\theta$ $\theta = 2\pi(n-1)/1000$

$\nu_y = 0.1772 - 0.005 \sin\theta$ n = turn number

$\Delta\nu = 0.01$

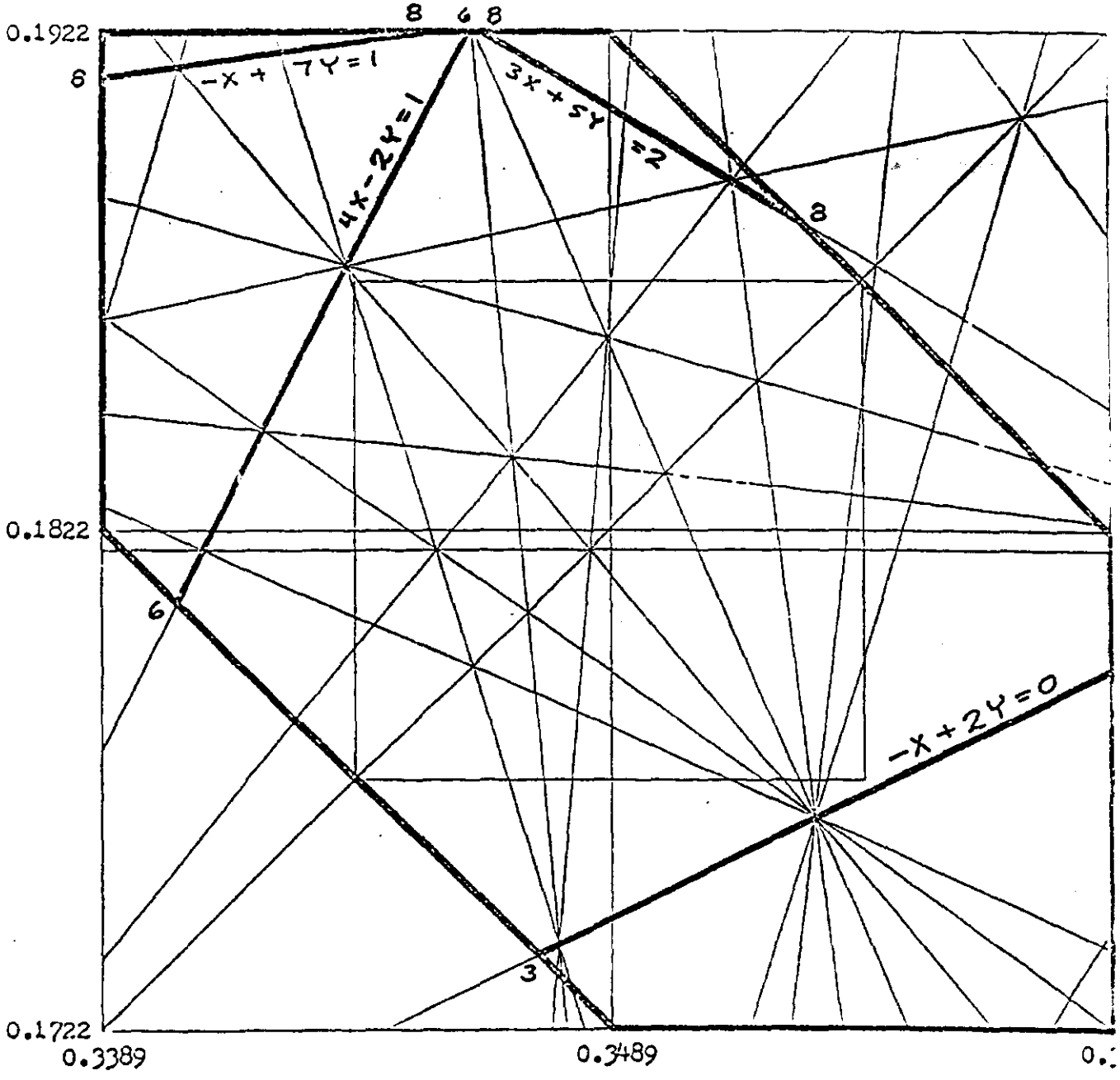


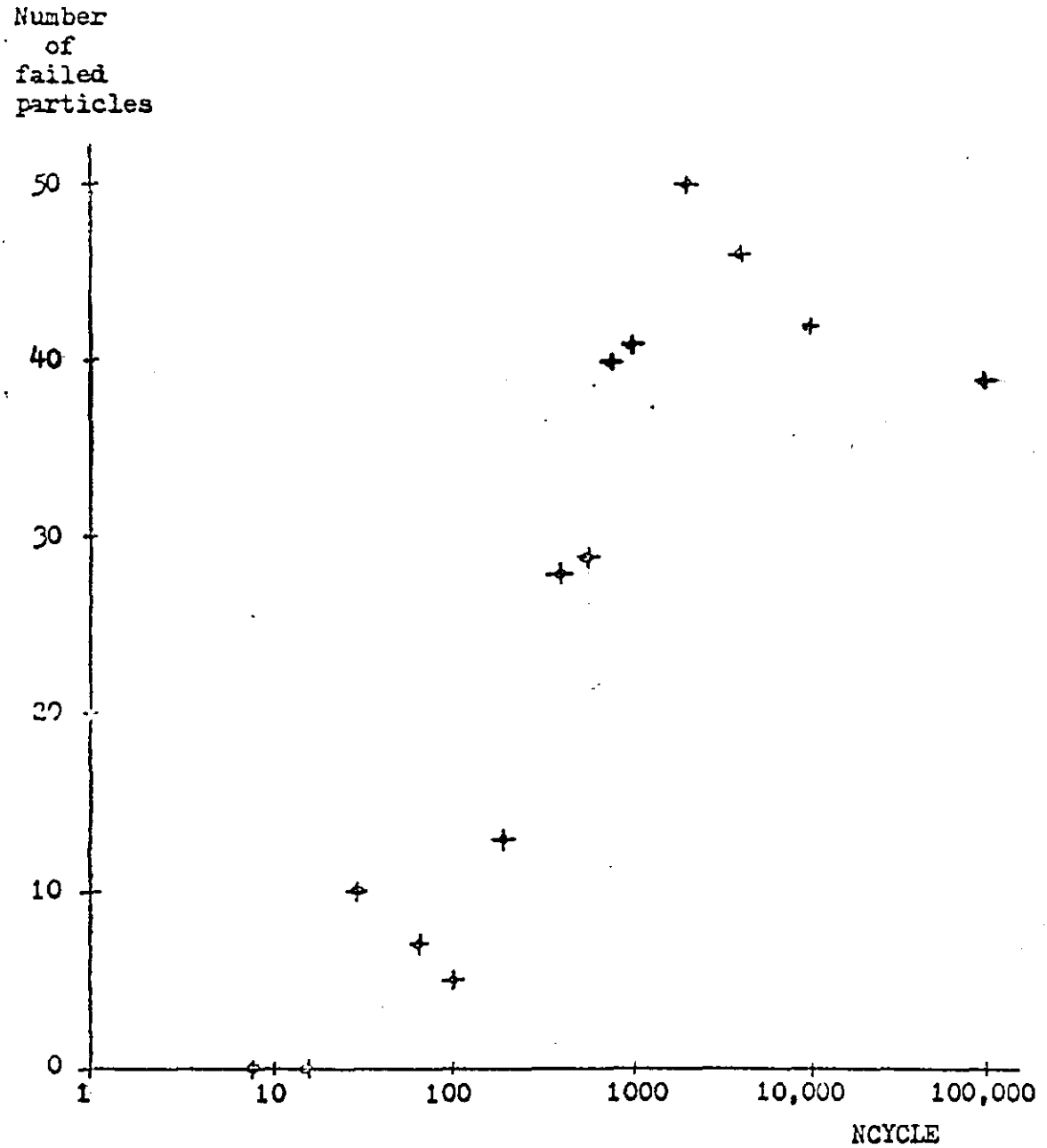
Figure 2

Reversibility test results; 100,000 turns forward, 100,000 turns reverse (except 10^6 forward and 10^6 back for NCYCLE = 100,000).

$\mu_x/2\pi = 0.3539 + 0.005 \sin(2\pi N/NCYCLE)$ N = Turn number

$\mu_y/2\pi = 0.1872 - 0.005 \sin(2\pi N/NCYCLE)$ = 0, 1, 2, ...

$GX = GY = 2\pi\Delta\nu, \quad \Delta\nu = 0.01$



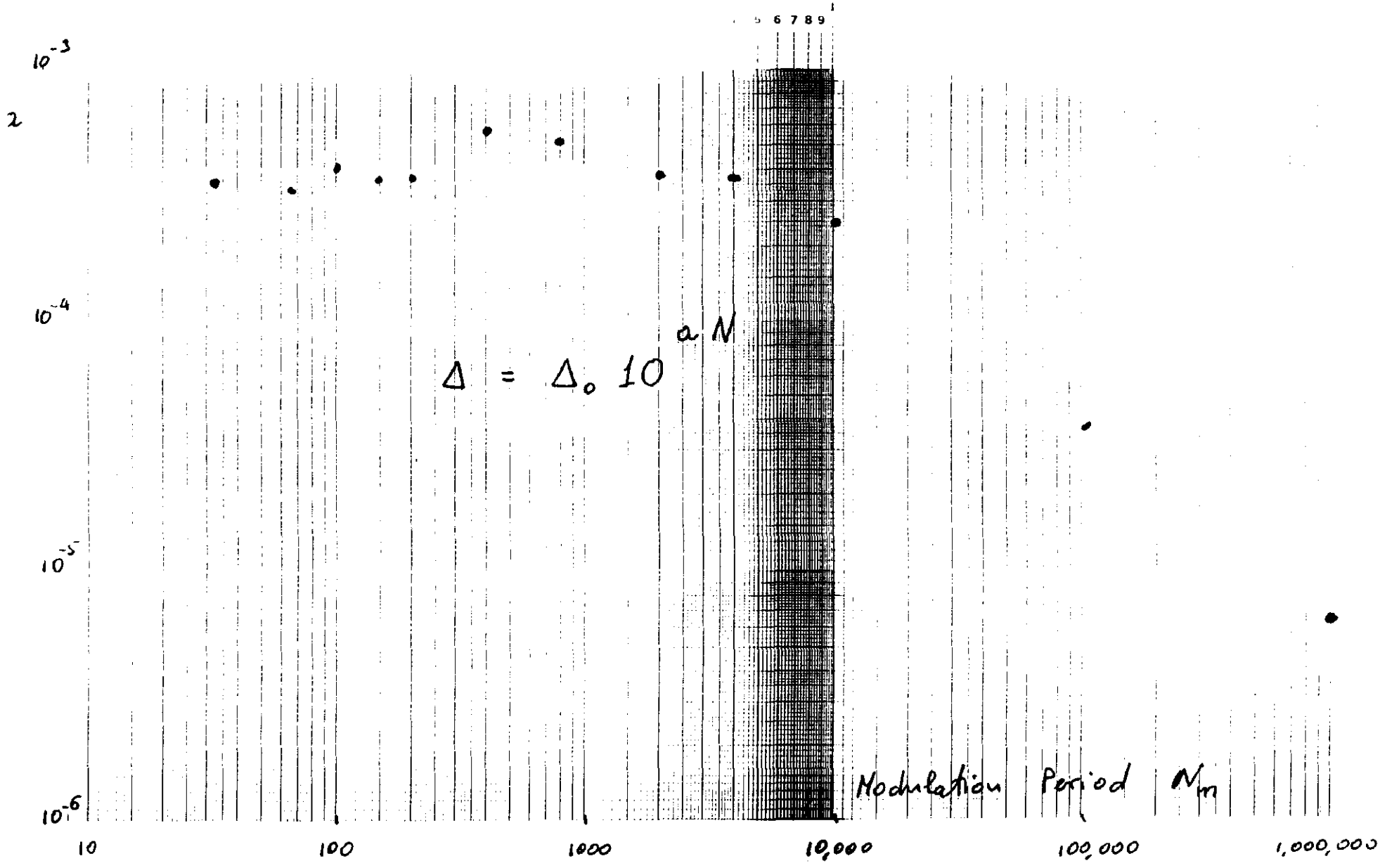


Fig. 3. Average Lyapunov exponent as function of modulation period

Figure 4

Slopes after 200,000 turns.

$\mu_x/2\pi = 0.3539 + 0.005 \sin(2\pi N/NCYCLE)$ N = Turn number

$\mu_y/2\pi = 0.1672 - 0.005 \sin(2\pi N/NCYCLE)$ = 0, 1, 2, ...

$GX = GY = 2\pi\Delta\nu, \quad \Delta\nu = 0.01$

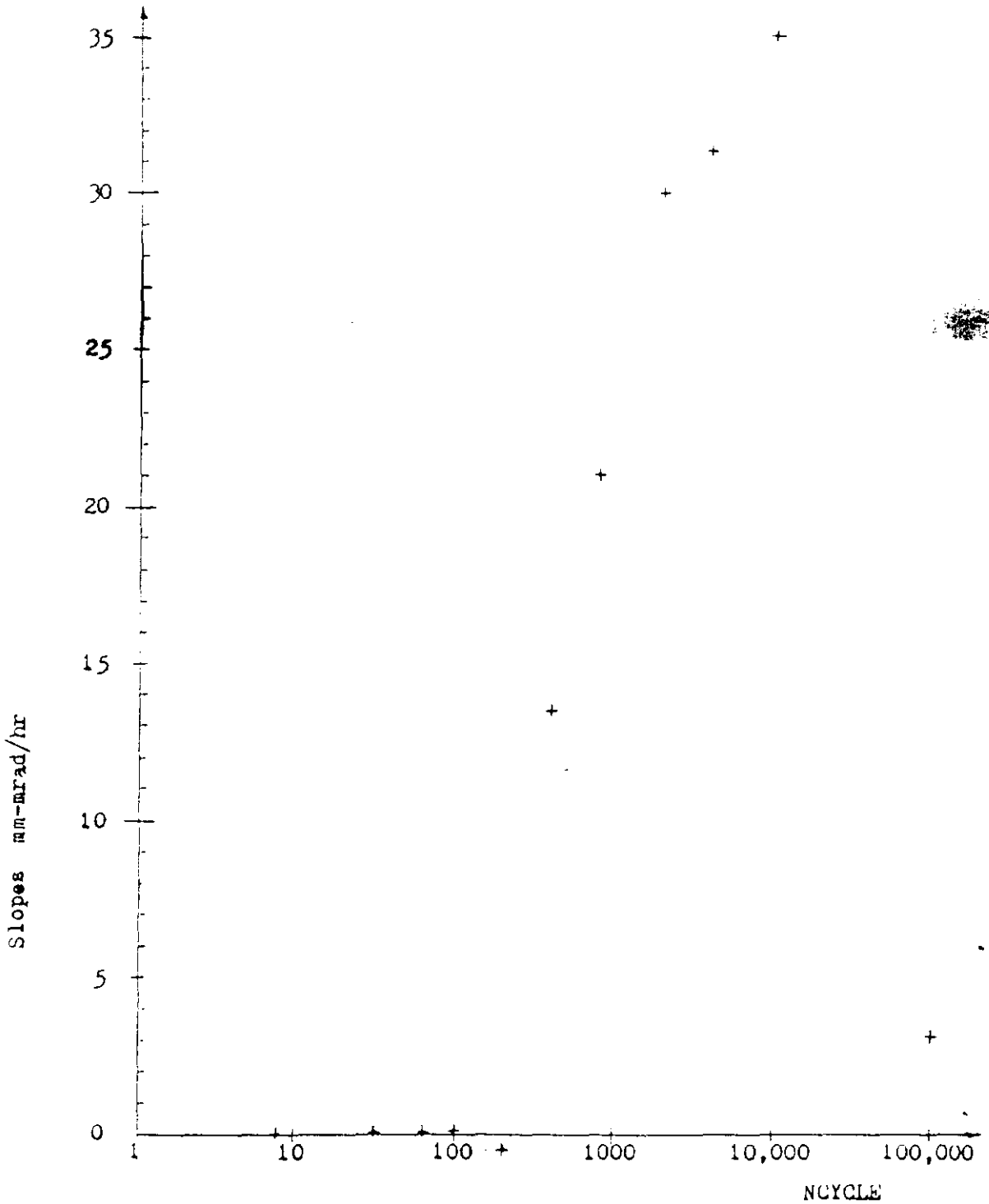


Figure 5

Tunes averaged over turns 0 to 800. NCYCLE = 800.

$\mu_x/2\pi = 0.3539 + 0.005 \sin\theta$ $\qquad \theta = 2\pi N/800$, N = Turn number.

$\mu_y/2\pi = 0.1872 - 0.005 \sin\theta$ $\qquad GX = GY = 2\pi\Delta\nu$, $\Delta\nu = 0.01$

The lower-left corner corresponds with $\mu_x/2\pi - \Delta\nu$, $\mu_y/2\pi - \Delta\nu$.

The upper-right corner corresponds with $\mu_x/2\pi$, $\mu_y/2\pi$.

Labels: A Reversible and small emittance after 3 million turns.

C Chaotic and small emittance after 3 million turns.

F Chaotic and large emittance after 3 million turns.

N Reversible and large emittance after 3 million turns.

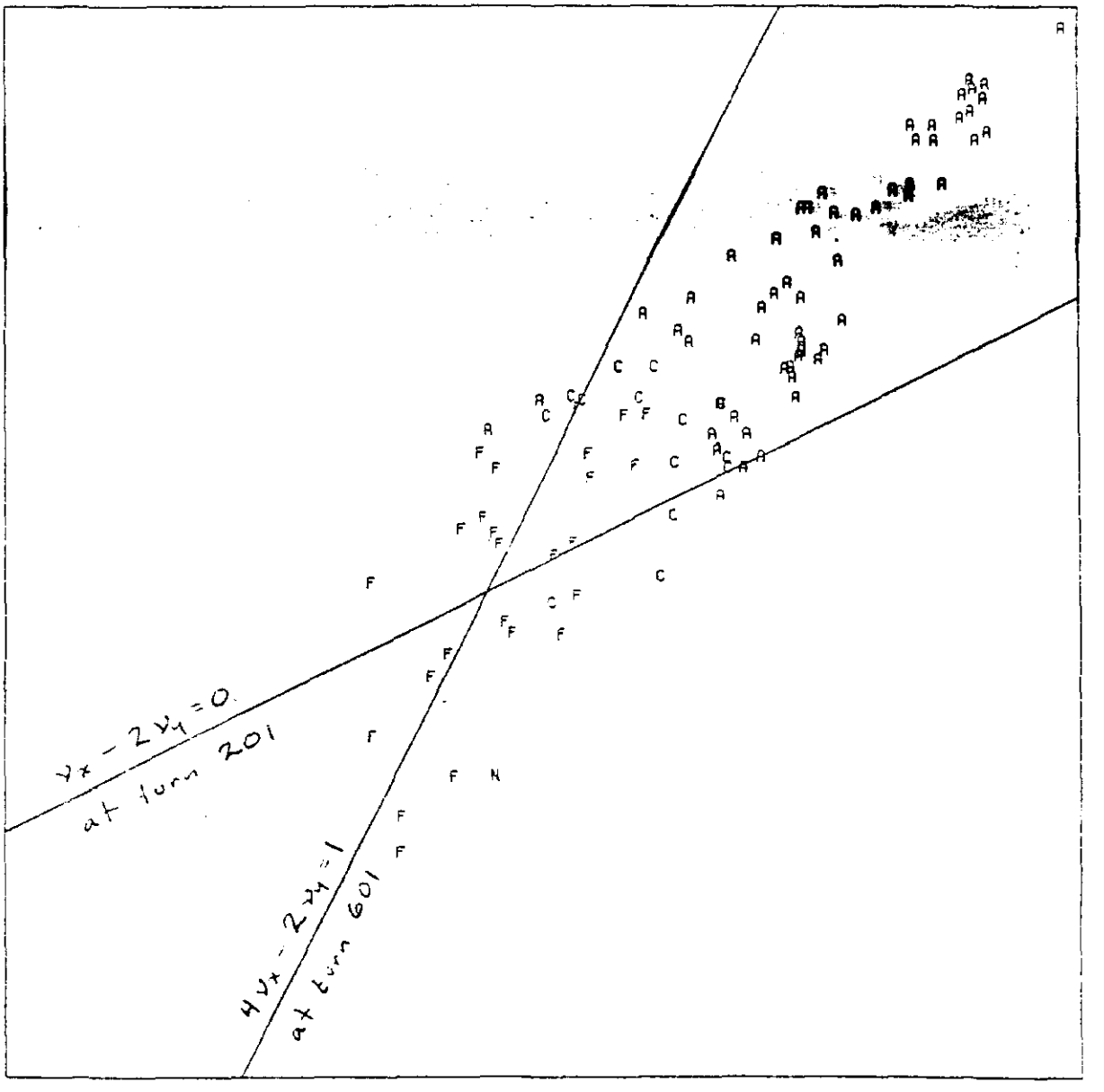


Figure 6

Tunes averaged over turns 3,000,000 to 3,000,800. NCYCLE = 800.

$\mu_x/2\pi = 0.3539 + 0.005 \sin\theta$ $\theta = 2\pi N/800$, N = Turn number.

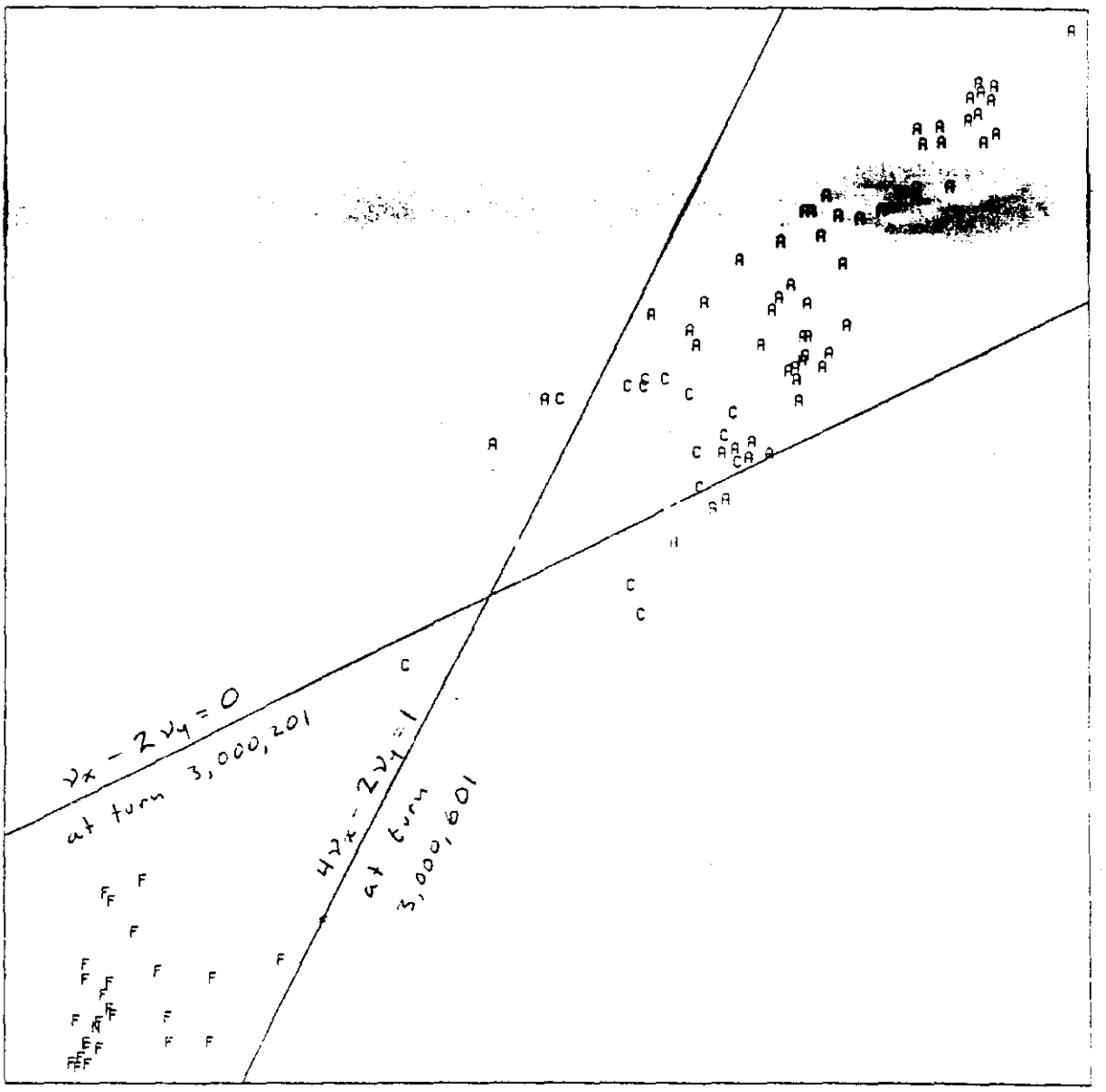
$\mu_y/2\pi = 0.1872 - 0.005 \sin\theta$ $GX = GY = 2\pi\Delta\nu$, $\Delta\nu = 0.01$

The lower-left corner corresponds with $\mu_x/2\pi - \Delta\nu$, $\mu_y/2\pi - \Delta\nu$.

The upper-right corner corresponds with $\mu_x/2\pi$, $\mu_y/2\pi$.

Labels: A Reversible and small emittance after 3 million turns.

C Chaotic and small emittance after 3 million turns.

F Chaotic and large emittance after 3 million turns.

N Reversible and large emittance after 3 million turns.