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Computer Simulation of the Main Ring Beam Colliding with Another Beam

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1. Introduction

In many instances several people have proposed to have the proton beam in the main ring colliding with another external beam for experiments at very high energy in the system of the center of mass. We recall: (1) the "electron target[1]" where an electron beam of few GeV is used; (2) a small proton storage ring[2,3] of few tens of GeV tangent to the main ring at one long straight section; (3) the energy doubler[4] colliding with the main ring; and (4) proton-antiproton colliding experiments[5].

A relatively easy approach to these experiments is to devote the main-ring operation exclusively to them, with no beam delivered to the external experimental areas. Nevertheless this solution can be found to be too drastic, and one would prefer to exploit the main ring by having its beam parasitically colliding with another beam, during the regular acceleration cycle. At the end of the cycle the beam would be extracted as usual and disposed down to the experimental areas or injected in the energy doubler as required.

This mode of operation would satisfy all the experimentalists, but the concern is that it could cause some deterioration of the main-ring beam quality which would make the handling of the beam more difficult during the acceleration and impede the extraction process and the injection in the energy doubler.

To put it in other words, accelerators and storage and colliding rings are designed and operated with different constraints. This is the first time that a machine like the main ring is demanded to operate in a hybrid fashion, and one wants to make sure that one side of the operation is not seriously damaging the other.

The main cause of the deterioration of the beam quality we want to investigate here is the beam-beam interaction itself. The other beam which is colliding with the main-ring beam exercises on the main-ring protons an effect equivalent to that of a nonlinear lens of some length, strength and shape. The strength of this lens changes during the acceleration cycle, and its location with respect to the main-ring protons also changes when the other beam is moved in and out.

The beam-beam effect causes severe limitations to the performance of a system of storage and colliding rings. Some of these limitations involve very short periods of time where beam sizes suddenly increase and affect the luminosity[6]. Other limitations appear in the range of long periods of time when a constant beam size increase reduces the beam lifetime.[7]

Some theories have been produced for the explanation of these effects.[8] Because of the roughness of the assumptions in the theories, numerical simulations have also been carried out on computer[9,10]. This is the technique we used to test the main-ring beam deterioration. We took the small storage ring proposed by J. Walker et al.[3] for example, because it is so far the best specified proposal. We assumed that the beam in the small storage ring is not affected by the interaction with the main ring. After all, our concern was entirely devoted to the behavior of the main-ring beam and we paid little or no attention to the survival of the "other" beam. Then we simulated the motion of the main ring protons during either all their accelerating cycle or a fraction of it where some of the crucial parameters are changed (for instance when the other beam is moved in or out).

Since the number of revolutions in the main ring is relatively small (fifty-thousand per second) we could track a particle motion every and each of the actual revolutions with not much computer time. Also we could take a very large sample of particles to simulate the sizes and the divergences of our beam.

No synchrotron oscillations were taken into account in our calculations. The main ring was assumed to be linear, namely made only of dipole bending field and quadrupole focusing field. No other linearity was added aside the nonlinear lens equivalent to the other beam. All the parameters (tune, beam size, tune shifts, etc. etc.) were taken constant or changing accordingly with the energy. No noise of any nature was added in our simulations.

The main results can be summarized as follows:

(i) The beam-beam effect causes the beam size to change. The change is sudden (within one-thousand turns). No long time effects have been found, considering the short period of the acceleration cycle (few seconds).

(ii) The two beams are to be separated by no less than 3 cm for energies lower than 100 GeV to avoid excessive size increases and moved together on top of each other as fast as possible (within 20 msec at 100 GeV).

(iii) Since the tune shifts are negative it is advisable to run at fractional tunes of 0.4. Lower values cause a higher increase of the beam size.

(iv) The vertical and horizontal tunes should be as close as possible within 0.01.

(v) The variation of the parameters with the energy due to the acceleration does not induce any further effect.

(vi) Extraction is not possible with the two beams on top of each other also at 400 GeV.

(vii) A beam separation of 0.75 cm in 20 msec at 400 GeV flattop does cause beam size increase and distortion.

(viii) We did not find appreciable closed-orbit distortion when the beams were moved together or separated.

Though we did not yet try a systematic verification of this, we believe that our results can be simply explained as an amplitude-dependent mismatch of the betatron emittances due to the nonlinear lens. We did not find any real indication of stochastic limit.[8]

2. Some Equations

Let x and y be respectively the horizontal and vertical displacement of a proton from the main-ring closed orbit. Let z denote either x or y.

Since we are interested in the ultra-relativistic case, in the following we shall always set, whenever possible, the particle velocity equal to the light velocity, c. The equation of motion in either of the two planes, then, is

$$z'' + k(s)z = \frac{2e}{E_o\gamma} E_z f(s) \qquad (1)$$

where s is the linear longitudinal coordinate along the closed orbit which has length $2\pi R$, the prime denotes derivative with respect to s, e is the proton charge, E_o is the energy at rest, $E_o\gamma$ is the total energy and f(s) is always zero except when the particle is traversing the "other" beam over a full length of ℓ in which case is unit. The ordinary lattice of the main ring is described by the focusing function k(s) which is periodic in s with period $2\pi R$. We assume to know the solution of the homogeneous equation associated to (1). This solution is described by the amplitude and angle functions $\beta(s)$ and $\alpha(s)$.

Finally $\mathcal{E}_z$ is the transverse electric field due to the "other" beam. In the special case the "other" beam is round with gaussian distribution in both transverse directions, one has[11]

$$\mathcal{E}_z = \frac{2I}{c}\,\frac{1 - e^{-\frac{r^2}{2\sigma^2}}}{r^2}\,z \qquad (2)$$

where I is the current and σ is the rms size of the "other" beam. Also

$$r^2 = x^2 + y^2 \ .$$

Eq. (2) is valid only for head-on collision and when the axis of the two beams coincide. In the case the two beams are separated by a distance x_o on the horizontal plane, Eq. (2) still applies with the provision of replacing x with $x-x_o$.

The effect of the beams separation at both ends of the interaction length is not included here.

Define the emittance which includes 95% of the beam

$$\varepsilon = 6\pi\frac{\sigma^2}{\beta} \ .$$

The beam size σ is a function of s. If we take s = 0 at the center of the collision length

$$\sigma^2 = \sigma^{*2}\left[1 + \left(\frac{s}{\beta^*}\right)^2\right]$$

where $\sigma^{*2} = \varepsilon\beta^*/6\pi$ and β^* are the values of σ^2 and β at s = 0.

The phase advance across the interaction length is

$$2\psi = \text{arctg}\ (\ell/2\beta^*).$$

In the main ring, this quantity is reasonably small (~15°), so that one can approximate the beam-beam effect as a lumped nonlinear kick which essentially leaves the displacement z unchanged and affects z' by the amount

$$\Delta z' = \frac{4\pi}{\beta^*_{MR}}\ (\Delta\nu)\, zF\left(\frac{r^2}{2\sigma^{*2}}\right) \tag{3}$$

where β^*_{MR} is the main ring value of β^*,

$$\Delta\nu = \frac{6Ir_p\beta_{MR}^*}{ec\gamma\varepsilon}\ \psi \tag{4}$$

is the tune shift in the limit of small amplitude due to the beam-beam effect, $r_p = 1.535\text{x}10^{-18}$ m is the classical proton radius, and

$$F(u^2) = \frac{1}{\psi}\int_0^{\ell/2\beta^*} \frac{1 - e^{-\frac{u^2}{1+t^2}}}{u^2}\, dt. \tag{5}$$

This function is plotted in Fig. 1 by using the small storage ring parameters.

Table I is a summary of the interaction region parameters we used in our calculations. We show the performance of the main ring colliding with the small storage ring in Table II. As one can see the tune shift on the Main Ring is quite large also at 400 GeV and one might expect some effect. On the other hand the tune shift on the small storage ring is reasonably small, and this could justify our assumption that the "other" beam remains unchanged. We believe this assumption is correct at low main ring energies but we have some questions about its validity at higher energies, where the tune shift on the "other" beam may not be small enough.

3. The Numerical Simulation

We take one-thousand particles for our computation. This number is a compromise between the needs of avoiding exceedingly large computer time and of giving a fair representation of the cross-section of the main ring beam. To each particle we associate the four initial conditions x, x', y and y'. These are taken randomly according to a four-dimensional gaussian distribution which describes the main-ring beam at the crossing location with the parameters specified in Table I. The size of the distribution is derived from the emittance values of Table II. This procedure of taking the initial distribution in a random way allows an accurate enough representation of the beam in any of the four-dimensional phase-space projection, by keeping, at the same time, the number of particles relatively small.

During the assignment of the initial conditions, values which exceeded three times the value of the standard deviations were rejected.

Observe that the beam shape in either x, x' or y, y' plane is not that of an upright ellipse because $\alpha^* \neq 0$ at the crossing point. Our beam representation by the 1000 particles takes this fact into account. Also we take a round beam with emittance decreasing linearly with the energy.

Our simulation consists in applying simultaneously to all the particles a series of a large number of cycles. When the energy is constant (namely, no acceleration) all the cycles are identical. When the energy is varied (acceleration), at each cycle the energy-dependent parameters are properly scaled in the way it is described further below. Each cycle simulates one revolution in the main ring. The dependence of the energy with the number of revolutions (cycles) is shown in Fig. 2. We start with a front-porch of about 50,000 turns; we accelerate from 8 GeV to 400 GeV in 200,000 turns, and we end with a flattop of 50,000 turns. The beam takes about one second to make 50,000 revolutions, thus in our simulation the ramp speed is of about 100 GeV/sec. Obviously our simulation takes the advantage of the relatively small number of revolutions involved (at most 300,000), whereas in other simulation projects[10] this number was much higher (several millions) because they are intended to simulate the behavior of a beam which circulates for hours in a storage ring. Thus we believe the results of our simulation reflect closely the behavior of the actual beam in the main ring for what time is concerned, though they are still certainly model depending.

Each cycle is made of two steps. The first step consists in applying simultaneously to all the particles a linear transformation to their coordinates x, x', y, and y', from the crossing point back to the same point one turn around. The transformation is represented by a 4x4 matrix which describes the main ring lattice. This matrix leaves the two pairs x, x' and y, y' uncoupled. For its determination we supply β_x, β_y, α_x and α_y at the crossing point (see Table I), and the two phase advances μ_x and μ_y per turn. In general it is sufficient to specify only the fractional part of the two betatron tunes ν_x and ν_y.

The second step consists in changing both x' and y' of each particle by the amount (3), where β^*_{MR} and $\Delta\nu$ are specified in Tables I and II. The shift $\Delta\nu$ decreases linearly with the energy.

In order to save computer time, the function $F(u^2)$ is calculated, with a fast and accurate integral subroutine, at the beginning of our simulation program at 4,000 values of u^2 between 0 and 100 and equally spaced by 0.025. The resulting quantities are stored in the memory as one vector. During the second step, and for each particle, the computer calculates first $u^2 = (x^2+y^2)/2\sigma^{*2}$ and then $F(u^2)$ by using the stored data and a linear interpolation between the calculated values. In case $u^2>100$, the asymptotic expression

$$F(u^2) = \frac{\ell/2\beta^*}{\psi u^2}$$

is used. We found that this expression is accurate at least for the first seven significant digits.

In case the two beams are separated by a distance x_0 on the horizontal plane, the quantity u^2 is calculated according to

$$u^2 = \frac{(x-x_0)^2 + y^2}{2\sigma^{*2}} .$$

In the most general case the separation x_0 is taken as a function of the number of revolutions (cycles) as shown in Fig. 3. In the case of Fig. 3a the two beams are on top of each other during the first n_1 revolutions, then are linearly separated during the next (n_2-n_1) revolutions, and kept at constant separation after n_2 revolutions. The reverse occurs in the case of Fig. 3b. During the separation, x_0 is added or subtracted the same amount every cycle. On the horizontal plane and when $x_0 \neq 0$, the factor z in Eq. (3) is replaced by $x - x_0$.

When acceleration is applied the momentum is changed every cycle before the second step. The momentum receives the same increment every revolution. Obviously no increment is given during front-porch and flattop. If we denote by δ the ratio of the new momentum value to the initial value, then, before performing the second step, the (actual) separation x_0 is multiplied by $\sqrt{\delta}$, the amplitude u^2 is divided by δ, and the tune-shift $\Delta\nu$ is calculated by dividing also the initial value by δ. Finally the second step of the cycle is applied to all particles. The first step of the next cycle is applied as usual.

With this procedure we always carry out normalized beam sizes and divergences, which, among other things, make comparisons easier.

Thus at the end of each cycle we have a new set of values x, x', y, and y' for each particle. Every 1000 turns these values are manipulated for output. First, four hystograms of 20 channels corresponding to the four coordinates are prepared and displayed. Then averages, standard deviations, minima and maxima are calculated and printed out. With exception of some special case, we always found that the hystograms fairly reproduce a gaussian distribution. Thus we take the standard deviation as a sort of the measure of the beam size and the average is the location of the main-ring beam center. The averages are assigned a significance only when they are larger than the standard deviation divided by the square root of the number of particles. Otherwise they are considered to be zero.

Another form of output is a plotting of two coordinates, one against the other (usually x and x').

As we said before the data in output correspond always to the end of a cycle, therefore we "observe" the main-ring beam at the crossing point. Nevertheless, when we wanted to do so, we could "observe" the beam at any other location of the ring (usually at the location diametrically opposite to the crossing point) by splitting the first step of the cycle where output is requested, in two partial linear transformations of the particle coordinates.

Our program has been run on the CDC-6600. To simulate the motion of a particle in one revolution, it takes about the actual time the particle does in the main ring (20 μs).

4. Results

We executed several runs which can be divided in nine groups. Each group of runs deals with a particular aspect of the simulation of the beam-beam effect.

A. We performed a test of our program by setting the magnitude ($\Delta\nu$) of the linear kick to zero. We tracked the particles over 50,000 turns with fractional tune 0.4 on both planes. We checked the beam every 1000 turns. We obtained an accuracy of at least eight digits.

B. We took a linear beam-particle kick. For this purpose we set $F(u^2)$ at the r.h.s. of (3) equal to 1 for any value of u^2. This is equivalent to replace the "other" beam with some sort of quadrupole which is defocusing on both planes. The effect of this special case can be also calculated analytically. Therefore this group of runs served the purpose of verification of our program as well as the need of analyzing the beam-beam effect by differential steps.

Each run included 50,000 turns. There was no acceleration. The beam was observed at the crossing point and at the location diametrically opposite to this. The fractional tunes were both 0.4.

As expected, we observed beam size oscillations due to the mismatch induced by the "quadrupole". The amount of mismatch was identical in the two planes. We show in Table III the average increase of the beam size. The oscillations are around these values and have an amplitude which equals the difference between the initial and the average values.

All the computer results check with the analytical results.

Observe that there is a beam size reduction at the crossing location, whereas there is a beam size increase at the diametrically opposite the point. This also was expected, since the phase advance across the kick is zero and the fractional tune between 0.25 and 0.50.

C. Other runs with the same conditions of the previous group were excuted, this time with a nonlinear kick, namely we used the actual function $F(u^2)$, Eq. (5), calculated as specified in the previous section. The results of these runs are also shown in Table III for an easy comparison with the previous results. We see now that the relative beam size reduction, locally, and the relative beam size increase, across, are both reduced. We can explain this fact by assuming that the main effect of the nonlinear lens is still that of a mismatch as in the previous linear case. Nevertheless, now, the amount of mismatch depends on the initial amplitude of the betatron oscillations. Particles with smallest amplitudes suffer most of the mismatch; particles with largest amplitudes are not essentially effected. One can express this in this other way: particles in the center of the beam have the largest tune shift. The shift decreases to zero for large amplitude. Thus there is also a tune spread in the beam and this has the effect of damping the coherent oscillations induced by the mismatch as we could also observe in our computer outputs (See Fig. 4). Most of the beam size variation occurs during the first 1,000 turns. After that the beam size remains constant, apart from fluctuations due to the small number of particles involved.

In conclusion we did not see any sign of stochastic limit even at low energy when the tune shift is large.

The effect so far described applies to both planes in the same manner.

In conclusion the beam size remains unchanged at the crossing point, and so the luminosity of the p-p colliding experiment does. There is a perturbation on the lattice which causes a β-value increase of 70% at 100 GeV. This increase is certainly compensated by the energy damping factor of 12 from 8 GeV, and we can reasonably handle it. But we do not believe it is the case to collide the two beams at energies lower than 100 GeV; the β-value increase would be too much to cope with.

D. We did one run which starts from 100 GeV, accelerates to 400 GeV and ends with one-second flattop. This run was done to check the effect of the acceleration. The kick was non-linear and the fractional tunes equal to 0.4. The result of the run is shown in Fig. 5, where the beam size at the location diametrically opposite to the crossing is plotted versus the number of revolutions. Observe that what is plotted is the normalized beam size. The actual beam size is obtained by dividing this by the square root of the energy.

Apart from the initial increase due to the nonlinear mismatch, in agreement with the previous findings, we see that the normalized size does not change during the rest of the cycle. Actually we believe to notice a little decrease during the acceleration which disappears on the flattop.

E. During the early part of the cycle the two beams are to be separated. The two beams are moved together on top of each other at some point, say at 100 GeV, and with some speed. To check the effect of this operation we ran a few cases at constant energy and for 50,000 revolutions. The initial separation, from center to center was taken to be 5 mm. The fractional tunes were 0.4. Fig. 6 shows the case the "other" beam is moved on top of the main-ring beam in 25,000 turns and linearly. Once the two centers coincide they are left unchanged. We show the horizontal and vertical beam size variation. During the displacement, the horizontal size increases faster than the vertical one. The horizontal increase picks to a maximum and then decreases to a constant value. We believe the vertical increase is due to the beam-beam nonlinear coupling.

All the results are shown in Table IV. As one can see, it is advisable to move the two beams as fast as one can to avoid excessive size increase.

We did not observe any significant main-ring closed orbit distortion during this operation.

F. In this group we have three runs which simulate closely one full acceleration cycle. They start at 8 GeV (without a front-porch) with the two beams separated by x_0 = 15 mm, 20 mm and 30 mm. The beams are accelerated to 400 GeV with fractional tunes 0.4. At 100 GeV the "other" beam is moved on top of the main-ring beam in 1000 turns (20 msec) and linearly. The runs end with a one-second flat-top at 400 GeV

We show in Fig. 7 the case of x_0 = 15 mm. There is a constant horizontal size increase up to 100 GeV. The overall increase is of a factor 6. Could this be stochasiticty? And does stochasticity require beam separation? During the same time the vertical size increases by only 50%. After the beam displacement, the horizontal and vertical size have again the same value which is about three times larger than the initial one. We think again that this fact is due to the nonlinear beam-beam coupling.

Again no significant closed-orbit distortion has been found.

One has the same picture with an initial separation of 20 mm. The horizontal beam size increase up to 100 GeV is of a factor two, whereas the vertical size increases only by 30%. Above 100 GeV the vertical size increases to the horizontal size which remained essentially unchanged.

Finally with an initial 3 cm separation the only beam size increase noticed occurs during the beam displacement operation and amounts to about 13% in both planes. Thus we suggest that 3 cm is the minimum distance one should consider for the initial beam separation with the displacement at 100 GeV to be executed in 20 msec.

G. We made two runs as described above with an initial separation of 3 cm. One run had fractional tunes of 0.4; the beam increase was of about 13% as also said above. The other run had fractional tunes of 0.1; the beam increase was now of 36%. This is consistent with the fact that the tune shifts are negative. Starting from tunes of 0.1 one gets closer to (or even traverses) a region which is considerably denser in resonances.

H. In this group we checked the dependence of the beam size increase on the difference between the two betatron tunes. The results are shown in Table V. The size increase occurs only during the beam displacement. Observe that the horizontal and vertical increases are unequal for large tune separation, thus the beam is no longer round. The vertical size increases more than the horizontal and that can be explained probably by observing that the vertical tune is lower.

When the two tunes are equal, and because both beams are round, we are essentially dealing with a one-dimensional problem. By splitting the tunes the problem becomes two-dimensional. We believe that every time a new degree of freedom is introduced the mechanical system we are investigating becomes more unstable.

The best one can do is to keep the tunes equal. In practice a separation of the tunes during the acceleration cannot be avoided. This will cause a loss of a factor 4 in the luminosity.

I. The last group contains miscellaneous runs intended to investigate the possibility of slowly extracting the main-ring beam with the "other" beam on top.

We considered a one-second flattop at 400 GeV (no acceleration).

(1) We set $\nu_x = 0.5$ and $\nu_y = 0.4$. No effects develop on the vertical plane. The horizontal size remains unchanged, but the horizontal divergence increases. When the beam-beam kick is linear (F = 1) the increase is of about 4 μrad/turn. For nonlinear kick the increase is 2.5 μrad/turn. Also in the first case the beam emittance is linearly stretched along the x'-axis, whereas in the second case the beam emittance is distorted to the shape of an S. This effect is peculiar to the half-integer resonance and can be analytically explained.[12]

(2) To compensate for the beam-beam tune shift we took $\nu_x = 0.547$ and $\nu_y = 0.4$. Again nothing really happened on the vertical plane. The horizontal size and divergence of the beam increase exponentially and very fast in the case of linear kick. After 1,000 turns the beam size is 10 times bigger and the divergence 20 times larger. But in the case of nonlinear kick the increase in size and divergence is of only a factor 2 and occurs

only during the first few milliseconds, being induced likely by the nonlinear mismatch.

(3) Since the previous results are indications that the main-ring beam cannot be extracted when the "other" beam is on top, we considered the possibility of using the first half of the flattop for the beam-beam colliding experiment and the second half for the external area experiments. For this purpose we have to separate again the two beams before starting extraction. We moved the "other" beam in 20 msec to a final separation of 7.5 mm. Again nothing unusual happens in the vertical plane. When the two tunes were equal to 0.4 a fast horizontal size increase of a factor 1.7 occurs during the beam displacement. When the horizontal tune was set to 0.5, we observed the same divergence increase as described earlier during the first half-second. But this increase stopped as soon as the two beams were separated.

5. Conclusions

We can draw two main conclusions from our calculations. First, the main-ring beam can be made to collide with another beam for energies larger than 100 GeV. If proper precautions are taken, one can expect, at most, an increase of the beam size of a factor 2 in both transverse directions. This increase, occuring at 100 GeV, can certainly be handled. Of course, the luminosity would decrease by a factor 4 at the same time, but this has been for us of less concern compared to the survival of the beam in the main ring. If the beams start to collide at higher energy, probably the size increase is less and the luminosity would be less affected.

The second conclusion is that during collision it is not possible to have the main-ring beam slowly extracted, at least not in a controlled fashion. One requires first a separation of the two beams, but this could cause a further beam size increase. At the end the beam might be three times bigger and it is not clear whether the present extraction beam can tolerate such large beam.

Of course our calculations were simplified by our assumptions. Next time we shall take into account phase oscillations and fast tune variations. These would make the situation worse but we do not know yet by how much.

Finally all these considerations apply also to the case where the main-ring beam is colliding with the energy doubler beam. A 1000-GeV beam can induce the same amount of tune shift we have been considering here.

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Table I

Interaction Region Parameters

	Main Ring	Small Storage Ring
Interaction length (ℓ)	18 m	
Crossing Angle	0 mrad (head on)	
$\beta_x^* = \beta_y^*$	70 m	4.65 m
α_x^*	-0.7	0
α_y^*	+0.7	0
Dispers. Function (η^*)	2.3 m	0 m
$\varepsilon_x = \varepsilon_y$ (95% beam)	(-)	$0.35\pi\ 10^{-6}$ m
$\sigma_x^* = \sigma_y^*$	(-)	0.521 m
Current, (I)	0.14 A	1.4 A
Energy (kinetic)	8-400 GeV	25 GeV

Table II

Performance

Energy (GeV)	$\varepsilon_{MR}^{(*)}$ ($\pi \cdot 10^{-6}$ m)	$\sigma_{MR}^{(*)}$ (mm)	$\Delta\nu_{MR}^{(*)}$	$\Delta\nu_{SR}^{(*)}$	L ($10^{31} cm^{-2} s^{-1}$)
8	1.0149	3.441	1.985	0.0004	0.11
100	0.0894	1.021	0.174	0.005	0.90
200	0.0449	0.724	0.087	0.009	1.35
300	0.0300	0.592	0.058	0.0138	1.64
400	0.0225	0.512	0.044	0.0185	1.86

(*) The same value applies to the horizontal and to the vertical plane.

Table III

Square of the Average Beam Size Variation(*)

Energy (GeV)	Linear Kick local	Linear Kick across	Nonlinear Kick local	Nonlinear Kick across
100	0.60	2.60	0.96	1.7
200	0.67	1.81	0.86	1.4
300	0.73	1.55	0.85	1.3
400	0.77	1.43	0.84	1.3

No acceleration

50,000 revolutions

$\nu_x = \nu_y = 0.4$

No separation between the two beams

(*) Ratio of average value to initial value.

Table IV

Beam Size Increase due to the Displacement of one Beam (Initial Separation, x_0 = 5 mm)

	Local		Across	
No. of Turns	max.	final	max.	final
25,000	2.2	1.9	2.5	2.0
5,000	2.1	1.8	-	-
1,000	-	1.5	2.3 (*)	2.0 (*)
100	-	1.1	-	-

No acceleration

E = 100 GeV

Fractional Tunes = 0.4

(*) Fractional Tunes = 0.8

Table V

Relative Size Increase vs. Tune Separation

ν_x	ν_y	horiz.	vertic.
.40	.40	1.1	1.1
.41	.39	1.9	1.9
.42	.38	1.8	2.0
.45	.35	1.8	2.4

Simulation of a normal accerlation cycle:

8-400 GeV

Initial Beam Separation: 3 cm

Beam Displacement in 20 msec

The beam size increase occurs solely during displacement.

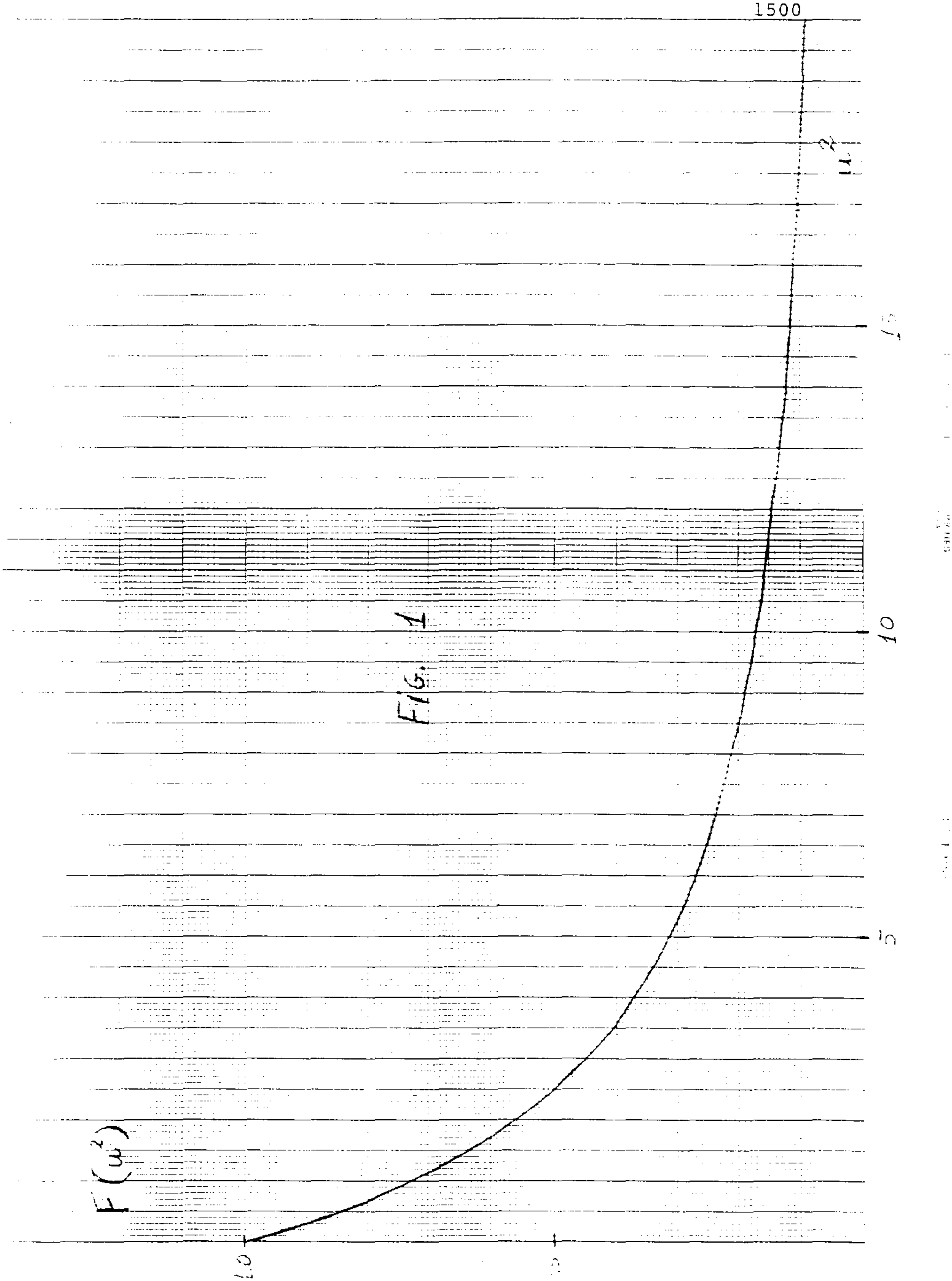

FIG. 1

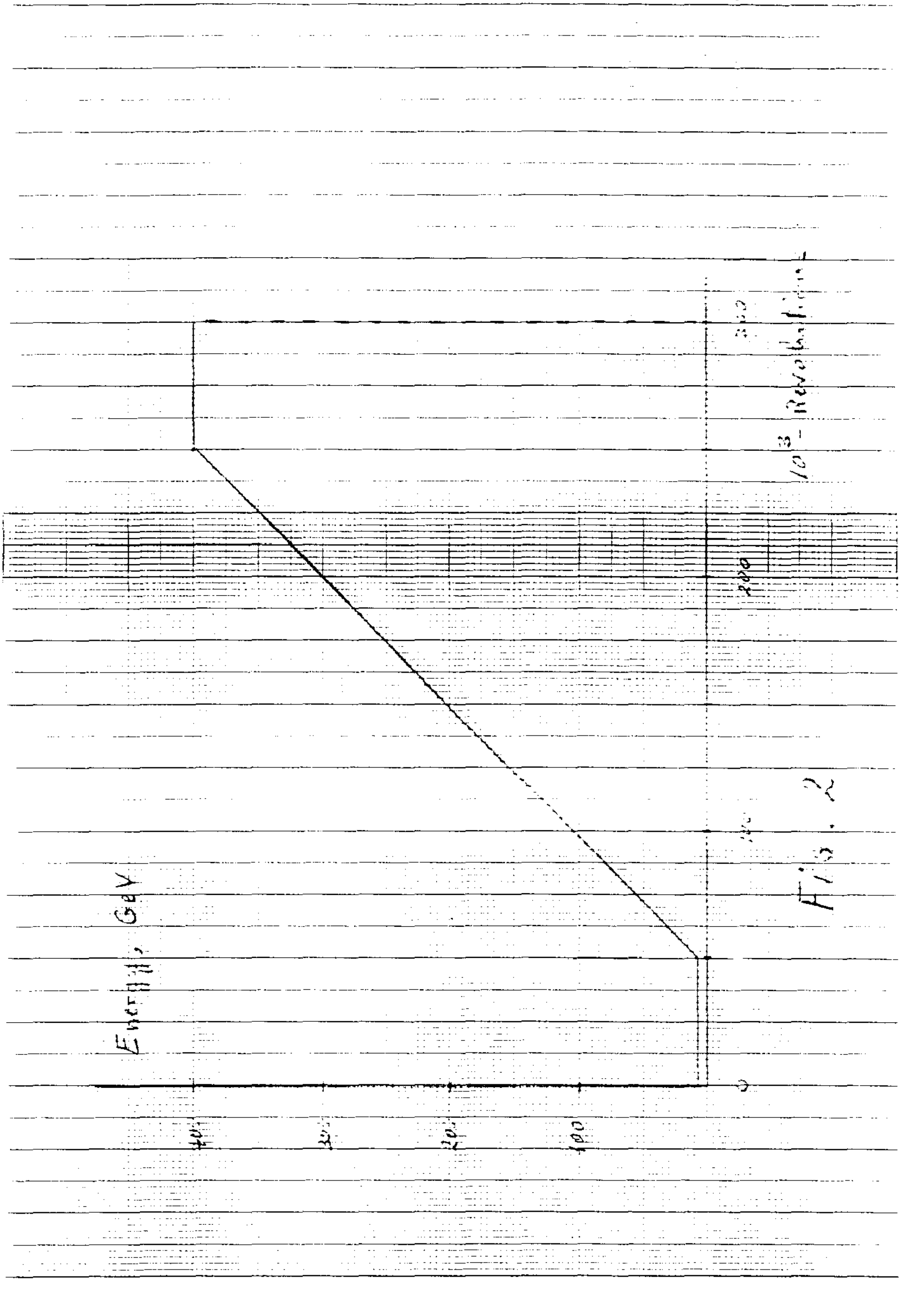

Fig. 2

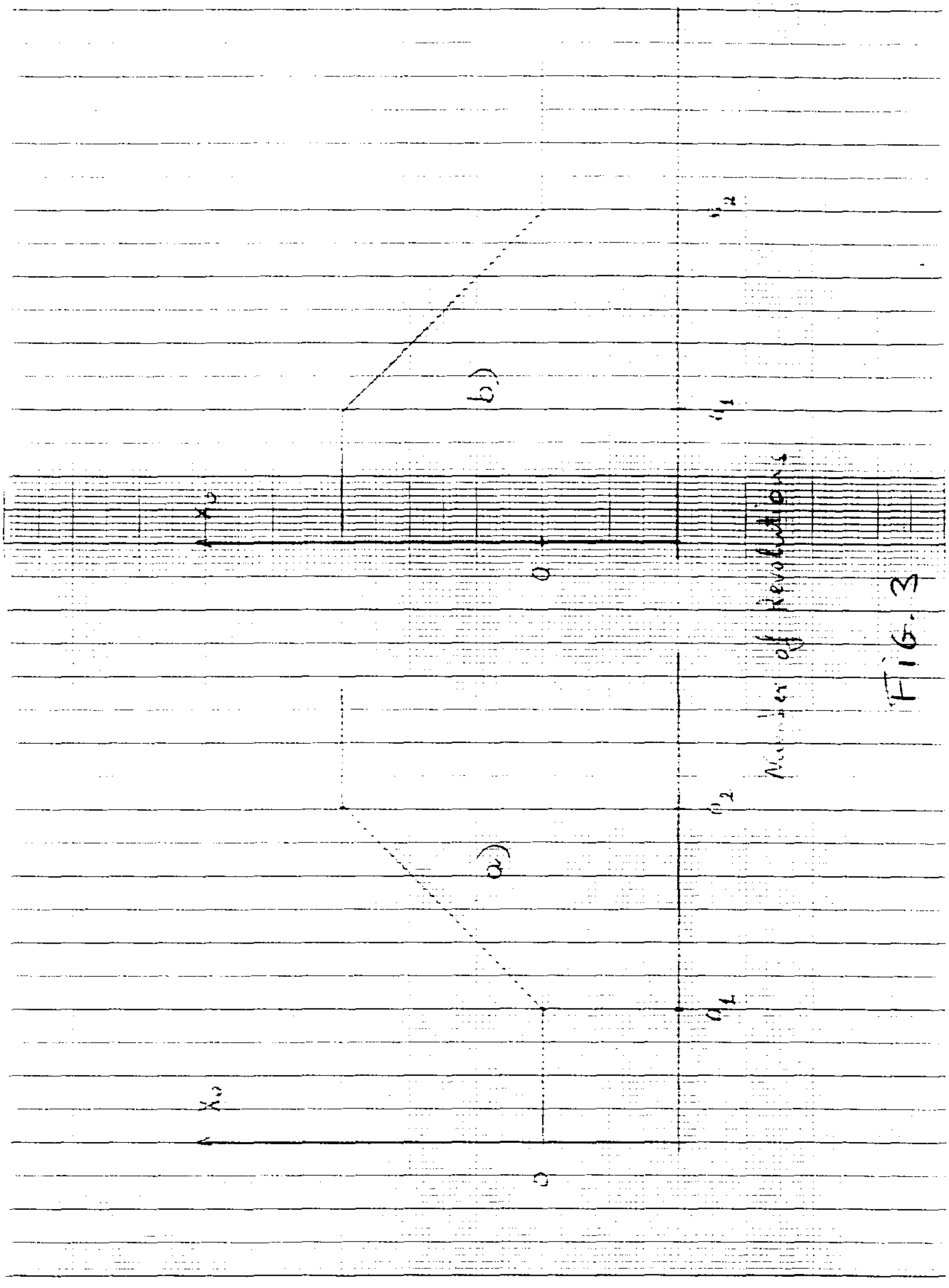

FIG. 3

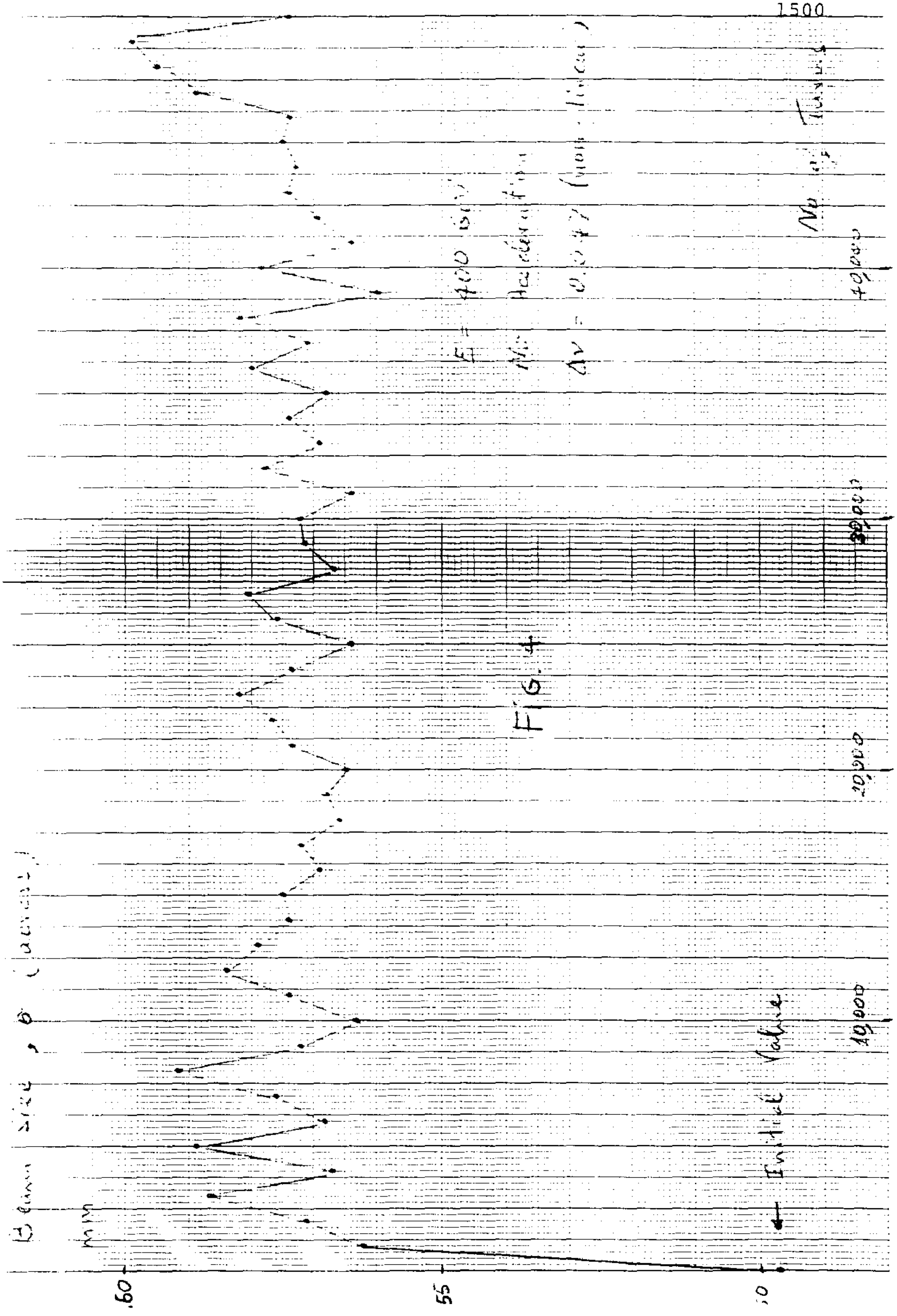

FIG. 4

FN-293
1500

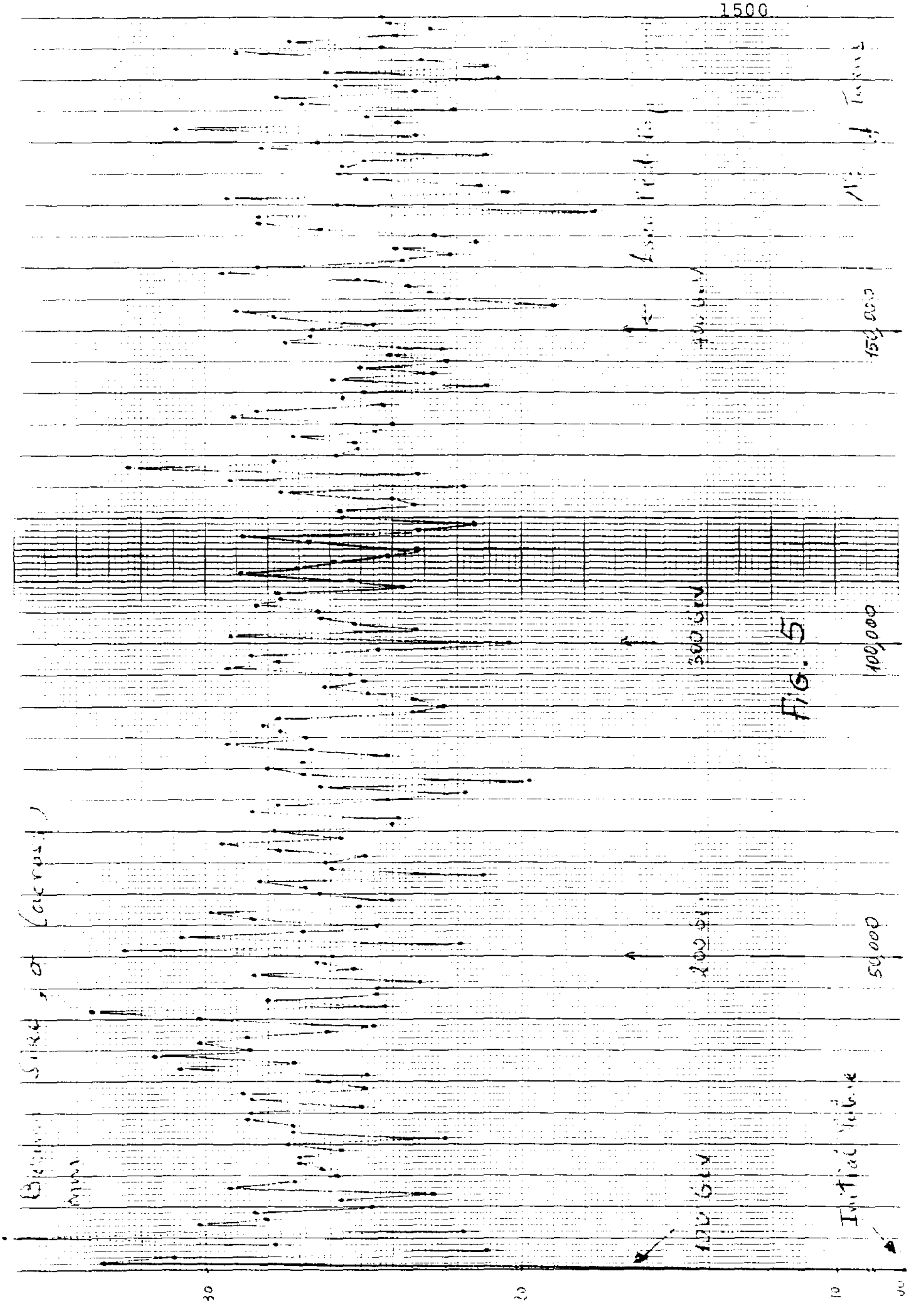

FIG. 5

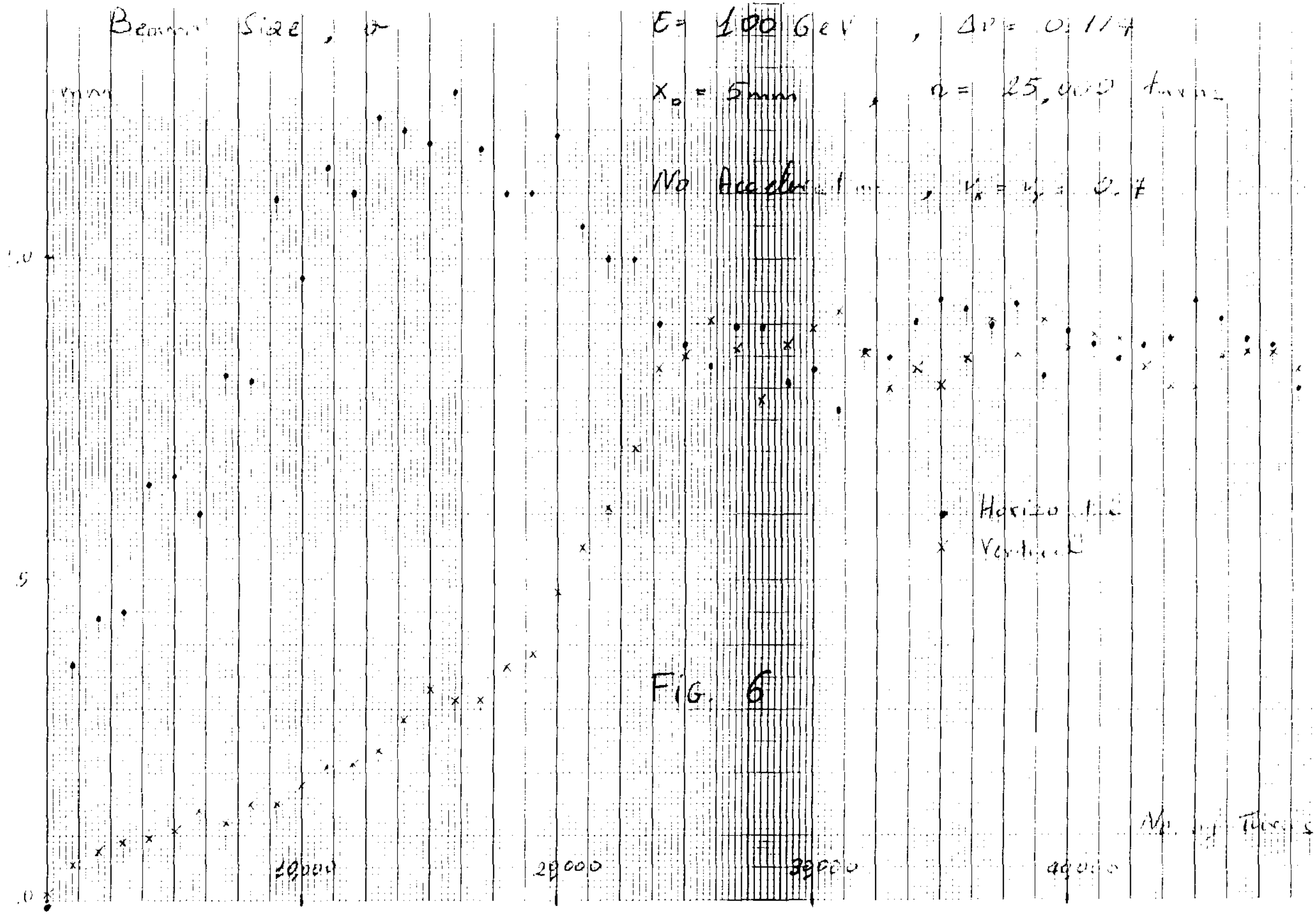
Beam Size, σ
mm
E = 100 GeV, Δν = 0.114
x_o = 5mm, n = 25,000 turns
No Acceleration, $\nu_x = \nu_y$ = 9.7
Horizontal
Vertical
No. of Turns
1.0
.5
.0
10000
20000
30000
40000

FIG. 6

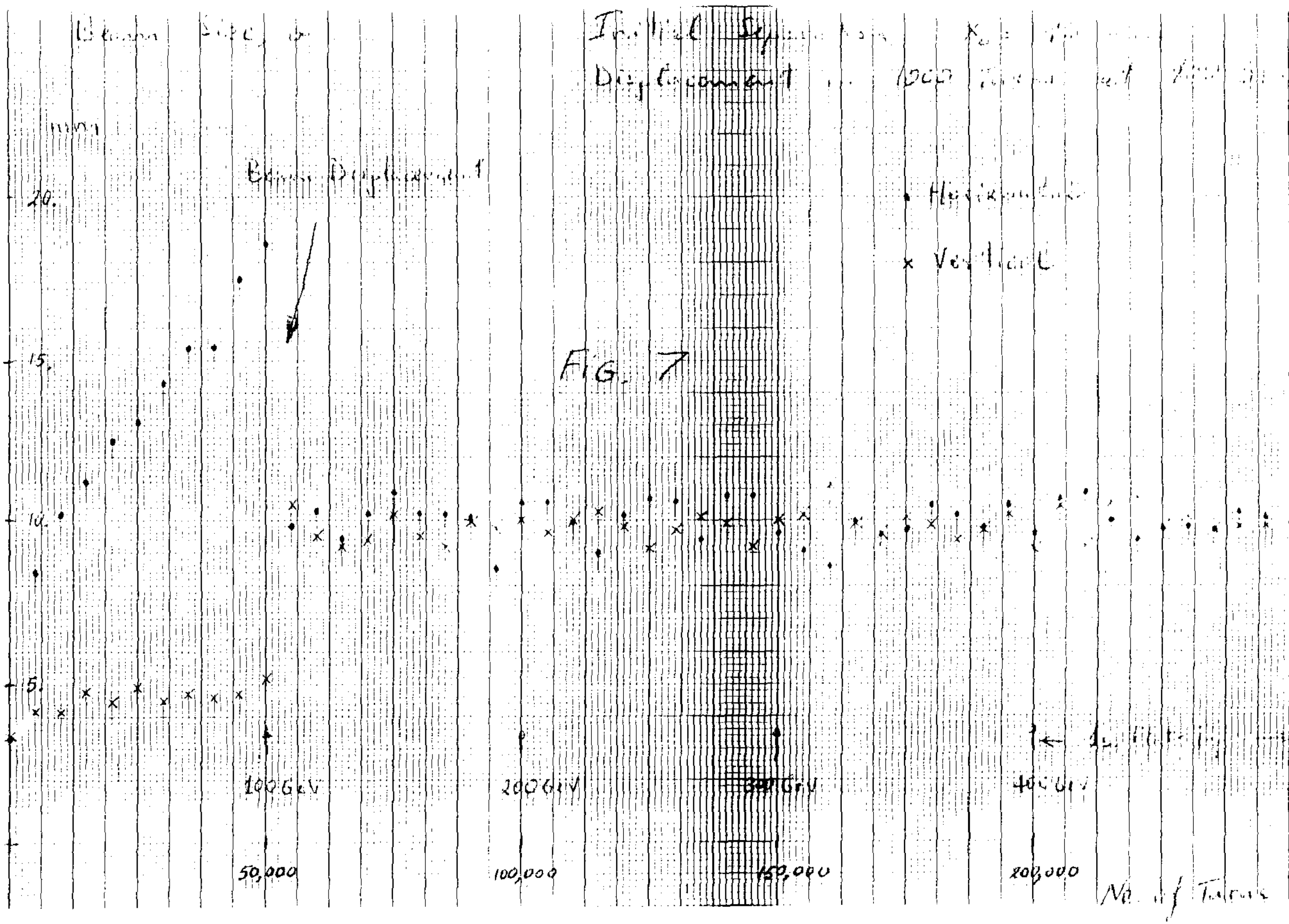
mm
20.
15.
10.
5.
Beam Displacement
• Horizontal
× Vertical
100GeV
200GeV
300GeV
400GeV
50,000
100,000
150,000
200,000
No. of Turns

FIG. 7