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A Model of Beam-Beam Interaction in e^+e^- Storage Rings with Dipole and Quadrupole Modes

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Abstract

A simple model of the coherent beam motions in e^+e^- colliding storage rings is proposed. The model is based on an approximation that the beam distribution is always Gaussian with variable barycenters and rms beam sizes. Most of the characteristic features of the multi-particle tracking results are reproduced by this model.

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In future high luminosity machines, such as flavor factories[1,2], the beam-beam effects[3] will be more serious than present. We, however, still do not understand these effects enough. At present, the multiparticle tracking (MPT) seems the only tool to evaluate them[4]. MPT, however, does not provide us an understanding of the effects: we need a simple model which permits us predicting most characteristic features of the effects with simple concepts. In this connection, the so-called coherent approach[5] seems useful. In particular, the Gaussian models have put forward some sound models.

In the rigid Gaussian model[6], the bunch distribution is assumed to be a Gaussian with fixed variances: only the barycenter variables can change. This model could explain the spontaneous beam separation (SBS) for the linear instabilities. The model was extended to the case with a primordial beam offset[7] in order to evaluate the closed orbit effects[8] due to the peripheral (parasitic) collisions [9]. This model, however, has the shortcomings that the beam size effects are overlooked.

In the soft Gaussian model[10], the beam variances can change but the barycenters are kept fixed. It could explain the beam-beam limit and the flip-flop effects. This, however, predicted too large beam sizes. To improve it, the linear-soft Gaussian model was proposed[11,12], where the beam-beam force is assumed to be linear. Since, although the force is linear, its focal length depends on the beam size of the encountering bunch, the total system is quite nonlinear. These models predict chaotic variation of the beam sizes, which, however, was not observed in MPT nor in experiments. Below the threshold of the chaotic motion, these model give quite reasonable predictions. In many cases, in addition, the threshold of this chaotic motion is either unpractically large or higher than that of dipole instability.

In this paper, we will extend these models: we make the rigid-Gaussian and linear-soft-Gaussian models merge into a unified model in order to get rid of shortcomings of each model. As we will see later, our model can reproduce the MPT results fairly well. This fact provides a hope that we can understand the

beam-beam effects rather simply.

Suppose an e^+e^- storage ring with only one interaction point (IP) and with only one bunch in each beam. In this paper, for simplicity, we consider the symmetric case only: both beams have the same parameters such as the number of particles N, the relativistic Lorentz factor γ, the nominal (i.e. without beam-beam effects) betatron functions β_0 and the nominal emittance ε. Also, for simplicity, we assume that beams are very flat and the horizontal force is negligible. We thus can restrict ourselves to the vertical motions. As canonical variables of an $e^\pm$, we use

$$Y_1^\pm = \frac{y_\pm}{\sigma_{0y}}, \qquad Y_2^\pm = \frac{\beta_{0y} y'_\pm}{\sigma_{0y}},$$

where σ_{0y} is the nominal vertical beam size.

Important coherent quantities are the barycenters $\bar{Y}_i = \langle Y_i \rangle$ (the $\langle\ \rangle$ is average over the distribution and i is 1 or 2) and the envelope matrix $M_{ij} = \langle (Y-\bar{Y})_i (Y-\bar{Y})_j \rangle$. For simplicity, we assume $M_+ = M_-$, that is, the envelope of both beams behave symmetrically. Technically, it is easy to introduce asymmetry in M as in Refs. [11,12]. In this case, however, the model inherits an abnormal property from the linear-soft-Gaussian model: the flip-flop and the chaotic instability occur too easily. By restricting to the symmetric case, we can avoid it.

A particle is kicked at the IP as

$$\Delta Y_1 = 0, \qquad \Delta Y_2 = -2\pi^{3/2}\eta \ \mathrm{erf}\left(\frac{Y_1 - \bar{Y}_1^*}{\sqrt{2M_{11}}}\right), \tag{1}$$

where η is the nominal beam-beam parameter

$$\eta = \frac{N r_e}{2\pi\gamma} \frac{\beta_{0y}}{\sigma_{0x}\sigma_{0y}}.$$

The quantity with $*$ refers to the encountering bunch. This induces a map for the barycenter variables as [13]

$$\Delta \bar{Y}_1 = 0, \qquad \Delta \bar{Y}_2 = -2\pi^{3/2}\eta \ \mathrm{erf}\left(\frac{\bar{Y}_1 - \bar{Y}_1^*}{2\sqrt{M_{11}}}\right). \tag{2}$$

For the envelope, we linearize Eq.(1) as

$$\Delta(Y_2 - \bar{Y}_2) = -K(\eta, \bar{Y}_1 - \bar{Y}_1^*, M_{11})(Y_1 - \bar{Y}_1),$$

$$K = 2^{3/2}\pi\eta \frac{1}{\sqrt{M_{11}}} \exp\left(-\frac{\left(\bar{Y}_1 - \bar{Y}_1^*\right)^2}{2M_{11}}\right).$$

Note that the both beams are focused to their own barycenters, like the tidal force. The focusing K depends on M_{11} as well as the difference between two barycenters. This makes the map quite nonlinear. Under this simplification, the change of the envelope M is simply as

$$\Delta M_{11} = 0, \qquad \Delta M_{12} = -KM_{11}, \qquad \Delta M_{22} = -2KM_{12} + K^2 M_{11}, \tag{3}$$

At the arc, i.e. from IP to IP for one turn, we have a betatron oscillation with radiation effects[14]:

$$\begin{pmatrix} \bar{Y}_1 \\ \bar{Y}_2 \end{pmatrix} \mapsto \lambda U \begin{pmatrix} \bar{Y}_1 \\ \bar{Y}_2 \end{pmatrix}, \qquad M \mapsto \lambda^2 U M U^t + \left(1 - \lambda^2\right) I, \tag{4}$$

where I is the unit matrix and

$$U = \begin{pmatrix} \cos\mu & \sin\mu \\ -\sin\mu & \cos\mu \end{pmatrix}.$$

Here $\mu = 2\pi\nu$ (ν being the tune) and $\lambda = \exp(-1/T)$ (T being the vertical damping time divided by the revolution time.)

Now the model is fixed. We study the behavior of $\bar{Y}_i^{\pm}$ and M_{ij}. In this paper, we are interested in the equilibrium distributions, represented by $\bar{Y}_\infty^{\pm}$ and M_∞. The most probable candidate is the period-one fixed point (P1FP): $\bar{Y}_\infty^{\pm}$ and M_∞ repeats the same values every turn. When the collision is head-on, the natural ansatz for the fixed point is $\bar{Y}_\infty^{\pm} \equiv 0$. The solution is given implicitly as

$$M_\infty = \frac{1 + k\cot\mu}{1 + 2k\cot\mu - k^2} \begin{pmatrix} 1 & k \\ k & 1 + 2k\cot\mu \end{pmatrix} + o\left(\frac{1}{T}\right),$$

where $k = \sqrt{2}\pi\eta/\sqrt{(M_\infty)_{11}}$. The solution is unique. Some examples of M_∞ are given in Fig.1. The M_{11}^∞ represents effects of the dynamic-beta[15] and the

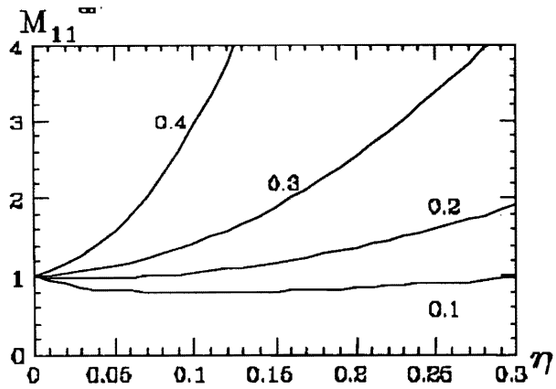

Figure 1: Examples of M_{11}^{∞} for various ν (written in the figure) in the limit $T \to \infty$. For T larger than 100, the behavior is almost completely the same.

dynamic-emittance[14]. In the figure, we showed ν only below 1/2. For ν larger than 1/2, the same pattern repeats periodically with period 1/2. This applies also for all the following figures.

The problem is the stability of this solution. We make the stability matrix:

$$S = \left.\frac{\partial(\bar{Y}'_+ - \bar{Y}'_-, M')}{\partial(\bar{Y}_+ - \bar{Y}_-, M)}\right|_{\mathrm{P1FP}},$$

where the prime refers to the quantities after one revolution. The quantity $\bar{Y}_+ + \bar{Y}_-$ is completely decoupled from other variables and we have $\bar{Y}_+^{\infty} + \bar{Y}_-^{\infty} = 0$. When one of its eigenvalues is larger than unity in absolute value, the P1FP is unstable.

The matrix S can be blockwise-diagonalized as $S = diag(S_d, S_e)$, where S_d is 2×2 and describes the stability of the barycenter motion and 3×3 matrix S_e corresponds to a subspace of M. The unstable regions of each mode are shown in Fig.2.

This diagram, however, predicts the stability of P1FP but nothing more. To study it, we perform the maps Eqs.(2), (3) and (4) iteratively starting from a certain initial state. (This is refereed to as a model tracking). The result is shown in Fig.3, where we start from $M = I$ and $\bar{Y}_1^+ - \bar{Y}_1^- = 0.1$. We show the average distance of two barycenters

$$D \equiv \langle |\bar{Y}_1^+ - \bar{Y}_1^-| \rangle_{av},$$

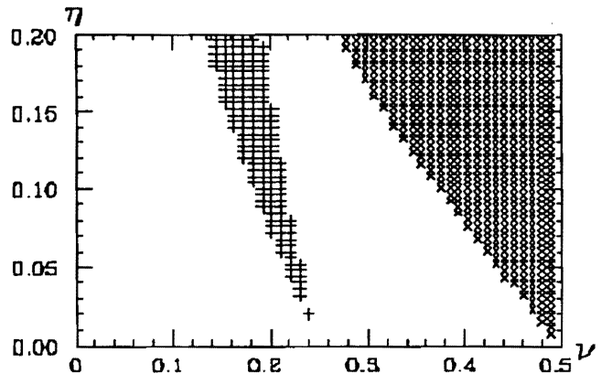

Figure 2: The unstable region of P1FP for $T \to \infty$. The branches extending from $(\nu, \eta) = (0.25, 0)$ and $(0.5, 0)$ correspond to the quadrupole and the dipole instabilities, respectively.

the average effective beam size $\langle\sqrt{M_{11}^{+} + M_{11}^{-}}\rangle_{av}$ (it is $\sqrt{2M_{11}}$ in the model) and the luminosity reduction factor R:

$$R = \langle\sqrt{\frac{2}{M_{11}^{+} + M_{11}^{-}}} \exp\left[-\frac{1}{2}\frac{(\bar{Y}_1^{+} - \bar{Y}_1^{-})^2}{M_{11}^{+} + M_{11}^{-}}\right]\rangle_{av},$$

where $\langle\ \rangle_{av}$ means average over many turns. The real luminosity L is $L_0 \times R$ where L_0 is the nominal luminosity. Both dipole and quadrupole instabilities reduce the luminosity.

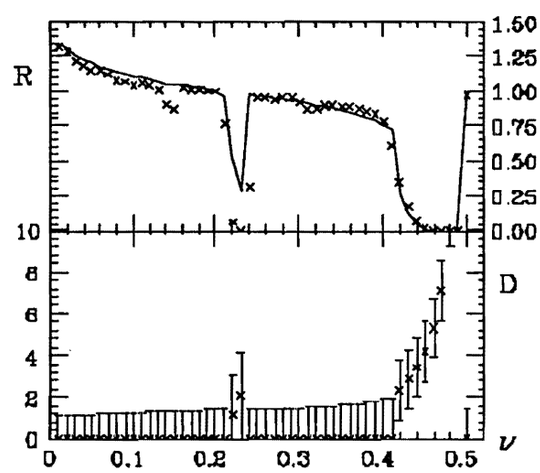

Figure 3: The tracking results of the model and MPT. (Above) the luminosity reduction factor R by the model (line) and MPT ($\times$). (Below) The barycenter difference D ($\times$) and the beam size $(2M_{11})^{1/2}$ (as error bars) by the model. MPT shows very similar results. Parameters: $\eta = 0.06$ and $T = 1000$. In MPT, 1000 particles were used for each beam and tracked for 10000 turns.

The model tracking shows that the resulting equilibrium is identical with

P1FP except for tunes around $\nu = 0.5$ and $\nu = 0.24$. The former is the half-integer resonance and can be understood in terms of the rigid-Gaussian model only[6]. As for the latter, the presence of instability is consistent with the stability diagram, Fig.2, but the diagram asserts that the beam size is enhanced while the model tracking indicates the barycenter is also affected. This does not imply the failure of the model: the stability analysis does not tell us what happens after the stability condition is broken. Dipole and quadrupole instabilities cooperate with each other. The unstable mode does not necessarily dominates the effects: the beam separation takes place even though the instability is triggered by the quadrupole mode. This phenomenon indicates that the present model is not a mere superposition of the rigid-Gaussian and linear-soft-Gaussian models.

At this tune, the 4-th order resonance occurs and the whole single bunch is trapped by the resonance: the 2-vector $\bar{Y}^+ - \bar{Y}^-$ repeats the same value every four turns with M enlarged but kept almost constant. In order to confirm that this observation is valid and not the consequence of the simplification of the present model, we perform a MPT: two beams are treated completely independent and the Gaussian approximation in calculating the beam-beam force is avoided by counting number of particles[7]. The result is shown in Fig.4. The prediction of the model is confirmed.

In this connection, we stress the usefulness of our method over the Vlasov technique[17]. The latter allows us to draw stability diagram like Fig.2. It is true that such approach can be more systematic and rigorous in predicting unstable regions. The shortcoming of this approach is that one cannot discuss the equilibrium beam distribution nor the luminosity. In addition, they are restricted usually to analyzing the stability of infinitesimal deviation from the nominal equilibrium state. Thus the Vlasov approach may predict only that the luminosity is L_0 in stable region and something happens in unstable region. As clear from the present example, $L \neq L_0$ even in stable region.

In Fig.3, we have also shown the evaluation of R by MPT. Considering its simplicity, the model agrees satisfactorily well with MPT. This supports the

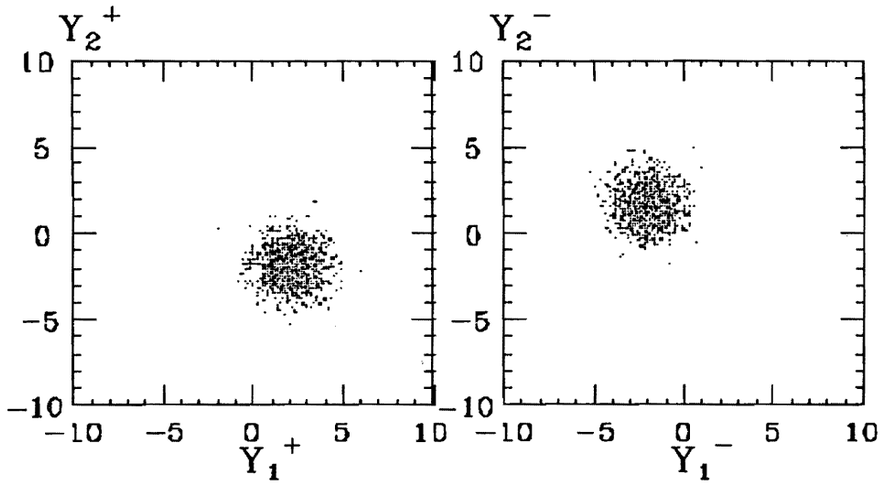

Figure 4: The phase space distribution of e^+ and e^- bunches $(Y_1^{\pm}, Y_2^{\pm})$ in an equilibrium evaluated by MPT. Both are caught by the fourth order resonance ($\nu = 0.21$). The same configuration repeats every 4 turns. To make the structure evident, the $\eta = 0.1$ and $T = 100$ are used. The barycenter and beam sizes are both affected.

validity of our model strongly. The nonlinear resonances seen in MPT around $\nu = 0.15$ and 0.32 and the detailed structure of the fourth order resonance are, however, not covered by the present model[16].

We have thus proposed a model which uses simple concepts only, permits us analytical calculations and gives practical and fairly reliable predictions of the beam-beam effects, though not perfect. The present model treats the simplest case but it can easily be extended to more general cases, including the asymmetry of the ring parameters between two beams, presence of the primordial offset and the peripheral collisions and two-dimensional (horizontal and vertical) motions with elliptic beam. In order to discuss the beam-beam limit (i.e. the case with large η), we should include the flip-flop mode (M_+ and M_- can act independently). This seems more difficult and challenging. More detailed and extended work will be published elsewhere.

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