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LENSING PROPERTIES OF LIGHTLIKE CURRENT CARRYING COSMIC STRINGS

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Abstract

The lensing properties of superconducting cosmic strings endowed with a time dependent pulse of lightlike current are investigated. The metric outside the core of the string belongs to the *pp*–wave class, with a deficit angle. We study the field theoretic bosonic Witten model coupled to gravity, and we show that the full metric (both outside and inside the core) is a Taub-Kerr-Shild generalization of that for the static string with no current. It is shown that the double image due to the deficit angle evolves in an unambiguous way as a pulse of lightlike current passes between the source and the observer. Observational consequences of this signature of the existence of cosmic strings are briefly discussed.

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Most of the gravitational properties of cosmic strings [1] and superconducting cosmic strings [2] have so far been calculated in the special framework of zero [3,4] or time independent currents [5,6] so that it is widely believed that observation of strings in the universe might be achieved through the now well known double image effect, i.e. the deflection associated with the missing angle in the conical metric surrounding a string. An interesting observation would therefore consist in two exactly identical stars (or galaxies) separated by a fixed angle (usually assumed of the order 10^{-6}rad $\sim 2.4''$, i.e., for strings generated at the grand unified phase transition), or perhaps an aligned series of such twin images [7]. However, because the probability that two nearly identical stars be separated in the sky by pure coincidence in exactly the way one expects for cosmic strings is rather high, there is in fact a demand for a more convincing signature. As we shall see, a lightlike current may in principle provide such a signature.

In this paper, we study the gravitational field created by a lightlike current-carrying string. For the case of timelike and spacelike currents, which have been studied before [5,6], all known solutions are stationary [8]. In the lightlike case, however, we shall see that the stationary solution has peculiar asymptotic behaviour, in the sense that all light rays are gravitationally bound by the string. Therefore, the problem of asymptotic light deflection is not well posed in this metric. This is just as well, since in realistic situations, we do not expect the current to be stationary. In particular we can consider a pulse of lightlike current travelling along the string at the speed of light. Such pulses may be generated when a long string interacts with a bounded external source. In this case, the motion of the double image with time due to the nonstationary gravitational field must be examined.

The article is organized as follows: in a first section, we derive the metric surrounding a lighlike current carrying cosmic string by boosting a spacelike current metric, and show how the resulting gravitational field can be generalized to a time-dependent exact solution of the Einstein-Maxwell equations. In section II we address the question of finding the corresponding solutions in the underlying field theoretic model. For definiteness, we use the bosonic Witten model [2] coupled to gravity. We show that the introduction of a lightlike current does not modify the vortex profile. Also, the effect of the current on the metric is accounted for by means of a single function which satisfies a linear differential equation. In order to understand the effect of a lightlike current on a possible observation of light deflection by a string, we use the fact that our solution can in fact account for time-varying fields, so we consider a finite pulse of such a current. We then work out analytically the characteristic features of the geodesic motion in this metric, and eventually calculate numerically, in the last section, the actual effective motion (meaning as seen by a geodesic observer) of a point source initially aligned with a string and an observer when a pulse of lightlike current passes between the source and the observer. We conclude by considering the astrophysical observational possibilities and discussing a possible mechanism for generating pulses of lightlike current in cosmic strings.

I. THE EXTERIOR METRIC.

In this section, we shall obtain the gravitational field surrounding a cosmic string carrying a lightlike current, treating the source as an infinitely thin distribution. One easy way to do so is by the limiting procedure [9] of boosting the metric of the spacelike current case – which is known [5,6,10] – to the speed of light in the direction parallel to the string (alternatively, one could also boost the timelike current metric, both procedures being equivalent).

In the case of spacelike current, for a static cylindrically symmetric configuration with the string lying along the z–axis, the metric outside the core of the string is [5,10]

$$ds^2 = g(-dt^2 + dr^2) + r^2 f \gamma^2 d\theta^2 + \frac{dz^2}{f}. \tag{1}$$

Here, $f \equiv [c_1(r/r_\sigma)^m + c_2(r/r_\sigma)^{-m}]^2$ and $g = (r/r_\sigma)^{2m^2} f$, where r_σ is the thickness of the core of the string. The nonvanishing component of the electromagnetic field is given by

$$F_{rz} = \frac{2m\sqrt{c_1 c_2}}{G^{1/2} r} f, \tag{2}$$

where G is Newton's constant. The various parameters c_1, c_2, m and γ should be determined, in principle, in terms of the microphysical parameters characterizing the vortex by matching the exterior metric (1) with the solution of Einstein's equations inside the core of the string (i.e., for $r < r_\sigma$). In practice, this can be difficult if the gravitational field is strong. However, for weak fields, one can arrive at the relations [6]

$$\begin{aligned} m^2 &= 4GI^2 + \mathcal{O}(G^2), \\ c_1 &= \frac{1}{2}[1 + \frac{G^{1/2}}{I}(U - T - I^2)] + \mathcal{O}(G), \\ c_2 &= \frac{1}{2}[1 - \frac{G^{1/2}}{I}(U - T - I^2)] + \mathcal{O}(G), \\ \gamma &= 1 - 4G(U + \frac{1}{2}I^2) + \mathcal{O}(G^2). \end{aligned} \tag{3}$$

Here, U, T and I are, respectively, the flat space values of the string's energy per unit length, tension and current (see, e.g., Refs. [4,6] for a discussion of the limiting procedure consisting in taking well-defined flat space integrals of the stress energy tensor and current as the source of the weak gravitational field).

Treating the string as an idealized distribution of zero thickness, the components of the string's energy momentum tensor are given by $T^S_{\mu\nu} = \tau_{\mu\nu}\delta(x)\delta(y)$, $[\mu, \nu = t, x, y, z]$, where [5]

$$\tau_{\mu\nu} = \mathrm{diag}\,(U, \frac{I^2}{2}, \frac{I^2}{2}, -T). \tag{4}$$

A boost in the z direction

$$\begin{aligned} z' &= z\cosh\Lambda + t\sinh\Lambda, \\ t' &= t\cosh\Lambda + z\sinh\Lambda, \end{aligned} \tag{5}$$

transforms Eq. (1) into

$$ds^2 = g dr^2 + r^2 f\gamma^2 d\theta^2 + \alpha(\frac{1}{f} - g)(dz^2 + dt^2 - 2\sqrt{\frac{\beta}{\alpha}}dz dt) + g dz^2 - \frac{1}{f}dt^2, \tag{6}$$

where $\alpha \equiv \cosh^2\Lambda$, $\beta \equiv \sinh^2\Lambda$ and we have dropped the prime on the coordinates. Also the current, which was purely spacelike $J^\mu = (0,0,0,I)\delta(x)\delta(y)$, $[\mu = t, r, \theta, z]$, transforms into $J'^\mu = (\sqrt{\beta}I, 0, 0, \sqrt{\alpha}I)\delta(x)\delta(y)$, and becomes null as $\alpha \to \infty$. The (t, z) components of the stress tensor (4) transform into

$$\tau'_{ab} = \mathrm{diag}\,(U, -T) + (U - T)\begin{pmatrix} \beta & -\sqrt{\alpha\beta} \\ -\sqrt{\alpha\beta} & \beta \end{pmatrix} \tag{7}$$

$(a, b = z, t)$, aquiring non diagonal pieces. It is clear that if we want the components of J'^μ and τ'_{ab} to be finite, the limit $\alpha \to \infty$ can only be taken by letting I and $(T - U)$ go to zero at the same time, in such a way that the products

$$I'_z \equiv \sqrt{\alpha}I \quad \text{and} \quad (U' - T') \equiv (1 + 2\beta)(U - T) \tag{8}$$

remain finite.

Upon so doing, we find

$$ds^2 = -du dv - V(r)du^2 + dr^2 + r^2\gamma^2 d\theta^2 \tag{9}$$

where we have used null coordinates $u \equiv t - z$ and $v = t + z$. Using Eqs. (3) and (8), the function $V(r)$ can be written as

$$V(r) = 8G(U' - T')\ln\frac{r}{r_\sigma} + 8GI'^2_z \ln^2\frac{r}{r_\sigma}; \tag{10}$$

also we can write

$$\gamma = 1 - 4G(U' + T'). \tag{11}$$

From Eq. (2), the electromagnetic tensor in the new frame is

$$F'_{ru} = -\frac{2I'_z}{r}, \tag{12}$$

and the energy momentum of the string is given by

$$\tau'_{\mu\nu} = \frac{U' + T'}{2}\mathrm{diag}\,(1, 0, 0, -1) + (U' - T')\delta_{\mu u}\delta_{\nu u}, \tag{13}$$

where we have used Eqs. (7) and (8).

Typically [11], the degeneracy $(U - T)$ is of order I^2/e^2, where e is the electromagnetic coupling. Therefore, from Eq. (8)

$$U' - T' \sim \frac{I'^2_z}{e^2}.$$

In cosmological applications, $\ln(r/r_\sigma) \sim 100$. Since $e^2 \sim 10^{-2}$, both terms in Eq.(10) will be of the same order of magnitude.

Although the metric (9,10) has been obtained using the linearized relations (3), we shall see that it is actually a solution of the full Einstein-Maxwell equations, with electromagnetic field given by Eq. (12). This fact, which is also encountered in the Aichelburgh-Sexl case [9], has a simple mathematical explanation. As $\alpha \to \infty$, we let I and $(T-U)$ to zero, and so we are boosting a metric for which (in the limit $\alpha \to \infty$) the linear approximation becomes exact.

For later use, it is convenient to generalize Eq. (9) to the case where, instead of having a stationary null current flowing along the string, we have a localized pulse of current which is travelling in the positive z direction at the speed of light. The metric (9) belongs to the class known as pp-waves [12]

$$ds^2 = -dudv - H(u,r)du^2 + dr^2 + r^2\gamma^2 d\theta^2. \tag{14}$$

For the metric (14), the only nonvanishing component of the Ricci tensor outside the core is

$$R_{uu} = \frac{1}{2}\Box H(u,r) = 8\pi G T_{uu}. \tag{15}$$

The only component of the electromagnetic energy momentum tensor is

$$T^{e.m.}_{uu} = \frac{1}{4\pi}F_{ur}F^{ur} = \frac{p^2(u)}{\pi}\frac{I_z'^2}{r^2}. \tag{16}$$

Taking the ansatz

$$H(u,r) = p^2(u)V(r), \tag{17}$$

with $V(r)$ given by Eq. (10), it is clear that the Einstein equation (15) is satisfied outside the core.

In the core of the string, our metric (14) should be matched with an interior solution. Instead of that, we shall treat the source as infinitely thin, with support only at $r=0$. In a space with a deficit angle $2\pi(1-\gamma)$, we have

$$\tilde{\Delta}\ln r = 2\pi\gamma\delta(x)\delta(y),$$

where $(x,y) = r(\cos\theta, \sin\theta)$ and $\tilde{\Delta}$ is the Laplacian in the (r,θ) plane. Then, one can see that the source corresponding to (14-17) has the form

$$\tau'_{\mu\nu} = \frac{U'+T'}{2}\mathrm{diag}\,(1,0,0,-1) + p^2(u)\gamma(U'-T')\delta_{\mu u}\delta_{\nu u}. \tag{18}$$

The first term, corresponding to the Goto-Nambu string, is responsible for the deficit angle. The second term, caused by the nondegeneracy $(U'-T')$, is modulated by the function $p(u)$ that gives the profile of the pulse of current. From $\nabla_\mu F^{\mu\nu} = 4\pi J^\nu$, we have

$$J^\nu = I_z'\gamma p(u)(1,0,0,1)\delta(x)\delta(y).$$

If $p(u)$ has compact support, the metric (14) represents the gravitational field of a pulse of current travelling in the positive z direction.

II. WITTEN MODEL COUPLED TO GRAVITY

In the previous section, we treated the superconducting string as an infinitely thin distribution along which a pulse of lightlike current flows. The question remains, however, of whether it is possible to find the corresponding solutions in the underlying field theoretic model coupled to gravity.

For definiteness, we shall consider the Witten bosonic model [2], consisting of two complex scalar fields and associated U(1) gauge fields (Φ, B_μ) and (Σ, A_μ). The Lagrangian is given by

$$\mathcal{L} = -\frac{1}{2}|D_\mu\Phi|^2 - \frac{1}{2}|D_\mu\Sigma|^2 - \frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu} - \frac{1}{16\pi}H_{\mu\nu}H^{\mu\nu} - \mathcal{V}(|\Phi|,|\Sigma|). \tag{19}$$

Here, $D_\mu\Phi = (\nabla_\mu + iqB_\mu)\Phi$, $D_\mu\Sigma = (\nabla_\mu + ieA_\mu)\Sigma$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $H_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$. The potential

$$\mathcal{V}(|\Phi|,|\Sigma|) = \frac{\lambda_\phi}{8}(|\Phi|^2-\eta^2)^2 + \zeta(|\Phi|^2-\eta^2)|\Sigma|^2 + \frac{\lambda_\sigma}{4}|\Sigma|^4 + \frac{m_\sigma^2}{2}|\sigma|^2$$

is chosen so that the field Φ undergoes spontaneous symmetry breaking. Since the vacuum manifold is nontrivial, Φ will admit vortex (string-like) solutions. Σ develops a condensate in the core of the string, but vanishes ouside. The field A_μ is then identified with electromagnetism, which is unbroken outside the string.

To find the gravitational field of a superconducting cosmic string with lightlike current in the model (19), we shall proceed in two steps. First, we shall consider the metric for the static and cylindrically symmetric configuration in which no current is flowing in the string. This configuration is also invariant with respect to boosts parallel to the symmetry axis [13]. Then, we shall see that the metric for the current-carrying case is just a Taub-Kerr-Schild generalization of the previous one.

In the case where there is no current, we can make the following ansatz for the fields and the metric [13,14]

$$\Phi = \varphi(r)e^{in\theta} \quad , \quad \Sigma = \sigma(r)e^{i\psi}, \tag{20}$$

$$ds^2 = -e^{a(r)}dudv + dr^2 + e^{b(r)}d\theta^2 \tag{21}$$

the only nonvanishing component of B_μ is taken to be

$$B_\theta = B_\theta(r), \tag{22}$$

and, for the time being, we set $A_\mu = 0$ and ψ =const. The equations of motion for the fields then reduce to

$$\begin{aligned} &B_\theta'' + (a' - \frac{b'}{2})B_\theta' = 4\pi q\varphi^2(n+qB_\theta) \\ &\varphi'' + (a' + \frac{b'}{2})\varphi' - e^{-b}\varphi(n+qB_\theta)^2 = \mathcal{V}_{,\phi} \\ &\sigma'' + (a' + \frac{b'}{2})\sigma' = \mathcal{V}_{,\sigma}. \end{aligned} \tag{23}$$

These have to be supplemented with the Einstein's equations for $a(r)$ and $b(r)$. The energy-momentum tensor's nonvanishing components are

$$T_{rr} = \frac{1}{2}\left(\varphi'^2 + \sigma'^2 + e^{-b}\frac{B_\theta'^2}{4\pi}\right) - \frac{1}{2}\left[e^{-b}(n + qB_\theta)^2\varphi^2 + 2\mathcal{V}\right] \tag{24}$$

$$T_{\theta\theta} = \frac{1}{2}\left[(n + qB_\theta)^2\varphi^2 + \frac{B_\theta'^2}{4\pi}\right] - \frac{1}{2}e^b\left(\varphi'^2 + \sigma'^2 + 2\mathcal{V}\right) \tag{25}$$

$$T_{uv} = g_{uv}\mathcal{L}(r), \tag{26}$$

where $\mathcal{L}(r)$ is the Lagrangian

$$\mathcal{L}(r) = -\frac{1}{2}\left[\varphi'^2 + \sigma'^2 + e^{-b}(n + qB_\theta)^2\varphi^2 + e^{-b}\frac{B_\theta'^2}{4\pi}\right] - \mathcal{V}. \tag{27}$$

Einstein's equations then take the form [13]

$$\begin{aligned} 4R_{uv}e^{-a} &= a'' + (a' + \frac{b'}{2})a' = 8\pi G(T_{rr} + e^{-b}T_{\theta\theta}) \\ -2R_{\theta\theta}e^{-b} &= b'' + (a' + \frac{b'}{2})b' = 8\pi G(-4e^{-a}T_{uv} + T_{rr} - e^{-b}T_{\theta\theta}) \\ -2R_{rr} &= 2a'' + b'' + a'^2 + \frac{b'^2}{2} = 8\pi G(-4e^{-a}T_{uv} - T_{rr} + e^{-b}T_{\theta\theta}). \end{aligned} \tag{28}$$

Actually, the third of Eqs. (28) is not independent of the other two [13]. Therefore, Eqs. (23) and (28) form a system of five coupled differential equations for the five unknown B_θ, φ, σ, a and b, with boundary conditions $\varphi(0) = 0$, $\varphi(\infty) = \eta$, $B_\theta'(0) = 0$, $B_\theta(\infty) = 0$, $\sigma'(0) = 0$, $\sigma(\infty) = 0$, $a'(0) = 0$, $a(\infty) = 0$ and $e^b/r^2 \to 1$ as $r \to 0$.

These equations are far too complicated to solve analytically, even in flat space ($a = b = 0$), where they have been studied only numerically [11]. However, it is generally believed that such solutions should exist, and that, at least for $G\eta^2 \ll 1$, their asymptotic properties can be inferred from what is known to be true in the flat space case [11]. In this case, the energy density is concentrated within a region of radius $r_c \sim \lambda_\varphi^{-1/2}\eta^{-1}$ and, for $r \gg r_c$, the energy density falls off exponentially. Therefore, when gravity is considered, we expect that the metric should behave as a vacuum solution for $r \gg r_c$. Static cylindrically symmetric vacuum solutions fall into the class given by Eq. (1), with $c_2 = 0$. If, in addition, we demand boost invariance along the axis, we must take $m = 0$ or $m = -2$. For $m = -2$, the metric is of Kasner type and has the property that a circle around the axis at $r \to \infty$ has zero length. Such metric is considered unphysical from the point of view of strings. For $m = 0$, the exterior metric is just flat space with a deficit angle which is given by [13]

$$\Delta\theta = 2\pi(1 - \gamma) = 8\pi G\mu + \mathcal{O}[(G\mu)^2], \tag{29}$$

where

$$\mu \equiv 4\pi\int_0^\infty e^{-2a}T_{uv}r\,dr. \tag{30}$$

The only difference with the Nielsen-Olesen case is that we now have the condensate σ in the core of the string, which will modify the value of T_{uv} [see Eq. (26)], and hence μ, but the metric is still basically flat space minus a small wedge.

Now, to introduce a lightlike current on the string, we take the ansatz (20) and (22) for Φ, Σ and B_μ, but we introduce a nonvanishing gauge field A_μ and u-dependent phase ψ.

$$A_\mu = A(r)p(u)\delta_{u\mu} \quad , \quad \psi = \psi(u). \tag{31}$$

As we shall see below, this is consistent with an ansatz for the metric of the form

$$d\hat{s}^2 = -e^{a(r)}\left(du\,dv + H(r,\theta,u)du^2\right) + dr^2 + e^{b(r)}d\theta^2. \tag{32}$$

The equation for A_μ following from (19) is

$$\nabla_\mu F^\mu_\nu = 4\pi e J_\nu, \tag{33}$$

where $J_\nu = \sigma^2(\psi_{,\nu} + eA_\nu)$ is the electromagnetic current. Eqs. (33) and (31) with the metric (32) require

$$\psi_{,u} = kp(u)$$

where k is a constant, and

$$A'' + (a' + \frac{b'}{2})A' = 4\pi e\sigma^2(k + eA). \tag{34}$$

It is interesting to observe that the introduction of the fields (31) and the new term g_{uu} in the metric do not affect the form of Eqs. (23). As we shall see, the equations for a and b are also unaffected and therefore, the null current does not change at all the profile of the vortex (unlike the case of spacelike or timelike currents).

To write down Einstein's equations, we use the fact that, as mentionned before, the metric (32) is the Taub-Kerr-Schild generalisation of (21). A direct application of Taub's formalism [15] (see, e.g. Eq.(3.23) of Ref. [16]) yields the new Ricci tensor

$$\hat{R}_{\mu\nu} = R_{\mu\nu} + \hat{R}_{uu}\delta_{\mu u}\delta_{\nu u}, \tag{35}$$

where $R_{\mu\nu}$ is the old Ricci tensor, given in terms of a and b by Eqs. (28), and

$$\hat{R}_{uu} = 2HR_{vu} + \frac{1}{2}e^a\Box H. \tag{36}$$

Also, it is easily verified that the new energy-momentum tensor can be written as

$$\hat{T}_{\mu\nu} = T_{\mu\nu} + \hat{T}_{uu}\delta_{\mu u}\delta_{\nu u},$$

where $T_{\mu\nu}$ is the old energy-momentum tensor, given by Eqs. (24-26), and the new component is given by

$$\hat{T}_{\mu\nu} = p^2[\sigma^2(k + eA)^2 + \frac{A'^2}{2}] - e^a H\mathcal{L},. \tag{37}$$

The trace T^{μ}_{μ} is unaffected, and so the Einstein equations (28) are unchanged, as promised. However, a new equation, corresponding to the uu component, has to be considered:

$$\frac{1}{2}e^{a}[\Box H + H(R_{rr} + e^{-b}R_{\theta\theta})] = 8\pi G\hat{T}_{uu}. \tag{38}$$

Using (28), we have

$$R_{rr} + e^{-b}R_{\theta\theta} = 32\pi G e^{-a}T_{uu} = -16\pi G\mathcal{L}.$$

Hence, from (37) and (38), one arrives at the simple linear equation for H

$$\frac{1}{2}e^{a}\Box H = 8\pi G p^2[\sigma^2(k + eA)^2 + \frac{A'^2}{2}]. \tag{39}$$

The general solution to this can be obtained as the sum of a particular solution plus the general solution of the homogeous equation

$$\Box H_h = 0. \tag{40}$$

To find the particular solution, we take the ansatz

$$H = p^2(u)V(r),$$

which implies

$$\frac{1}{2}e^{a}\bar{\Delta}V = 8\pi G[\sigma^2(k + eA)^2 + \frac{A'^2}{2}], \tag{41}$$

where $\bar{\Delta} = e^{-(a+b/2)}\partial_r(e^{a+b/2}\partial_r)$ is the Laplacian in the transverse plane. Note that (41) is just an ordinary differential equation. Note also that outside the core, $a \to 0$, $\sigma \to 0$ and Eq. (39) reduces to (15) of the previous section. Also, if the thickness of the core is small with respect to all other relevant length scales, then it is justified to replace the term $\sigma^2(k + eA)^2$ in the right hand side of (39) by a delta function distribution.

To summarize the results of this section, we can say that the lightlike current actually decouples from the background fields forming the static string, since the introduction of the null gauge field has no effect on σ, φ, a and b. Its only effect on the space time geometry can be accounted for by solving the linear ordinary differential equation (41).

As mentionned before, to the particular solution given by (41), we can add any solution of the homogeneous equation (40), $H_h(r, \theta, u)$. Such solutions can accomodate an additional gravitational wave, as well as a cosmic string transverse wave travelling in the same direction as the null current [16].

III. GEODESICS.

In the stationary case, i.e., when the metric takes the form (9), one can find three constants of motion for geodesic test particles

$$\begin{aligned} p_- &\equiv \frac{\dot{u}}{2} \\ p_+ &\equiv \frac{\dot{v}}{2} + 2p_-V(r) \\ L_\mu &\equiv (1 - 4G\mu)r^2\dot{\theta} \end{aligned} \tag{42}$$

where $\mu \equiv \frac{1}{2}(U' + T') = \frac{1}{2}(U + T)$ [μ can be expressed in terms of underlying fields through (30)]. In (42), a dot denotes derivative with respect to an affine parameter λ. For convenience, we take $\lambda \equiv \tau/m$, where τ is the proper time, and m is the mass of the particle. Using the constraint $\dot{x}_\mu\dot{x}^\mu = -m^2$, we have

$$\dot{r}^2 + \frac{L_\mu^2}{r^2} + 4p_-^2V(r) = -m^2 + 4p_+p_-, \tag{43}$$

and the problem reduces to the motion of a Newtonian particle in a potential $V(r)$ given by (10).

Note, however, that the stationary metric has peculiar asymptotic properties. In particular, since the potential grows logarithmically with radius, the motion in the r direction will be bounded even for light rays ($m^2 = 0$). Therefore, the problem of asymptotic light deflection is not well posed because we do not have asymptotic regions where light rays would propagate along straight lines.

Of course, this extreme behaviour will not arise if, instead of the stationary metric, we consider a finite pulse of current, because then the metric is flat before and after the passage of the wave. For simplicity, we can take a pulse with a step function profile. The metric is given by (14) with

$$H(u, r) = \Theta(u)\Theta(a - u)V(r), \tag{44}$$

where a is the duration of the pulse (not to be confused with the metric function in the previous section). The geodesic equation for the v coordinate reads

$$\frac{d}{d\lambda}(4p_-H + \dot{v}) = 4p_-^2[\delta(u) - \delta(a - u)]V(r).$$

Therefore, p_+ defined in (42) is only piecewise constant in this case, undergoing jumps when the particle enters and exits the wave.

Denoting by r_1 the coordinate radius where the wave hits the particle (at $u = 0$) and by r_2 the radius at which the particle emerges from the wave (at $u = a$), we have

$$\Delta p_+ \equiv p_+^{(2)} - p_+^{(1)} = -p_-\Delta V, \tag{45}$$

where $\Delta V \equiv V(r_2) - V(r_1)$ and $p_+^{(1)}$ and $p_+^{(2)}$ are the values of $(\dot{v}/2)$ before and after the particle has interacted with the wave. Without loss of generality, we take $dz/dt = 0$ as an initial condition. If $v_z \equiv dz/dt \neq 0$ initially, we can always perform a Lorentz transformation to a new frame where such component of the velocity will vanish [under this Lorentz transformation, $H(u, r) \to \kappa H(u, r)$ and $a \to \kappa^{-1/2}a$, where $\kappa \equiv (1 - v_z)/(1 + v_z)$]. Then, in the new frame, $p_+^{(1)} = p_-$, and, from (45), we find that the velocity of the particle parallel to the string (after the wave has past) is

$$\left(\frac{dz}{dt}\right)_2 = \frac{-\Delta V}{2-\Delta V} \approx \frac{1}{2}[V(r_1) - V(r_2)], \tag{46}$$

(typically, $\Delta V \ll 1$). Hence, the particle receives a boost in the z direction whose sign and magnitude are completely determined by r_1 and r_2. In particular, if the particle is initially at rest, it will be attracted towards the string by the passing wave, so that $r_2 \leq r_1$ [i.e., $V(r_2) \leq V(r_1)$] and the boost will be in the positive z direction. On the other hand, a light ray in the (x, y) plane will be deflected in the z direction by the wave, but the sign of the deflection depends on whether the interaction with the wave happens when the ray is approaching the string or when the ray has already surpassed the string and is moving away from it. In the former case, the deflection is in the positive z direction, whereas in the latter it is in the negative z direction.

In the next section, we shall numerically treat the motion of null geodesics in the spacetime given by (44), but for the rest of this section, we shall concentrate on a special case which can be treated analytically, namely the shock wave case [12]

$$H(u, r) = a\delta(u)V(r). \tag{47}$$

This can be considered as limiting case of (44) when the thickness of the wave is much smaller than other relevant dimensions in the problem (such as the impact parameter or the distance of the wave and of the observer to the string).

It will be convenient to use cartesian coordinates in which the metric (14) reads

$$ds^2 = -dudv - Hdu^2 + dX^2 + dY^2, \tag{48}$$

where $X = r\cos(\gamma\theta)$, $Y = r\sin(\gamma\theta)$, with $-\pi \leq \theta \leq \pi$, so that there is a wedge of angle $2\pi(1-\gamma)$ missing in the (X, Y) plane (see fig. 1). We shall take the source to be sufficiently far away so that the incident rays are parallel to the X axis. The geodesic equations read

$$\begin{aligned} \dot{u} &= 2p_- = \text{cte} \\ \dot{X} &= -\frac{1}{2}\int \frac{\partial H}{\partial X}\dot{u}du \\ \dot{Y} &= -\frac{1}{2}\int \frac{\partial H}{\partial Y}\dot{u}du \end{aligned} \tag{49}$$

and $\dot{v}$ can be obtained from the constraint

$$\dot{x}_\mu \dot{x}^\mu = -m^2. \tag{50}$$

Using (47), we have

$$\begin{aligned} \dot{X} &= \dot{X}_1 - ap_- V'(r_0)\frac{X_0}{r_0}\Theta(u) \\ \dot{Y} &= -ap_- V'(r_0)\frac{Y_0}{r_0}\Theta(u). \end{aligned} \tag{51}$$

Here, $\dot{X}_1$ is the initial velocity, $V' = dV/dr$, X_0 and Y_0 are the coordinate where the particle hits the wave, and $r_0 = \sqrt{X_0^2 + Y_0^2}$. For light rays lying in the (X, Y) plane, we have $\dot{X}_1 = -2p_-$. From (50),

$$\dot{v} = \frac{1}{2p_-}[\dot{X}^2 + \dot{Y}^2 - 4ap_-^2\delta(u)],$$

with $\dot{X}$ and $\dot{Y}$ given by (51).

Denoting by θ the deflection in the (X, Y) plane and φ the deflection in the z direction, we have, to first order in V,

$$\varphi \approx \frac{1}{2}aV_0'\frac{X_0}{r_0} \tag{52}$$

$$\theta \approx \frac{1}{2}aV_0'\frac{Y_0}{r_0} + \pi(1-\gamma)\text{sign}(Y_0), \tag{53}$$

where the last term in the r.h.s. of (53) corresponds to the deficit angle created by the static string.

An interesting question, from the observational point of view, is to find out what is the effect of the wave on the double image caused by the static string. As we shall see, the two images undergo apparent motion, describing (almost) closed trajectories in the sky.

For simplicity, we shall take

$$V(r) = \alpha \ln \frac{r}{r_\sigma}, \tag{54}$$

where $\alpha = 8G(U' - T') + 8GI_z'^2 \ln(\bar{r}/r_\sigma)$, with $\bar{r}$ a cosmological scale. This is a good approximation to (10), since in the course of the scattering, $\ln(r/r_\sigma)$ changes only by a small percentage.

From the geometry of fig. 1, it is clear that the deflection angle is given by

$$\theta = \frac{Y_0 + \pi(1-\gamma)X_0}{\ell + X_0},$$

where ℓ is the distance of the observer to the string. Comparing with (53), we have

$$\frac{a\alpha}{2}\frac{Y_0}{r_0^2} = \frac{Y_0 - \ell\pi(1-\gamma)}{\ell + X_0}. \tag{55}$$

This equation gives the locus of points at which the rays received by the observer have been scattered by the wave. From it, we can obtain $Y_0 = Y_0(X_0)$, and, substituting in (52) and (53), X_0 can be eliminated to obtain the curve $\varphi = \varphi(\theta)$. Since (55) is a complicated expression, it is difficult to carry out this procedure in general. However, we can easily obtain the answer in two limiting cases.

The first limiting case is when

$$a\alpha \ll \ell\pi^2(1-\gamma)^2.$$

In that case, we can replace Y_0 by $\ell\pi(1-\gamma)$ in (52) and (53) to find

$$\varphi^2 + \left[\theta - 4G\mu\pi - \frac{a\alpha}{16G\mu\ell\pi}\right]^2 \approx \left[\frac{a\alpha}{16G\mu\ell\pi}\right]^2, \tag{56}$$

that is, the double images will describe circles of angular radii $a\alpha/[16G\mu\ell\pi]$.

The second limiting case which can be solved is when the deficit angle can be neglected, $\gamma = 1$. In that case, we have

$$\varphi^2 + \theta^2 \approx \frac{1}{2}\frac{a\alpha}{\ell}. \tag{57}$$

The image describes the following trajectory. Initially, it moves upwards to an angular distance $(a\alpha/2\ell)^{1/2}$ from its initial position without moving sideways. Then it splits into two images which describe semicircles of radius $(a\alpha/2\ell)^{1/2}$. Finally, the two images merge at the bottom of the circle and eventually go back to the original position. However, for all its beauty, this pattern will not arise in situations involving realistic cosmic strings. Typically, the effect of the deficit angle is always comparable, if not much larger, than the effect due to the current. The reason is that, although a cosmic string can in principle support an arbitrarily large lightlike current, it is difficult to think of a mechanism to generate currents much stronger than the "critical current", defined by the condition $\alpha \sim G\mu$. We shall return to this question in the concluding section.

Equations (56) and (57) correspond to the case of a shock wave. If the duration of the wave is $a \gg \mathrm{Max}[G\mu\ell, (a\alpha\ell)^{1/2}]$, then the shock wave approximation is no longer a good description, since the duration is large compared with the impact parameter b. In such a case, one can see from Eqs. (46) and (49) that to first order in α the deflection angles are of order $\Delta\theta \sim \alpha$ and $\Delta\varphi \sim \alpha \ln(a/b)$.

IV. MOTION OF A POINT SOURCE.

In the previous section we considered the apparent motion of double images in the sky due to the light deflection caused by an impulsive wave. However, we neglected the fact that due to the gravitational field, the observer and the source will undergo geodesic motion, which will also contribute to the apparent motion. Here, we shall include this effect, studying the actual motion of a point source as seen from the observer. Also, we shall consider a wave of finite duration a. We shall assume that initially the source and the observer are both in a plane orthogonal to the string, and that neither of them is at infinite distance. We are therefore looking at the particular configuration shown in Fig. 2, where the angles of observation Θ and ϕ are defined as the usual azimuthal and polar angles. It turns out that it will be simpler to use, instead of the forms (42) and (43) for the geodesic equations where the constant of the motions are explicitely exhibited, the full second order differential equations directly derived from the metric (14) with $H(u,r)$ given by (17):

$$\ddot{r} = r\gamma^2\dot{\theta}^2 - \frac{1}{2}\frac{\partial H}{\partial r}(\dot{t} - \dot{z})^2, \tag{58}$$

$$\ddot{\theta} = -\frac{2\dot{r}}{r}\dot{\theta}, \tag{59}$$

$$\ddot{t} = \ddot{z} = -\frac{1}{2}\frac{\partial H}{\partial u}(\dot{t} - \dot{z})^2 - \frac{\partial H}{\partial r}\dot{r}(\dot{t} - \dot{z}). \tag{60}$$

Here a dot still means differentiation with respect to the affine parameter λ along the geodesic under consideration and we have explicitely made use of the conservation of $\dot{u}$. We consider the case where the null current is of constant amplitude over a compact support, i.e., the function $H(u,r)$ is given by (44).

Now, there is a problem with the set of coordinates we have been working with to derive Eqs. (58), (59) and (60). Indeed, as the wave reaches the observer, the latter will experience motion in the z direction [see e.g., Eq. (62) below]. To avoid this unphysical "frame dragging" effect, it is convenient to pick a particular coordinate system in which the observer and the source are at rest. Assuming a free-falling observer of mass M for which we neglect the motion in the radial direction, we have, according to Eqs.(14)

$$\dot{u}\dot{v} + H\dot{u}^2 = M^2, \tag{61}$$

and since Eqs. (42) tell us that u is proportional to the proper time $\tau = M\lambda$, we are free to choose $u = \tau$ so that Eq. (61) transforms into

$$\frac{dv}{du} = 1 - H. \tag{62}$$

Incidentally, this equation also tells us that $dz/du = -(1/2)H$ so the motion in the z direction after the wave has past is

$$\Delta z \approx -aV(\ell)/2, \tag{63}$$

where ℓ is the distance from the observer to the string. We see that setting

$$\tilde{v} = v + \int H du \quad \text{and} \quad \tilde{z} = \frac{1}{2}(\tilde{v} - u)$$

(u being unchanged in the new frame) implies

$$\frac{d\tilde{z}}{d\tau} = \frac{d\tilde{v}}{du} - 1 = \frac{dv}{du} + H - 1$$

which vanishes at the observer's and at the source's locations according to Eq. (62).

Dropping the tildes, we now have the following equation to solve numerically:

$$\ddot{t} = \ddot{z} = \frac{1}{2}\frac{\partial^2 P}{\partial r^2}\dot{r}^2 + \frac{1}{2}\frac{\partial P}{\partial r}\ddot{r}, \tag{64}$$

where

$$P \equiv \int H(u,r)du, \tag{65}$$

while Eqs. (58) and (59) still hold.

That the approximation of neglecting radial motion is valid can be checked by considering the actual radial displacement experienced by the observer. According to Eq. (58) with the previous choice $u = \tau$, and assuming the observer to be initially at rest, one has

$$\dot{r} = -\frac{1}{2}\int^u \frac{\partial H}{\partial r}\dot{u}du \simeq -\frac{1}{2}\frac{\partial V}{\partial r}\begin{cases} u\Theta(u) & \text{if } u \le a \\ a & \text{for } u \ge a \end{cases},$$

where we neglected variations of $V(r)$ with u, since this effect is of higher order. Thus, at most, the correction in r reads

$$r \sim \ell - 2G(U' - T')\frac{a^2}{\ell} - 4GI_z'^2 \ln(\ell/r_\sigma)\frac{a^2}{\ell} \sim \ell(1 - G\mu\frac{a^2}{\ell^2})$$

with $G\mu \sim 10^{-6}$ as discussed earlier. Therefore, since in practical application, one expects $a \ll \ell$, the motion in the radial direction can be confidently neglected.

We have investigated numerically the solutions of Eqs. (58), (59) and (64) for various values of the parameters describing the pulse, and the results are displayed on Figs. 3, 4 and 5. On theses figures, we have assumed a GUT string of core radius $r_\sigma = 10^{-60}$Mpc, and the internal string parameters have been enhanced to $4G(U' - T') = 4G\mu = 10^{-4}$ and $8GI_z^2 = 10^{-6}$ in order to magnify the corresponding lensing effect. The string is located at $\ell = 25$Mpc from the observer and $d = 100$Mpc from the source (see Fig. 2), and the duration of the pulse is $a = 10$Mpc. This particular example provides an illustration of a generic situation.

We are mainly interested by what the observer actually sees, that is, with the notation of Fig. 2, we want to know the variations of the angles Θ and ϕ with time, i.e., we calculate the values of $\Theta \equiv \arctan(\ell\dot{\theta}/\dot{r})|_{obs}$ and $\phi \equiv \pi/2 - \arctan(\sqrt{\dot{r}^2 + \ell^2\dot{\theta}^2}/\dot{z})|_{obs}$ as a function of the time of arrival of each light ray. The predicted observations are the following. Initially, the observer sees the usual double image (the two parallel straight lines on Fig. 4) until the wave reaches the first deflected ray. Then, the two images appear to move away from each other, while in the same time moving upwards (Fig. 3). After a time characteristic of the pulse duration (or of the impact parameter if this is larger than the pulse duration), the images begin to go down and eventually towards each other, and finally they reach a new position in the sky as the wave has passed. Put together, Figs. 3 and 4 yield Fig. 5 which shows the actual apparent trajectory of the source.

The source is seen to follow two open symmetric curves, i.e., the final location of the source in the sky is not the same as the initial one. This can be understood as follow: as the wave passes through the observer and the source, they both reach new positions in space, but because they were not initially at the same distance to the string (only case for which we expect closed curves at this level of approximation), they have not travelled the same distances in the z direction, the variation in z being $\Delta z \approx -aV(r)/2$ as discussed earlier. Assuming the source to be farther than the observer implies that the latter has travelled more than the former which is thus seen below its initial location. More precisely, the actual angle of observation will be $\phi_0 \sim a\alpha \ln(d/\ell)/(d+\ell)$, with α as defined previously on Eq. (54). In addition to this effect there is a frame rotation effect: since the observer moves in both radial and z directions, its frame is being rotated (parallel transported), so that he will see the source at an angle $\phi_{final} \sim \phi_0 + a\alpha/\ell$ because of this rotation. In practice, ϕ_{final} will be negligible due to the large denominators, and the curves will be almost closed.

Another possible way to measure the effect of the wave is through the redshift of the source: since each light ray experiences a different gravitational potential, one would expect to observe the source with a frequency that should vary while the wave passes. For the purpose of calculating this effect, let us define the redshift $\mathcal{Z}$ by the relation

$$\mathcal{Z} \equiv \frac{\nu_{emitted} - \nu_{observed}}{\nu_{emitted}} = \frac{u_\mu p^\mu\Big|_{emitted} - u_\mu p^\mu\Big|_{observed}}{u_\mu p^\mu\Big|_{emitted}}. \tag{66}$$

Here u_μ is the four-velocity of the observer or of the source, whereas p^μ is the four-momentum of the light ray. We have $u_\mu p^\mu = \dot{t} - \frac{1}{2}\dot{r}\int du \partial H/\partial r$. Fig. 6 shows this redshift as a function of the time of arrival of the light rays with, for comparison, the corresponding angles Θ and $\phi - \pi/2$, and assuming the same parameters (d, ℓ, a, U, T, I and r_σ) as on previous figures. The redshift (solid line) is seen to be strongly correlated with the deflection in the ϕ direction. It should be clear that simultaneous measurements of effective motion of double images (Figs. 3 to 5) together with the reshift variability shown on Fig. 6 would provide an unambiguous signature of the existence of cosmic strings endowed with superconducting currents.

The characteristic time scale during which the source is actually moving in the sky can be evaluated as roughly the wave duration if this one is large enough (see Fig. 7). However, one may expect astrophysical situations in which a is much smaller than any other length involved in the problem. In particular, for the idealized shock wave case of Eq. (47), the observation time should be of the order of the impact parameter $b \sim G\mu\ell$. In practice, the actual time during which the motion of the source takes place is thus

$$t_{motion} \sim \text{Max}\,[a, b],$$

which, in the case of a GUT string located at a few Mpc from us, would be of a few years. This can be seen more precisely on Fig. 7.b where the time variability of the angle ϕ is shown for various values of the wave duration a: on these curves, it is clear that the characteristic time in which the signal reaches its maximum is essentially independent of a, and in fact reflects only the distance of the string to the observer.

Finally, let us ask the question of the dependence of these results with the various geometric parameters d, ℓ and a. As long as $d \gg \ell$, the results do not depend strongly on d and we can take $d \to \infty$ as we did in the previous section. For fixed d and a, the effect decreases with increasing ℓ. Also displayed on Fig. 5 is the previously discussed result for $\ell = 25, 50$ and 100 Mpc, with a fixed value $a = 10$ Mpc of the wave duration. The effect decreases with increasing ℓ, although only by a factor of order 1. Since we are in the case in which the duration of the wave is much longer than the impact parameter $b \sim G\mu\ell$, the angular deflection is of order α in the Θ direction and of order $\alpha \ln(a/b)$ in the ϕ direction, as discussed at the end of the last section. If the source is much closer to the string than the observer is, i.e., in the limit $\ell \gg d$ the angular deflection would be suppressed by a geometrical factor of order d/ℓ.

CONCLUSIONS.

We have derived an exact solution of the Einstein's equations that describes the gravitational field surrounding a lightlike current carrying cosmic string. The current can be of arbitrary shape and time dependence. In the thin string limit, the exterior metric belongs to the general class of pp-wave solutions. This exterior metric can be matched with the solution

of the Witten model coupled to gravity in the core of the string. The resulting metric is just a Taub-Kerr-Shild generalization of the metric of a cosmic string with vanishing current.

An interesting feature of the solution is that both the function $A(r)$ characterizing the electromagnetic field and the function $H(u,r,\theta)$ describing the gravitational disturbance obey linear equations [see Eqs. (34) and (39)]. This fact can be understood heuristically as a consequence of the Lorentz invariance of the "seed" solution and the lightlike nature of the perturbation. Indeed, as mentioned before, under a Lorentz transformation of velocity v_z parallel to the string we have $H \to \kappa H$ and $A \to \kappa^{1/2} A$, where $\kappa = (1 - v_z)/(1 + v_z)$. Physically, this reflects the fact that the source is redshifted for $v_z > 0$. Therefore, by making v_z sufficiently close to 1, we can make H and A as small as we wish. Thus, it is not surprising that these quantities should obey linear equations. This also explains why the profile of the vortex and the metric functions $a(r)$ and $b(r)$ of section II are unchanged by introducing the lightlike current.

We calculated the motion of light rays for the case of a pulse current of finite duration a and constant amplitude characterized by the parameter α given in Eq. (54). In addition, we have the parameter $G\mu$ characterizing the mass scale of the cosmic string (typically we have $\alpha \lesssim G\mu \sim 10^{-6}$ for GUT strings), and the geometric parameters d and ℓ, which give the distance from the string to the source and the observer respectively. We restricted attention to the case when source and observer are colineal with the string and are initially in a plane orthogonal to the string axis, obtaining the following results.

The initial image is the usual double image before the lightlike current passes, and each one of the double images describes an (almost) closed loop in the sky as the wave passes. For $d \gg \ell$, the angular size of the loop is of order α in the direction perpendicular to the string and of order $\alpha \ln(a/b)$ in the direction parallel to the string. Here $b \sim G\mu\ell$ is the typical impact parameter and we are assuming $a \gtrsim b$. We have also seen that the images experience a blueshift and a redshift which is strongly correlated with the apparent motion in the direction parallel to the string, and which is also of order $\alpha \ln(a/b)$. For sources very close to the string, such that $d \ll \ell$, the magnitude of the angular deflection is decreased by a geometrical factor roughly of order d/ℓ, although for obvious reasons, the redshift effect is unaffected by this factor.

These results only apply if the duration of the pulse is large compared with the impact parameter, that is, if $a \gg \text{Max}\,[G\mu\ell, (a\alpha\ell)^{1/2}]$. In the opposite limit, the pulse can be approximated by a delta function shock wave. Sending the source to infinite distance, the apparent motion of the double images can be calculated analytically. When the effect of the deficit angle is dominant with respect to the effect of the passing wave (more precisely, when $a\alpha \ll \ell(G\mu)^2$), the double images describe circular motion in the sky, of angular radius given by $a\alpha/(16\pi G\mu\ell)$.

The time scale needed for the image to go around the curve reflects the impact parameter as well as the duration of the pulse. The effect grows to an angular size of order α in a characteristic time of order $\ell G\mu$. If $a \gg \ell G\mu$ the effect persist for a time of order a, and then decreases again in a time scale of order $\ell G\mu$. For instance, a GUT string endowed with a short pulse of GUT lightlike current located at a few Mpc from us, would deflect an infinitely far source in a few years time, and along a curve of angular aperture of the order 10^{-6}rad, a phenomenon which should be within the reach of observational detection limits. In fact, an observation of this kind requires quite particular conditions due to the

scarcity in the number of strings that we expect out to such distances [17]. Nonetheless, we believe that the low observational probability is largely compensated by the uniqueness of the signal: such an observation would provide a clear proof of the existence of cosmic strings, and show that they are of the superconducting current-carrying kind.

Let us estimate this observation probability more precisely by proposing a mechanism through which lightlike currents can build in cosmic string, namely the interaction of such a string with a galaxy, or any region having an electromagnetic field. In the original Witten model [2], the currents built in such a way are small compared with the GUT scale. However, in models with spontaneous current generation [18] one may be able to build up saturation currents in such a way. If this interaction region is finite, and if the string can be approximated as straight across it, then a current will be induced [2], which can be represented by means of a phase function ζ along the string as $J^\mu \propto (-\zeta_{,t}, 0, 0, \zeta_{,z})$. Now, away from the interaction region, where the external electromagnetic field vanishes, the general solution for ζ (to first order in e) is [19]

$$\zeta = \zeta_1(z - t) + \zeta_2(z + t),$$

and must be constant if no interaction takes place. So, far from the interaction region, one has two waves of null current, one travelling in the positive z direction and the other in the negative z direction, both at the speed of light, and with a duration of the order of the size of the interaction region. As mentioned before, the null current does not affect the profile of the string, and so there is in principle no limit to the amount of lightlike current that a string can carry. However, since in the interaction region the current will be typically spacelike or timelike, and such currents achieve saturation, it is clear that we cannot build up lightlike currents whose strength α is much larger than $G\mu$.

We see therefore that a requirement for observing a string's lightlike current pulse is that there exist string loops crossing galaxies say, and whose size would exceed that of the interaction region. Now we know that the string loop distribution is such that [17]

$$N \sim \frac{\nu \ell^3}{R t^2},$$

for the number N of loops of size R located in a sphere of radius ℓ. Here, t is the age of Universe (10^{10} yr) and $\nu \approx 0.01$ is a parameter obtained by numerical simulations. The coherence length of a string due to wiggles is $\Gamma G\mu t$, where $\Gamma \approx 100$ [20] and so we want R to be at least this value. Moreover, the impact parameter $b \sim G\mu\ell$ being the typical duration of the effective motion of the source, we require it to be $b \lesssim 100$ yr so as to be able to actually measure the effect. This means that we require $\ell \lesssim 100$ yr $/G\mu$. Thus, the expected number of events can be estimated as

$$N \approx \frac{\nu}{\Gamma} \frac{10^{-24}}{(G\mu)^4} \approx \frac{10^{-28}}{(G\mu)^4},$$

which can be quite small for GUT strings, but otherwise increases tremendously for lower mass strings, and is of order one for $G\mu \sim 10^{-7}$. This shows that lighter strings would perhaps produce a detectable (at the level of arc seconds) double image moving according to our calculations with a correspondingly varying redshift.

It should be mentioned that this work has been involved with a lightlike current wave separating two regions of the string where there was no current. Lightlike currents may also exist at the boundary between spacelike and timelike currents. The time evolution of such lightlike currents and their gravitational effect is left for further research. We should also say that for cosmic strings without currents, time dependent exact solutions have been studied in the past, e.g. in Refs. [16,21]. In these cases, the positions of the double images are also time dependent, although the actual apparent motion is qualitatively different from our case.

Finally, one may consider solutions describing the collision of two pulses of lightlike current travelling in opposite directions. Before the collision the metric is a trivial superposition of two solutions like the one we have considered in this paper, but during and after the collision the solution will contain interesting non-linear effects. In particular, since the metric in the impulsive case resembles very much the Aichelburg-Sexl metric [9] (representing the gravitational field of an ultrarelativistic body), and it is known that the head on collision of two such shock waves generates trapped surfaces [22], it would be interesting to study the possibility that the collision of such lightlike currents would produce black holes.

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FIGURES

FIG. 1. Light deflection in the transverse plane in the shock wave case with the source at infinity.

FIG. 2. The configuration under study as a lightlike-current wave of duration a passes along a string located at a distance ℓ of the observer and d of the source.

FIG. 3. Azimutal angle ϕ (see Fig. 2) of the source as seen from the observer as a function of the time when the light ray reaches the observer. The string parameters [i.e., the parameters appearing in the potential (10)] have been set to $8GI_z^2 = 10^{-6}$, $4G(U' - T') = 4G\mu = 10^{-4}$.

FIG. 4. Polar angle Θ (see Fig. 2) of the source as seen from the observer as a function of the light ray arrival time with the same string's parameters as on the previous figure. The presence of two similar curve simply reflects the double image expected in the non-carrying case.

FIG. 5. Effective observation of the source in the $\varphi - \Theta$ plane (see Fig. 2). Both images in the positive and negative Θ directions behave symmetrically, and the source is seen as a function of time to move upwards at first, then down and up again until it reaches a final location slightly below its initial location. This figure has been obtained with a distance from the source to the string $d = 100$ Mpc with a wave duration $a = 10$ Mpc, and the distance from the observer to the string takes the values $\ell = 25$ Mpc (full lines), $\ell = 50$ Mpc (dashed lines), and $\ell = 100$ Mpc (dotted lines).

FIG. 6. Redshift of the source (full line) as a function of the time of arrival of the light rays. The dashed line represents the angle Θ and the dotted line the angle $\phi - \pi/2$.

FIG. 7. Influence of the wave duration a on the resulting observation. (a), the angle Θ versus time, and (b), the azimuthal angle ϕ also versus time; this represents various values for a, namely $a = 1$ Mpc (full lines), $a = 5$ Mpc (dashed lines), and $a = 10$ Mpc (dotted lines). It can be seen that although variations in the Θ direction are strictly correlated with the wave duration, the same is not true in the ϕ direction for which the rising time is in fact comparable in all cases, reflecting basically the value of the impact parameter as discussed in the last section.

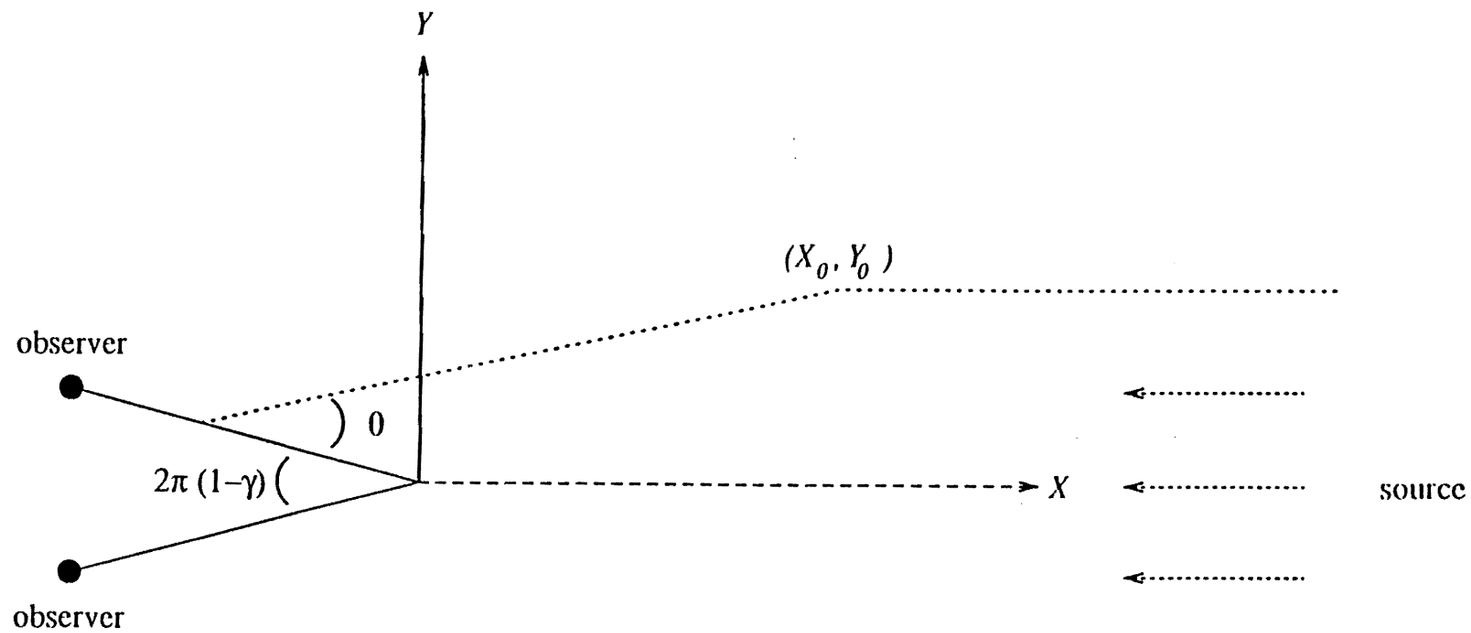


Figure 1

Figure 2

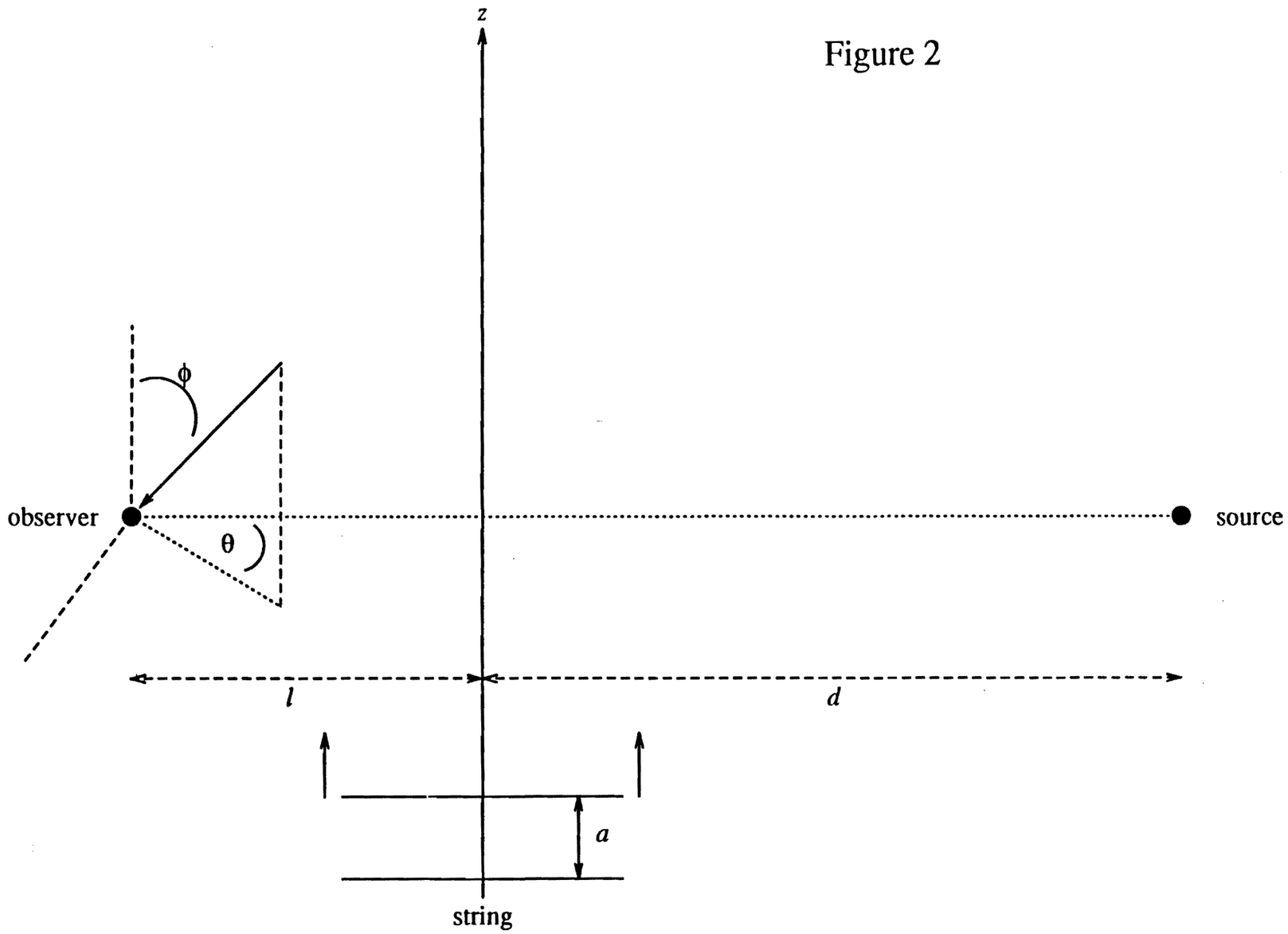
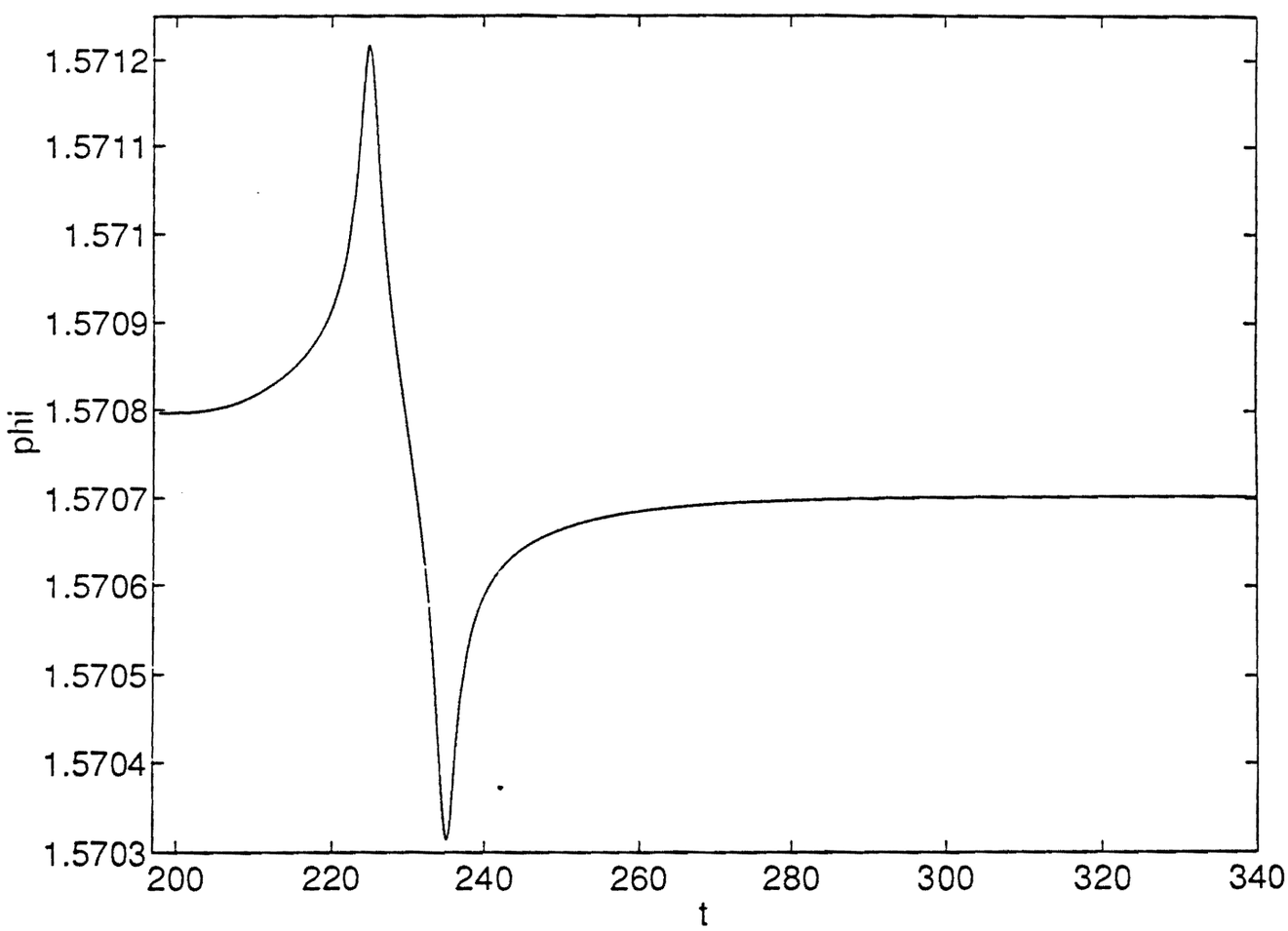


Figure 3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1.5711
1.571
1.5709
1.5708
1.5707
1.5706
1.5705
1.5704
1.5703
phi
200
220
240
260
280
300
320
340
t

Figure 4

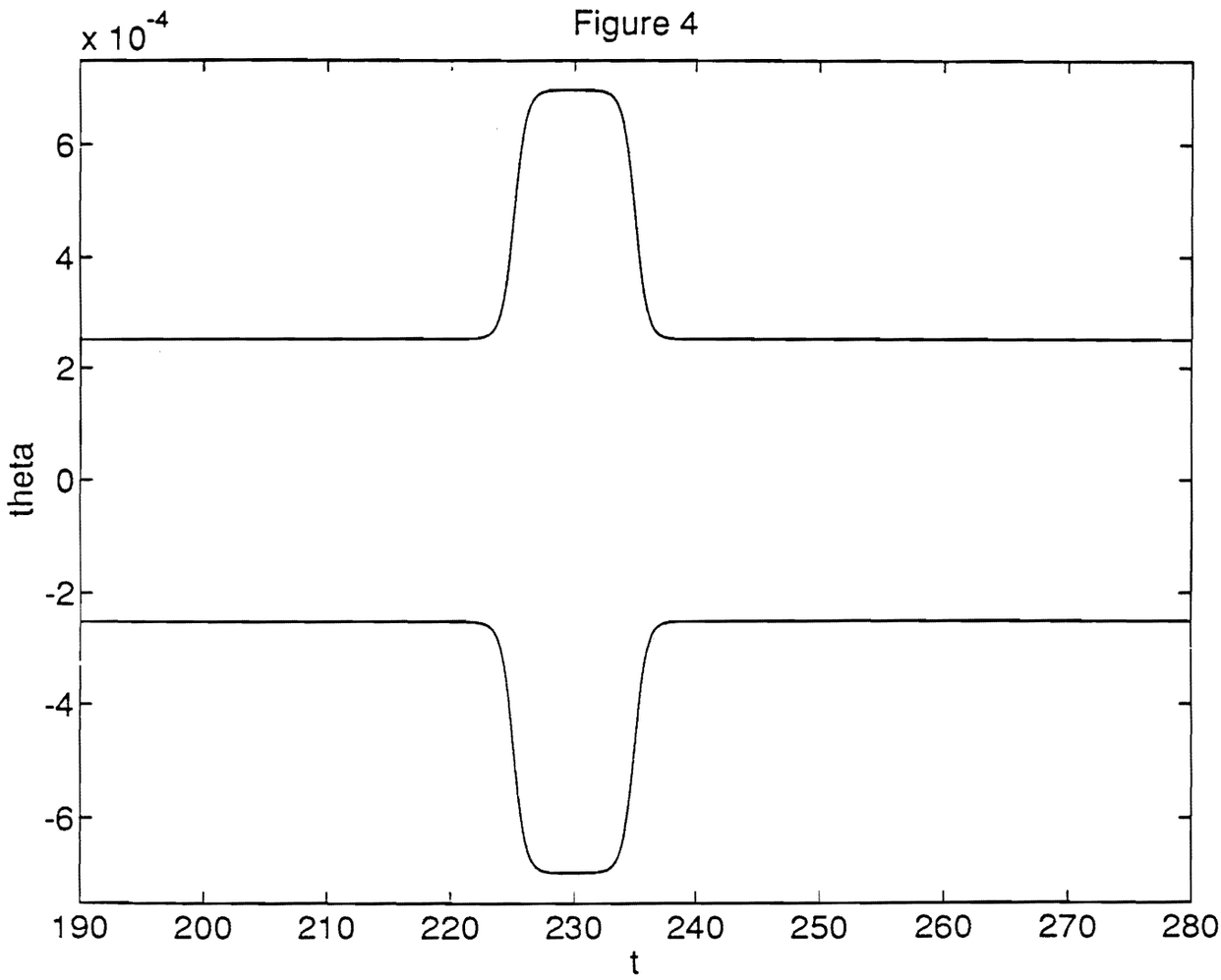
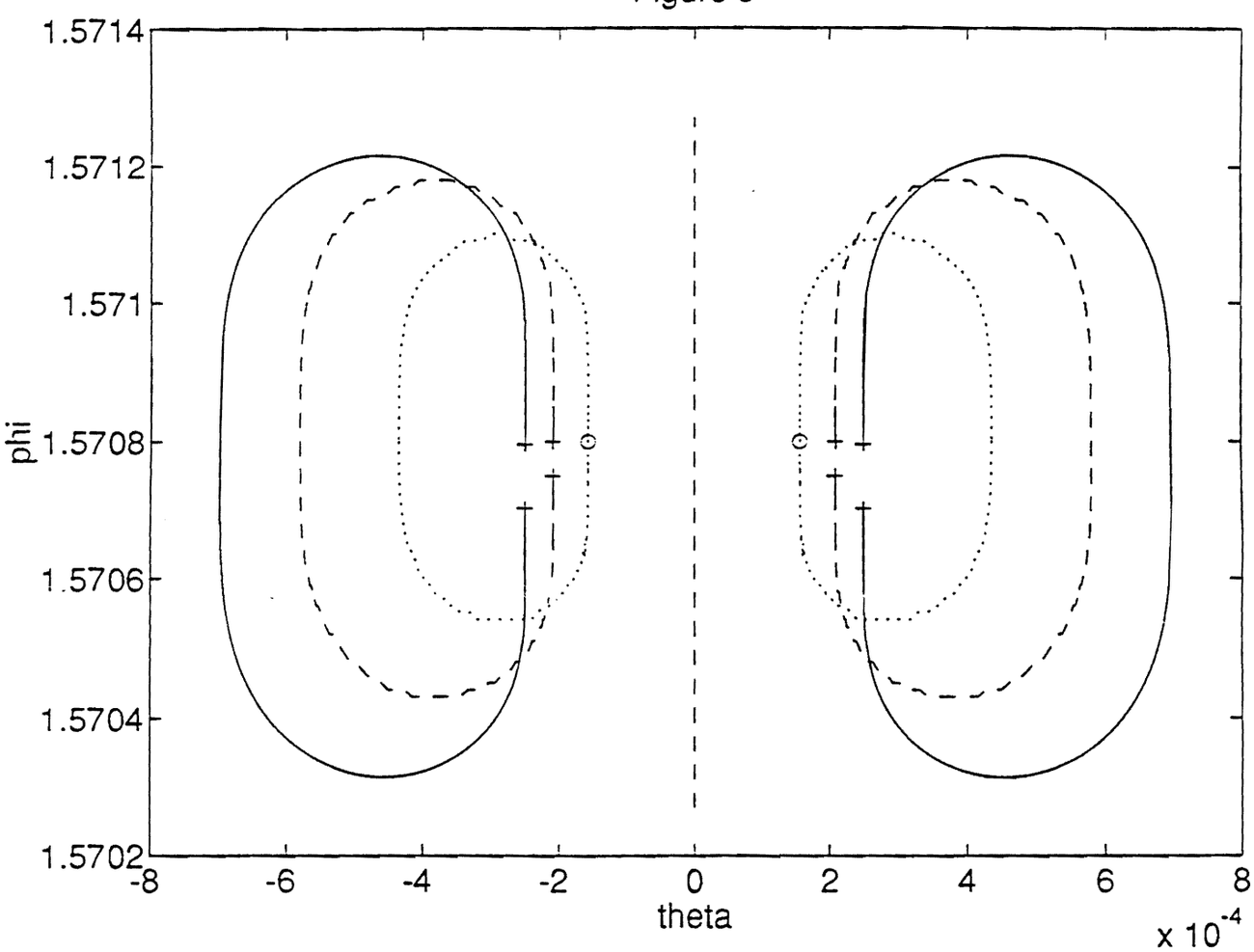


Figure 5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1.571
1.5708
1.5706
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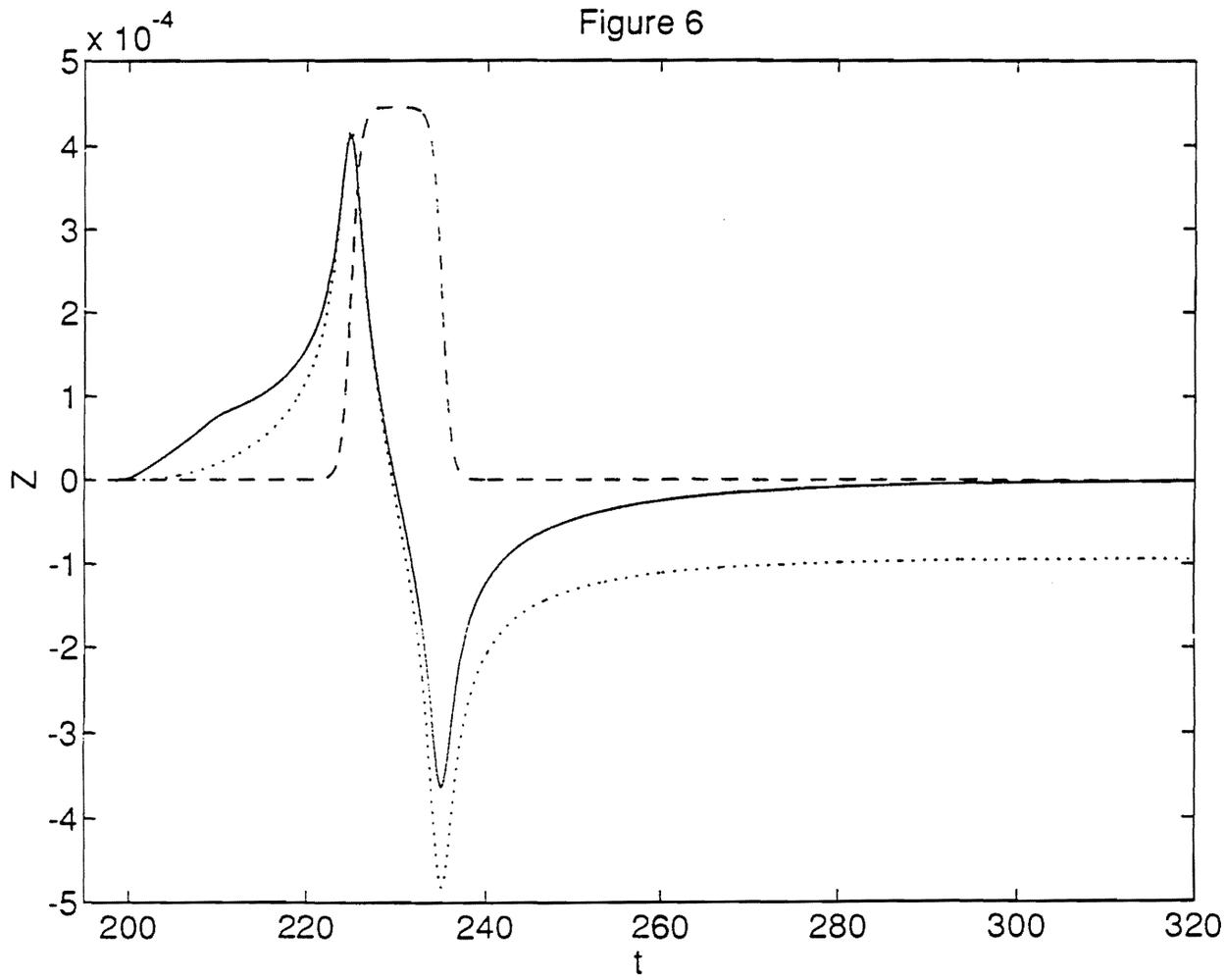
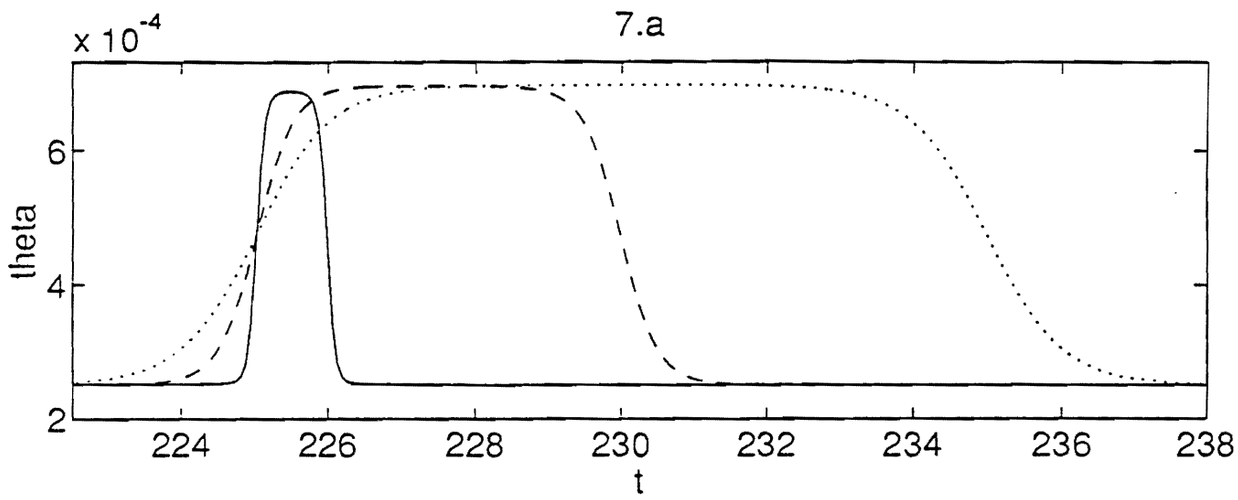
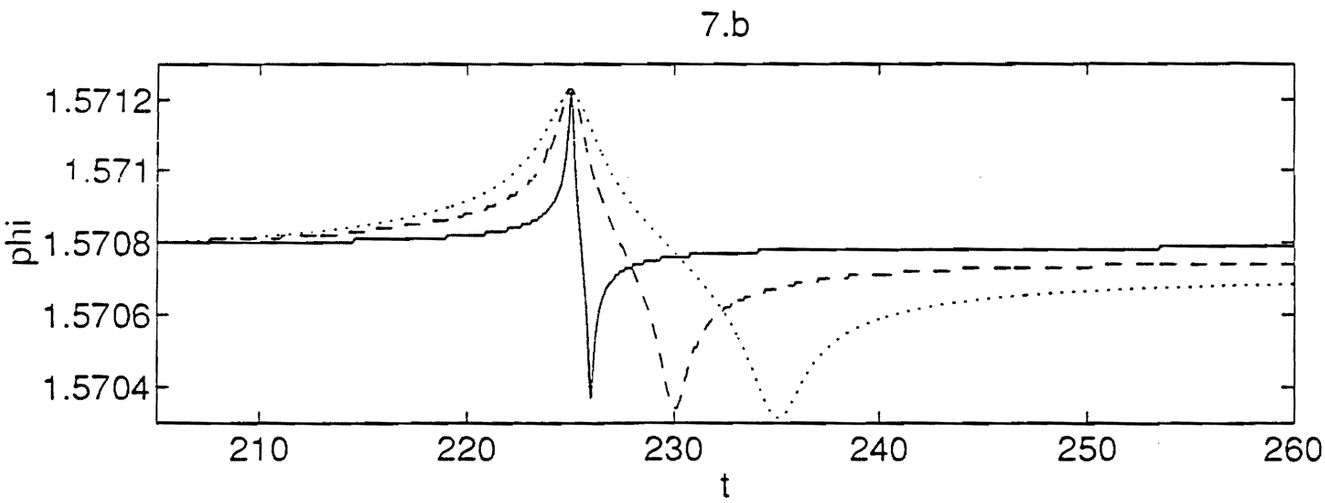
1.5702
phi
-8
-6
-4
-2
0
2
4
6
8
theta
x 10^-4

Figure 6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