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COMMENTS ON THE INTERNAL STRUCTURE OF COSMIC STRINGS

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Abstract

The formation of cosmic strings requires a symmetry-breaking phase transition which can be first or second order. Assuming the underlying microscopic theory to be the abelian Higgs-Kibble model, the influence of the order of the phase transition and the dependency of the energy per unit length with the winding number are explicitely calculated.

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It has long been known [1] that spontaneous symmetry breaking in the early Universe can lead to the formation of topological defects, and in particular cosmic strings [2]. The general cosmological interest in these objects lies in the fact that they can be responsible for large scale structure formation [3], the observed [4] anisotropies in the microwave background [5], and their ability to deflect light [2,6] in an unambiguous way should provide in principle a clear astrophysical signature. These defects are topologically stable because the phase of a scalar field (the order parameter in condensed-matter theory [7]) whose vacuum expectation value (VEV) does not vanish (a Higgs field) winds around a line an integer number of time. This integer, appropriately called the winding number, is usually assumed to be unity.

Similarly to the superconductor case, there are basically two types of cosmic strings, namely those for which the interaction between vortices is repulsive (type II) or attractive (type I), i.e., regardless of a possible relative velocity of the two colliding string and of the geometry of the interaction, the total energy per unit length U of two strings having unit winding number n, i.e., $2U(n=1)$, is less (type II) or greater (type I) than the energy per unit length of a string having a winding number $n=2$.

As for most phase transitions, those leading to the formation of cosmic strings can be of first or second order. The main difference between these two types of phase transition lies in the effective potential for the self-interaction term of the Higgs field, at the one loop order, even in the limit of vanishing temperature. Let us consider the abelian Higgs-kibble model [1] whose dynamics is described by the bare Lagrangian density

$$\mathcal{L}_{bare} = -\frac{1}{2}(D_\mu\Phi)^*(D^\mu\Phi) - \frac{1}{16\pi}H_{\mu\nu}H^{\mu\nu} - V(\Phi), \tag{1}$$

$$V_{bare}(\Phi) = \frac{\lambda_\varphi}{8}(|\Phi|^2-\eta^2)^2 \tag{2}$$

where $D_\mu \equiv \partial_\mu + iqB_\mu$ and $H_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$. A second order phase transition is characterised by $q^4 \ll \lambda_\varphi$, and the effective potential, at the one loop order and zero temperature is (formally) exactly the same as in Eq. (2) with renormalised parameters. In the case $q^4 \gtrsim \lambda_\varphi$, the phase transition is first order and the effective potential transforms into (again at the one loop order) [8]:

$$V_{eff}(\Phi) = \frac{\lambda_\varphi}{8}(|\Phi|^2-\eta^2)^2 + \frac{\lambda_\varphi\beta}{4}|\Phi|^4\left(\ln\frac{|\Phi|^2}{M^2} - \frac{25}{6}\right), \tag{3}$$

where the constant β is related to λ_φ and q by $16\pi^2\beta = 5\lambda_\varphi/2 + 48\pi^2q^4/\lambda_\varphi$.

In Eq. (3), the renormalisation mass M can be chosen such that $\delta V_{eff}/\delta\Phi = 0$ and $V_{eff} = 0$ for $|\Phi| = \langle|\Phi|\rangle$ =Cte, which is equivalent to saying that we require the energy per unit string length, i.e., the energy density integrated over a string section $U = \int d^2x\, T^{tt}$, to converge. These conditions give the Higgs field VEV (which is modified by the inclusion of the logarithmic self-interaction term in the potential) and the renormalisation mass M, namely

$$\langle|\Phi|^2\rangle = \frac{\eta^2}{2\beta}(\sqrt{1+4\beta}-1), \tag{4}$$

$$M^2 = \frac{\eta^2}{2\beta}(\sqrt{1+4\beta}-1)\exp\left[\frac{1}{2\beta}-\frac{11}{3}+\frac{1}{1-\sqrt{1+4\beta}}\right]. \tag{5}$$

It should be noted that the second-order phase transition (SOPT) limit, which is obtained by $\beta \to 0$, gives back the usual value $\langle|\Phi|^2\rangle = \eta^2$.

We now consider strings, i.e., linear topological defects present at the phase transition occuring in the model (1), whose structure we investigate through the Nielsen-Olesen vortex solution [9]. This means that we set

$$\Phi = \varphi(r)e^{in\theta}, \tag{6}$$

in polar coordinates, the vortex defect being aligned along the z axis, n being the winding number. It may be seen [10] that, due to the form of the solution (6), only B_θ is nonzero, so that any solution is given by the field functions $\varphi(r)$ and $Q(r) \equiv (n + qB_\theta)/n$, satisfying

$$\frac{d^2\varphi}{dr^2} + \frac{1}{r}\frac{d\varphi}{dr} = n^2\frac{\varphi Q^2}{r^2} + \frac{1}{2}\lambda_\varphi\varphi(\varphi^2-\eta^2) + \frac{1}{2}\lambda_\varphi\beta\varphi^3\left(\ln\frac{\varphi^4}{M^4} - \frac{22}{3}\right), \tag{7}$$

$$\frac{d^2Q}{dr^2} - \frac{1}{r}\frac{dQ}{dr} = 4\pi q^2 Q\varphi^2, \tag{8}$$

subject to the boundary conditions $\varphi(0) = 0$, $\varphi(\infty) = \sqrt{\langle|\Phi|^2\rangle}$, $Q(0) = 1$ and $dQ/dr(0) = 0$. In order to investigate the solutions of Eqs. (7) and (8), we rescale the Higgs field, the radial coordinate and the parameter q through

$$\varphi(r) = \sqrt{\langle|\Phi|^2\rangle}X(\rho) \quad , \quad \rho = r\sqrt{\lambda_\varphi}\eta \quad , \quad \alpha = \frac{2\pi q^2}{\lambda_\varphi\beta}(\sqrt{1+4\beta}-1), \tag{9}$$

and we now have to solve the set of equations

$$X'' + \frac{1}{\rho}X' = \frac{n^2}{\rho^2}XQ^2 + \frac{1}{2}X(X^2-1) + \frac{1}{4}X^3(\sqrt{1+4\beta}-1)\ln X^4, \tag{10}$$

$$Q'' - \frac{1}{\rho}Q' = \alpha Q X^2, \tag{11}$$

(a prime meaning differentiation to the rescaled radial variable ρ). Fig. 1 shows some typical solutions of Eqs. (10) and (11) obtained by means of a successive over relaxation method [11] for various values of the parameters α and β and for unit winding number (variations on the winding number yield similar conclusions). It is clear from these plots that the effect of the parameter β is to make the vortex thinner, but that the characteristic features of the microscopic structure are otherwise unaltered. As a result, Kibble vortices generated at a first or second order phase transition will behave in essentially the same way (although the initial network may be slightly different [12] because of differences in the dynamics of the phase transition).

The effect of the first order transition parameter β may be more clearly exhibited by the explicit calculation of the energy per unit string length U, given by

$$U \equiv \int r\,dr\,d\theta T^{tt} = \pi\langle|\Phi|^2\rangle\int_0^\infty \rho\,d\rho\left[X'^2 + n^2\left(\frac{Q'^2}{\alpha\rho^2} + \frac{X^2Q^2}{\rho^2}\right) + \frac{\beta(1-X^4)}{2(\sqrt{1+4\beta}-1)}\right.$$
$$\left. + \frac{1}{2}X^2(X^2-1) + \frac{1}{8}(\sqrt{1+4\beta}-1)X^4\ln X^4\right]. \tag{12}$$

Once the solutions X and Q are known, the effect of the parameter β can be computed: on Fig. 2 the energy per unit length U/π is shown as a function of β for $\alpha = 1$ (a case for which we know [7] that $U = \pi\langle|\Phi|^2\rangle$). It is to be remarked that although the parameter β takes values in a wide range of 7 orders of magnitude, the resulting energy per unit length varies by a factor hardly exceeding 2. It seems thus safe to assume, for macroscopic purposes, that the phase transition is second order and to allow small variations (by numerical factors of order unity) in the various quantities involved. We believe that this result should apply also in the case of a superconducting cosmic strings [10,13], although no similar study has yet been performed.

We shall now, according to our previous result, be interested in the special case $\beta = 0$, i.e., the SOPT limit in which we wish to exhibit explicitely the dependence of the energy per unit length U with the winding number n. The reason for this calculation is that once the underlying theory has been given, i.e., once the parameter α has been assigned a numerical value, e.g., by some approximation [1] of a string-forming grand unified theory (GUT), or by some extension of the standard electroweak model [14], then, when the string forms at the phase transition, n is the only arbitrary parameter in the simple model of Eq. (1). It has usually been assumed that when the string network forms, then two different possibilities may take place as far as the vortex defects are concerned, i.e., the vacuum is of type I or type II. In other words, either N strings with unit winding number are energetically favored, or one string with $n = N$. In the latter case, all strings would tend to collapse into one. Explicitely, the energy per unit length per winding number as function of the winding number would be such that the state with $n = 1$ would represent a global minimum (type II) or a global maximum (type I).

Fig. 3 shows the actual behaviour of the energy per unit length with the winding number, rescaled to its values for $n = 1$, namely U_1. As expected, we see that for $\alpha > 1$, the vacuum is in fact of type I, with $U_n < nU_1$, whereas the type II vacuum having $U_n > nU_1$ occurs for $\alpha < 1$. It should be remarked that for $\alpha = 1$, as may be shown analytically, one has $U_n = n\pi\langle|\Phi|^2\rangle$, a relation which has been used as a precision check of our numerical calculations.

For the type I strings, there exist a velocity threshold below which colliding strings can anti-peel: in the simplest configuration where two parallel strings, having winding numbers n_0 and n_v, collide with a relative velocity v (assumed small enough to neglect relativistic correction), the total energy per unit length, calculated in a plane perpendicular to the strings axes, is (in the rest frame of the $n = n_0$ string)

$$U_T = U(n_0) + U(n_v)[1 + \frac{v^2}{2}], \tag{13}$$

and is to be compared to the static energy of a $n = n_0 + n_v$ state. The relative velocity provides a kinetic energy which must not balance the stability gap $\Delta = U(n_0) + U(n_v) - U(n_0 + n_v) > 0$. The anti-peeling condition can then be rephrased into

$$v \le v_{threshold} = \left\{\frac{2}{U(n_v)}[U(n_0) + U(n_v) - U(n_0+n_v)]\right\}^{1/2}. \tag{14}$$

It has often been assumed in previous litterature that the vacuum in which the strings evolve is a type II vacuum, so that only strings with unit winding number exist. It was also

shown that since the initial distribution of strings in a network is relativistic, one expects large-winding-number strings to peel through the thermal collisions, so that, again, the variation of the macroscopic quantities with the winding number seem unimportant. However, for the type I strings, it is possible that after a few expansion times during which the strings in the network will decelerate, they eventually reach a relative velocity below the threshold velocity and then would tend to anti-peel, and merge. Before any definite conclusion can be stated, more work has to be done, including in particular the investigation of the condensation dynamics, i.e., the mechanism through which two colliding cosmic string may "anti-peel" to form a larger winding number string. This is closely related to the mechanism of intercommutation which seems to have been studied mainly for low winding numbers [15]. Knowledge on this dynamics as well as on the string-generating phase transition itself would allow a more precise calculation of the probability that high winding number strings form in the early Universe, and eventually estimate their cosmological consequences.

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REFERENCES

[1] T.W.B. Kibble, J. Math. Phys. **A 9**, 1387 (1976), Phys. Rep. **67**, 183 (1980)
[2] A. Vilenkin, in *300 years of Gravitation*, Edited by S. W. Hawking and W. Israel, (Cambridge University Press, Cambridge, England, 1987)
[3] R. H. Brandenberger, preprint BROWN-HET-906
[4] J. F. Smoot *et al.*, Astrophys. J. **396**, L1 (1992)
[5] T. Hara, P. Mähönen, S. Miyoshi, Astrophys. J. **414**, 421 (1993) and references therein; D. Coulson, P. Ferreira, P. Graham, N. Turok, preprint HUP-TH-93/1393
[6] P. Peter, Class. Quantum Grav. **11**, 131 (1994); P. Peter, D. Puy, Phys. Rev. D **48**, 5546(1993) and references therein
[7] L. Jacobs, C. Rebbi, Phys. Rev. B **19**, 4486 (1979)
[8] L. Dolan, R. Jackiw, Phys. Rev. D **9**, 3320 (1974); S. Weinberg, Phys. Rev. D **9**, 3357 (1974); D. Bailin, A. Love, *Introduction to gauge field theory*, Adam Hilger Ed. (1986)
[9] H. B. Nielsen, P. Olesen, Nucl. Phys. B **61**, 45 (1973)
[10] A. Babul, T. Piran, D. N. Spergel, Phys. Lett. B **202**, 307 (1988); P. Peter, Phys. Rev. D **45**, 1091 (1992); *ibid* **46**, 3335 (1992); *ibid* **47**, 3169 (1993)
[11] S. L. Adler, T. Piran, Rev. Mod. Phys. **56**, 1 (1984)
[12] M.Hindmarsh, A.-C. Davis, R. H. Brandenberger, preprint DAMTP-93-13, BROWN-HET-902
[13] E. Witten, Nucl. Phys. B **249**, 557 (1985)
[14] P. Peter, Phys. Rev. D **46**, 3322 (1992) and references therein; W. B. Perkins, A.-C. Davis, Nucl. Phys. B **406**, 377 (1993)
[15] P. Laguna, R. Matzner, Phys. Rev. D **41**, 1751 (1990)

FIGURES

FIG. 1. Field configurations as functions of the free parameters α and β. The full line represents the case $\alpha = 10^{-2}$, $\beta = 10^{-2}$, (with $U/\pi = 2.54754$), the dashed line stands for the case $\alpha = 10^{-2}$, $\beta = 1$, (and $U/\pi = 2.83861$), the dotted line is $\alpha = 1$, $\beta = 1$, ($U/\pi = 1.14358$), and the dot-dashed line is $\alpha = 1$, $\beta = 10^{-2}$, ($U/\pi = 1.00282$). Plots -a- and -b- represent the Higgs field X, whereas plots -c- and -d- show the gauge vector field Q.

FIG. 2. Variations of the energy per unit length U/π as function of β for $\alpha = 1$. The limit $\beta \to 0$ gives back the SOPT result $U = \pi$ (corresponding to a regular Ginzburg-Landau vortex).

FIG. 3. Plot of the rescaled energy per unit length $U(n)/(nU_1)$ as function of α and the winding number n in the SOPT limit $\beta = 0$.

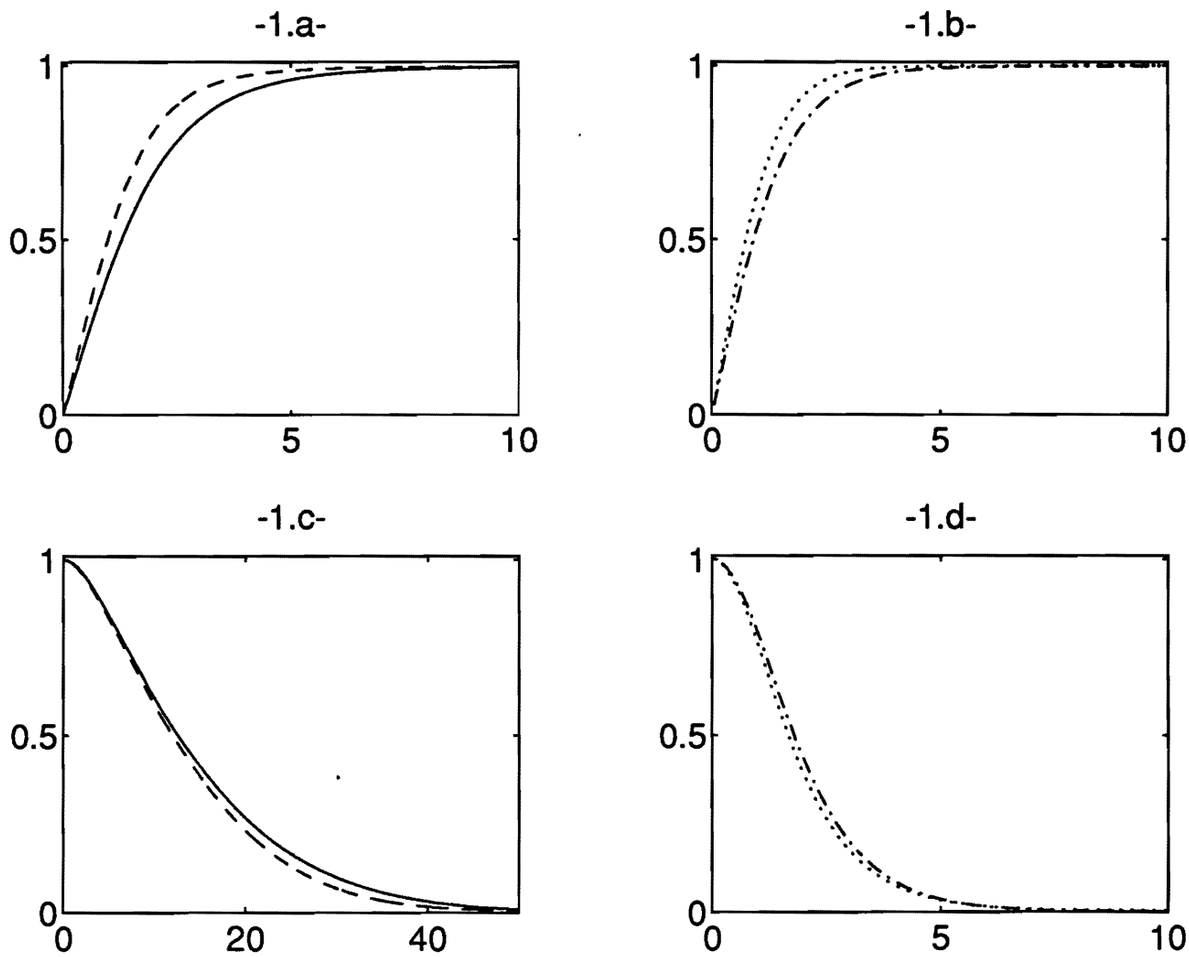
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Figure 2

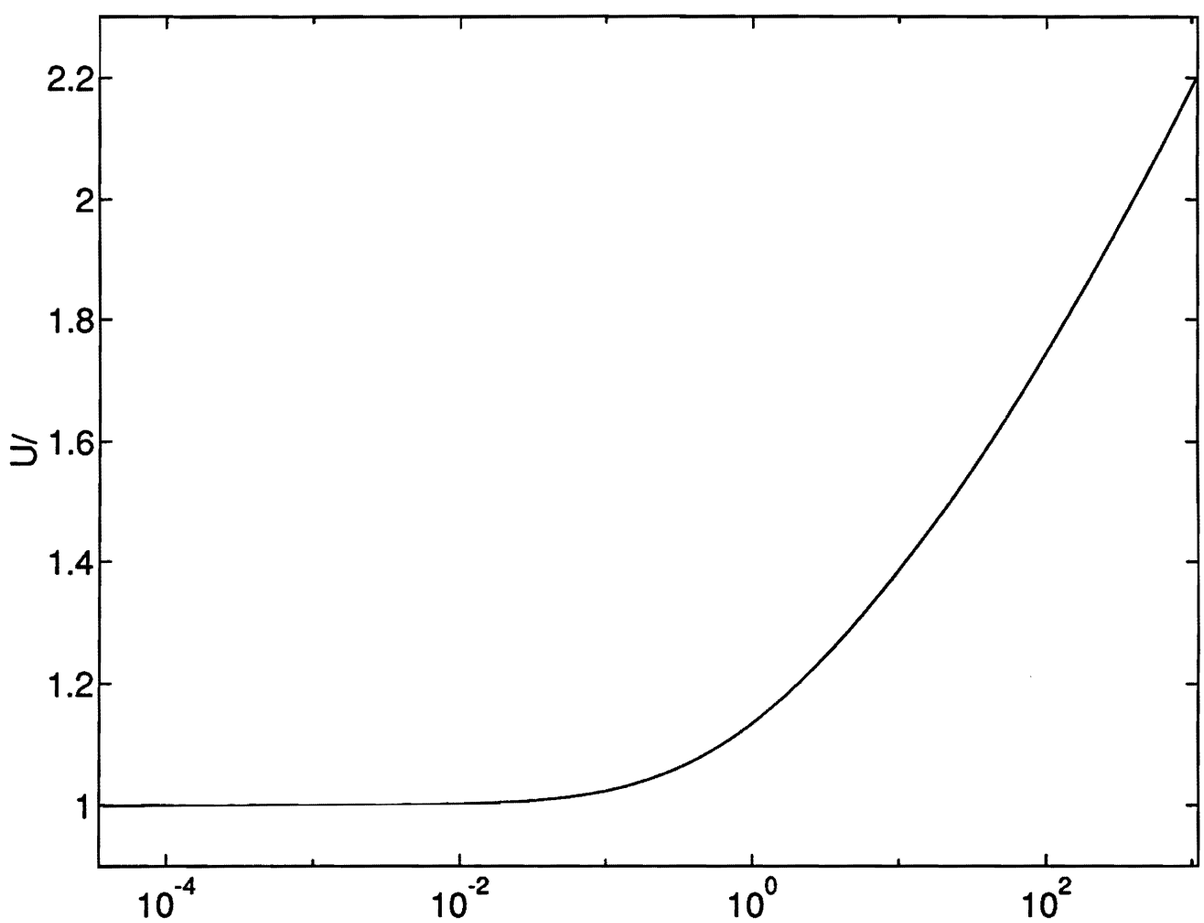
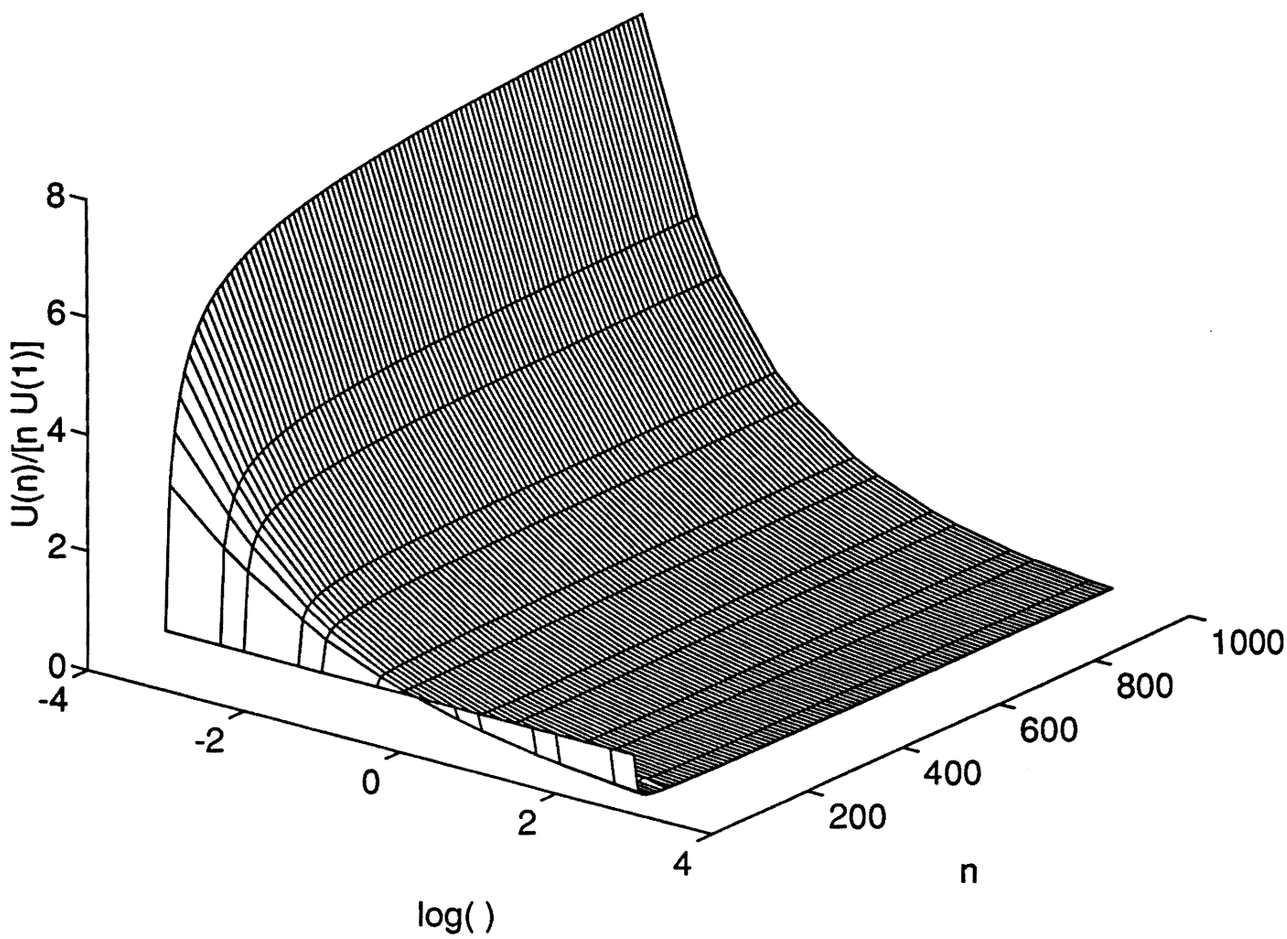
Figure 3.