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# DETECTING THE NON-MINIMAL SUPERSYMMETRIC HIGGS BOSON AT THE NLC

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## Abstract

We discuss the perspectives of discovering a non-minimal supersymmetric Higgs boson at the next linear collider (NLC). The analysis includes the leading logarithmic radiative corrections.

The cancellation of quadratic divergences in the unrenormalized Green functions is one of the main motivations of supersymmetry (SUSY). It stabilizes any mass scale under radiative corrections and thus allows the existence of different mass scales such as the electroweak scale [ $\mathcal{O}(10^2 \text{ GeV})$ ] and the Planck scale [ $\mathcal{O}(10^{19} \text{ GeV})$ ]. The minimal supersymmetric standard model (MSSM) is the most popular model of this kind due to its minimal particle content. Here, the Higgs sector contains only two electroweak doublets with all the self couplings determined by SUSY. While this model, with the Higgs sector determined by only two input, is very desirable from an experimental point of view, it faces various problems if one tries to unify the  $SU(3)_c \times SU(2)_L \times U(1)_Y$  in a single simple gauge group. The minimal versions of such grand unified theories (GUT) [1] predict the existence of two color-triplets, which are members of the same representations as the Higgs doublets. These color triplets mediate baryon number violating processes. Therefore, their masses have to be of the order of the GUT-scale,  $M_{\text{GUT}}$ , in order to satisfy the constraints coming from the lower limit of the proton lifetime [2]. A doublet-triplet mass splitting of the order of  $M_{\text{GUT}}$  can be achieved through the coupling to the adjoint representation that breaks the GUT gauge symmetry. However, a severe fine-tuning, known as the  $\mu$  problem of minimal SUSY-GUT models [3], is required in order to keep the mass of the doublets,  $\mu$ , at the electroweak scale while moving the Higgs triplets to the GUT scale.

The dynamical generation of the  $\mu$  term is the most economical attempt to solve this problem [4]. Here one eliminates the explicit Higgs mass term of the superpotential by imposing a  $Z_3$  symmetry. It is replaced by an electroweak singlet (the so-called "sliding singlet") whose vacuum expectation value (VEV) is not fixed, but can adjust itself such that it cancels the mass of the doublets coming from the coupling to the VEV of the adjoint representation. Therefore, it is natural in this model for  $\mu$  to be of the order of the SUSY breaking scale,  $M_{\text{SUSY}}$ .

Note, that such an extension of the MSSM enters the renormalization group equations

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(RGE) of the gauge couplings only at the two-loop level

$$\frac{d\alpha_i}{dt} = \beta_i^{NMSSM} = \beta_i^{MSSM} + \frac{a_i}{(4\pi)^3} \alpha_i^2 \kappa^2, \quad (1)$$

where  $a_i = 6/5, 2, 0$  for  $i = 1, 2, 3$  and  $t$  is the logarithm of the squared energy scale [5]. Hence, the successful unification of the gauge coupling constants of the MSSM [6] will not be modified significantly.

The Higgs sector of this next-to-minimal supersymmetric model (NMSSM) described above has been studied at tree-level in ref. 7-9. Radiative corrections have also recently been presented using an effective potential formalism [10] and renormalization group techniques [11]. These corrections are quite significant and we shall include them in our analysis using the latter approach. Thus, we will begin with a brief review of the renormalization group technique used to obtain the leading log corrections.

Let  $\Phi_1$  and  $\Phi_2$  denote two complex  $Y = 1$ ,  $SU(2)_L$  doublet scalar fields and let  $N$  denote a  $SU(2)_L \times U(1)_Y$  complex singlet. We introduce the notation

$$\Phi_n = \begin{pmatrix} H_n^+ \\ (H_n^0 + iA_n^0)/\sqrt{2} \end{pmatrix}, \quad N = \frac{1}{\sqrt{2}} (H_3^0 + iA_3^0). \quad (2)$$

The most general scalar potential that is invariant under the transformation  $N \rightarrow \exp(i\theta)N$ ,  $\Phi_1^\dagger \Phi_2 \rightarrow \exp(-i\theta)\Phi_1^\dagger \Phi_2$  and the  $SU(2)_L \times U(1)_Y$  gauge symmetry is given by

$$\begin{aligned} \mathcal{V}_q &= \frac{1}{2}\lambda_1(\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\ &+ (N^* N)[y_1(\Phi_1^\dagger \Phi_1) + y_2(\Phi_2^\dagger \Phi_2)] + y_{12} [(\Phi_1^\dagger \Phi_2)N^{*2} + \text{h.c.}] + y(N^* N)^2, \\ \mathcal{V}_t &= -\lambda A_\lambda [(\Phi_1^\dagger \Phi_2)N + \text{h.c.}] - \frac{1}{3}\kappa A_\kappa [N^3 + \text{h.c.}], \\ \mathcal{V}_m &= m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + m_N^2 N^* N, \end{aligned} \quad (3)$$

The  $SU(2)_L \times U(1)_Y$  gauge symmetry is broken by a non-zero vacuum expectation value of a Higgs doublet. The CP invariant and  $U(1)_{EM}$  gauge symmetry preserving minimum of the potential can be written as

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad \langle N \rangle = \frac{x}{\sqrt{2}}, \quad (4)$$

where the  $v_i$  can be chosen to be real. The VEVs have been normalized so that  $m_w^2 = \frac{1}{4}g_2^2(v_1^2 +$

$v_2^2$ ). In the NMSSM the coupling constants at scales equal or above  $M_{\text{SUSY}}$  are given by

$$\begin{aligned}
\lambda_1 &= \frac{1}{4}(g_2^2 + g_1^2), & y &= |\kappa|^2, \\
\lambda_2 &= \frac{1}{4}(g_2^2 + g_1^2), & y_1 &= |\lambda|^2, \\
\lambda_3 &= \frac{1}{4}(g_2^2 - g_1^2), & y_2 &= |\lambda|^2, \\
\lambda_4 &= |\lambda|^2 - \frac{1}{2}g_2^2, & y_{12} &= -\lambda\kappa^*.
\end{aligned} \tag{5}$$

Three of the ten degrees of freedom of the original Higgs fields are eaten by the  $W^\pm$  and  $Z$  as a result of the spontaneous breaking of the gauge symmetry. The remaining seven physical Higgs particles are: three CP-even scalars,  $H_i$  ( $i = 1, 2, 3$ ; with  $m_{H_1} \leq m_{H_2} \leq m_{H_3}$ ), two CP-odd scalar,  $A_i$  ( $i = 1, 2$ ), and a charged Higgs pair ( $H^\pm$ ). The mass parameters  $m_{11}$ ,  $m_{22}$  and  $m_N$  can be eliminated by imposing the minimization conditions. The resulting charged Higgs mass is

$$m_{H^\pm}^2 = \mathcal{A}_\Sigma x / (s_\beta c_\beta) - \frac{1}{2}\lambda_4 v^2 \tag{6}$$

and the neutral mass matrices are

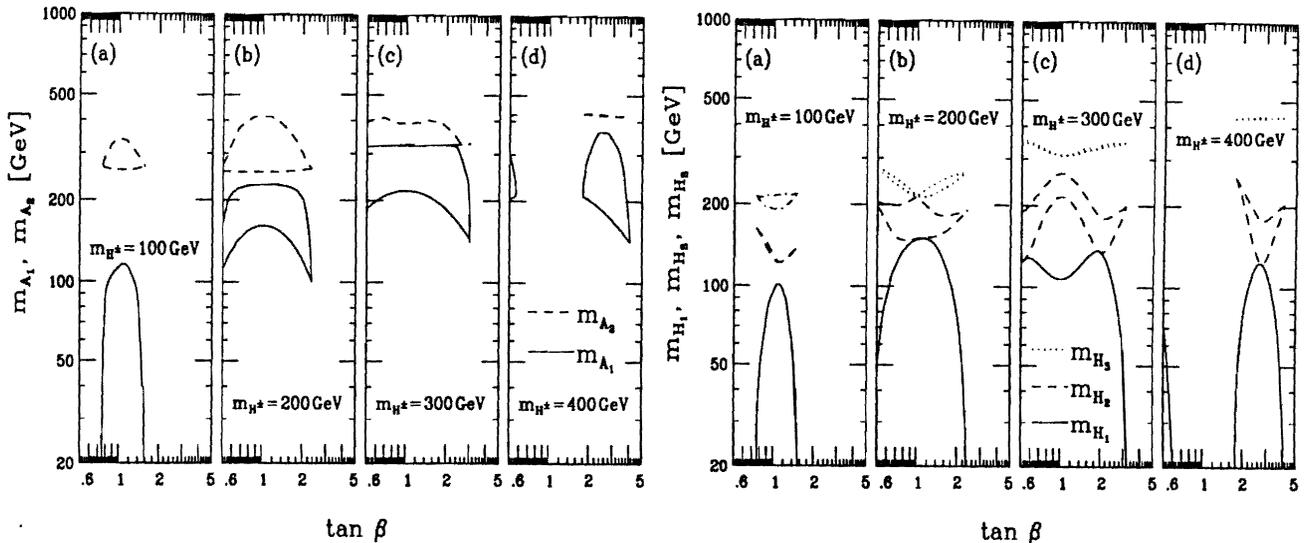
$$\begin{aligned}
\mathcal{M}_{A^0}^2 &= \mathcal{A}_\Sigma \begin{pmatrix} x/(s_\beta c_\beta) & v \\ v & s_\beta c_\beta v^2/x \end{pmatrix} + \frac{3}{2} \begin{pmatrix} 0 & y_{12} v x \\ y_{12} v x & \kappa A_\kappa x - s_\beta c_\beta y_{12} v^2 \end{pmatrix}, \\
\mathcal{M}_{H^0}^2 &= \mathcal{A}_\Sigma \begin{pmatrix} t_\beta x & -x & -s_\beta v \\ -x & t_\beta^{-1} x & -c_\beta v \\ -s_\beta v & -c_\beta v & s_\beta c_\beta v^2/x \end{pmatrix} \\
&+ \begin{pmatrix} \lambda_1 c_\beta^2 v^2 & (\lambda_3 + \lambda_4) s_\beta c_\beta v^2 & v x (y_1 c_\beta + \frac{1}{2} s_\beta y_{12}) \\ (\lambda_3 + \lambda_4) s_\beta c_\beta v^2 & \lambda_2 s_\beta^2 v^2 & v x (y_2 s_\beta + \frac{1}{2} c_\beta y_{12}) \\ v x (y_1 c_\beta + \frac{1}{2} s_\beta y_{12}) & v x (y_2 s_\beta + \frac{1}{2} c_\beta y_{12}) & 2y x^2 + \frac{1}{2} [s_\beta c_\beta y_{12} v^2 - \kappa A_\kappa x] \end{pmatrix},
\end{aligned} \tag{7}$$

where we have already eliminated the Goldstone modes. For convenience we have introduced the following abbreviations

$$\mathcal{A}_\Sigma = \frac{\lambda A_\lambda}{\sqrt{2}} - \frac{y_{12} x}{2}, \tag{8}$$

$$v^2 \equiv v_1^2 + v_2^2, \quad t_\beta \equiv \tan \beta \equiv v_2/v_1, \tag{9}$$

and  $s_\beta \equiv \sin \beta$ ,  $c_\beta \equiv \cos \beta$ . We now obtain the mass eigenstates and eigenvalues by diagonalizing the mass matrices in eq. (7). The main difference of eq. (5) to the corresponding conditions in the MSSM is the dependence of  $\lambda_4$  on the a priori arbitrary parameter  $\lambda$ . As a result, the



**Fig. 1** The CP-odd and CP-even Higgs masses as a function of  $\tan \beta$  for four choices of  $m_{H^\pm}$  and  $x = v$ . All the soft SUSY breaking mass parameters are equal to  $M_{\text{SUSY}}$  and we take  $\kappa = 0.63$ ,  $\lambda = 0.87$  and  $m_t = 150$ .

lightest Higgs mass or even its upper limit are undetermined, similar to the SM. However, if we require that the theory remains perturbative up to  $M_{\text{GUT}}$  we can constrain the allowed region in the  $\lambda$ - $\kappa$  plane. It was demonstrated in ref. 7 that the RGEs possess an infrared fixed point at  $(\lambda, \kappa) \sim (0.87, 0.63)$ . This yields an upper limit of  $m_{H_1} \lesssim 150$  GeV if we use the tree-level relations in eq. (5). However, it has been shown that the SUSY Higgs sector acquires large radiative corrections due to an incomplete cancellation of the top and stop contributions to the Higgs self energies [12;11;10]. These corrections can easily be included at the leading log level in eq. (7) by using the running couplings evaluated at the electroweak scale. This was discussed in detail in ref. 13 in the case of the MSSM and generalized to the NMSSM in ref. 11. This approach assumes that all soft SUSY breaking terms can effectively be parameterized by one common scale,  $M_{\text{SUSY}}$ . This might seem oversimplified, but note that the Higgs phenomenology depends on  $\lambda$  already at tree-level. However, since the only constraints on  $\lambda$  come from RGE arguments it is quite unclear, whether the accuracy of the result can indeed be significantly improved by a more precise treatment of the SUSY threshold effects.

In fig. 1 we present the allowed region of CP-odd and CP-even Higgs masses as a function of  $\tan \beta$  for  $m_t = 150$  GeV and four choices of  $m_{H^\pm}$ . We have varied  $0 \leq \kappa A_\kappa \leq 1$  TeV and we have fixed  $\mathcal{A}_\Sigma$  by eq. (6) and (8). In fig. 2 we present the allowed region of Higgs masses as a function of  $m_{H^\pm}$  for  $x/v = 0.4, 1$ . We have varied  $\kappa A_\kappa$  and  $\mathcal{A}_\Sigma$  as described above.

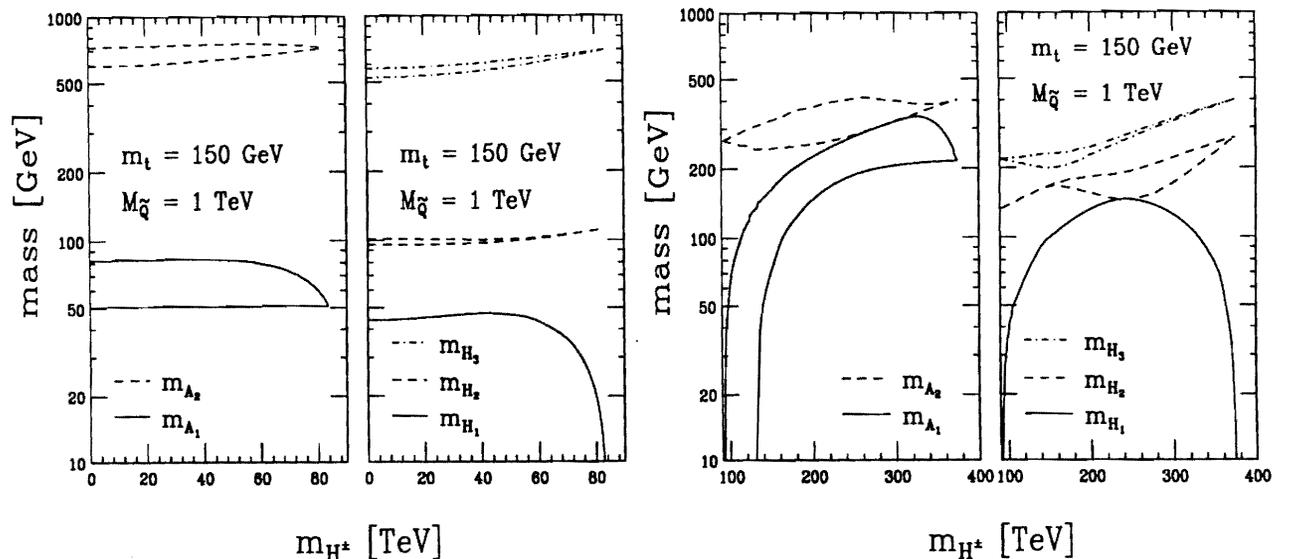


Fig. 2 The Higgs masses as a function of  $m_{H^\pm}$  for  $\tan\beta = 1.5$  and  $x/v = 0.4$  (left) and 1 (right). All the soft SUSY breaking mass parameters are equal to  $M_{\text{SUSY}}$  and we take  $\kappa = 0.63$ ,  $\lambda = 0.87$  and  $m_t = 150$ .

The phenomenologically interesting  $H_i Z Z$  and  $H_i A_j Z$  couplings ( $i = 1, 2, 3$  and  $j = 1, 2$ ) are obtained by diagonalizing the mass matrices in eq. (10) and expressing the interaction Lagrangian in terms of the mass eigenstates [8]. With these trilinear vertices at hand we readily obtain the Higgs production rates  $\sigma(e^+e^- \rightarrow H_i Z)$  and  $\sigma(e^+e^- \rightarrow H_i A_j)$ . The coupling of the charged Higgs boson to the  $Z$  boson is determined by gauge invariance and thus the rate  $\sigma(e^+e^- \rightarrow H^+H^-)$  is model independent.

In fig. 3 we present the region in the  $m_{H^\pm} - m_{H_1^0}$  plane where a deviation of the SM (SM) can be detected. We assume, that the analysis requires 50 events at the NLC with a CM energy of  $\sqrt{s} = 500$  GeV and a luminosity of  $10 \text{ fb}^{-1}$ . We have varied  $0.5 \leq \tan\beta \leq 5$ ,  $0 \leq \kappa A_\kappa \leq 1$  TeV,  $200 \text{ GeV} \leq M_{\text{SUSY}} \leq 1$  TeV and  $0.1 \leq x/v \leq 10$ .  $A_\Sigma$  is again fixed by eq. (6) and (8). Here a (+) denotes the region where a deviation from the SM can be detected in all the sets of parameters under consideration. A (-) denotes the region where no deviation from the SM can be detected for any choice of parameters under consideration and (0) denotes the region where a deviation from the SM depends on the choice of parameters. As a deviation of the SM counts the detection of two neutral Higgs bosons or one charged Higgs boson or a deviation of  $\sigma(e^+e^- \rightarrow H_i Z)$  by more than two standard deviation from the SM Higgs production rate.

We see that we can always detect a CP-even Higgs boson. In addition, for  $m_{H^\pm} \lesssim 180$  GeV we can always detect a deviation of the SM via  $e^+e^- \rightarrow H^+H^-$ . However, whether a deviation from the SM can be observed for  $m_{H^\pm} \gtrsim 180$  GeV depends on the choice of SUSY parameter,

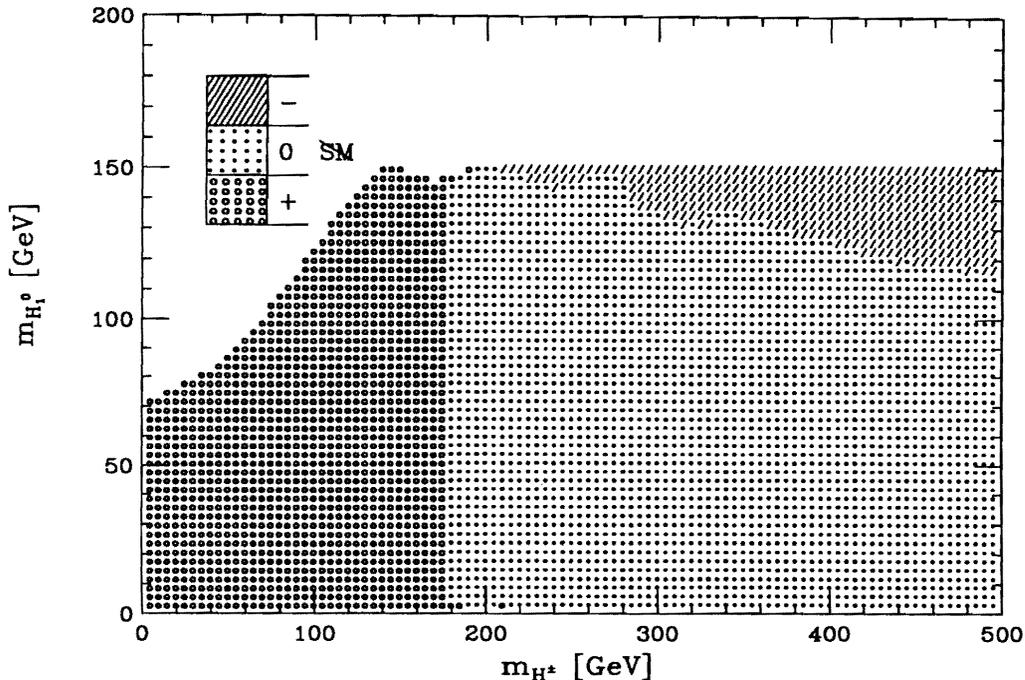


Fig. 3 The region in the  $m_{H^\pm}$ - $m_{H^0}$  plane where a deviation of the SM can be detected always, sometimes or not be detected (denoted by +, 0, -, respectively). We take  $m_t = 150$  GeV and vary all the SUSY parameter as described in the text

except for large values of the lightest CP-even Higgs boson,  $m_{H_1} \gtrsim 130$  GeV where a detection of SM is impossible for any choice of parameters. This can be understood as follows: for large values of  $m_{H^\pm} \gtrsim 300$  GeV the second Higgs doublet decouples and is out of kinematic reach of a 500 GeV collider. In this scenario the only chance of detecting a SM is if a large mixing of the light Higgs doublet with the Higgs singlet causes a significant reduction of the Higgs production rate. Such a mixing will decrease the mass of the lightest Higgs boson.

In conclusion we can say that a non-minimal SUSY Higgs boson will be detected at NLC if we assume that 50 events are sufficient for the analysis and if we require that the theory remains perturbative up to  $M_{\text{GUT}}$ . Furthermore, there is a good chance of detecting a deviation from the SM via direct production of more than one Higgs boson, or by measuring the Higgs production rate.

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