

AN ESTIMATE OF LUMINOSITY AND  $\nu$  SHIFTS

E. Keil, B. W. Montague, W. Schnell, and A. M. Sessler

August 27, 1968

1. Number of protons in  $\Delta p/p = \pm 0.1\%$

At booster injection energy the longitudinal phase-plane area of one bunch in a  $(\Delta p/m_0c, \Delta\phi_{rf})$  plane is given by (from the June 25, 1968,

$$A = 6.85 \times 10^{-3} \text{ rad}$$

booster parameter list).

There are three obvious points in the acceleration cycle where dilution of phase-plane density is likely to occur viz. transition in the booster, transfer from the booster to the main ring and transition in the main ring. At the nominal intensity ( $5 \times 10^{13}$  particles per pulse in the main ring) the transition space-charge parameter  $\eta_0(0)$  equals 2.5 in the booster and 4.7 in the main ring.\* The effect of this is not known exactly but will be rather severe; especially in the main ring. In the absence of better knowledge we assume that the total blow-up of phase-plane area from 200 MeV to 100 GeV is by a factor of 8 viz. a factor of 2 at each of the 3 points mentioned above. This assumption may be somewhat pessimistic for the beam transfer, but it is fairly optimistic for the two transition points and presupposes a certain degree of compensation. It should be noted that the CERN PS has a blow-up by about a factor of two with  $\eta_0(0) = 1.3$  and compensation has been found to be difficult so far.

---

\* Scaled from E. D. Courant's note of November 6, 1967.

No large blow-up is expected to occur at the transfer from main ring to storage ring since no synchronization problem exists at this point and phase-lock beam-control will center the bunches within the buckets with very good precision.

With an eight-fold blow up one finds

$$A = 5.48 \times 10^{-2} \text{ rad}$$

at 100 GeV. The corresponding full width of an adiabatically debunched beam is given by

$$\left( \frac{\Delta p}{p} \right)_0 = 0.82 \times 10^{-4}$$

Assuming a stacking efficiency of about 82% one can stack 20 pulses in a momentum bite of  $\pm 10^{-3}$ . With  $5 \times 10^{13}$  particles per pulse (i. e. transferring the full charge of the main ring into the storage ring of 1/3 radius by 3-turn injection) one can stack

$$N = 10^{15}$$

particles.

## 2. Beam emittance in SR

$$\text{Assume } E_v = 0.2 \pi \text{ } \mu\text{rad m}$$

$$E_h = 0.5 \pi \text{ } \mu\text{rad m}$$

at 100 GeV/c in main ring.

Add errors in main ring ejection, transfer tunnel, SR injection. We take a factor of 2 in emittance.

Hence in SR for one turn

$$E_v = 0.4 \pi \text{ } \mu\text{rad m}$$

$$E_h = 1.0 \pi \text{ } \mu\text{rad m}$$

Assume 3 turn injection into SR, injection scheme as shown in figure, no septum thickness assumed. Get a blow-up factor B in emittance of

$$B = \frac{49}{27} = 1.8$$

hence have for circulating beam in SR

$$E_v = 0.4 \pi \quad \mu\text{rad m}$$

$$E_h = 5.4 \pi \quad \mu\text{rad m}$$

This is a very flat beam.

We find for the luminosity in one intersection

$$\mathcal{L} = \frac{c}{\alpha b} \left( \frac{N}{2 \pi R} \right)^2 ,$$

where  $\alpha$  is the crossing angle

$b$  is the vertical beam radius, assuming horizontal crossing.

$$E_v = \frac{\pi b^2}{\beta_v}$$
$$b = \left( \frac{E_v \beta_v}{\pi} \right)^{1/2} .$$

Put in numbers:

$$N = 10^{15}$$

$$R = 333 \text{ m}$$

$$\alpha = 0.1$$

$$\beta_v = 10\text{m} .$$

We find

$$b = 0.2 \text{ cm}$$

$$\mathcal{L} = 0.38 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1} .$$

Hence a reduction of  $\beta_v$  is essential for reaching the design aim  $\mathcal{L} = 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$ .

3. Beam-beam  $\nu$  shift

$$\Delta\nu = \frac{Nr \beta}{2\pi R \alpha \gamma b} \nu$$

$$\Delta\nu = 4 \times 10^{-4}$$

4. Single beam  $\nu$  shift

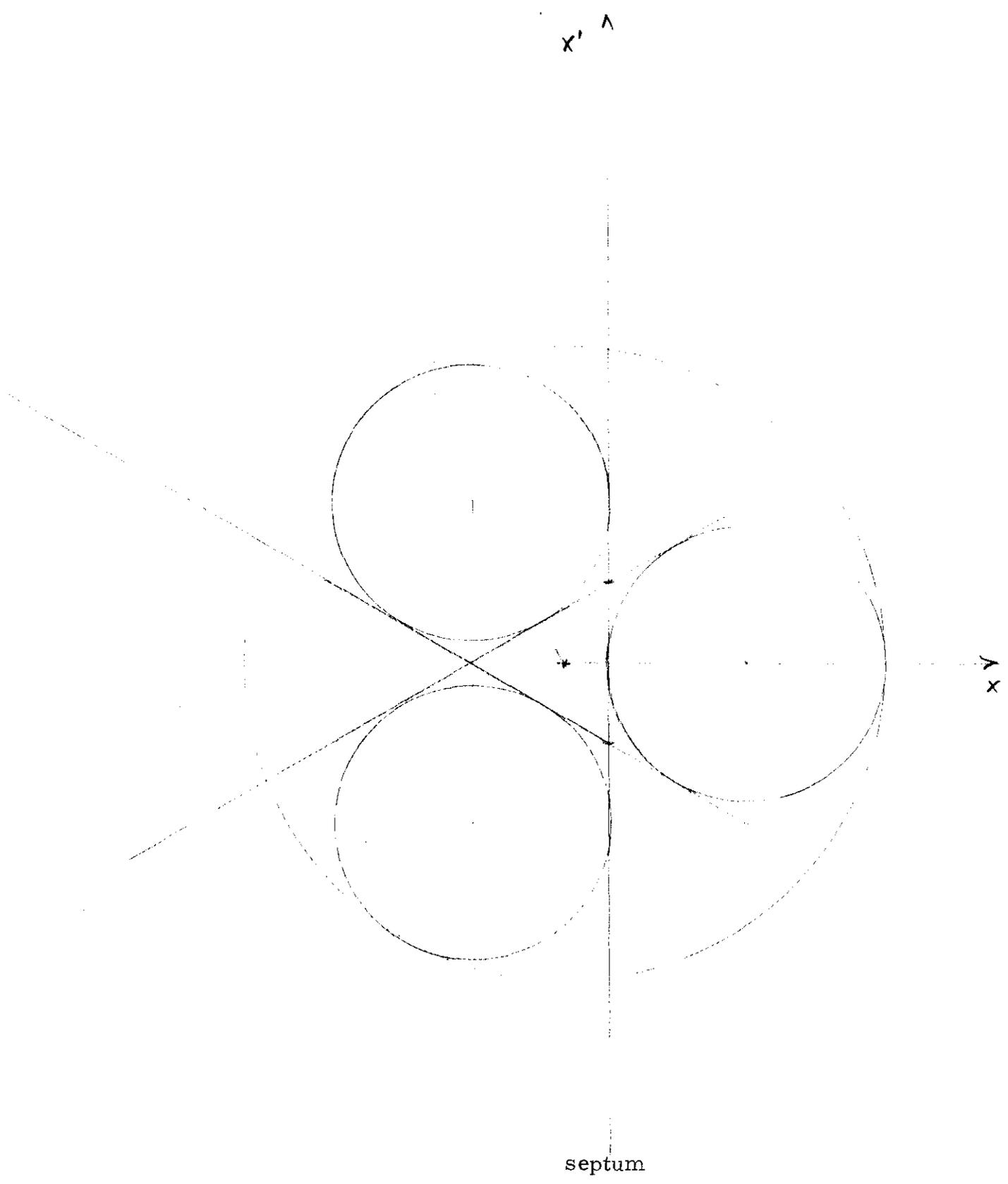
$$\Delta\nu = \frac{Nr R}{\pi \gamma \nu h} \left[ \epsilon_1 + \frac{h^2}{g^2} \epsilon_2 \right]$$

$h, g$  are the half heights of the vacuum chamber and of the magnet gap, respectively.

Find for  $\epsilon_1 = 0.16$

$$\Delta\nu = \frac{0.2}{h}$$

Thus with  $h = 1$  cm the SR is close to the single beam space charge limit.



Clean 3-turn injection.