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THE IMPREGNATION OF SUPERCONDUCTING WINDINGS WITH EPOXY RESINS

Part 1. THEORETICAL CONSIDERATIONS

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ABSTRACT

The causes of stresses in magnet windings are discussed; thermal contraction stresses and temperature gradient stresses are examined in detail. The initiation and spreading of cracks are considered. The improvement of properties by addition of fillers and flexible polymers is discussed.

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December 1972

CONTENTS

		Page
1.	Foreword	1
2.	Introduction	1
3.	The Causes of Stresses in Windings	3
4.	Thermal Contraction Stresses	5
5.	Temperature Gradient Stresses	8
6.	The Initiation and Spreading of Cracks	13
7.	The Modification of Epoxy Resins	18
8.	Acknowledgement	23
9.	References	24
Appendix I.	Stress and Thermal Contraction	25
" II	Modified Outwater's Theory	26
" III	Quench stresses	31
" IV.	Weibull's theory	32
Fig.1	Properties of FMNA 65	33
" 2	Thermal Contraction Stresses	34
" 3	Bond Fracture	35
" 4	Trapped resin	36
" 5	Effects of Additional Tensile Stress	37
" 6	Conductor and Resin Temperatures during Quenching	38
" 7	Thermal Contractions of Silica-loaded resins	39
" 8	$E\alpha$ graph for FMNA 65	40
" 9	Modified Outwater's Method. Diagrams	41
" 10	Stress and Thermal Contraction	42

1. FOREWORD

The purpose of this report is to summarize our work on epoxy resin impregnants. Some data for material other than epoxies are, however, included; and much of the theoretical work is, of course, applicable to any suitable impregnant. The potential field of research is enormous; we have been able merely to touch on some of the problems. Much of our theoretical work is hypothetical in nature and would require greater effort and more knowledge than we have available to reach sound conclusions.

The reader will find numerous references to an epoxy resin known as "FMNA". He should avoid interpreting this as a recommendation for this particular material. We used this resin to impregnate early magnets since we had some considerable experience in its use for other purposes. It is an easily prepared material which gives reasonably reproducible results, and for these reasons we have tended to retain it as a sort of standard against which other resins may be judged. There are many other resins suitable for impregnating coils.

This work has been carried our primarily to aid the development of dipole and quadrupole beam bending and focussing magnets. This is the reason why so little consideration is given to stresses caused by electromagnetic or Lorentz forces, since these magnets are invariably surrounded by heavy support structures. The information is nevertheless relevant to solenoids. In a sense, the designer of solenoids is faced with an additional problem, since the high stresses developed from the Lorentz forces are imposed on a material which may be already highly stressed as a result of the cooling process.

The report is in two parts. This is the first part and deals with theoretical considerations. The second part is a summary of the properties data which have been obtained.

2. INTRODUCTION

Movement of the conductors in a superconducting winding must be prevented, since it gives rise to two deleterious effects: incorrect positioning of conductors will cause errors in magnetic field, and the rubbing of conductors against each other or against other parts of the structure will cause frictional heating which may degrade the performance

of the superconductor.

The simplest method of preventing movement is to impregnate or "pot" the winding with a suitable material. This material should satisfy the following requirements:

(a) it must be electrically insulating, but possess adequate thermal conductivity so that heat generated within the windings may be easily removed without excessive temperature rise,

(b) it must be solid at the operating temperature (about 4K). Preferably it should be solid at room temperature also, since this eases the problem of handling the windings during assembly work. Coils impregnated with substances liquid at temperatures above the operating temperature eg oil, water or liquid nitrogen, have been tried and gave excellent results (ie quench currents greater than 90% of short sample current), but the lack of mechanical rigidity at room temperature has prevented the adoption of this technique,

(c) the material must be able to withstand the stresses developed during operation without undergoing excessive distortion. It must withstand the stresses which are applied at room temperature by preloading of the clamping system around the magnet: this clamping system is necessary since the winding is rarely of itself strong enough to withstand the electromagnetic forces during operation, nor is it often sufficiently rigid not to deform excessively. These stresses and their effects are discussed in the next section,

(d) the material should not enhance the possibility of distortion of the winding. For example, during cooling to operating temperature a material with thermal contraction significantly different to that of the superconductor may cause distortion if sufficiently large concentrations of the material occur at places within the magnet. The occurrence of stress relaxations, dependent on temperature, time and stress level, may also cause distortions,

(e) the material should be free from long term effects which may cause distortion, eg creep at room temperature, moisture absorption, degradation due to chemical change,

(f) the material should not cause the windings to distort during manufacture, eg because of shrinkage or expansion during solidification or curing,

(g) the material should be radiation-resistant if required to be used in the presence of ionizing radiations,

(h) finally, it must be easy (or at least, possible) to apply the material to the impregnation of windings.

It is not easy to find a material which satisfies all these requirements. Present technology makes use of epoxy resins, since these have advantages of strength and relative ease of application: they possess many disadvantages, as will be seen hereafter. Other materials have been suggested (and tried) from time to time; for example, cements. But the use of such alternatives would require large amounts of development effort, and they have some significant disadvantages: cement, for example, has very little tensile strength, and although recent work[1] has shown that this might be improved by adding fibrous materials, yet a disproportionate amount of development was felt to be necessary to evolve suitable mixtures and methods of application. Since the impregnant problem is only one of several major problems in superconducting magnet technology, and because of the limited effort available for solving these problems, we have tended to restrict our investigations to epoxy resins.

3. THE CAUSES OF STRESSES IN WINDINGS

The chief causes of stresses are as follows:

(a) Thermal contraction differences. Resins have large thermal contractions between room temperature and 4K. Stress is developed in the resin because it is restrained by the metallic conductors embedded in it which have much lower contraction.

(b) Temperature gradients. These may develop during cooling or warming. Of more interest, however, are the stresses which result from quenching. When a magnet begins to quench the normal region, which is hot and therefore expanded, causes tensile stresses in the remainder of the winding surrounding it. Within the hot region itself, the conductors, since they are the sources of heat, are hotter than the resin surrounding them, so they expand and increase the stresses. When all the energy has been dissipated, the warm magnet is cooled rapidly by contact with liquid helium, a process which also generates tensile stresses at surfaces.

(c) Electromagnetic forces. In dipole and quadrupole magnets it is

necessary to provide rigid clamping systems, to react the Lorentz forces developed along the straight sides. As a result, the development of tensile stresses in the windings due to these forces is usually avoided, and the windings are subjected instead to radial and circumferential compressive stresses: typical values are less than 5000 lbf/in^2. In solenoids, however, it may be necessary to allow the conductors to resist these forces, or some porportion of them: this will give additional tensile stresses in the resin.

(d) Preloading of clamping system. Usually the clamp material has less thermal contraction than the windings. It is therefore necessary to pre-strain the clamps and windings so that adequate support is still available when cold. Reasonably low stresses are applied eg 1000-2000 lbf/in^2 in the windings. These may be sufficient to cause distortion or creep.

(e) Shrinkage stresses which result from curing. It is probable that these stresses are due only to the thermal contraction which occurs when the resin is cooled from cure temperature to room temperature, as has been demonstrated for fibres embedded in polyester resin by Haslett and McGarry.[2] They found an axial stress of 7 MN/m^2 (1000 lbf/in^2) in resin cured at 50°C above room teperature. We have not measured the elastic modulus and expansion coefficients for epoxy resins above room temperature, but if we assume that the product $E\alpha$ maintains a constant value for FMNA resin up to the glass/rubber transition temperature T_g, we obtain a stress of 33 lbf/in^2 per degree Celsius difference between T_g and room temperature for restraint in two dimensions by metal conductors. This compares with the results for polyester resin mentioned above, bearing in mind the difference in properties between this and epoxy resin. Thus for a temperature difference of 100°C, we might expect a room temperature tensile stress of 3300 lbf/in^2 in our FMNA resin, when restrained in two dimensions. This is probably an overestimate; the stress may be reduced by stress-relaxation. Nevertheless, it indicates that moderately high stresses are already present in the resin before the winding is cooled to liquid helium temperature.

In addition to the foregoing, stresses may result from the method of support, eg supporting a long horizontal magnet at each end will develop bending stresses.

Usually the most serious stresses are those due to different thermal

contractions, followed perhaps by quenching stresses. Electromagnetic stresses may require consideration in large solenoids.

As will be seen hereafter, thermal contraction stresses may be a significant proportion of the ultimate strength of a resin. Unfortunately, the situation is made worse by the presence of stress-raisers, eg sharp edges or vapour bubbles, causing local stresses to be much higher than the average or nominal stress. Even a spherical bubble or rigid spherical inclusion will double the average stress.[3] It is at such stress concentrations that cracks may begin, and may spread through the winding with dissipation of strain energy as heat which may degrade the magnet's performance.

Little attempt has been made to measure stresses in operating magnets. Understandably, magnet designers have been loth to incorporate strain gauges and their associated wiring into windings; large superconducting magnets are very expensive to build, and one hesitates to jeopardize performance by including 'foreign bodies' in the windings. Strain gauges would give no information about stress concentrations, although they would, of course, act as undesirable stress-raisers themselves. However, now that superconducting magnet technology has developed to the point where predictable and reliable operation is common, attempts to measure operational stresses would seem worthwhile and the results would be of great value to materials development.

In default one is forced to estimate stresses theoretically. This is a somewhat unsatisfactory approach since the windings themselves are far too complex for stress analysis and one has to resort to simple models. The results should be treated with caution, regarding them as guides to the better choice of resins, and not as indications of the actual stresses existing in superconducting magnets during operation.

In the following two sections we shall consider at greater length the stresses which arise from thermal contraction differences and temperature gradients. After that we will go on to discuss the initiation of cracks and the factors which control their development.

4. THERMAL CONTRACTION STRESSES

Consider a bar of resin of unit length. Let du = change in length

due to change in temperature dT and change in stress $d\sigma$.

Then[4]
$$du = \left(\frac{\partial u}{\partial T}\right)_\sigma dT + \left(\frac{\partial u}{\partial \sigma}\right)_T d\sigma \qquad (1)$$

$$= \alpha dT + \frac{d\sigma}{E} \qquad (2)$$

where α = thermal contraction coefficient

E = modulus of elasticity

From (2) we obtain

$$\int d\sigma = \int E du - \int E\alpha dT$$

If the ends of the bar are fixed so as to prevent contraction completely when cooling from 300K to 4K, ie $du = 0$, then

$$\int_{300K}^{4K} d\sigma = -\int_{300K}^{4K} E\alpha dT = \int_{4K}^{300K} E\alpha dT \qquad (3)$$

We have measured E and α over the temperature range 4 to 300K for one epoxy resin (FMNA 65), see fig 1 (Fig 8 shows the variation of $E\alpha$). The tensile stress calculated from equation (3) for this resin is $\sim$ 8200 lbf/in^2.

If the contraction of the bar is also prevented in another direction perpendicular to the direction of the length, the stress increases to

$$\frac{\int_{4K}^{300K} E\alpha dT}{1-\mu} \qquad (4)$$

where μ = Poisson's ratio,

which, with $\mu = 0.35$, gives a stress of 12600 lbf/in^2.

Complete restraint in three directions at right angles would give a stress of

$$\frac{\int_{4K}^{300K} E\alpha dT}{1-2\mu} \qquad (5)$$

or 27300 lbf/in^2 when $\mu = 0.35$.

If the restraint is provided by a metal substrate which also contracts, the stress will be less. Typically, the integrated contraction of the metal between 300K and 4K is about 25% of that of the FMNA resin. If the elastic modulus and volume of the metal are much larger than those of the resin, we would expect the stress to be $\sim$ 9500 lbf/in^2 for restraint in two dimensions only, and $\sim$ 20500 lbf/in^2 for complete restraint.

During the experiments to measure E and α, we also noted the effect of stress on thermal contraction (see Appendix I).

Another approach is to consider a circular cylinder of metal (representing a conductor wire) surrounded by a layer of resin of even thickness. Infinite lengths may be assumed and stresses calculated from thin or thick cylinder theory. In the limit, as resin thickness tends to zero, the stress tends towards the value given by equation (4) above. However, it is more interesting to consider the resin bounded by a crack perpendicular to the cylinder axis, and to attempt to calculate the way in which tensile stress is developed in the resin from the shear bond between resin and metal. The problem has some resemblance to the problem of a fibre embedded in a matrix, for which several solutions have been proposed, notably by Cox, Dow, Rosen and Outwater.[5] These theories consider the application of a general strain to both matrix and fibre, and only the theory by Outwater is easily adaptable to the case where strain is generated by differential contractions within the system.

The adaption of the method is given in Appendix II: some results are shown in fig 2. The theory is unable to consider the effects of an adhesive bond between resin and metal, and assumes that the shear is transmitted by friction, but it provides a picture of the stress distribution adjacent to a crack. The axial tensile stress in the resin is built up from zero to its maximum value in a very short distance from the crack face, typically of the order of one wire (or metal cylinder) diameter, depending on the value assumed for the friction coefficient. The friction force is provided by the contraction of the resin onto the metal. Thinner resin layers give higher tensile stresses, but the shear stress between metal and resin falls rapidly as resin thickness is reduced.

Although the stresses may be much modified when an adhesive bond is present between wire and resin, the pattern will be similar. The bond will be highly stressed adjacent to a crack; the shear stress is likely to be high if the resin layer is thick. When a crack, more or less perpendicular to the conductors, develops suddenly in a winding, it is necessary for the stress distribution to be set up in the resin around each wire adjacent to the crack on both sides. If the shear stress exceeds the adhesive bond strength, the bond will break. The bond fracture will travel along the wire away from the crack, and the resin will move until the necessary equilibrium has been set up between frictional forces and axial tension. It is not easy to observe this behaviour in actual windings and we have not had success; probably more sophisticated techniques are needed than are available to us. It was possible, however, to produce the effect with a large cylinder of copper embedded in transparent flexible resin, which was cooled to 77°K and re-warmed, see fig 3. This possibility of bond failure is one reason for not surrounding conductors with excessive thicknesses of resin.

The method of Appendix II assumes that the elastic modulus of the resin remains constant during cooling. Maximum axial and circumferential tensile stresses of 12000 lbf/in^2 are obtained for FMNA resin in a very thin layer, when a value of 10^6 lbf/in^2 is taken for the elastic modulus. This value is only attained at 4K, at room temperature the modulus is only 5×10^5 lbf/in^2. Correction for variation of modulus and expansion coefficient would give a stress value similar to that obtainable from equation (4), modified to take account of the contraction of the conductor.

In many magnets there will be regions where the resin is restrained in all three dimensions, thus giving rise to stresses according to equation (5), with due allowance for the contraction of the (restraining) conductors. Such a region is shown in fig 4a, where resin is trapped between the wires of a cabled conductor; but such regions can also occur between conductors, see fig 4b. Certainly FMNA resin would be likely to fail at the stress of nearly 20000 lbf/in^2 which would be developed.

5. TEMPERATURE GRADIENT STRESSES

The temperature gradients developed during cooling and warming should

be small. It is uneconomical to cool a large magnet rapidly. In most cases the windings are surrounded by heavy clamps or iron shielding which tend to prevent direct contact with the refrigerant (liquid nitrogen initially, then liquid helium). Care might be needed in the case of magnets with channels cooled by forced flow of refrigerant; the sudden contact of low temperature fluid with room temperature windings should be avoided. It is probably impossible to warm a magnet at an excessive rate without providing some form of artificial heating, eg by passing current through the conductors, or by forcing warm gas through the cooling channels.

The problem of the calculation of stresses developed during the spread of the normal region when quenching cannot be solved generally, since the shape of this region and the temperature distribution within it are very dependent on the design of the magnet and its protection system. For a particular magnet an approximate solution might be obtained with the aid of a computer program to simulate the effects of quenching, eg QUENCH[6]. Allowance must be made for the clamping system, which if sufficiently rigid, will prevent a general tensile straining of the winding around the normal region, and only compressive stresses will be developed. However, it is interesting to consider the effect of a general tensile stress on a cold conductor/resin composite, and also to estimate the increase of local stress in the resin due to the sudden heating of a conductor embedded in it.

Let us consider a number of conductors embedded in a region A, see fig 5a. Due to the difference of thermal contractions of the two materials, each conductor is surrounded by resin under tensile stress parallel to the conductor surfaces. Region A is now subjected to an additional uniaxial tensile stress, as for example, might occur in a magnet provided with an elastic clamping system during a quench. This stress is unable greatly to increase the strain in the resin near the conductors, as the following simple calculation will demonstrate, see fig 5b.

Let $\Delta\sigma_m$ = change of stress in resin matrix

$\Delta\sigma_c$ = change of stress in metal

E_m = elastic modulus of resin matrix

E_c = Elastic modulus of metal

A_m = cross-sectional area of resin

A_c = cross-sectional area of metal

σ = applied tensile stress

Increase of strain in metal = increase of strain in resin, ie

$$\frac{\Delta\sigma_m}{E_m} = \frac{\Delta\sigma_c}{E_c} \tag{6}$$

The load exerted by the applied stress = the sum of the changes in loads exerted by resin and metal, ie

$$\sigma(A_m + A_c) = \Delta\sigma_m . A_m + \Delta\sigma_c . A_c \tag{7}$$

From (6) and (7)

$$\Delta\sigma_m = \sigma . \frac{A_m + A_c}{A_m + \frac{E_c . A_c}{E_m}} \tag{8}$$

Usually $A_c > A_m$ and $E_c > E_m$, hence $\Delta\sigma_m$ is only a fraction of σ. In fact, as $A_c/A_m \to \infty$, so

$$\Delta\sigma_m \to \sigma(E_m/E_c) \tag{9}$$

E_c/E_m is usually about 10, in which case $\Delta\sigma_m$ varies from 18% of the general stress when $A_c/A_m = 1$ to a minimum of 10% when $A_c/A_m = \infty$.

Of course, the resin between ocnductors in the direction normal to the stress (B-B, see fig 5a) is subjected to the whole stress σ. Before the application of the additional tensile stress, the resin was, apart from the effects of stress concentration and the transfer of stress around the corners, unstressed in this region. If σ is high enough, failure may well occur along the surface B-B, or more likely the adhesive bond between resin and conductors parallel to B-B will be broken. This type of failure is probably more likely to occur where the resin layers are very thin.

An upper bound can be calculated for the general stress σ, for the

case where the outside of the winding is unconstrained, see Appendix III. If, during quenching, the normal region rises to room temperature, this stress is of the same magnitude as the value given by equation (4) above for the differential contraction stress. In practice temperature gradients will be much less severe than that assumed, and stresses will be correspondingly less, since adequate electrical protection requires a rapid spread of the normal region to as much of the magnet as possible.

The problem of the increased stress due to the sudden heating of a conductor will now be considered. If we assume that the thermal conductivity of the resin is negligibly small, then the stress will tend to rise towards the value given by equation (4) for a thin layer completely restrained in two dimensions, for the case when the conductor reaches room temperature (300K). Higher stresses will occur in regions which are restrained in all three dimensions. However, the fact that the resin does conduct heat, albeit poorly, will limit the stress to a lower value, since the increased resin temperature will reduce the contraction already present in the resin. Some very simple calculations, using results given by Jakob[7] for an infinitely wide plate whose surface temperature increases proportionally to time, gave the curves shown in fig 6. Only with very short quench times, or with very thick resin layers (which may occur between conductors, see fig 5b) will the resin temperature lag appreciably behind the conductor temperature. Indeed, the lateral spread of the normal region through the winding requires an ability in the resin to conduct heat and increase in temperature.

Finally, consider what happens after all the stored energy has been dissipated during the quench, and the (now) hot magnet is in contact with liquid helium. If we assume that the temperature of the magnet has risen to room temperature (300K) and that its surface (including surfaces of cooling channels, if present) is placed immediately in contact with helium at 4K, then the maximum surface stress cannot exceed that given by equation (4) above. It might be less: the maximum surface stress for a flat plate of thickness 2a cooled on both sides, or of thickness a cooled on one side only, according to Manson[8], is given by

$$\sigma_{max} = \frac{E\alpha T_0}{1-\mu} \cdot \sigma^*_{max} \qquad (10)$$

Where E = elastic modulus, assumed constant

α = thermal contraction coefficient, assumed constant

T_o = initial uniform temperature of plate above ambient temperature

μ = Poisson's ratio

σ^*_{max} is a function of the Biot number β and is given by

$$\frac{1}{\sigma^*_{max}} \simeq 1.5 + \frac{3.25}{\beta} - 0.5e^{-16/\beta} \qquad (11)$$

where $\beta = ah/k$

h = heat transfer coefficient

k = thermal conductivity

As β goes to infinity, σ^*_{max} approaches unity

Assume a = 1 mm

k = 1.5×10^{-3} w/cm K (an average value between 4 and 300K)

h = 10 w/cm^2 K (a typical value for film boiling of helium on an open surface with ΔT between 100 and 300K).

Then β = 667

and σ^*_{max} = 1.0

For variable elastic modulus and contraction coefficient, the maximum surface stress will, in this case, be given by equation (4), ie $\int_{4K}^{300K} E\alpha\, dT/(1-\mu)$. Manson has shown that an index of merit for rating materials under conditions of thermal shock is given by

$k\sigma_b/E\alpha$ for <u>small</u> values of β

or $\sigma_b/E\alpha$ for <u>large</u> values of β

where σ_b = breaking stress in a bend test,

since the maximum temperature to which a plate specimen can be heated, if

it is not to crack when quenched, is given by

$$T_{o,max} = \frac{k\sigma_b}{E\alpha} \cdot \frac{3.25(1 - \mu)}{ah} \quad \text{for small } \beta$$

$$\text{or} \quad T_{o,max} = \frac{\sigma_b}{E\alpha} \cdot (1 - \mu) \quad \text{for large } \beta$$

Such an index of merit might be of some use for comparing resins, since there is a tendency for the product $E\alpha$ to remain reasonably constant over a wide temperature range for many resins (eg for FMNA resin, $E\alpha = 28$ lbf/in^{2}K between 300 and 80K, over which range occurs the bulk of the thermal contraction when cooling).

6. THE INITIATION AND SPREADING OF CRACKS

The most troublesome stresses are those which arise from differences in thermal contraction between the resin and the metal conductors. Volumes of resin which are "trapped" ie are restrained in all three dimensions, are highly stressed, and suffer worse during quenching. In FMNA resin, for example, the stress in these regions after cooling from 300 to 4K would be about 20000 lbf/in^2, to which should be added the stress generated by cooling from the cure temperature. It is important to ensure that these trapped regions are as small as possible in size (in a plane perpendicular to the axes of the conductors) to avoid large increases of stress during quenching. When the trapped resin is in the form of a plate or layer cooled on both sides, a maximum thickness of 0.5 mm should ensure that quenching does not cause appreciable rise in stress, in the case of FMNA. Trapped resin of other cross-sections, eg square or cylindrical, could be larger since it would be more effectively cooled; 1 mm might be a reasonable upper limit. But every attempt should be made to avoid trapped resin, or to keep it as thin as possible. The stress in a very thick trapped layer of FMNA resin might rise towards 27300 lbf/in^2 during quenching, if cracking did not occur. This resin usually breaks at a tensile stress of about 15000 lbf/in^2 when in a very thin layer; thick layers will fail at lower stresses (see below). One would therefore expect a considerable amount of cracking to occur during cooling from room temperature to liquid helium temperature, and this has been observed.

At first sight it would appear that if we can avoid trapped volumes,

a resin should be satisfactory if it will withstand a stress equal to that given by equation (4), ie $\int_{4K}^{300K} E\alpha dt/(1 - \mu)$, without cracking. (The upper temperature limit may be replaced by the cure temperature or the glass transition temperature, in order to allow for the stresses developed during cooling to room temperature after the curing process). If trapped volumes are present, the resin must withstand $\int_{4K}^{300K} E\alpha dt/(1 - 2\mu)$. Most epoxy resins will withstand the lower of these two values, but fail at stresses less than the higher. We have assumed that the bonds between resin and conductors are strong enough to resist the stresses which tend to break them. We have seen very little evidence of bond failure in dipole and quadrupole magnets, although it must be acknowledged that the number of windings which have been cut up for examination has been very small.

Additional stresses may arise from the lack of homogeneity of a winding, eg regions rich in resin may, when contracting, exert tensile forces on neighbouring regions with lower resin content. High local stresses will develop around sharp edges or corners on the conductors, or around inclusions or bubbles in the resin. From the work of Neuber,[9] we can deduce that the resin between conductors should be kept as thin as possible in order to reduce the effects of stress concentration. It is also known that the probability of rupture at a given stress is dependent on the size and volume of a brittle material, see Weibull[10]. Thicker layers of resin will fail at lower stresses than thin layers. Weibull has also shown that the effect of polydimensional stresses is to cause failure at a lower stress than the one-dimensional tensile strength: a trapped volume of resin, for example, which is subjected to approximately equal stresses in the three directions, will probably fail at half to three-quarters of the uniaxial tensile strength, measured on a specimen of the same volume. Fortunately, breaking stresses are often measured on specimens of larger colume than those which are present in windings, thus yielding a lower value for the uniaxial tensile strength. A brief further consideration is given in Appendix IV.

It is clear that in practical windings there is no shortage of highly stressed regions where cracks may begin. Probably the first stage is the development of a flaw which, when it is large enough, precipitates brittle

fracture in accordance with Griffith's Theory. Fairly large flaws are required in glassy polymers if they are to develop into cracks, eg flaws 0.004 cm long are required in FMNA resin at 4K. This resin is transparent and such flaws would be easily visible if they were present before application of stress. Since they are not, we must assume the existence of some mechanism for growing flaws of sufficient size. That usually proposed is crazing: a brief summary is given by McCrum[11], who says that "a craze is a thin plate-like region (formed under the action of stress) containing craze material which is composed of orientated polymer and void, the void content being approximately 50% by volume. The normal to the craze is parallel to the tensile stress". It is stated that crazes can develop at stresses appreciably lower than the short time strength, causing failure known as stress-cracking.

According to Berry,[12] when a crack propagates through a polymer, the crack front is preceded by a region containing crazes. Work is done on this crazed region in a manner analogous to the plastic working of a metal around a crack front. The energy involved in this process is much greater than the surface energy of the newly formed crack surface, and must cause a considerable rise in temperature locally. Usually most of the necessary energy comes from the stored strain energy in the stressed material surrounding the crack. Some idea of the amounts involved can be got by considering FMNA resin, which requires work of fracture equal to 10^5 erg/cm^2 of crack free face to propagate a crack, whereas its true surface energy is probably not more than 10^3 erg/cm^2. Consider a volume of resin, one square centimetre in area, and of thickness h. Assume that when a crack propagates across this area, it dissipates heat energy $\simeq 2\gamma_p$, where γ_p is the work of fracture, and since there are two surfaces to the crack. This heat is absorbed by the resin, hence

$$\rho h \int_{T_1}^{T_2} c \, dT = 2\gamma_p \qquad (12)$$

where ρ = resin density

C = resin specific heat

T_1 = temperature before cracking (4.2K)

T_2 = temperature after cracking

Debye's theory shows that, at low temperature, the specific heat of a non-metallic solid will tend to vary as the cube of the temperature. The specific heat of Araldite[13], between 4 and 11K is given approximately by

$$C = 300\ T^3 \text{ erg/g K} \tag{13}$$

which, upon substitution into (12) gives

$$T_2^{\ 4} = T_1^{\ 4} + \frac{8\gamma_p}{300\rho h} \tag{14}$$

Thus, when $\gamma_p = 10^5$ erg/cm^2

$\rho = 2$ g/cm^3

$h = 0.1$ cm

the temperature reached during cracking is $T_2 = 10.8$K. This could initiate quenching, if the temperature were similarly increased in adjacent conductors. It should be remembered that the specific heats of copper and niobium-titanium are small compared with that of resin at these temperatures: in fact, the specific heat of copper is only one-thirtieth of that of Araldite between 4 and 10K.

Of course, not all the work of fracture is necessarily converted into heat. Some is converted to surface energy, and some may be locked-up in regions adjacent to the crack which remain permanently strained. On the other hand, Griffith's theory specifies the minimum amount of work required to extend the crack, and strain energy in addition to this may be dissipated during the cracking process. The total strain energy stored in our layer of resin before cracking may be estimated, it is

$$U = \frac{\sigma^2}{2E} h \tag{15}$$

or about 7×10^5 erg/cm^2 when $E = 7 \times 10^{10}$ dyne/cm^2 (10^6 lbf/in^2), $\sigma = 10^9$ dyne/cm^2 (15000 lbf/in^2) and $h = 0.1$ cm. This is about 3.5 times more than is required to form the crack. It is interesting to note that there is a lower limit of thickness for the cracking of a layer of resin subjected to a given tensile stress between two <u>rigidly fixed</u> platens,

since there must initially be sufficient strain energy to provide the work of fracture, ie

$$\frac{\sigma^2}{2E}.h \geqq 2\gamma_p \quad \text{or} \quad h \geqq \frac{4E\gamma_p}{\sigma^2} \tag{16}$$

Using the values which were applied to equation (5) gives $h \geqq 0.028$ cm. From equation (14) the corresponding maximum temperature is 15K. If the layer of resin is thinner than the value given by (16), the crack is unable to form at the given stress. Consideration of equation (14) shows that T_2 rises rapidly as h is reduced below 0.1 cm (with the given values of γ_p and ρ). There must be a lower limit to the thickness, presumably that which is involved in the work process. In practice, of course, heat is conducted into the surrounding resin, although this process is slow compared with the velocity of a crack in a more or less brittle material (which, according to Mott[14], may rise to $0.38\sqrt{E/\rho}$, or $\sim 7 \times 10^4$ cm/s for the resin being considered).

What is important is that, if $T_2^4 >> T_1^4$, T_2 depends on the fourth root of (γ_p/h). Griffith's equation, for plane strain fracture, as modified by Orowan and Irwin, states that a necessary condition for a crack to extend is:

$$\sigma(\pi c)^{1/2} \geqq \left(\frac{2E\gamma_p}{1-\mu^2}\right)^{1/2} \tag{17}$$

where σ = nominal stress

c = semi-crack length (for an elliptical crack)

μ = Poisson's ratio

Thus, for a given initial crack length, the stress required to extend the crack depends on $\gamma_p^{\frac{1}{2}}$, or more precisely $(E\gamma_p)^{\frac{1}{2}}$. Alternatively, the critical crack length at a given stress depends on $E\gamma_p$. The improvement of a resin by increasing its elastic modulus and work of fracture is thus seen to be well worthwhile, since it does not lead to significantly higher temperatures during cracking, provided, of course, that there is no accompanying large decrease in the thickness of the layer undergoing work during the cracking process. It has been assumed that the specific heats of the modified and unmodified resins are similar. This may not be the case, since certain additives, eg silica and alumina, have very low

specific heats at low temperatures.

So far very little has been said about the strength of the bond between the resin and the conductor, except to indicate the causes of bond breakage. This process will also cause heating of the conductors. It does not seem to have caused difficulties in magnet development, which is the reason why we have not felt justified in devoting time to the problem. In one respect, a reasonably weak bond may be advantageous, since , as Cook and Gordon[15] have shown, it may rupture before an approaching crack front reaches it, thus relieving local stresses and stopping the crack.

In the next section we shall consider ways in which resins might be modified in order to reduce the probability of cracking.

7. THE MODIFICATION OF EPOXY RESINS

Before considering the addition of modifying agents the question should be asked whether certain straight resin systems might possess superior properties, ie contract less or have higher strengths or works of fracture. By "straight" we mean that the system is composed of an epoxy resin, or a mixture of epoxy resins, together with a cross-linking agent and any small amount of accelerator or similar additive which may be required. Measurement has shown that the thermal contractions of all such systems tend to be similar, about 1.1% between 300 and 4K. Tensile strengths lie in the range 10 000 to 20 000 lbf/in^2, and it is not obvious that any system has a particular advantage. Adequate information on works of fracture is not yet available, although certain amine cured resins are tougher than MNA cured resins at 4K.

From the above discussion on stresses it can be seen that a most effective method of reducing or preventing cracking would be to add low-contraction fillers to the resin system. The fillers could be in the form of particles, or they might be fibrous. The difficulty with such mixtures is that they are very viscous and do not easily lend themselves to vacuum impregnation techniques. To some extent a solution is to fill the voids in the coil with particles or fibres while winding (or after winding) and then to vacuum impregnate with an unmodified resin. Alternatively, it is possible to put the filled resin into the coil during

the winding process, a method which is described as "wet lay-up". This technique is very messy, and there is a definite risk that voids will be left: in particular, the stiff resin mixture may fail to penetrate into superconducting cables to fill the voids therein. Another approach is to attempt to increase the fracture toughness, or work of fracture, of the resin. This is, to some extent more or less, done when particles or fibres are added. However, large increases in toughness can be obtained by adding suitable rubbery substances to epoxy resins; for example, a polyurethane known as Adiprene L100 has given excellent results. Unfortunately there are penalties. The thermal contraction is considerably increased, although this does not necessarily result in high stresses when cold: this will be discussed hereafter. A more serious disadvantage is the decrease in pot-life which results from the additive and the cross-linking agent (MOCA) used with it. Nevertheless, satisfactory techniques have been devised for impregnating windings with these mixtures.

Until now, the tendency has been to adopt this second approach, since it lends itself to vacuum impregnation. However, of late some windings have been made employing glass fibre filler wherever possible, together with a straight resin system, vacuum impregnated. This appears to be a very satisfactory method, but it does impose some limitations on the design of the winding. Perhaps it may be possible to combine some of these methods, eg by pre-impregnating the cable or by using a heat-activated adhesive enamel on the cable wires to prevent wire movement, and then filling the voids with a suitable low-contraction filled resin. We will consider first the effects of fillers, and then discuss the addition of polyurethane.

A suitable low-contraction filler is require. We have used substances such as silica, alumina, talc and zirconium silicate in powder form. Silica has the lowest thermal contraction in this group, being practically zero between 300 and 4K. Resins containing similar fillers are available commercially, eg Stycast 2850 from Emerson and Cummings. There is a limit to the amount of filler which can be added, if the mixture is to remain workable. It may be necessary to use a colloid mill to disperse the powder in the resin adequately. With silica flour, for example, the maximum volume fraction is less than 70%, and the thermal contraction (300 - 4K) is reduced to 0.5%, which is still larger than that of the

conductor ($\sim$ 0.25%).

It would be useful to be able to predict the thermal contraction for a given proportion of filler. Fig 7 shows the results obtained for a number of samples of FMNA resin containing different proportions of silica flour. The points do not lie on the straight line predicted by the simple rule of mixtures, although the estimates based on this may often be sufficient. An attempt to fit the data by using the theory of Rosen and Hashin[16] was unsuccessful; this is not surprising in view of the wide variation of E and α for the resin over the temperature range. It is possible that an attempt to calculate α by this method at a given temperature would be more successful. The semi-empirical approach of Tsai[17] is more useful. He suggests that a property could be represented by an equation

$$\frac{p}{p_m} = \frac{1 - \xi \chi V_f}{1 - \chi V_f} \qquad (18)$$

where p = a property of the matrix

p_m, p_f = corresponding properties of matrix and filler

V_f = volume fraction of filler

$$\chi = \frac{(p_f/p_m) - 1}{(p_f/p_m) + \xi}$$

The value of ξ is between 0 and ∞.

When $\xi = 0$: $\frac{1}{p} = \frac{V_m}{P_m} + \frac{V_f}{P_f}$

and when $\xi \to \infty$: $\frac{1}{p} = V_f P_f + V_m P_m$

In practice, a value of ξ is determined from a single data point (or more, if available), and then used to predict properties at other concentrations. Lines are shown in fig 7, corresponding to values of ξ = 3 and 5 for total contraction between 300 and 4K. One point at 4K, and two points at 77K (they are circled in fig 7) do not conform. These points would seem to indicate a change in behaviour at about 50% volume percentage of silica. More data are required to confirm the validity of this approach.

This method could also be applied to estimate elastic modulus. We have not measured E for a wide range of filler proportions and so we are unable to test the result. In general, the simple rule of mixtures

$$\frac{1}{E} = \frac{V_m}{E_m} + \frac{V_f}{E_f} \tag{19}$$

will give a conservative estimate. The same equation may be applied to conductors embedded in resin to estimate the composite transverse modulus for the winding as a whole, just as Tsai's method could be applied to estimate the transverse thermal contraction.

A reasonably accurate method for estimating E would be useful, since together with values of α it can be used to calculate thermal contraction stresses. Ideally, one requires values over the whole temperature range, but to obtain these for every resin would be expensive and time-consuming. Fortunately, for resins which are glassy at room temperature, the product $E\alpha$ tends to remain constant, certainly down to liquid nitrogen temperature, by which time most of the thermal contraction has occurred (see fig 8). This is not surprising, since E and α are both dependent on the inter-atomic forces. So it is possible to determine the product $E\alpha$ at any one temperature and apply this to stress calculation, or more useful, the average value of E can be multiplied by the integrated thermal contraction to estimate the stress in a bar restrained at both ends, ie

$$\frac{1}{2}\left(E_{300K} + E_{4K}\right)\int_{4K}^{300K} \alpha \, dT \tag{20}$$

provided that E is a nearly linear function of T.

The addition of fillers may have other advantages. Tensile strength and work of fracture can be increased. Unfortunately, these properties are often decreased, perhaps because of poor adhesion between resin and filler. Increased strength and increased toughness may result in the manner proposed by Nielson[18], who suggests that the particles act as stress concentrators, so that cracks start at these particles, and since there are many of them, there are many sources of weakness. The stress fields of these sources interact and stop the growth of individual cracks. Increased work of fracture may be due to the crack being forced

to take a devious route in between and around the particles.

In addition to finely powdered fillers, we have made use of relatively large particles, for example, glass ballotini (small glass spheres) and sintered aluminium oxide rumblings (approximately spherical in shape). These lend themselves to a particular technique, since they can be vibrated into a dry winding, and the whole subsequently vacuum impregnated. They fill the gross voids and reduce contraction and stress (on the large or "macro" scale, at least. On the "micro" scale, the stresses around particles of any sort are presumably high). Ballotini up to about 800 μm and aluminium oxide rumblings up to about 3 mm diameter have been tested.

The other approach to the modification of resins is, as mentioned earlier, the addition of rubbery material. Its effect on low temperature toughness was first described by Soffer and Molho[19], who obtained excellent results with a system described as "4A", containing two epoxy resin precursors, Adiprene L100, and MOCA as cross-linking agent. We have also studied this material, together with some similar compositions. It is quite flexible and relatively soft at room temperature, but its modulus of elasticity, tensile strength and work of fracture are all high at low temperatures. These properties may, perhaps, be explained by Nielson's theory (see above). Disadvantages are a very high thermal contraction, at least until the temperature has fallen below the rubber/glass transition, and a tendency to creep at room temperature. These disadvantages can be mitigated by adding fillers, but the unfilled mixture is already quite viscous, hence this treatment has limits. Alternatively, the technique using ballotini or aluminium oxide rumblings may be used. The system also suffers from a short pot-life. At present we are unable to estimate thermal contraction stresses for this type of material. Its rubber/glass transition is well below room temperature, and visco-elastic behaviour occurs until the resin is glassy. Subsequent contraction may lead to lower stresses than in the case of FMNA. Eventually we hope to measure these stresses.

So far, we have said little about fibrous fillers. Generally it is not practicable to mix these with resin before application to a winding, since even very small proportions of fibres give extremely stiff mixtures. Excellent results may be obtained by incorporating the fibres

into a winding before impregnation. It is often difficult to get enough fibre into voids. Fibre in the form of cloth or felt is preferred, since there is a tendency for loose fibre wool to migrate to one part of a void, leaving the remainder unreinforced. So far only glass fibre has been used for this purpose, but it is possible that stronger fibres such as carbon may find application, particularly in large magnets. Carbon fibre, incidentally, has practically zero contraction in the direction of the fibres. The properties of fibrous composites have been studied extensively, and the results and attempts at prediction are well documented: see, for example, McCullough[17], McCrum[11], Holister and Thomas[5]. The low temperature properties of some composites have been measured by Keys[20].

The preliminary consideration of the problem of impregnation has now been completed. The next part will begin with a brief discussion on methods of testing, after which some results will be presented.

8. ACKNOWLEDGEMENT

Thanks are due to A J Middleton, for discussions and valuable criticisms: figures 1 and 10 were provided by him.

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APPENDIX I. STRESS AND THERMAL CONTRACTION

The behaviour of certain resin specimens under test forced us to observe more closely the effects of stress and change in elastic modulus on thermal contraction. We loaded a tensile specimen of resin to a known stress at room temperature. It was then cooled to 4K under load. The load was then removed and the specimen was warmed to room temperature. Change in length was measured at each step. This process was repeated at several different stresses. The behaviour is shown in fig 10. The same contraction occurs irrespective of the stress imposed, but the expansion during warming is reduced by an amount proportional to the stress imposed during cooling.

Consider a specimen of unit cross-sectional area, cooled under a stress σ.

Change in length during loading			=	σ/E_{RT}
"	"	"	cooling =	$-\int_{4K}^{RT} \alpha \, dT$
"	"	"	unloading =	$-\sigma/E_{4K}$
"	"	"	warming =	δ

where α = thermal contraction coefficient

T = absolute temperature

E_{RT} = room temperature elastic modulus

E_{4K} = 4K elastic modulus

The sum of these changes must be zero, ie

$$\delta = \int_{4K}^{RT} \alpha \, dT - \sigma\left(\frac{1}{E_{RT}} - \frac{1}{E_{4K}}\right) \qquad (1)$$

It is possible to apply Maxwell's equations and derive expressions for changes in energy. This has been done, but the results are of no relevance here.

APPENDIX II. MODIFIED OUTWATER'S THEORY

We consider a metal cylinder of radius r_1 surrounded by a thick uniform layer of resin of outside radius r_2, see fig 9. It is required to determine the manner in which tensile stress is developed in the resin, due to thermal contraction, from the shear stress between metal and resin at the ends of the cylinder. Assume plane strain at all cross-sections.

First, calculate the stresses at positions remote from the ends.

Let σ = stress (tensile stress is positive)

ϵ = strain

p = interface stress, normal to interface

E = elastic modulus

m = reciprocal of Poisson's ratio

r = radius

Subscripts r, θ, z refer to radial, circumferential and axial directions respectively.

Subscripts c and m refer to metal cylinder and resin respectively.

Axial strain in resin

$$\epsilon_{zm} = \frac{\sigma_{zm}}{E_m} - \frac{1}{m_m E_m}\left(\sigma_{rm} + \sigma_{\theta m}\right) \qquad (1)$$

but σ_{zm} is constant across the cross-section, hence $(\sigma_{rm} + \sigma_{\theta m})$ = constant

From Lame's theorem

$$\sigma_{rm} = a_m - \frac{b_m}{r^2} \quad : \quad \sigma_{\theta m} = a_m + \frac{b_m}{r^2} \qquad (2)$$

where a_m and b_m are constants to be determined from the boundary conditions.

Addition of these two equations gives

$$\sigma_{rm} + \sigma_{\theta m} = 2a_m$$

At r_2, $\sigma_{rm} = 0$; and at r_1, $\sigma_{rm} = p$

Substituting in (2) gives

$$b_m = \frac{p}{\frac{1}{r_2^2} - \frac{1}{r_1^2}} \quad : \quad a_m = \frac{p}{r_2^2\left(\frac{1}{r_2^2} - \frac{1}{r_1^2}\right)}$$

Therefore, from (1)

$$\epsilon_{zm} = \frac{\sigma_{zm}}{E_m} - \frac{1}{m_m E_m} \cdot \frac{2p}{r_2^2\left(\frac{1}{r_2^2} - \frac{1}{r_1^2}\right)} \tag{3}$$

Similarly, the axial strain in the conductor is found to be

$$\epsilon_{zc} = \frac{\sigma_{zc}}{E_c} - \frac{1}{m_c E_c} \cdot 2p \tag{4}$$

But $$\epsilon_{zm} - \epsilon_{zc} = A_m - A_c \tag{5}$$

where $$A_m = \int_{4K}^{RT} \alpha_m \, dT \quad \text{and} \quad A_c = \int_{4K}^{RT} \alpha_c \, dT$$

α_m, α_c are thermal contraction coefficients of resin and metal respectively, T is absolute temperature.

Also $$\sigma_{zc} = -\sigma_{zm} \cdot \frac{r_2^2 - r_1^2}{r_1^2} \tag{6}$$

From (3), (4), (5) and (6)

$$\left[\frac{\sigma_{zm}}{E_m} - \frac{1}{m_m E_m} \cdot \frac{2p}{r_2^2\left(\frac{1}{r_2^2} - \frac{1}{r_1^2}\right)}\right] - \left[-\frac{\sigma_{zm}}{E_c} \cdot \frac{r_2^2 - r_1^2}{r_1^2} - \frac{2p}{m_c E_c}\right] = A_m - A_c \tag{7}$$

In a similar way, we may determine the radial displacements of resin and metal at the interface r_1.

Radial displacement of resin

$$u_{m1} = \frac{r_1}{E_m}\left[p \cdot \frac{\frac{1}{r_2^2} + \frac{1}{r_1^2}}{\frac{1}{r_2^2} - \frac{1}{r_1^2}} - \frac{p}{m_m} - \frac{\sigma_{zm}}{m_m} \right] \tag{8}$$

Radial displacement of metal

$$u_{c1} = \frac{r_1}{E_c}\left[p - \frac{p}{m_c} - \frac{\sigma_{zc}}{m_c} \right] \tag{9}$$

But $$u_{m1} - u_{c1} = r_1 \left(A_m - A_c \right) \tag{10}$$

From (6), (8), (9) and (10)

$$\frac{1}{E_m}\left[p \cdot \frac{r_1^2 + r_2^2}{r_1^2 - r_2^2} - \frac{p}{m_m} - \frac{\sigma_{zm}}{m_m} \right] - \frac{1}{E_c}\left[p - \frac{p}{m_c} + \frac{\sigma_{zm}}{m_c} \cdot \frac{r_2^2 - r_1^2}{r_1^2} \right] = A_m - A_c \tag{11}$$

Equations (7) and (11) can be solved simultaneously to give

$$\sigma_{zm} = \frac{K_1 (K_5 - K_3)}{K_3 K_4 + K_2 K_5} \quad : \quad p = \frac{K_1 + K_4 \sigma_{zm}}{K_5} \tag{12}$$

where

$$K_1 = A_m - A_c$$

$$K_2 = \frac{r_2^2 - r_1^2}{E_c r_1^2} + \frac{1}{E_m}$$

$$K_3 = 2\left[\frac{1}{m_c E_c} - \frac{r_1^2}{m_m E_m (r_1^2 - r_2^2)} \right]$$

$$K_4 = \frac{1}{m_m E_m} + \frac{1}{m_c E_c} \cdot \frac{r_2^2 - r_1^2}{r_1^2}$$

$$K_5 = \frac{r_1^2 + r_2^2}{E_m (r_1^2 - r_2^2)} - \frac{1}{m_m E_m} - \frac{1}{E_c} + \frac{1}{m_c E_c}$$

Now let us consider the equilibrium of the element of length dz near the end.

$$\frac{d\sigma_{zm}}{dz} = -\frac{2\mu_F r_1}{r_2^2 - r_1^2} \cdot p \tag{13}$$

when μ_F = coefficient of friction between resin and metal. Equations (8), (9), (10) and (11) are also applicable to the resin near the end. Since the value of p given in (12) was obtained from these four equations, we may substitute in (13) to give

$$\frac{d\sigma_{zm}}{dz} = K_6 \cdot \frac{K_1 + K_4\sigma_{zm}}{K_5}$$

where we have written $K_6 = -2\mu_F r_1/(r_2^2 - r_1^2)$

Solving this equation, and obtaining the constant of integration from the boundary condition $\sigma_{zm} = 0$ at $z = 0$, yields

$$\sigma_{zm} = \frac{K_1}{K_4}\left[\exp(K_4 K_6 z/K_5) - 1\right] \tag{14}$$

Shear stress is calculated as follows:

Shear stress $$\tau = \frac{d}{dz}\left[\frac{\sigma_{zm}(r_2^2 - r_1^2)\pi}{2\pi r_1}\right]$$

Substituting from (14) for σ_{zm} gives

$$\begin{aligned}\tau &= \frac{r_2^2 - r_1^2}{2r_1} \cdot \frac{K_1 K_6}{K_5} \exp(K_4 K_6 z/K_5) \\ &= -\mu_F \frac{K_1}{K_5} \exp(K_4 K_6 z/K_5)\end{aligned} \tag{15}$$

σ_{zm} reaches its maximum value at z_o, when, from (14) and (12):

$$\frac{K_1}{K_4}\left[\exp(K_4 K_6 z_o/K_5) - 1\right] = \frac{K_1(K_5 - K_3)}{K_3 K_4 + K_2 K_5}$$

or

$$z_o = \frac{K_5}{K_4 K_6} \ln\left[\frac{K_4(K_5 - K_3)}{K_3 K_4 + K_2 K_5} + 1\right] \tag{16}$$

These equations were used to calculate the stresses shown in fig 2.

APPENDIX III. QUENCH STRESSES

Consider a cylindrical region of radius b at low temperature. In this region is embedded a concentric hot region of radius x, which is growing in size. According to Timoshenko and Goodier[21] the stresses in an isotropic cylinder subjected to a radial temperature distribution are given by

$$\sigma_r = \frac{E}{1-\mu}\left[-\frac{1}{r^2}\int_0^r \alpha T r\,dr + \frac{1}{b^2}\int_0^b \alpha T r\,dr\right] \tag{1}$$

$$\sigma_\theta = \frac{E}{1-\mu}\left[\frac{1}{r^2}\int_0^r \alpha T r\,dr + \frac{1}{b^2}\int_0^b \alpha T r\,dr - \alpha T\right] \tag{2}$$

$$\sigma_z = \frac{E}{1-\mu}\left[\frac{2}{b^2}\int_0^b \alpha T r\,dr - \alpha T\right] \tag{3}$$

where μ = Poisson's ratio

E = elastic modulus

α = thermal expansion coefficient

T = absolute temperature

r = radius

If we assume a step temperature distribution, with the region inside x at T_2, and the region outside x at T_1, such that $T_2 > T_1$, then equations reduce to

$$r \geqq x \begin{cases} \sigma_r = \dfrac{E\alpha(T_2-T_1)}{2(1-\mu)}\left[\dfrac{x^2}{b^2} - \dfrac{x^2}{r^2}\right] & (4) \\[2ex] \sigma_\theta = \dfrac{E\alpha(T_2-T_1)}{2(1-\mu)}\left[\dfrac{x^2}{b^2} + \dfrac{x^2}{r^2}\right] & (5) \\[2ex] \sigma_z = \dfrac{E\alpha(T_2-T_1)}{1-\mu}\cdot\dfrac{x^2}{b^2} & (6) \end{cases}$$

Provided that E and α remain constant. When x is very small compared with b:

$$\text{Maximum } \sigma_r = -\frac{E\alpha(T_2 - T_1)}{2(1-\mu)} \tag{7}$$

$$\text{Maximum } \sigma_\theta = \frac{E\alpha(T_2 - T_1)}{2(1-\mu)} \tag{8}$$

$$\sigma_z = 0 \tag{9}$$

But when x is nearly equal to b :

$$\sigma_r = 0 \tag{10}$$

$$\text{Maximum } \sigma_\theta = \frac{E\alpha(T_2 - T_1)}{1-\mu} \tag{11}$$

$$\text{Maximum } \sigma = \frac{E\alpha(T_2 - T_1)}{1-\mu} \tag{12}$$

Equations (11) and (12) represent the worst case. The values of E, α and μ are those of the magnet winding as a whole. The winding is not isotropic and the temperature distribution will be less severe than was assumed. Despite the crudeness of this model, it does give an appreciation of the way in which the stresses change during the spread of the hot region.

APPENDIX IV. WEIBULLS'S THEORY

Weibull[10] suggests that the probability of rupture of a material sample at given stress σ is given by

$$S = 1 - \exp\left(-\int n(\sigma)\, dv\right) \tag{1}$$

where dv is a small element of volume and $n(\sigma)$ is a function of stress for an isotropic brittle material. Good agreement with measured data can often be obtained by putting

$$n(\sigma) = k\sigma^m \tag{2}$$

where k and m are constants for a material.

Instead of k, put $\sigma_o = k^{-1/m}$

Then
$$S = 1 - \exp\left[-V\left(\frac{\sigma}{\sigma_o}\right)^m\right] \tag{3}$$

when stress is uniform over the whole volume V. σ_o is that stress which for the unit of volume gives the probability of rupture S = 0.63.

The quantity $B = \int n(\sigma)dv$ he calls the risk of rupture. He goes on to show that, in the case of three-dimensional stresses, all equal to σ; $B = (2m + 1)\, k.\, \sigma^m$ for unit volume. Comparing this with $B = k\sigma_u$ for uniaxial stress gives

$$\frac{\sigma}{\sigma_u} = \left(\frac{1}{2m+1}\right)^{1/m} \tag{4}$$

from which the following values were obtained

m	=	1	2	3	5	7	10	∞
σ/σ_u	=	0.33	0.45	0.52	0.62	0.68	0.74	1.00

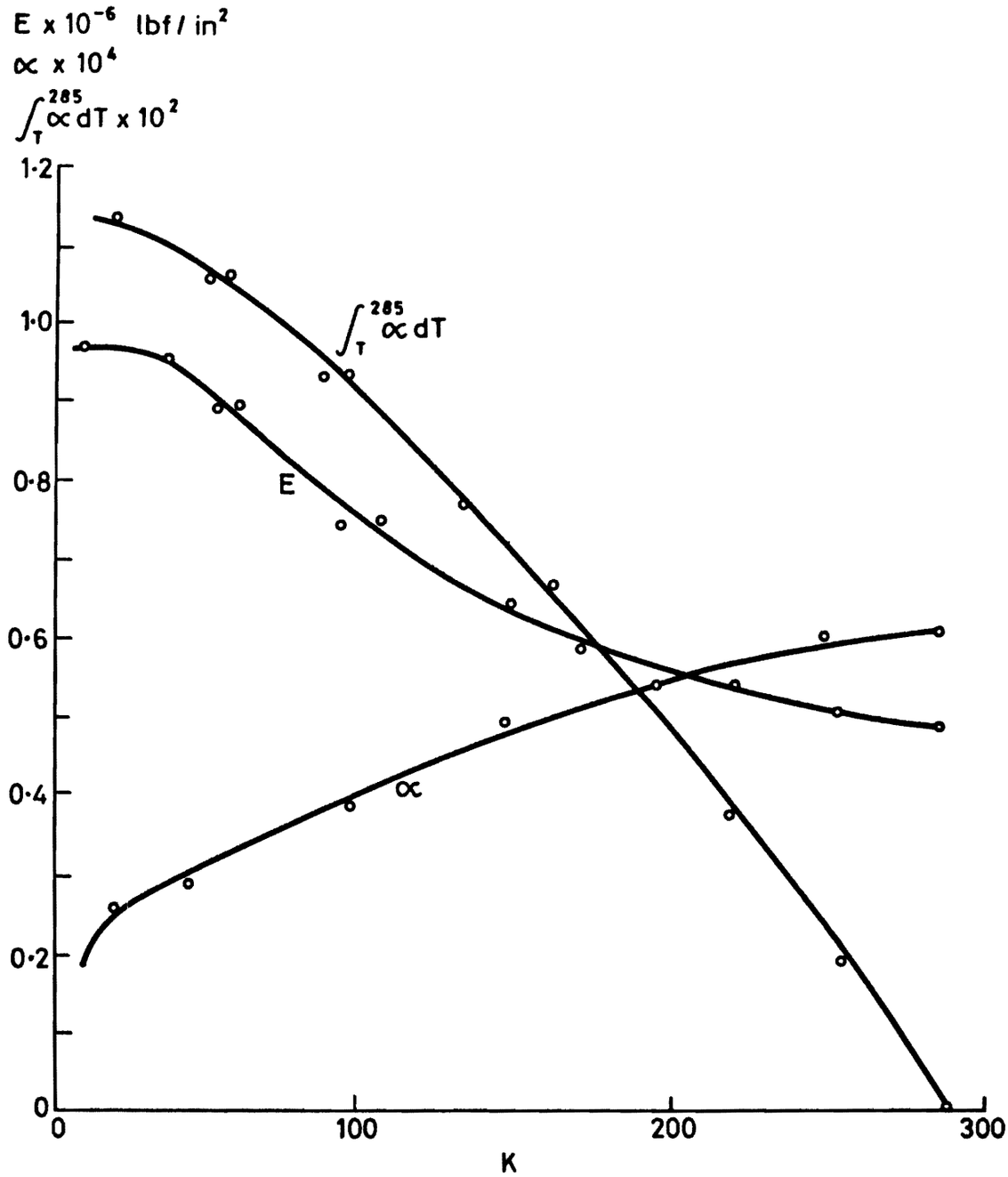
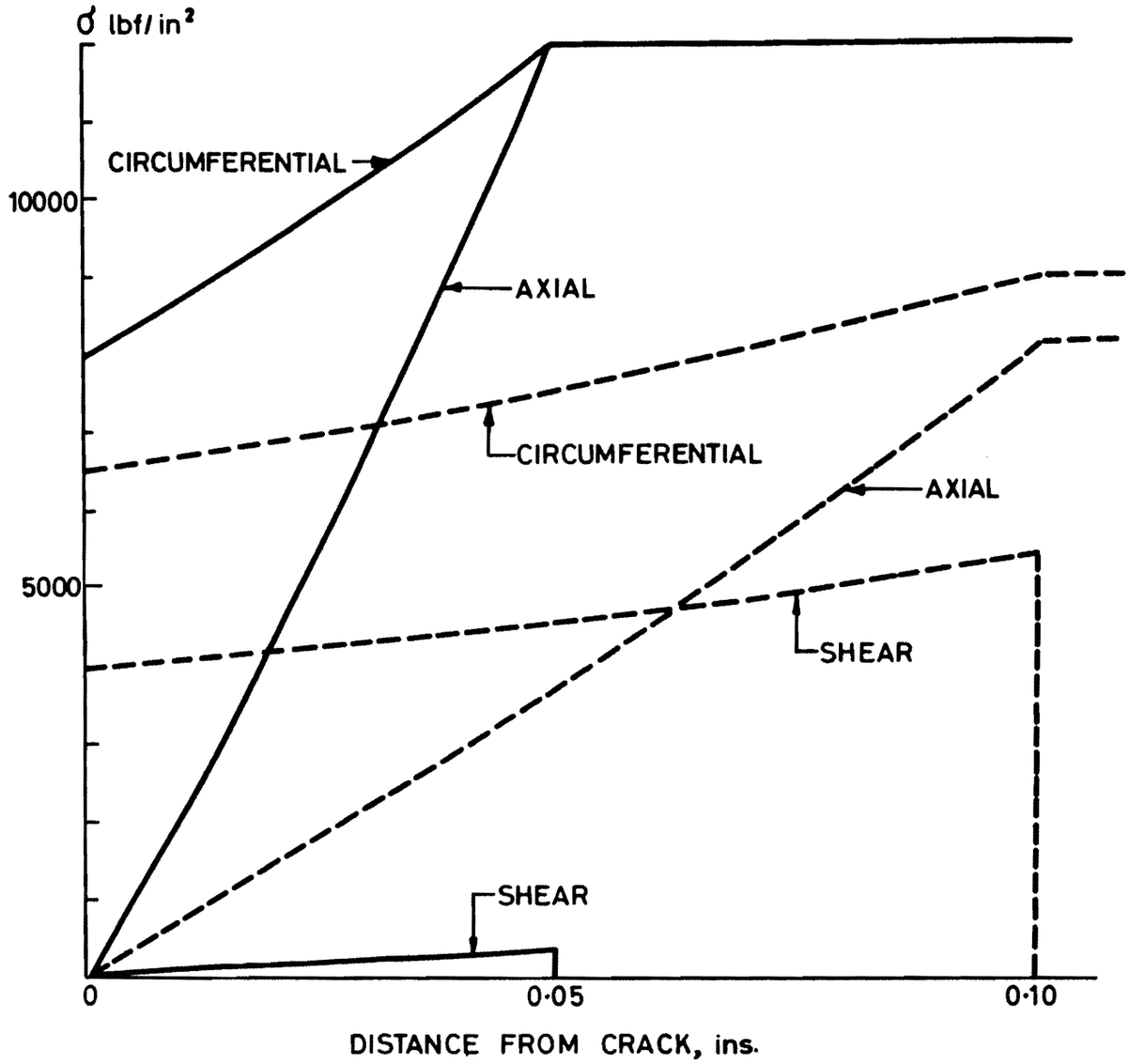

E x 10⁻⁶ lbf / in²
α x 10⁴
∫ₜ²⁸⁵ α dT x 10²
1·2
1·0
0·8
0·6
0·4
0·2
0
0
100
200
300
K
∫ₜ²⁸⁵ α dT
E
α

FIG. 1. FMNA 65



$r_2/r_1 = 1{\cdot}025$

$r_2/r_1 = 2$

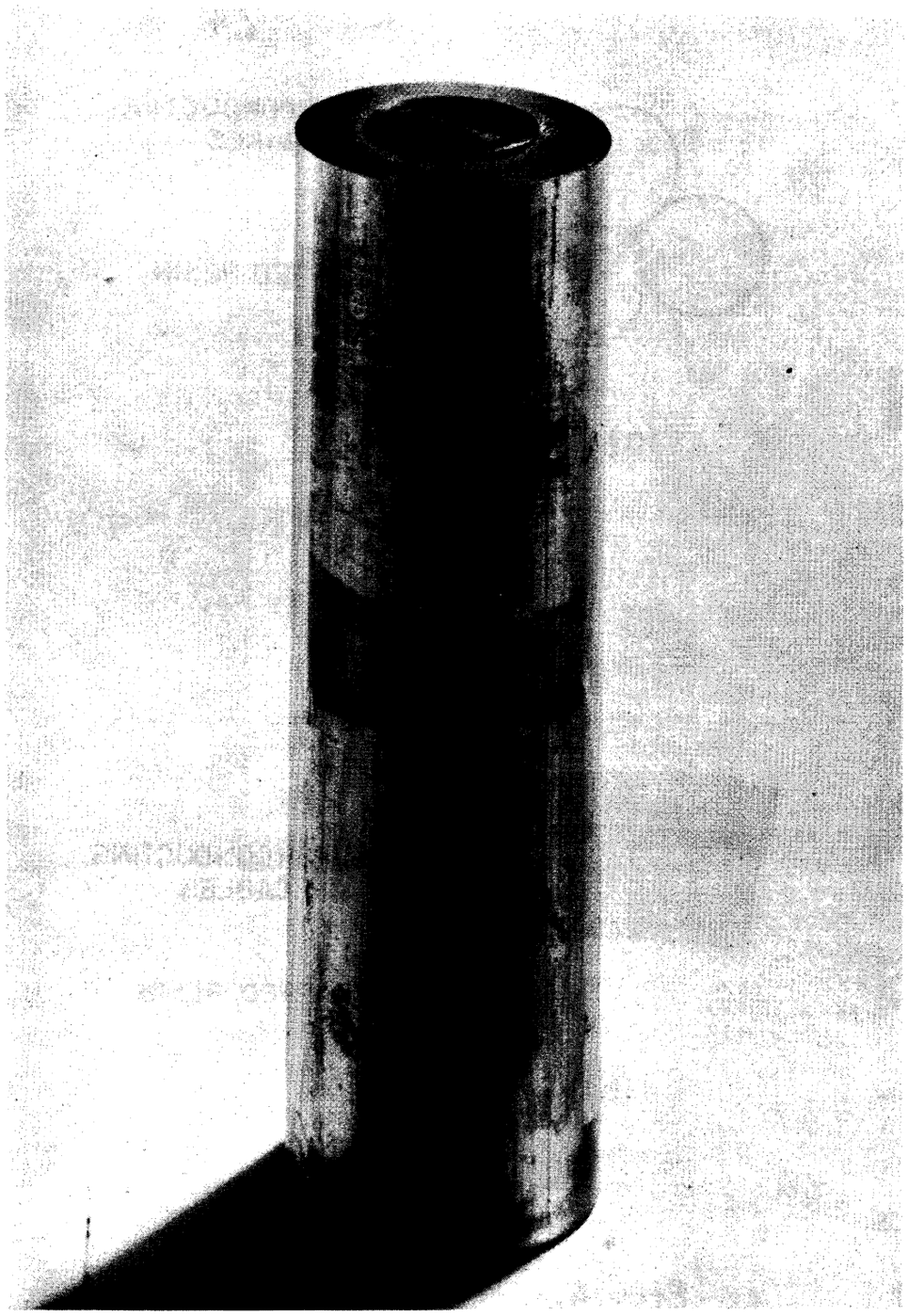
$r_1 = 0{\cdot}04''$

$E_m = 10^6$ lbf/in²; $E_c = 16 \times 10^6$ lbf/in²

$\mu_F = 1{\cdot}0$

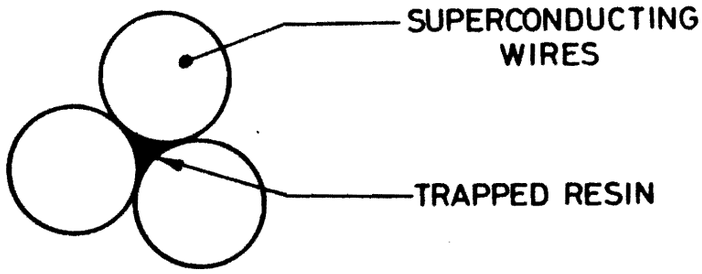
$m_c = 4$; $m_m = 3$

FIG. 2. THERMAL CONTRACTION STRESSES. FMNA

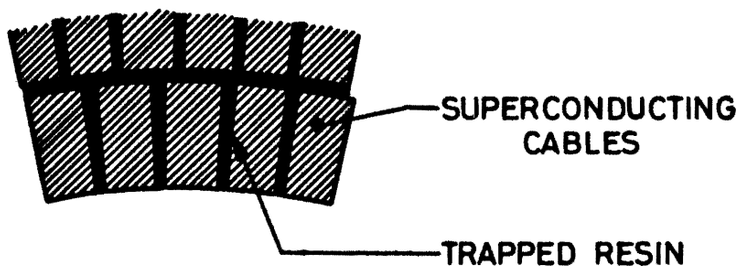


A copper cylinder 0.5 in dia. x 3 in. long embedded in flexible epoxy resin. Cooling to 77K has produced bond fractures which have travelled towards the middle. The dark band at the middle is still bonded. Note that the resin, which was initially level with the ends of the cylinder, has not returned to its original length; probably this is because the resin was initially restrained from contracting when cooled from the cure temperature to room temperature, although to some extent it may be due to frictional forces still existing between metal and resin.

Fig. 3

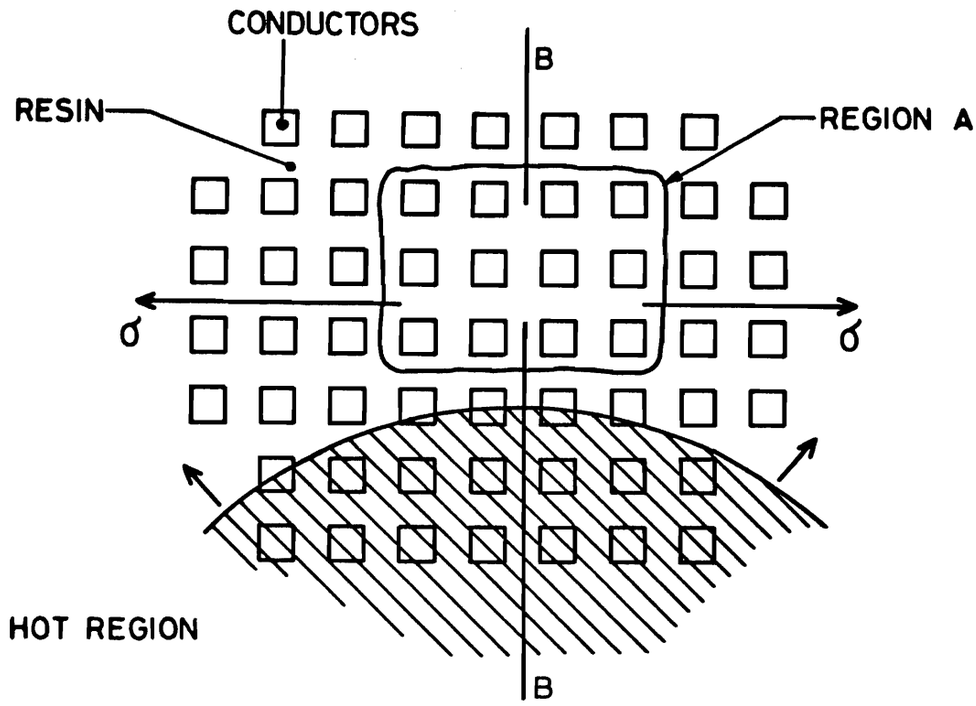

SUPERCONDUCTING
WIRES
TRAPPED RESIN

(a)

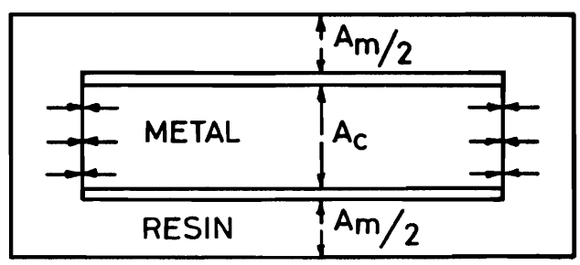

SUPERCONDUCTING
CABLES
TRAPPED RESIN

(b)

FIG. 4.


CONDUCTORS
B
RESIN
REGION A
σ
σ
HOT REGION
B

(a)


Am/2
METAL
Ac
RESIN
Am/2

(b)

FIG. 5.

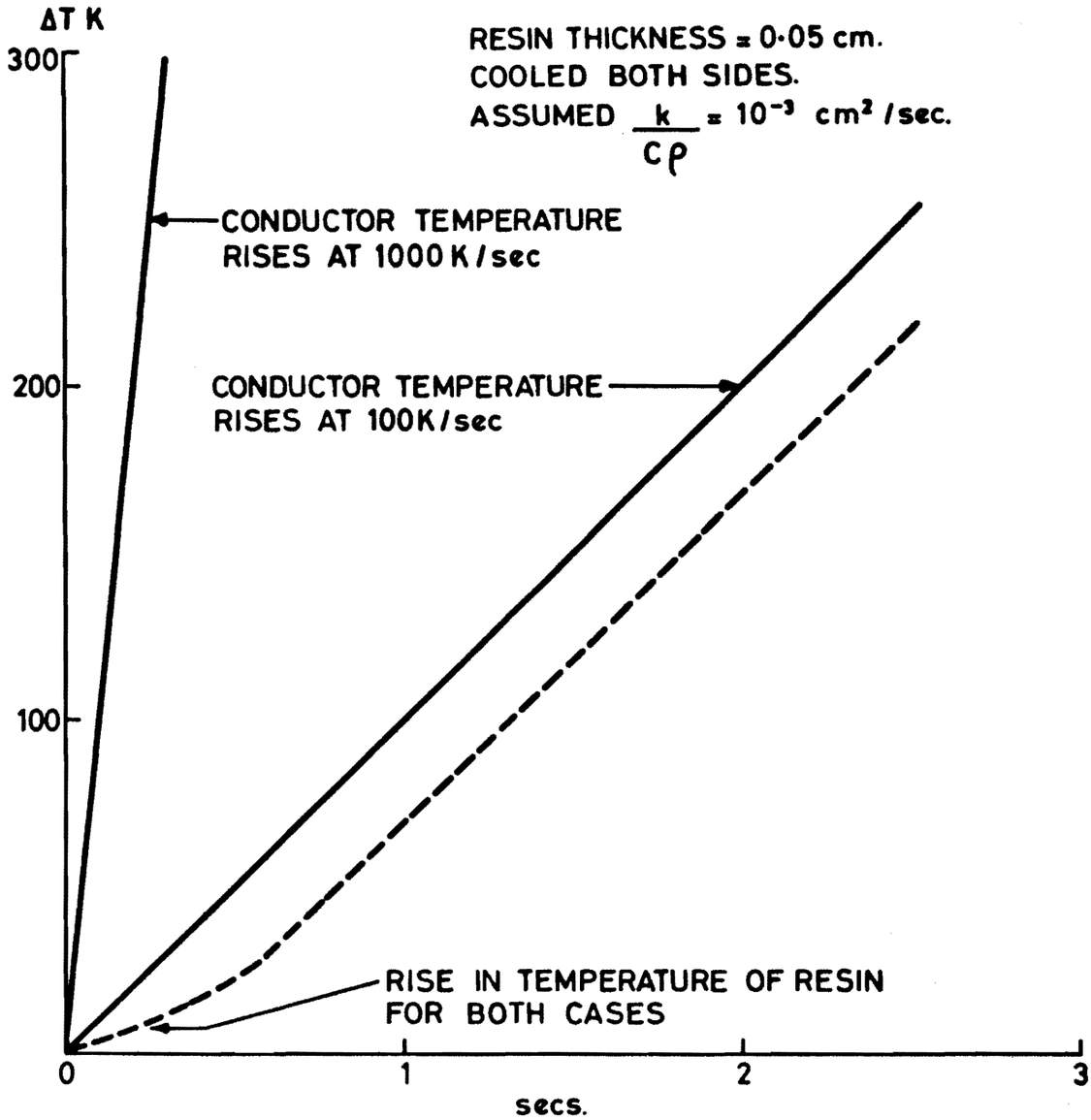

ΔT K
300
200
100
0
1
2
3
secs.
RESIN THICKNESS = 0·05 cm.
COOLED BOTH SIDES.
ASSUMED k/cρ = 10⁻³ cm²/sec.
CONDUCTOR TEMPERATURE RISES AT 1000 K/sec
CONDUCTOR TEMPERATURE RISES AT 100K/sec
RISE IN TEMPERATURE OF RESIN FOR BOTH CASES

FIG. 6.

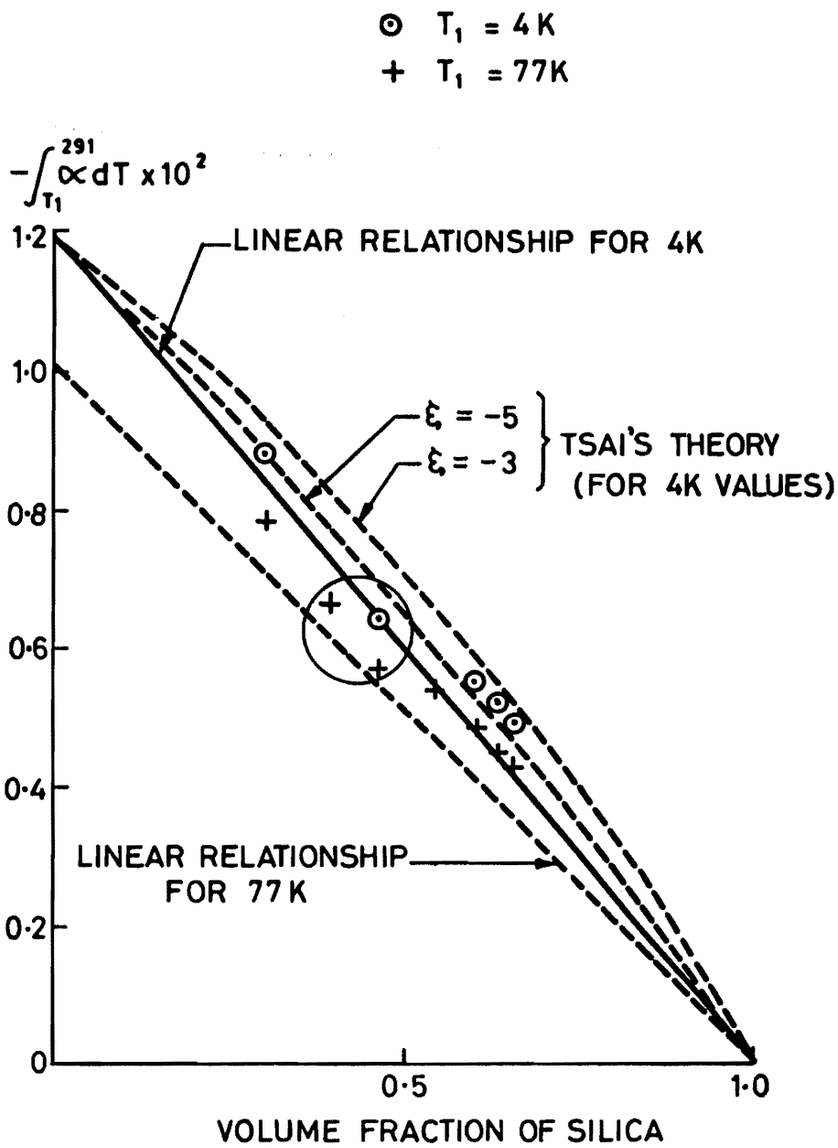


FIG. 7. FMNA 88 + SILICA

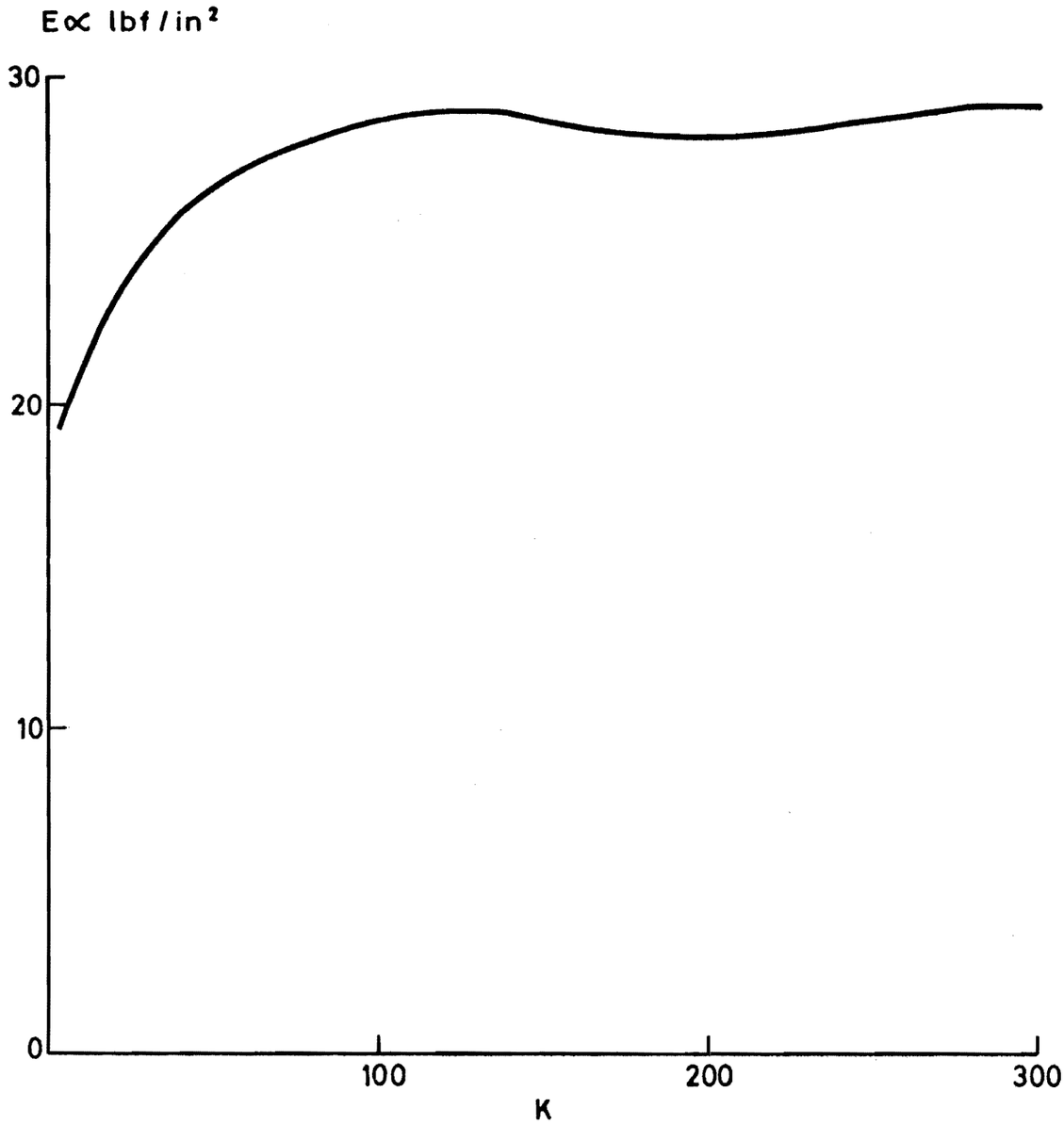


FIG. 8. E∝ FOR FMNA 65

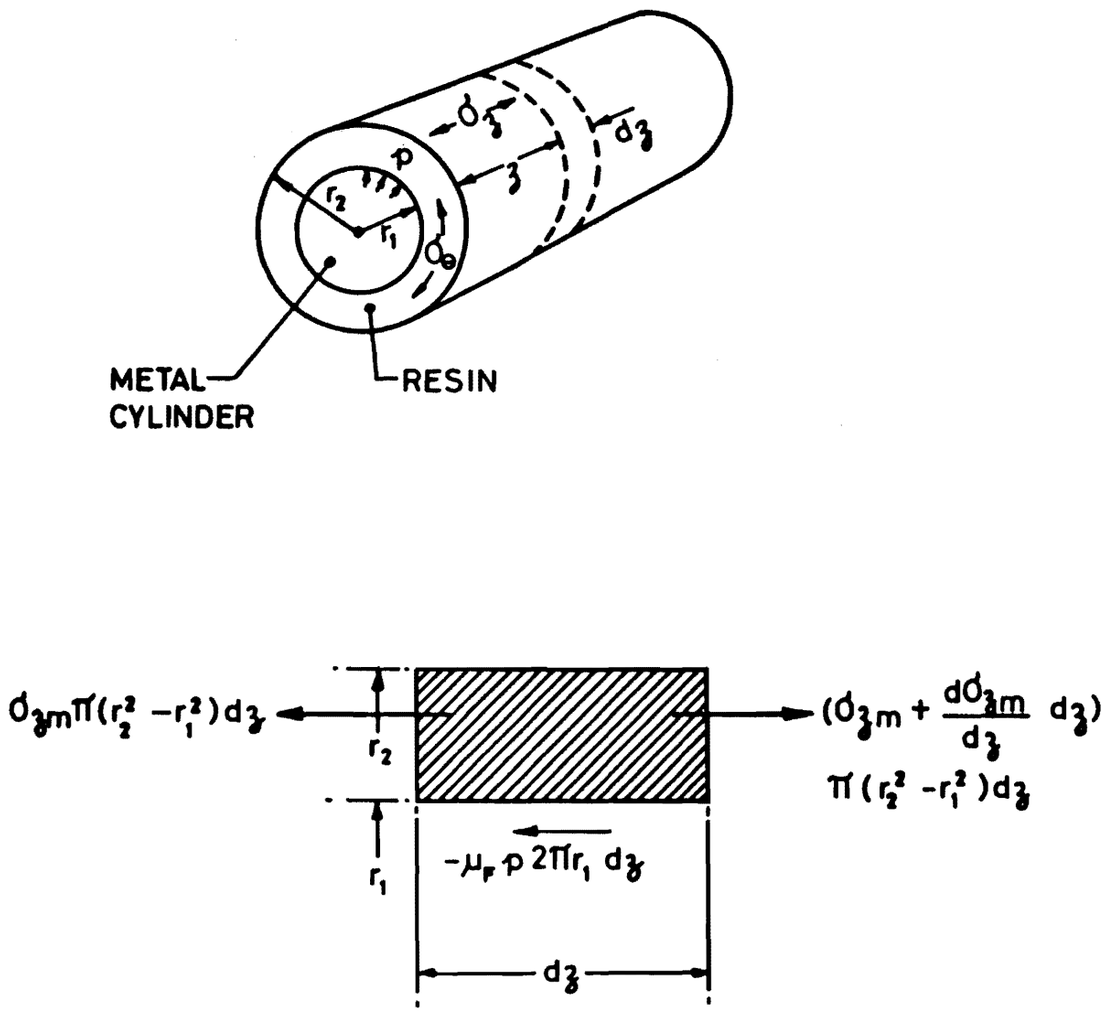

p
r_2
r_1
σ_z
dz
z
σ_θ
METAL CYLINDER
RESIN
$\sigma_{zm}\pi(r_2^2 - r_1^2)dz$
$(\sigma_{zm} + \frac{d\sigma_{zm}}{dz} dz)$
$\pi(r_2^2 - r_1^2)dz$
$-\mu_F p 2\pi r_1 dz$
dz

FIG. 9.

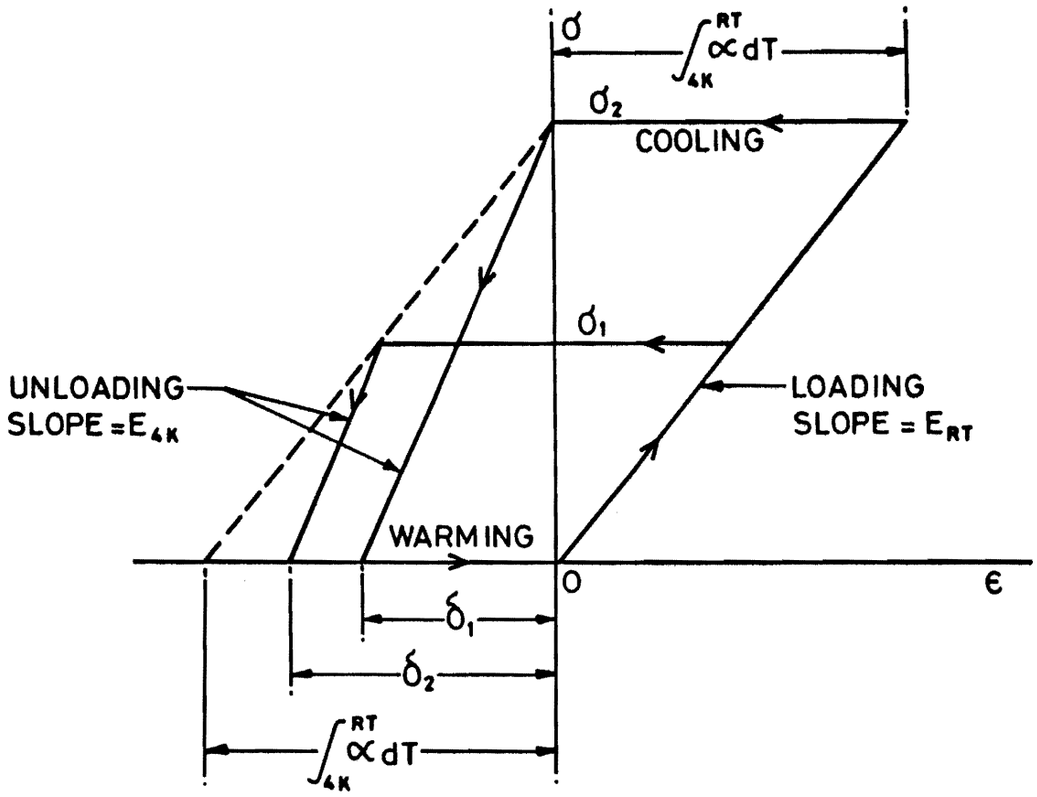


FIG. 10. STRESS AND THERMAL CONTRACTION