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ENERGY LOSS DUE TO THE RESISTIVE MAGNET LAMINATION IN THE NAL BOOSTER

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SUMMARY

A sizeable fraction of the energy gained from the accelerating RF cavities is lost by dissipation through the resistive magnet lamination by the proton beam circulating in the NAL booster. The energy losses per turn and per particle depend linearly on the local beam current around the position of the particle in consideration, and on the resistivity of the material used for laminating.

Numerical calculation at full intensity (3.5×10^{12} protons per pulse) for a beam bunch with a total length of 34 cm (corresponding to the transition energy and in absence of space charge forces) gives an energy loss of more than 100 keV per turn per particle corresponding to about 1/6 of the energy gained from the accelerating cavities.

1. INTRODUCTION

The magnet core is approximated by a stack of annular laminations shorted on the outer surface and open on the side facing the beam. The cracks between laminations act like resonating cavities with a very low figure of quality Q. The value of Q is very close to unity, which is evident from the fact that for almost all frequencies of interest the skin depth in the laminations is larger than the width of the cracks. The energy in the field induced in these crack-cavities by the circulating beam is dissipated as heat. This dissipation results in a loss of the beam energy. Since the field energy varies as the square of the beam intensity the rate of energy loss per particle is expected to increase linearly with the beam intensity.

The equivalent impedance per unit length of the booster lamination has already been calculated in another paper.[1] Besides, we have by now the mathematical tools[2,3] to construct the longitudinal self-fields within a bunched beam, starting with the Fourier analysis of the charge distribution associated to a bunched beam, and for very general beam surroundings.

We shall again take cylindrical geometry and symmetry and a circular beam centered in the pipe. The beam is supposed to consist of a single bunch with a parabolic charge distribution.

2. THE SPACE CHARGE FORCES

Let

z = longitudinal rectified coordinate

v = bunch velocity

t = time

R = closed orbit radius

N = number of particles in the bunch

e = particle charge

$\beta = v/c$, with c the light velocity.

Introducing the distribution function $f(z-vt)$, the charge per unit length λ and the current I are

$$\lambda = Nef(z-vt)$$

$$I = \beta cNef(z-vt)$$

with

$$\int f(x)\,dx = 1.$$

As the motion is periodic with period $2\pi R$ the function $f(x)$ can be decomposed in a Fourier series

$$f(z-vt) = \frac{1}{2\pi R} \sum_{n=-\infty}^{+\infty} \tilde{f}_n \, e^{in\theta}, \quad \theta = \frac{z-vt}{R}$$

$$\tilde{f}_n = \int_{-\pi R}^{+\pi R} f(x) \, e^{-in\frac{x}{R}} \, dx, \qquad x = z-vt.$$

The same can be done for λ and I

$$\lambda = \sum_{n=-\infty}^{+\infty} \tilde{\lambda}_n e^{in\theta}, \qquad I = \sum_{n=-\infty}^{+\infty} \tilde{I}_n e^{in\theta}$$

$$\tilde{\lambda}_n = \frac{Ne}{2\pi R} \tilde{f}_n, \qquad \tilde{I}_n = \beta c \frac{Ne}{2\pi R} \tilde{f}_n.$$

The electric properties of the boundary are described by the normalization coefficient ζ_n between the longitudinal electric field $\tilde{E}_n$ and the transverse magnetic field $\tilde{H}_n$ associated to $\tilde{\lambda}_n$

$$\tilde{E}_n = -\zeta_n \tilde{H}_n. \tag{1}$$

This relation is valid only in first approximation in ζ_n and, hence, ζ_n must be a small quantity. This will be verified a posteriori.

We shall work in the following in terms of impedance per unit length

$$\tilde{Z}_n = \frac{2\zeta_n}{bc}$$

where b is the inner radius of the pipe. It could be easily shown that $\tilde{Z}_n$ is the impedance one can measure when the pipe is excited by a surface current at the frequency $nv/2\pi R$.

We can write for the total space charge forces[3] on a particle sitting at θ

$$F_{tot} = F^{I} + F^{II} = \sum_{n=-\infty}^{+\infty} \tilde{F}_n e^{in\theta}$$

$$\tilde{F}_n = \tilde{F}_n^{\ I} + \tilde{F}_n^{\ II}$$

where

$$\tilde{F}_n^{\,I} = -i\frac{n/R}{\gamma^2}\,\frac{Ne^2}{2\pi R}\,g\,Q_n\,\tilde{f}_n \tag{2}$$

$$\tilde{F}_n^{\,II} = -\frac{Ne^2}{2\pi R}\,\frac{\beta c\,\tilde{Z}_n\,R_n\,\tilde{f}_n}{1 - i\frac{bc}{2}\beta\gamma\tilde{Z}_n\,\frac{I_1\left(\frac{nb}{\gamma R}\right)}{I_0\left(\frac{nb}{\gamma R}\right)}} \tag{3}$$

and

$$Q_n = \frac{1 - \frac{na}{\gamma R}\,\frac{K_1\left(\frac{na}{\gamma R}\right)I_0\left(\frac{nb}{\gamma R}\right) + I_1\left(\frac{na}{\gamma R}\right)K_0\left(\frac{nb}{\gamma R}\right)}{I_0\left(\frac{nb}{\gamma R}\right)}}{\frac{g}{4}\left(\frac{na}{\gamma R}\right)^2}$$

$$R_n = \frac{I_1\left(\frac{na}{\gamma R}\right)\Big/\frac{na}{2\gamma R}}{\left[I_0\left(\frac{nb}{\gamma R}\right)\right]^2}$$

a = beam radius

$\gamma = (1-\beta^2)^{-1/2}$

I_m, K_m = modified Bessel functions

$g = 1+2\ \lg(b/a)$.

The Fourier transform $\tilde{f}_n$ has been plotted in Fig. 1 vs $n\Delta$ where Δ is the half of the angular bunch length in the physical space. Observe that $\tilde{f}_n = \tilde{f}_{-n}$ and that the first zero occurs for $n\Delta = \pm 4.5$. In order to make easy comparisons between different cases we can assume the "beam spectrum" is ranging in the interval $-4.5/\Delta < n < + 4.5/\Delta$ and neglect the contributions of the higher modes.

The two form factors Q_n and R_n have been plotted, respectively, in Fig. 2 and 3 vs $nb/\gamma R$ and for several ratios a/b. Also, Q_n and R_n are even functions of n. We can see that

$$Q_n \sim R_n \sim 1$$

is a good approximation for $\frac{|n|b}{\gamma R} < 0.5$. Thus we can drop the form factors Q_n and R_n at the right-hand side of (2) and (3) if the following condition is satisfied.

$$L = \text{half length of the bunch} = \Delta R < 9b/\gamma .$$

In the following we shall consider only the cases where the above condition is fulfilled.

Besides, as $I_1(x)<I_0(x)$, we drop also the denominator at the right-hand side of (3) if

$$bc\beta\gamma|\tilde{Z}_n|/2 \ll 1$$

from which we also infer the validity of (1).

Antitransforming back we have from (2)

$$F^{I} = -Ne^2(1-\beta^2)\left(1+2\ \lg\frac{b}{a}\right)\frac{df}{dx} \tag{4}$$

which is the well known result for the case of perfectly conductive wall. We have from (3)

$$\begin{aligned} F^{II} &= -\frac{Ne^2}{2\pi R}\beta c \sum_{n=-n_0}^{+n_0} \tilde{Z}_n \tilde{f}_n e^{in\theta} \\ &= -e \sum_{n=-n_0}^{+n_0} \tilde{I}_n \tilde{Z}_n e^{in\theta} \end{aligned} \tag{5}$$

with n_0 the closest integer to $4.5/\Delta$.

3. THE LAMINATION IMPEDANCE

The lamination impedance per unit length $\tilde{Z}_n$ has already been calculated in a previous paper.[1] The computation has been carried out assimilating each crack to a radial transmission line and by solving the equivalent "telegraph equations."

The real and the imaginary parts of $\tilde{Z}_n$ are reproduced in Fig. 4 vs angular frequency ω. The following quantities are also given in Fig. 4.

$$\left.\begin{aligned} \sigma_c &= \text{conductivity} \\ \epsilon_c &= \text{dielectric constant} \\ \mu_c &= \text{magnetic permeability} \end{aligned}\right\} \text{ of the insulator in the crack}$$

$$\left.\begin{aligned} \sigma_w &= \text{conductivity} \\ \mu_w &= \text{magnetic permeability} \end{aligned}\right\} \text{ of the conducting sheet}$$

and

- $b_1 = b$, inner pipe size
- b_2 = outer pipe size
- d = crack width
- D = conducting sheet width
- α = percentage of the accelerator circumference occupied by the lamination.

By varying the values of these parameters we found the following behaviors of $\tilde{Z}_n$:

a. Very weak dependence on b_2 and

$$Z_n \sim \frac{d}{b_1} .$$

b. Very weak dependence on σ_c if σ_c is less than 5×10^7 sec, and

$$Z_n \sim \mu_c/\varepsilon_c.$$

c. σ_w and μ_w enter always as the ratio μ_w/σ_w. We observed

$$Z_n \sim \sqrt{\mu_w/\sigma_w}.$$

In addition, we calculated $\tilde{Z}_n$ with different termination at the bottom of the cracks. The solid lines in Fig. 4 refer to the case that the conducting laminating material is also used as termination. We did not find any change when we replaced it with a perfect shorting, and we observed a change only at low frequencies when there was no termination (cracks open on both sides. Dashed curves in Fig. 4).

Writing

$$\tilde{Z}_n = \tilde{X}_n + i\tilde{Y}_n$$

with $\tilde{X}_n$ and $\tilde{Y}_n$ real, it can be proved that

$$\tilde{X}_n = \tilde{X}_{-n} \qquad \text{and} \qquad \tilde{Y}_n = -\tilde{Y}_{-n}$$

so that (5) can be written as

$$F^{II} = -2e \sum_{n=1}^{n_0} \tilde{I}_n (\tilde{X}_n \cos n\theta - \tilde{Y}_n \sin n\theta). \tag{6}$$

In this formula we are neglecting the contribution of the DC current (n = 0) for we expect it to be very small.

4. THE ENERGY LOSS

The energy loss per particle per turn is

$$\begin{aligned} E &= -2\pi R F^{II} \\ &= 2\pi \mathrm{Re} \sum_{n=1}^{n_0} \tilde{I}_n (\tilde{X}_n \cos n\theta - \tilde{Y}_n \sin n\theta) \end{aligned} \qquad (7)$$

with

$$\tilde{I}_n = \beta c \frac{Ne}{2\pi R} \tilde{f}_n .$$

For fixed bunch half length L and β we used the computer to calculate $\tilde{X}_n$, $\tilde{Y}_n$ and $\tilde{f}_n$ for all the modes

$$n = 1,2,3,\ldots,n_0 .$$

The results have been stored in separated files and then displayed to perform the summation (7).

The energy loss per particle per turn in the NAL booster with $N = 4.2 \times 10^{10}$ particles in a bunch is given in the Figs. 5, 6 and 7 for different β (T is the kinetic energy). The number labelling the curves is the bunch half-length L. In the abscissa we have the internal normalized coordinate.

We notice the following:

1. The energy loss increases linearly with β and there is no other dependence with the energy.
2. The energy loss increases linearly with the negative power of the bunch length.
3. The energy loss function vs the internal coordinate reproduces the particle distribution (a parabola).

In fact $\tilde{Y}_n$ is negligible compared to $\tilde{X}_n$, and $\tilde{X}_n$ is almost constant in the frequency range of interest. Then replacing $\tilde{Z}_n$ at the right-hand side of (5) by its approximated constant value $\tilde{Z}$ we have as a good approximation

$$F^{II} \simeq -Ne^2 \beta c \tilde{Z} f(x).$$

Thus in normal conditions of operation the protons circulating in the NAL booster are expected to lose about 100 keV per turn at the transition energy and at the design intensity. This loss is about 1/6 of the energy gain per turn with normal acceleration.

REFERENCES

1. A. G. Ruggiero, "Longitudinal Space Charge Forces within a Bunched Beam in the presence of Magnetic Lamination," FN-220 (0402) (January, 1971).
2. A. M. Sessler and V. G. Vaccaro, "Longitudinal Instabilities of Azimuthally Uniform Beams in Circular Vacuum Chambers with Walls of Arbitrary Electrical Properties," CERN Report 67-2 (February, 1967).
3. A. G. Ruggiero, "Longitudinal Space Charge Forces within Bunched Beams," FN-219 (0402), (December, 1970).

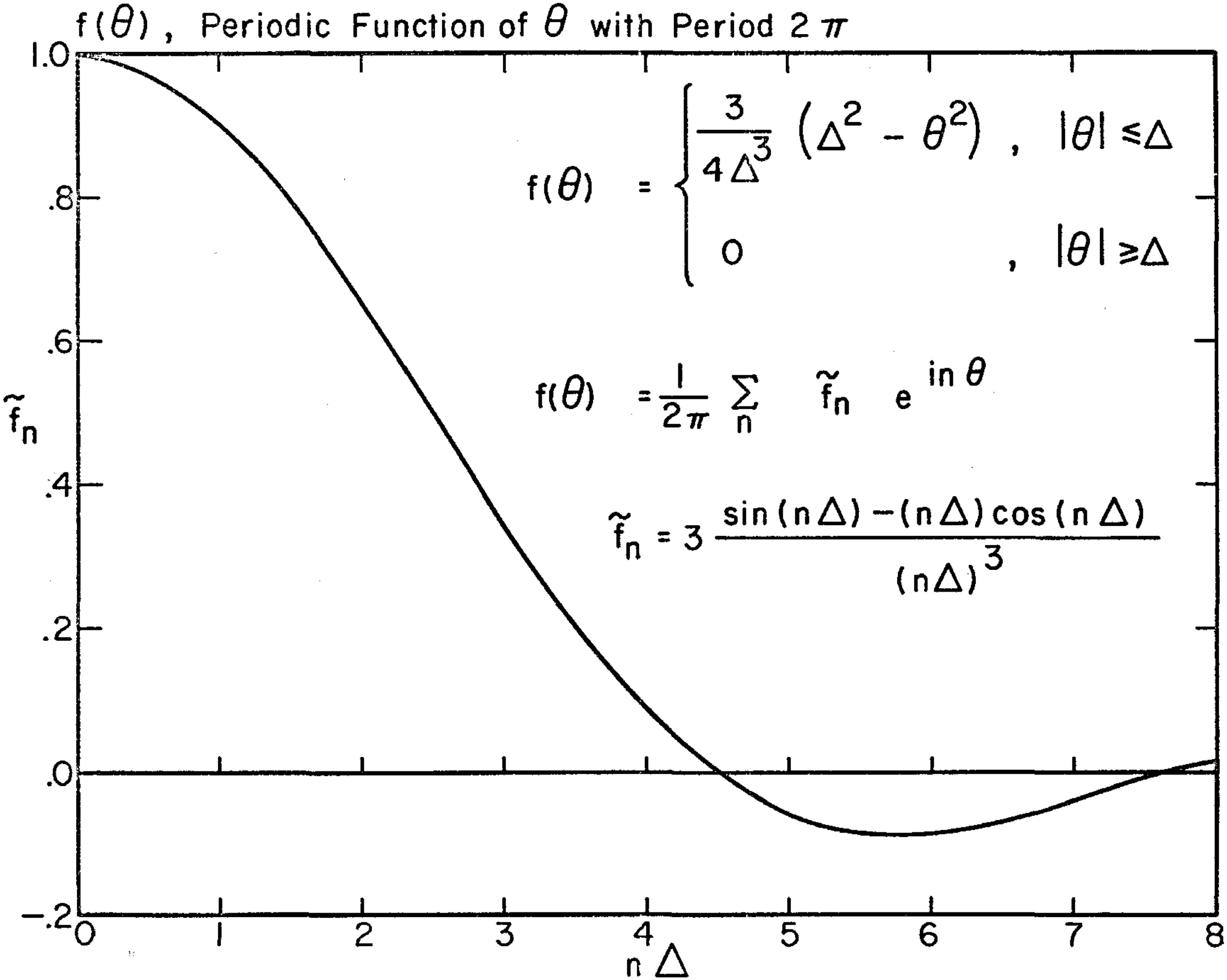
f(θ), Periodic Function of θ with Period 2π
1.0
.8
.6
.4
.2
.0
-.2
f̃n
0
1
2
3
4
5
6
7
8
nΔ
$f(\theta) = \begin{cases} \frac{3}{4\Delta^3}\left(\Delta^2 - \theta^2\right), & |\theta| \le \Delta \\ 0, & |\theta| \ge \Delta \end{cases}$
$f(\theta) = \frac{1}{2\pi}\sum_n \tilde{f}_n e^{in\theta}$
$\tilde{f}_n = 3\,\frac{\sin(n\Delta) - (n\Delta)\cos(n\Delta)}{(n\Delta)^3}$

Figure 1

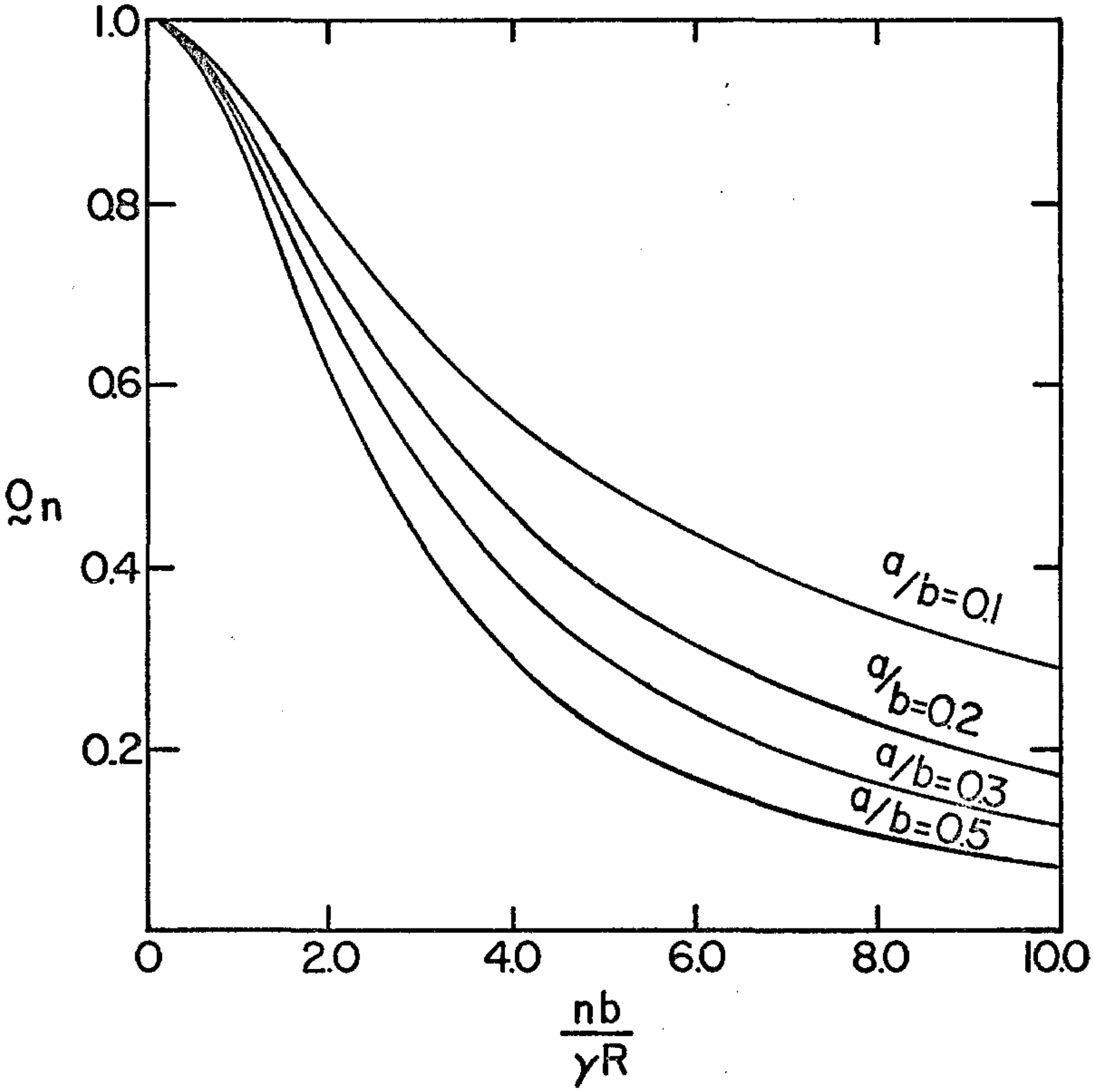

Figure 2

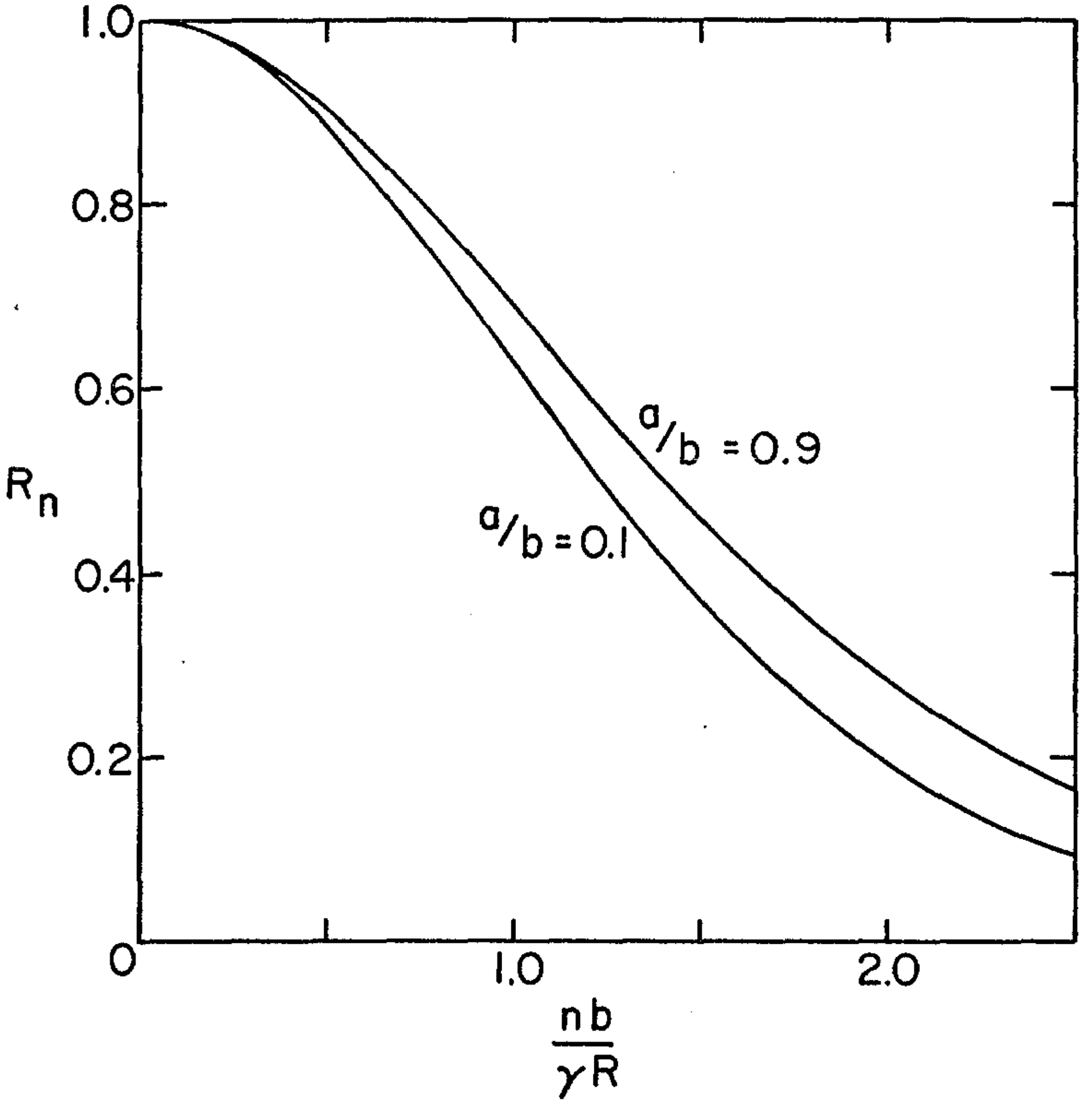

Figure 3

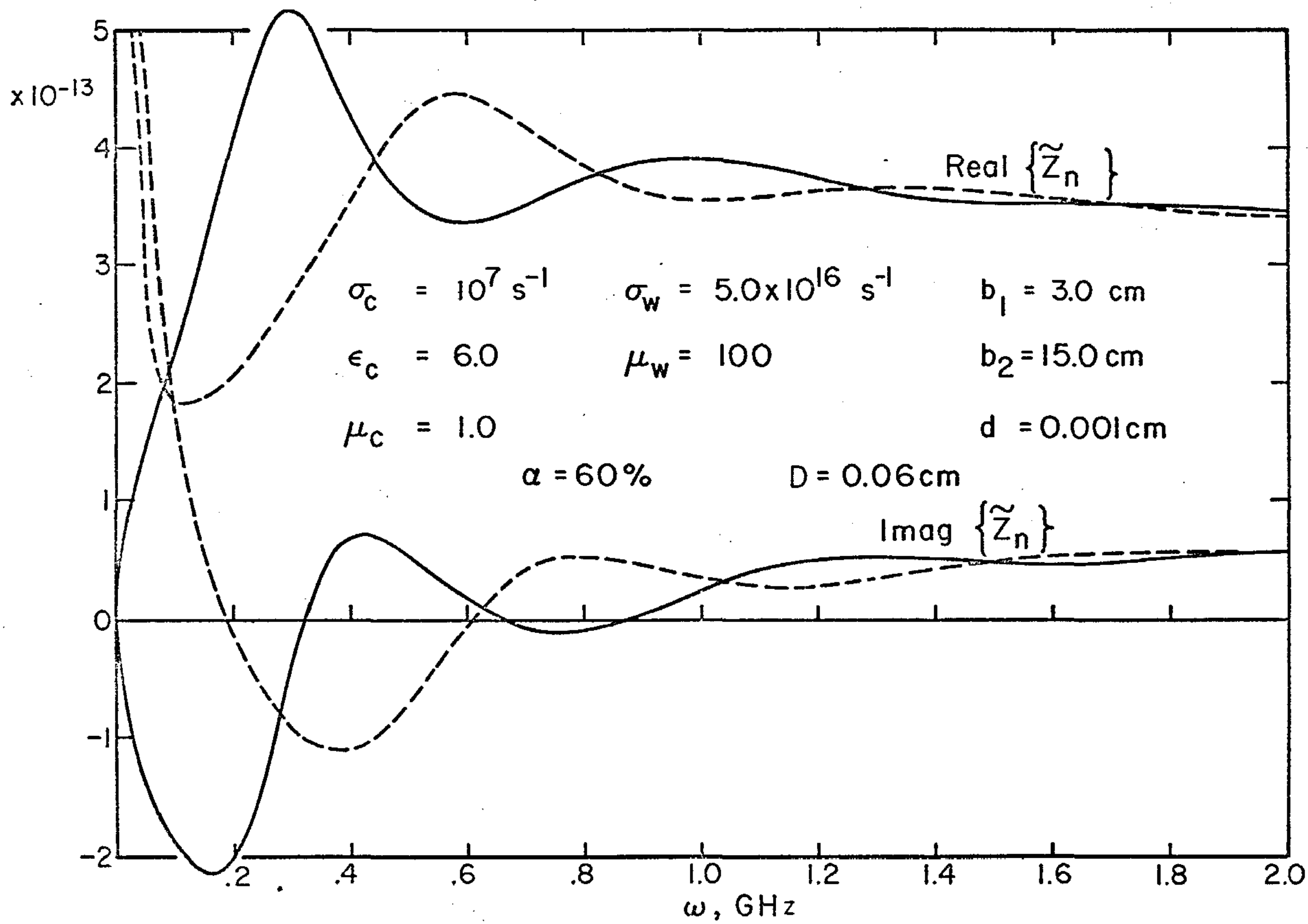

Figure 4

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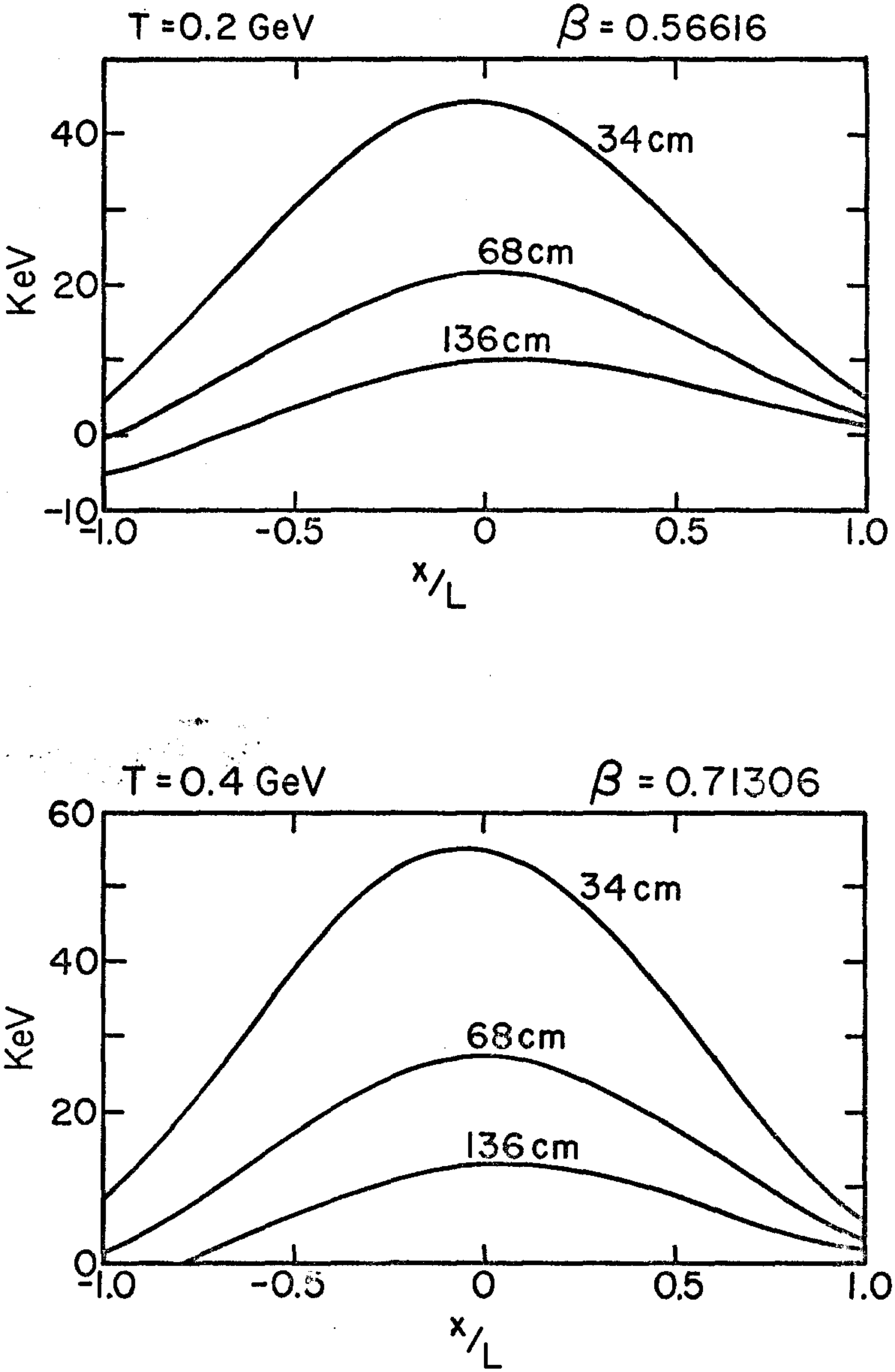
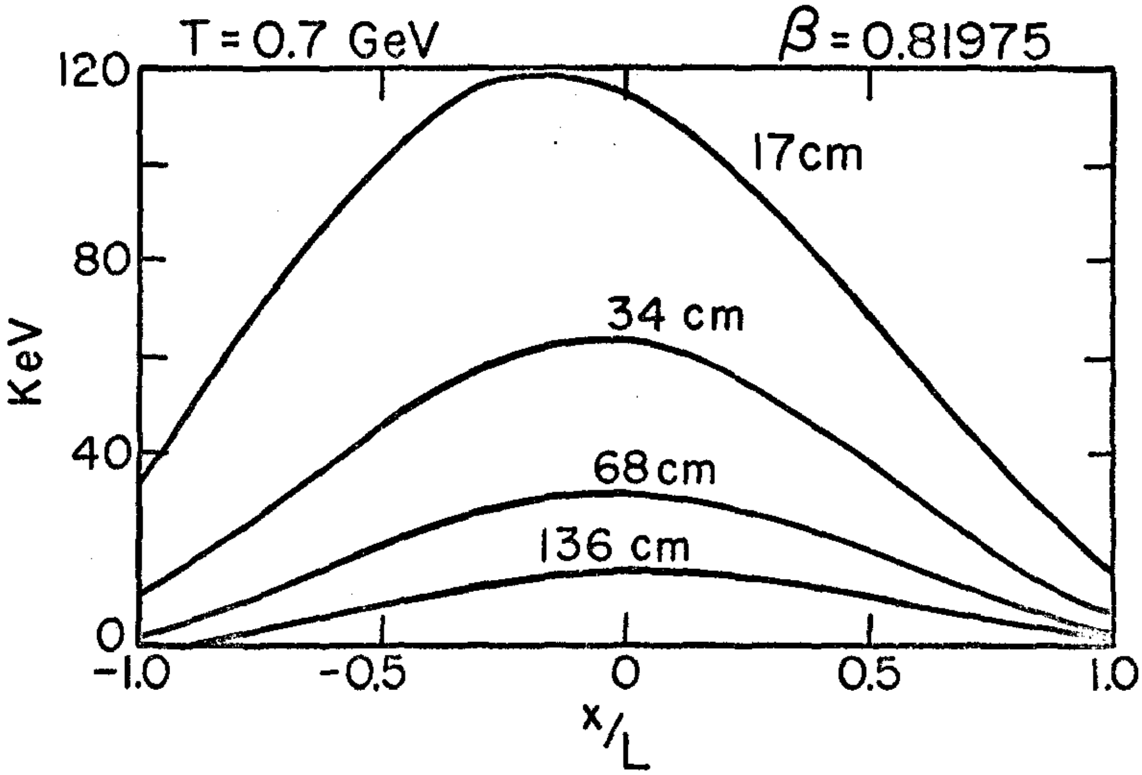

T = 0.2 GeV
β = 0.56616
34 cm
68 cm
136 cm
KeV
40
20
0
-10
-1.0
-0.5
0
0.5
1.0
x/L
T = 0.4 GeV
β = 0.71306
60
40
20
0
34 cm
68 cm
136 cm
KeV
x/L

Figure 5


T = 0.7 GeV
β = 0.81975
120
80
40
0
KeV
17cm
34 cm
68cm
136 cm
-1.0
-0.5
0
0.5
1.0
x/L

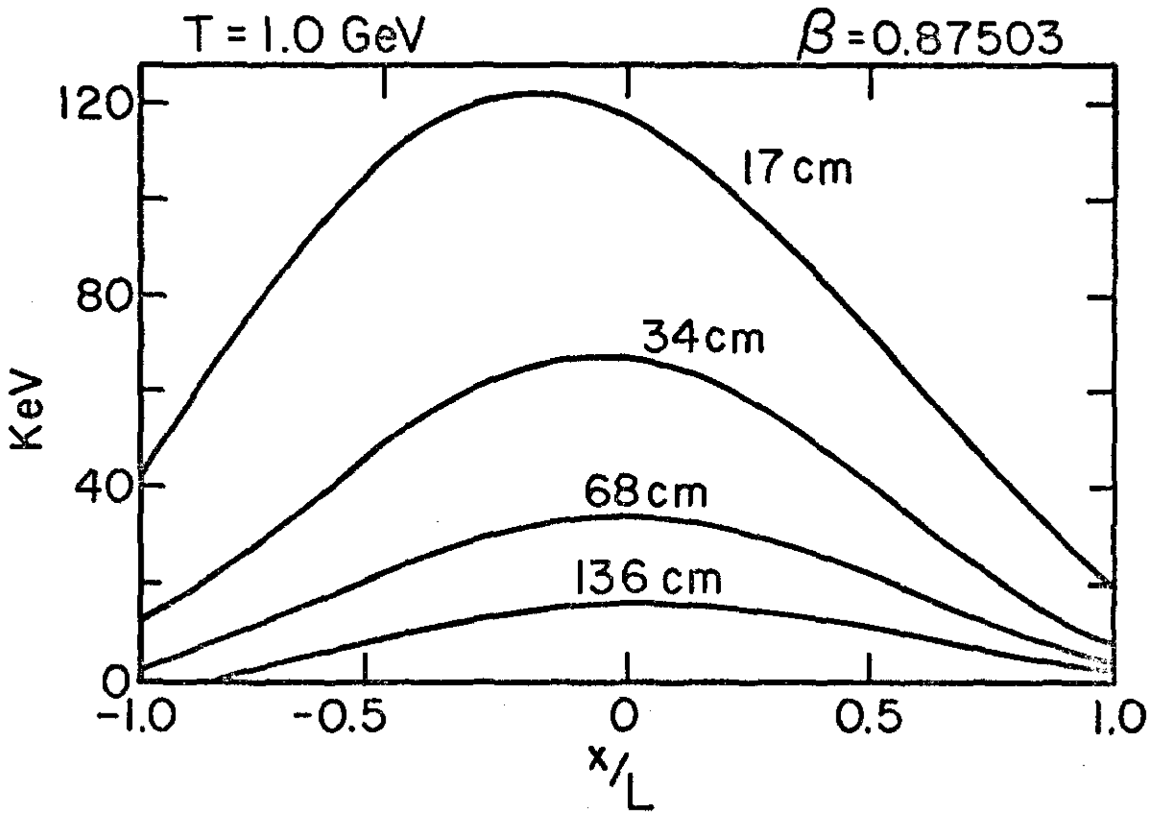

T = 1.0 GeV
β = 0.87503
120
80
40
0
KeV
17 cm
34cm
68cm
136 cm
-1.0
-0.5
0
0.5
1.0
x/L

Figure 6

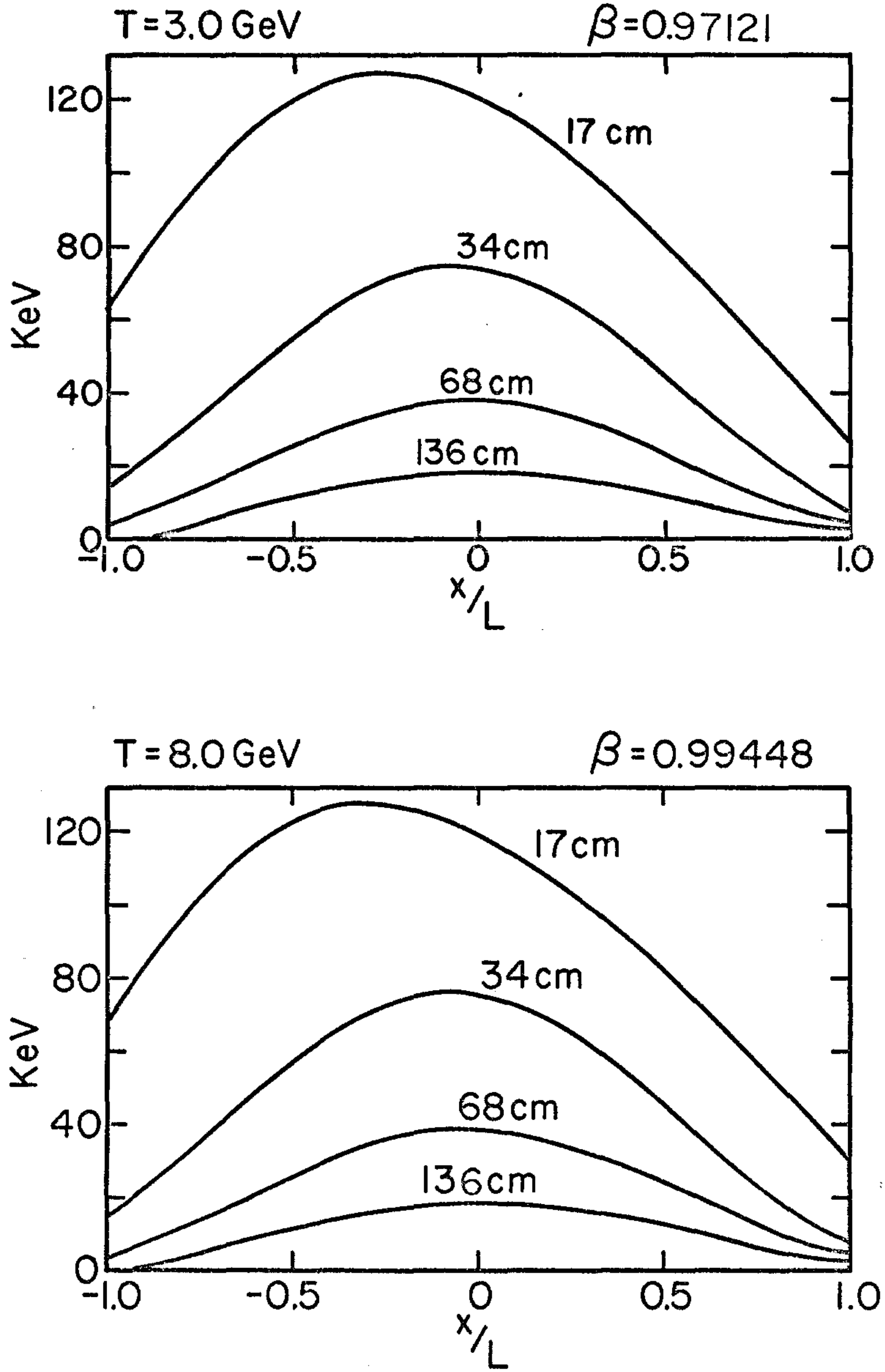

T=3.0 GeV
β=0.97121
120
80
40
0
KeV
17 cm
34cm
68cm
136cm
-1.0
-0.5
0
0.5
1.0
x/L
T=8.0 GeV
β=0.99448
120
80
40
0
KeV
17cm
34cm
68cm
136cm
-1.0
-0.5
0
0.5
1.0
x/L

Figure 7