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STATUS OF INFLATIONARY SCENARIO AFTER DETECTION OF COSMIC MICROWAVE BACKGROUND ANISOTROPY

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ABSTRACT

The recent detection of the microwave background anisotropy by the Cosmic Background Explorer Satellite (COBE) is in accord with the the inflationary scenario. However, there are many inflationary models and the goal now is to to resolve which is the correct one. The most promising chaotic inflationary models are compared and their impact on structure formation are described. Surprisingly, the contribution of gravitational radiation to $\Delta T/T$ can dominate over the usual scalar perturbations for power-law inflation models. In addition, it is shown how inflation may be incorporated into a particle physics theory. Strategies for observing new physics at the highest energy scales $10^{14} - 10^{19}$GeV are discussed.

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1. INTRODUCTION

After a decade of intensive study, the inflationary scenario[1] remains the simplest and most promising mechanism for generating fluctuations for structure formation.

On the observational side, several groups utilizing the IRAS (Infrared Astronomical Satellite) data base find that it is consistent to assume that the Universe is at critical density, $\Omega \equiv \rho/\rho_{crit} \sim 1$, which is the strongest evidence for cosmological inflation. Ω can be measured by comparing peculiar velocity measurements with the observed distribution of matter in the Universe. Frenk and collaborators use the QDOT redshift catalogue[2] to show that $\Omega^{0.6}/b_{IRAS} = 0.86 \pm 0.15$. On smaller distance scales, Dekel[3] finds that $\Omega^{0.6}/b_{IRAS} \sim 1.35 \pm 0.25$. Kaiser and Lahav[4] claim that the IRAS galaxy dipole together with the large scale streaming velocity imply that $\Omega^{0.6}/b_{IRAS} \sim 0.6$. If the IRAS biasing parameter $b_{IRAS} \gtrsim 1$, then $\Omega \sim 1$. As was discussed by Lynden-Bell in this meeting, one problem that remains is why dynamical estimates from optical surveys yield a lower value of $\Omega \sim 0.2$.[5,4] It is important to resolve this point, but even then this value is close to unity. (The discrepancy may be the result of systematic errors. For example, optical surveys suffer from dust obscuration in the plane of the Galaxy, and they also combine data from different telescopes in the northern and southern hemispheres.) The argument for $\Omega = 1$ is more persuasive if one notes that Ω is a very unstable parameter in the radiation and matter dominated epochs. If it deviates slightly from unity, it quickly moves to zero or infinity. The fact that Ω now is close to unity is a strong indicator that in the past it was unity to very high accuracy.

The detection of the microwave background anisotropy by the Differential Microwave Radiometers (DMR) on the Cosmic Explorer Satellite (COBE)[6] is in accord with inflation because its determination of the spectral index $n = 1.1 \pm 0.5$ is consistent with the simplest inflation models where $n < 1$. Its measurement of the rms temperature variation $\sigma_{sky}(10^0) = 30 \pm 5\mu K$ on the scale of the 10^0 is almost exactly that predicted by Peebles[7] in 1982 using the Cold Dark Matter model with a flat Zeldovich spectrum (see also Bond with Efstathiou[8] and Vitorrio with Silk[9]). The beauty of the COBE results are that its error bars for $\sigma_{sky}(10^0)$ are the order of ~15%, whereas galaxy clustering data gives results with errors greater than 30%. (For example, the determination of the Hubble parameter is uncertain to a factor of two.) With more data, one can envisage discriminating tests of the numerous inflationary models. In addition, COBE can probe new physics at the highest imaginable energy scales $10^{14} - 10^{19}$GeV.

After Guth's[1] initial paper on the subject, the most important theoretical advances have been:

(1) Proposal of the chaotic inflationary scenario.[10] In these models proposed by Linde, a very weakly coupled scalar field begins with values greater than the

Planck scale m_P. Here we have our first clue of the enormous energy scales being probed. New inflation[11] actually was the first viable inflationary model but it fell into disfavor because if required a very special potential— a Coleman-Weinberg potential— with initial values of the scalar field much smaller than the GUT scale. In addition, if the scalar field was identified as the GUT Higgs, the temperature fluctuations $\Delta T/T \sim 1$ were much too large.

(2) Calculation of density perturbations from quantum fluctuations. [12–15] In order to agree with the measurement of $\Delta T/T$, one finds that the Hubble parameter was the order of $H \sim 10^{-5} m_P$ when comoving scales of observational interest left the Hubble radius during inflation. This introduces a second energy scale to the problem.

(3) Realization that inflation requires a quantum theory of the gravitational field.[16,17] Halliwell with Hawking[17] pointed out that one should solve the Wheeler-DeWitt equation.[18] They considered linear quantum fluctuations on a homogeneous background. In linear theory, one can improve this scheme by using the second order action formalism of Mukhanov, Feldman and Brandenberger.[19]

(4) Reconciling the inflation model with particle physics. This is a non-trivial problem because the scalar field potential must be extremely flat. For example, chaotic inflation with a quartic potential $V(\phi) = \lambda\phi^4/4$ requires $\lambda = 1.1 \times 10^{-13}$ to agree with COBE measurements whereas radiative corrections to the GUT Higgs potential are of the order of $\lambda \sim 10^{-3}$. In the *Variable Planck Mass Model* proposed by myself, Bond and Bardeen,[20] we identified the inflaton field with the GUT Higgs and the Brans-Dicke scalar. This is perhaps the most economical model. Other viable models that mesh inflation with particle physics are extended inflation[21] which is a refinement of power-law inflation proposed by Lucchin and Matarrese,[22] and natural inflation[23] where the inflaton is a pseudo-Goldstone boson.

However, problems have appeared in the structure formation scenarios. Observations indicate that there may be more power at cluster scales than predicted by the CDM model[24–26] with a flat Zeldovich spectrum. Can one adjust the primordial fluctuation spectrum to accommodate this result? Unfortunately, recent work has shown that the promising possibility of power-law inflation with CDM cannot yield enough large scale power at large scales (the gravity wave contribution to $\Delta T/T$ proves important here[27]). It is indeed possible to fit the data by constructing inflation models which yield a mountain in the fluctuation spectrum but these proposals do require some tuning.[20] If large scale power is confirmed by future observations, it may be the result of some astrophysical effect (see Couchman with Carlberg[28] and Frenk in these proceedings).

A likely extension of the inflationary scenario is the generation of non-Gaussian fluctuations with a flat spectrum. Although there is no strong observational evidence, non-Gaussian fluctuations may signal the presence of nonlinearities,[29] quantum gravity effects,[30] cosmic strings[31–32] or even superstrings. (For a discussion of superstrings in an inflationary setting, consult N. Sanchez *et al.*[33] For a review of inflation, see Kolb with Turner[34] or Linde.[35]) Hopefully, COBE will provide a definitive test of these models although their errors are currently too large.

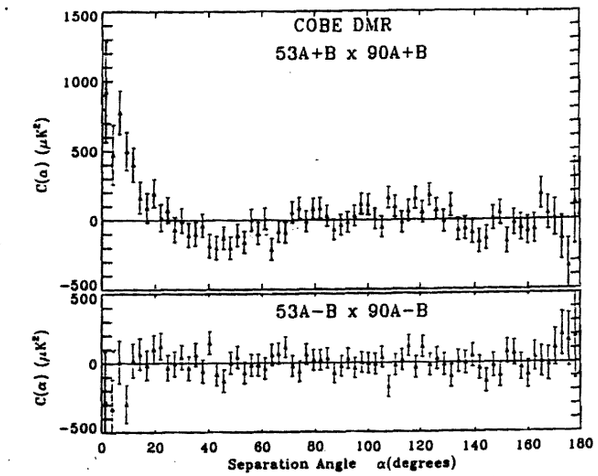

Fig. (1): The strongest evidence that COBE has indeed detected the cosmic microwave background anisotropy is its measurement of the angular correlation function (top figure) with the monopole, dipole and quadrupole removed. Below about 20^0, the measurements are clearly much larger than the error bars. In fact, COBE claims a 7-σ detection. The bottom figure is an estimate of their errors. (Taken from Smoot *et al.*[6])

In Sec. 2, I review the calculation of the large angle $\Delta T/T$ arising from inflation. Particular care is given to the contribution from gravity waves[27] which can be dominant in some models. In Sec. 3, I discuss some of the most promising inflation models in the context of COBE's results. One can compute the coupling constants and the resulting fluctuation spectra that enter in each model. Galaxy data together with COBE can put stringent limits on the spectral index for power-law inflation. In Sec. 4, I discuss the implications of the various inflation models

for structure formation. In Sec. 5, the possibility that new physics can alter the distribution of $\Delta T/T$ is considered. A simple concrete model is given where one employs a nonlinear semi-classical treatment of the gravitational field. Improvements of this model are suggested with the ultimate aim being a fully quantum treatment. I give my conclusions in Sec. 6.

(I will assume that the present Hubble parameter is $H_0 = 50 km\, s^{-1} Mpc^{-1}$, and that the fraction mass density of baryons $\Omega_B \equiv \rho_B/\rho_{crit} = 0.03$ is consistent with nucleosynthesis. Throughout, $\hbar = c = 1$, and Newton's constant $G = m_{\mathcal{P}}^{-2}$ is expressed in terms of the Planck scale $m_{\mathcal{P}} = 1.221 \times 10^{19}$GeV.)

2. $\Delta T/T$ FROM INFLATION

Since the errors for some of COBE's results are of the order of 15%, it is important that all theoretical calculations be performed at least to that level of accuracy. Effects which were neglected before must be considered now. In Sec. 2.1, I show how to calculate the deviation from a Zeldovich spectrum that occurs even in the simplest models. In Sec. 2.2, I calculate the contribution to $\Delta T/T$ from gravity waves.

2.1 Contribution from Scalar Perturbations

Classically, inflation makes the Universe homogeneous and flat. However, inhomogeneities are continually produced by quantum fluctuations that are spontaneously produced. Since $\Delta T/T$ is small, it seems reasonable then to use linear perturbation theory about a homogeneous Friedmann Universe (see, for example, ref.[20])

$$\phi(\tau, x) = \bar{\phi}(\tau) + \delta\phi(\tau, x)$$

$$g_{\mu\nu} = a^2(\tau)\left[\eta_{\mu\nu} + h_{\mu\nu}(\tau, x)\right]$$

to describe the evolution of Einstein gravity with a scalar field self-interacting through a potential $V(\phi)$. Misner, Thorner, Wheeler[36] sign conventions are used throughout and hence $\eta_{\mu\nu} = \mathrm{Diag}[-1, 1, 1, 1]$. $\delta\phi(\tau, x)$ and $h_{\mu\nu}(\tau, x)$ are quantum Heisenberg operators. In the inflationary scenario, it is essential that scalar field quantum fluctuations transform themselves into metric fluctuations. This is perhaps the strongest motivation for quantizing the gravitational field. This becomes even clearer when one discusses the tensor modes describing gravitational radiation (Sec. 2.2). Scalar perturbation are complicated by the freedom in the gauge. I will employ longitudinal gauge,

$$h_{\mu\nu}(\tau, x) = 2\Phi_H(\tau, x)\, \mathrm{Diag}[1, 1, 1, 1]\,, \tag{2.1}$$

because Bardeen's[37] gauge-invariant variable $-\Phi_H$ can be interpreted as the generalization of the Newtonian potential which satisfies the Newton-Poisson relation

$$a^{-2}\nabla^2\Phi_H = -\frac{4\pi}{m_{\mathcal{P}}^2}\delta\rho_{com}\,.$$

Here $\delta\rho_{com}$ is the comoving density perturbation. The gauge choice is taken for mathematical convenience, *i.e.* the resulting equations are easier to solve. However, when a solution is found, one may have to perform a gauge transformation in order to compare it with observations.

One can decompose the scalar field and the metric into comoving Fourier modes, *i.e.*,

$$\delta\phi(\tau, x) = \int \frac{d^3k}{(2\pi)^3}\, a(k)\delta\phi(\tau, k)e^{ik\cdot x} + h.c.\,. \tag{2.2a}$$

In linear theory, there is no mode-mode coupling. Here $a^\dagger(k)$ and $a(k)$ are creation and annihilation operators which satisfy the quantum commutation relations

$$[a(k), a^\dagger(k')] = (2\pi)^3\delta^3(k - k')\,. \tag{2.2b}$$

If one temporarily neglects the effect of metric perturbations, one finds that each Fourier amplitude $\delta\phi(\tau, k)$, which is just a complex-valued function, satisfies an equation analogous to a damped harmonic oscillator

$$\delta\phi''(\tau, k) + 2\frac{a'}{a}\delta\phi'(\tau, k) + k^2\delta\phi(\tau, k) = 0\,. \tag{2.2c}$$

Here $' = \partial/\partial\tau$ is a derivative with respect to conformal time τ, whereas $\dot{} = \partial/\partial t$ denotes a derivative with respect to proper time: $dt = a d\tau$. The background Hubble parameter is $H = \dot{a}/a$. In the quantum treatment one chooses the positive frequency solution corresponding to the Bunch-Davies[38] vacuum

$$\delta\phi(\tau, k) \sim \frac{e^{-ik\tau}}{\sqrt{2k}a(\tau)}$$

which is an excellent approximation when the physical wavelength is much shorter than the Hubble radius, $ak^{-1} << H^{-1}$. In this regime, this solution yields the adiabatic ground state giving the 'minimal energy fluctuations.' This solution is normalized by the equal-time quantum commutation relation

$$[\phi(\tau, x), \pi^\phi(\tau, x')] = i\delta^3(x - x')\,.$$

When the physical wavelength exceeds the Hubble radius, the oscillations stop, and the amplitude freezes out at the Hawking temperature

$$\mathcal{P}_\phi{}^{1/2}(\tau,k) \equiv \left[\frac{k^3}{2\pi^2}|\delta\phi(\tau,k)|^2\right]^{1/2} \sim \frac{H(k)}{2\pi}. \tag{2.3}$$

The power spectrum $\mathcal{P}_\phi(\tau,k)$ gives the variance at a single point $< \delta\phi(\tau,x)\delta\phi(\tau,x) >$ per unit $\ln k$ interval.

A full discussion of metric fluctuations is unfortunately complicated[20,19] although the final results are quite simple. One defines a quantity zeta[12]

$$\zeta \equiv \frac{2}{1+p/\rho}\left(\Phi_H + H^{-1}\dot{\Phi}_H\right) + 3\Phi_H \tag{2.4}$$

where p and ρ are the background values of the pressure and density. Zeta is the perturbation in $\ln(\gamma^{1/2})$ on a uniform Hubble slice (γ is the determinant of the full 3-metric $\gamma_{ij} = g_{ij}$.) This combination of Φ_H and its derivative is independent of time when the wavelength exceeds the Hubble radius. This is true during inflation for a single scalar field, through heating of the Universe, and even during the radiation- and matter-dominated epochs.[20] This simple behavior can be attributed to the fact that different spatial points are no longer in causal contact, and they each evolve as independent, homogeneous and flat Universes.[39]

For the metric of the form (2.1), the G^0_i Einstein equations (the momentum constraints) yield[37]

$$H\Phi_H + \dot{\Phi}_H = -\frac{4\pi}{m_P^2}\dot{\phi}\delta\phi \tag{2.5}$$

in linear perturbation theory. It can be shown that when a comoving scale leaves the Hubble radius that the term $3\Phi_H$ is small compared to the remainder in eq.(2.4); the value of ζ is then given by

$$\zeta(k) = -3H(t_H)\,\delta\phi(t_H,k)/\dot{\phi}(t_H) \tag{2.6}$$

where all functions of the scalar fields are evaluated at the time t_H that the comoving scale k^{-1} crosses the Hubble radius. This formula is easy to interpret. $\delta t = \delta\phi/\dot{\phi}$ is the variation in the initial starting time; as a result $-\zeta/3 = H\delta t$ is just the variation in the expansion factor.

For most models, it is adequate to assume the slow-roll approximation for the background evolution

$$\dot{\phi} = -\frac{\partial V}{\partial\phi}/(3H), \quad H = \left[8\pi V(\phi)/(3m_P^2)\right]^{1/2} \tag{2.7}$$

and the final expression for the power spectrum in ζ is

$$\mathcal{P}_\zeta = \frac{81}{4\pi^2}H^6/\left(\frac{\partial V}{\partial\phi}\right)^2. \tag{2.8}$$

It proves convenient to parametrize this in terms of the number of e-foldings that the comoving scale k^{-1} undergoes outside the Hubble radius during the inflationary era

$$N_I(k) \equiv \int_{\phi_{reh}}^{\phi(k)} d\ln(a) = \int_{\phi_{heat}}^{\phi(k)} d\phi'\, 3H^2(\phi')/\frac{\partial V(\phi')}{\partial\phi'}. \tag{2.9a}$$

Although the function $N_I(k)$ does depend on the details of heating where the physics is poorly understood, the dependence is rather weak and it is sufficient for most models to apply the simple expression

$$N_I(k) = 60 - \ln(k/10^{-4}Mpc^{-1}). \tag{2.9b}$$

For example, the comoving scale $k^{-1} = 10^4$Mpc typically undergoes 60 e-foldings during inflation. The spectral index n for scalar perturbations is defined as

$$n \equiv 1 + d\ln\mathcal{P}_\zeta(k)/d\ln(k). \tag{2.10}$$

In general, it is a function of wavenumber, but it typically varies slowly and one may approximate the power spectrum in ζ by a power-law

$$\mathcal{P}_\zeta(k) = \mathcal{P}_\zeta(k_0)\,(k/k_0)^{n-1}. \tag{2.11}$$

Explicit examples will be given in Sec. 3.

The Sachs-Wolfe[40] formula yields the large angle temperature anisotropy from a line integral over the perturbation in the 3-metric $\tilde{h}_{ij}$ computed in *synchronous gauge* where $g_{00} = -1$ and $g_{0i} = 0$:

$$\Delta T/T(x) = \frac{1}{2}e^i e^j \int_{t_i}^{t_f} d\tau \frac{\partial\tilde{h}_{ij}}{\partial\tau} \tag{2.12}$$

The line integral follows the path of a photon path from the surface of last scattering to the present epoch; e^i is a unit vector giving the direction of the photon's velocity. For angles of interest to COBE, we are concerned with those comoving scales that reenter the Hubble radius during the matter-dominated era. The gravitational potential $\Phi_H \equiv \Phi_H(x)$ is independent of time in the matter dominated era, for scales both outside and inside the Hubble radius. Its value is determined by

setting $p = 0$ in (2.4): $\Phi_H(k) = \zeta(k)/5$. Note also that in comoving synchronous gauge

$$\tilde{h}_{ij} = \delta_{ij}\left(1 + \frac{10}{3}\Phi_H(x)\right) - \frac{1}{3}\tau^2\,\Phi_{H,ij}(x)\,.$$

Hence, one derives the famous result for the temperature anisotropy in a flat, matter-dominated Universe:

$$\Delta T(x)/T = -\Phi_H(x)/3 = -\zeta(x)/15\,. \tag{2.13}$$

Using a plane wave decomposition into spherical harmonics,[41]

$$e^{ik\cdot x} = 4\pi\sum_{l=0}^{\infty}\sum_{m=-l}^{m=l} i^l j_l(kr) Y^*_{lm}(\Omega_x) Y_{lm}(\Omega_k)$$

one finds that scalar contribution to the angular correlation function is[7,8,27]

$$C_S(\alpha) \equiv < \Delta T(x)\Delta T(x') >_S = \sum_{l=0}^{\infty} < \Delta T_l^2 >_S P_l(\cos\alpha)\,, \tag{2.14a}$$

$$\begin{aligned} < \Delta T_l^2 >_S &\equiv \left(\frac{T_\gamma}{15}\right)^2 (2l+1)\int_0^\infty \frac{dk}{k}\,\mathcal{P}_\zeta(k)\, j_l^2(kr) \\ &= A^2\,\frac{(2l+1)}{5}\,\frac{\Gamma(l+(n-1)/2)}{\Gamma(l+(5-n)/2)}\,\frac{\Gamma((9-n)/2)}{\Gamma((3+n)/2)}, \end{aligned} \tag{2.14b}$$

$$A^2 = T_\gamma^2\,\mathcal{P}_\zeta(k_0)(k_0 r)^{1-n}\,\frac{\pi}{45}\,2^{n-4}\,\frac{\Gamma(3-n)}{\Gamma^2(2-n/2)}\,\frac{\Gamma((n+3)/2)}{\Gamma((9-n)/2)} \tag{2.14c}$$

where $T_\gamma = 2.735 \pm 0.06K$.[42] Here x and x' are two points separated by an angle α on the surface of last scattering where the radius is $r = 11,700$Mpc.

2.2 Contribution from Tensor Perturbations

The tensor perturbation describes the traceless transverse part of the 3-metric h_{ij}. In linear perturbation theory, for each comoving wavenumber k there are two polarization modes which behave like a *massless scalar field* $\psi(\tau, k)$ with evolution equation identical to eq.(2.2c):

$$h_{ij}(\tau, x) = \left(\frac{32\pi}{m_{\mathcal{P}}^2}\right)^{1/2}\sum_{q=1}^{2}\int\frac{d^3k}{(2\pi)^3}\, b^{(q)}(k)\, P_{ij}^{(q)}(k)\,\psi(\tau,k)\, e^{ik\cdot x} + h.c.\,. \tag{2.15}$$

The $b^{(q)}(k)$ are the annihilation operators for each polarization state $q = 1, 2$, and they satisfy commutation relations directly analogous to eq.(2.2b). When a comoving scale crosses the Hubble radius during inflation, the amplitude $\psi(\tau, k)$ reaches a fixed values given by the Hawking temperature eq.(2.3): $\psi(\tau, k) = H(k)/\sqrt{2k^3}$ for $k < Ha(\tau)$. This amplitude is normalized using the canonical commutation relations for the 3-metric. (If $k = [0, 0, 1]$ is aligned about the z-axis, then the first polarization tensor is $P_{ij}^{(1)} = 2^{-1/2}\,\mathrm{Diag}[1, -1, 0]$, whereas the second is given by $P_{12}^{(2)} = P_{21}^{(2)} = 2^{-1/2}$, with all other components vanishing.)

When a comoving scale reenters the Hubble radius during the matter-dominated era, it begins to oscillate

$$\psi(\tau, k) = 3\frac{j_1(k\tau)}{(k\tau)}\,\frac{H(k)}{\sqrt{2k^3}} \tag{2.16}$$

where j_1 is a spherical Bessel function. Once again, the temperature anisotropy is calculated using the Sachs-Wolfs formula eq.(2.12). For perturbations arising from pure DeSitter space with Hubble parameter H, one may verify the following formula for the tensor contribution to the correlation function,

$$C_T(\alpha) = \sum_{l=0}^{\infty} < \Delta T_l^2 >_T P_l(\cos\alpha)$$

$$\begin{aligned} < \Delta T_l^2 >_T = &\frac{9}{\pi}T_\gamma^2\frac{H^2}{m_{\mathcal{P}}^2}(l-1)l(l+1)(l+2)(2l+1) \\ &\int_0^{2k_{max}H_0} dw\,w\left\{\int_0^{1-\tau_1/(2H_0)} dv\left[3\frac{\cos(w(1-v))}{w^3(1-v)^3} + \frac{\sin(w(1-v))}{w^2(1-v)^2} - 3\frac{\sin(w(1-v))}{w^4(1-v)^4}\right]\right. \\ &\left.\left[\frac{j_{l+2}(wv)}{(2l+1)(2l+3)} + 2\frac{j_l(wv)}{(2l-1)(2l+3)} + \frac{j_{l-2}(wv)}{(2l+1)(2l-1)}\right]\right\}^2 \end{aligned} \tag{2.17}$$

which was first computed by Abbott and Wise.[43] Here τ_1 is the value of conformal time at the time of decoupling, and $k_{max} = 10^{-2}\mathrm{Mpc}^{-1}$ corresponds to the Hubble radius at matter-radiation equality. In deriving this result, it is useful to note that

$$\sum_{q=1}^{2} P_{ij}^{(q)}(k)\,P_{kl}^{(q)}(k) = \frac{1}{2}\left[\tilde{\delta}_{ik}\tilde{\delta}_{jl} + \tilde{\delta}_{il}\tilde{\delta}_{jk} - \tilde{\delta}_{ij}\tilde{\delta}_{kl}\right] \quad \text{where} \quad \tilde{\delta}_{ij} = \delta_{ij} - k_i k_j/k^2\,.$$

Even then, it takes many pages of algebra to verify eq.(2.17). The quadrupole amplitude is given by $< T_2^2 >_T = 0.078\,T_\gamma^2 H^2/m_{\mathcal{P}}^2$. This result can be applied for most inflation models with the exception of power-law inflation where the

Hubble parameter changes quite significantly when the perturbations of interest are generated. For this case, the results are shown in Fig.(4).

Since the scalar and tensor contributions are independent Gaussian random fields, they add in quadrature

$$< \Delta T_l^2 >=< \Delta T_l^2 >_S + < \Delta T_l^2 >_T .$$

To determine coupling constants in any inflationary model, one computes the root-mean-square temperature fluctuation with dipole removed

$$\sigma_{sky}^2(10^0) = \sum_{l=2}^{\infty} < \Delta T_l^2 > \exp\left[-l(l+1)/13.5^2\right] , \quad \sigma_{sky}(10^0) = 30 \pm 5\mu K .$$

The exponential factor corresponds to a Gaussian window function with full width at half maximum of 10^0.

3. INFLATION MODELS

3.1 Chaotic Inflation: $V(\phi) = \lambda\phi^4/4$.

The simplest chaotic models using polynomial potentials provide a simple illustration of the principles of Sec.2. For a quartic potential, the number of e-foldings of inflation as a function of ϕ is (eq.(2.9a))

$$N_I(\phi) = \pi \left(\frac{\phi}{m_P}\right)^2 .$$

The power spectra as a function of N_I is then (see Fig.(2))

$$\mathcal{P}_\zeta(k) = \frac{6\lambda}{\pi^2} N_I^3(k),$$

$$\lambda = 1.12 \times 10^{-13} \pm 30\% \quad b_\rho = 1.07 \pm 15\% .$$

Using linear perturbation theory, the biasing parameter is computed using the mass fluctuations within a sphere of radius $8h^{-1}Mpc = 16Mpc$:

$$< \left(\frac{\Delta M}{M}(r = 16Mpc)\right)^2 > = 1/b_\rho^2 .$$

Gravity waves contributed about 20% to $\sigma_{sky}^2(10^0)$. The consequences for structure formation are discussed in Sec.4. For comparison, a flat Zeldovich spectrum with no gravity waves gives a biasing parameter of $b_\rho = 0.84 \pm 0.15\%$.

3.2 Chaotic Inflation: $V(\phi) = m^2\phi^2/2$.

$$\mathcal{P}_\zeta(k) = \frac{12}{\pi}\left(\frac{m}{m_P}\right)^2 N_I^2(k),$$

$$\frac{m}{m_P} = 1.07 \times 10^{-6} \pm 15\% \quad b_\rho = 0.98 \pm 15\%$$

Gravity waves contributed about 10% to $\sigma_{sky}^2(10^0)$.

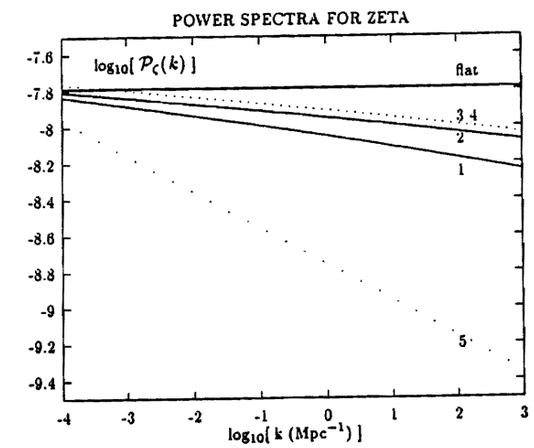

Fig.(2): Different inflation models (normalized using $\sigma_{sky}(10^0)$) may be distinguished according to their power spectra although the current experimental accuracy is insufficient. Model 1 is $V(\phi) = \lambda\phi^4/4$, Model 2 is $V(\phi) = m^2\phi^2/2$, Model 3 is the Variable Planck Mass Model, and Model 4 is R^2 inflation. For power-law inflation (Model 5), $n = 0.8$ is shown which yields a biasing parameter of $b_\rho = 2.0$.

3.3 Variable Planck Mass Model

One of the harshest criticisms of the inflation model is that it does not mesh well with particle physics. What is this weakly interacting field ϕ? As a possible resolution of this question, I together with Bond and Bardeen[20] have constructed a model where the inflaton is identified with the GUT Higgs and the Brans-Dicke scalar. The trick is to alter the gravitational sector and leave the particle sector essentially intact.

Since the scalar field hovers around the Planck scale in chaotic inflation, one should really include the term $-\xi R\phi^2/2$ in the action:

$$\mathcal{I} = \int \sqrt{-g}\left\{\frac{m^2}{16\pi}R - \frac{1}{2}\xi\phi^2 R - \frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - \frac{\lambda}{4}(\phi^2-\sigma^2)^2\right\}. \tag{3.1}$$

Such a term must occur in order to cancel infinities when one considers a $\lambda\phi^4/4$ self-interaction in curved space-time.[44] This theory can be interpreted in a simple way. The coefficient of R can be interpreted in terms of the *effective* value of the Planck mass

$$G_{eff}^{-1}(\phi) \equiv m_{\mathcal{P}_{eff}}^2(\phi) \equiv m^2 - 8\pi\xi\phi^2. \tag{3.2}$$

Here m^2 is the bare contribution to $m_{\mathcal{P}_{eff}}^2$ whereas $-8\pi\xi\phi^2$ is the Higgs contribution. In order that the Universe inflate sufficiently, one requires that[45]

$$\xi < 0.002 \quad \text{if} \quad m \sim m_{\mathcal{P}}. \tag{3.3}$$

Although this is a rather crude constraint, it is nonetheless very vexing for chaotic inflation. If this constraint is violated, Newton's constant G_{eff} can become negative. Classically, it would have to remain negative because it must cross a singularity in order to reach positive values.

The simplest way to satisfy eq.(3.3) is to take ξ to be negative. Then there are essentially two cases to consider. In induced gravity, one legislates that the bare value m vanish, and that the present value of the Planck scale is determined when the scalar field rolls to its minimum σ in the potential. This is a particularly attractive model requiring the fewest number of parameters, although it may not produce a radiation-dominated Universe after inflation (which would be a disaster). In the second case which is adopted here one assumes that $m \sim m_{\mathcal{P}}$, and that the present value of the Planck scale obtains only a small contribution from ϕ at the minimum of its potential, although it was much much larger during the inflationary era when $\phi >> \sigma$. This situation is called the Variable Planck Mass Model.

Since the gravitational sector is quite complicated, it proves very convenient to employ a conformal transformation of the metric

$$g_{\mu\nu} = \Omega^2(\phi)\tilde{g}_{\mu\nu}, \quad \Omega(\phi) = m_{\mathcal{P}}/m_{\mathcal{P}_{eff}}(\phi). \tag{3.4}$$

This transformation suggested by Brans and Dicke[46] is easy to interpret— it is just a change of units from a variable ruler $m_{\mathcal{P}_{eff}}(\phi)$ to a uniform ruler $m_{\mathcal{P}}$ (which is the current value of the Planck scale). Much of the physics that is discussed below is similar to changing units in the same way that one converts from feet to metres. However, one should remember that the Ricci curvature transforms in a more complicated way

$$R(\Omega^2\tilde{g}_{\mu\nu}) = \Omega^{-2}R(\tilde{g}_{\mu\nu}) - 6\Omega^{-3}\tilde{g}^{\mu\nu}\Omega_{;\mu\nu}. \tag{3.5}$$

The new Lagrangian density

$$\tilde{\mathcal{L}} = \frac{m_{\mathcal{P}}^2}{16\pi}\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\chi_{,\mu}\chi_{,\nu} - V_{eff}(\phi(\chi)) \tag{3.6}$$

describes standard Einstein gravity for the new metric $\tilde{g}_{\mu\nu}$. The effective potential given by

$$V_{eff}(\phi) = \Omega^4(\phi)\,V(\phi) \tag{3.7}$$

is very flat as $|\phi| \to \infty$ which is very desirable for inflation. χ is a function of ϕ, and it is given in ref.[20]. For induced gravity, its expression is quite simple

$$\phi = \sigma\, e^{\alpha\chi}, \quad V_{eff}(\chi) = \frac{\lambda}{4}\left(\frac{m_{\mathcal{P}}^2}{8\pi|\xi|}\right)^2(1-e^{-2\alpha\chi})^2, \quad \alpha = \left(\frac{8\pi|\xi|}{1+6|\xi|}\right)^{1/2} m_{\mathcal{P}}^{-1}. \tag{3.8}$$

The fluctuation spectrum for the Variable Planck Mass model is essentially the same as for induced gravity:

$$\mathcal{P}_{\tilde{\zeta}}(k) = \frac{1}{8\pi^2}\frac{\lambda}{\xi^2}N_f^2(k), \tag{3.9}$$

Normalizing according to the COBE observations, one obtains

$$\frac{\lambda}{\xi^2} = 3.8\times 10^{-10} \pm 30\% \quad b_\rho = 0.93 \pm 15\%.$$

Gravity waves contribute about 0.3% to $\sigma_{sky}^2(10^0)$. If ϕ is the GUT Higgs, then $\lambda \sim 0.05$ is consistent with radiative corrections and one can fit COBE if $\xi \sim -10^4$. (A formula analogous to eq.(3.9) was first suggested by Spokoiny.[47] The precise expressions were given in ref.[20]; compare with the independent results of ref.[48].)

The main advantage of having large negative ξ is that radiative corrections do not destroy the required flatness in the effective potential.[20] For example, one can couple a gauge boson A_μ through the minimal prescription replacing

$$\partial_\mu \to \partial_\mu - ieA_\mu \,,$$

and by adding kinetic terms $F_{\mu\nu}F^{\mu\nu}$ to the original action (3.1). After the conformal transformation is performed, the gauge field A_μ does not transform— it is conformally invariant. Its mass is given by the Higgs mechanism

$$m_A(\phi) = e\phi\,\Omega(\phi)$$

which is scaled by the additional factor $\Omega(\phi)$. If $m \neq 0$, then for small values of ϕ, one recovers the usual coupling expression, but for large values of ϕ, the gauge boson mass is independent of ϕ: the gauge boson decouples from the Higgs field. As a result the 1-loop radiative potential[49]

$$V_{rad}(\phi) = \frac{3}{64\pi^2} m_A^4(\phi)\,\ln\left(\frac{m_A^2(\phi)}{\mu^2}\right)$$

remains very flat as $\phi \to \infty$ (μ is the renormalization scale.) The above argument works for fermions as well. By altering the gravitational sector, one has a robust method of producing a flat potential that proves favorable for the inflationary scenario.

For induced gravity with $m = 0$, the mass of the gauge boson $m_A = em_{\mathcal{P}}/\sqrt{8\pi|\xi|}$ is independent of the scalar field, and hence the scalar field does not couple directly to radiation. Hence, there may be a problem with producing a radiation-dominated era at the end of inflation.

However, for the Variable Planck Mass model, heating is efficient, and the maximum temperature

$$T_{max} \sim \epsilon\left(\frac{15}{128\pi^4} g_{eff}^{-1}\frac{\lambda}{\xi^2}\right)^{\frac{1}{4}} m_{\mathcal{P}} = 1.1\times 10^{-4} m_{\mathcal{P}}\,,$$

reached after inflation is rather high and in order not produce magnetic monopoles one requires that the GUT scale satisfy

$$\sigma \gtrsim 1.5\times 10^{-4} m_{\mathcal{P}}\,.$$

This result is computed for the SU(5) model, and it is slightly model dependent (the effective number of degrees of freedom was taken to be $g_{eff} = 160.75$ and the efficiency factor for heating was $\epsilon \sim 0.5$).

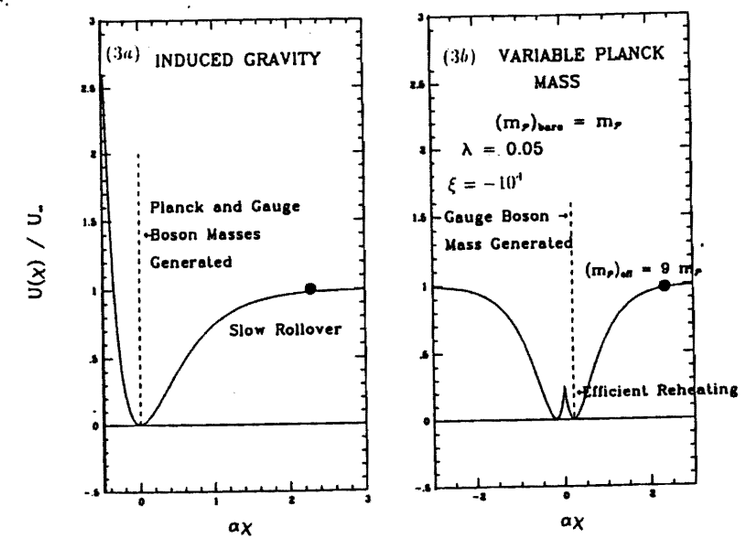

Fig.(3a,b): Inflation can be reconciled with particle physics using the Variable Planck Mass model which is closely related to Induced Gravity. The effective potentials are plotted as a function of $\chi \equiv \chi(\phi)$. For large positive values of χ, the effective potential is extremely flat as required by inflation. One-loop radiative corrections do not destroy the flatness of the potential.

3.4 R^2 Inflation

This model proposed by Starobinsky[50] was one of the first inflationary models, and I discuss it for the sake of completeness. The Lagrangian is given by

$$\mathcal{L} = \frac{m_{\mathcal{P}}^2}{16\pi}\left[R + R^2/(6M^2)\right]\,.$$

In order to solve this system, one performs a conformal transformation proposed by Whitt.[51] If one defines a new field[20]

$$\chi = \left(\frac{3m_{\mathcal{P}}^2}{16\pi}\right)^{1/2}\,\ln\left[1 + \frac{R}{3M^2}\right],$$

the transformed Lagrangian takes the form

$$\mathcal{L} = \frac{m_{\mathcal{P}}^2}{16\pi}\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - U(\chi),$$

where the effective potential is

$$U(\chi) = \frac{3m_{\mathcal{P}}^2 M^2}{32\pi}\left(1 - \exp\left[-\left(\frac{16\pi}{3m_{\mathcal{P}}^2}\right)^{1/2}\chi\right]\right)^2.$$

This Lagrangian is virtually the same as that for induced gravity (see eq.(3.8)). The fluctuation spectrum is

$$\mathcal{P}_\zeta(k) = \frac{3}{\pi}\left(\frac{M}{m_{\mathcal{P}}}\right)^2 N_I^2(k), \quad \frac{M}{m_{\mathcal{P}}} = 2.2\times 10^{-6} \pm 15\%.$$

This value of M is much smaller than one may have expected from computations of quantum fields in curved space-time. For a more extensive discussion of higher derivative theories, consult Maeda *et al*.[52]

3.5 Power-law Inflation: $V(\phi) = V_0 \exp[-(16\pi/p)^{1/2}(\phi/m_{\mathcal{P}})]$.

This model yields power-law growth of the scalar factor[22] $a(t) \propto t^p$ leading to inflation if $p > 1$. It is interesting because it shows that arbitrary power laws with $n < 1$ can arise from inflation.

$$\mathcal{P}_\zeta(k) = \mathcal{P}_\zeta(k_0)\left(\frac{k}{k_0}\right)^{-2/(p-1)}, \quad \mathcal{P}_\zeta(k_0) = \frac{9p}{\pi}\left(\frac{H(k_0)}{m_{\mathcal{P}}}\right)^2. \tag{3.10}$$

The spectral index for scalar perturbations is $n = 1 - 2/(p-1)$ and hence a naive application of the COBE limit $n > 0.5$ would yield $p > 5.0$. However for $n = 0.5$, the contribution of gravitational radiation is dominant contributing about 70% to $\sigma^2_{sky}(10^0)$. The power spectrum $\mathcal{P}_\zeta(k) = 1.18\times 10^{-8}(kr)^{-0.5}$ yields a huge biasing parameter $b_\rho \sim 5.7$, making this model unlikely since observations favor $b_\rho < 2$ [20,2,61]. In fact, this requirement sets a limit of $n \gtrsim 0.8$, $p \gtrsim 11$ (assuming a best-fit normalization for the COBE data). For $n = 0.8$, the fluctuation spectrum is

$$\mathcal{P}_\zeta(k) = 1.14\times 10^{-8}(kr)^{-0.2}, \quad b_\rho = 2.0, \tag{3.11}$$

and gravity waves contribute about 50% to $\sigma^2_{sky}(10^0)$. Hence, it is difficult for power-law inflation to account for extra power[53,27,54] at scales of $20 - 40$Mpc, although this model is still interesting for structure formation because it can yield a large biasing parameter.

In Fig.(4), I have computed the relative contribution of the tensor component $< \Delta T_l^2 >_T /(< \Delta T_l^2 >_S + < \Delta T_l^2 >_T)$ for various values of $n = 1 - 2/(p-1)$. For smaller values of n, it increases quite dramatically. These results are in agreement with the calculations of Lucchin, Matarrese and Mollerach[55] who use the method of Starobinsky.[56]

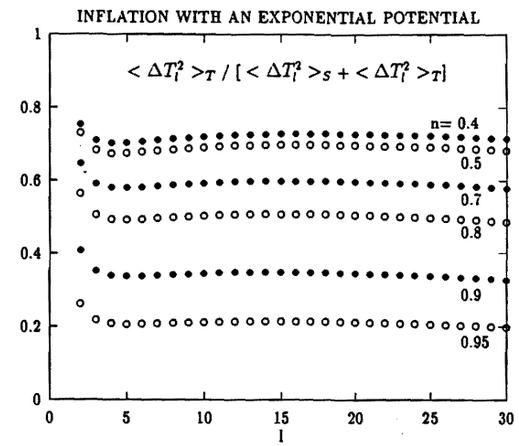

Fig.(4): The gravity wave contribution to large angle $\Delta T/T$ can be dominant for power-law inflation which employs an exponential potential, $V(\phi) = V_0\exp[-\sqrt{16\pi/p}(\phi/m_{\mathcal{P}})]$. As a function of the spherical harmonic l, the relative contribution of the gravity waves $< \Delta T_l^2 >_T$ to the total contribution $< \Delta T_l^2 >_S + < \Delta T_l^2 >_T$ is plotted for various potential parameters $n = 1 - 2/(p-1)$. If one normalizes to COBE, then $n > 0.8$ is required to give fluctuations large enough to produce galaxies.

Davis et al[57] have pointed out that one may measure the contribution of gravity waves from a detection of the anisotropy using the South Pole microwave background experiment.[58] At the smaller angular scales $\sim 1^0$ of this experiment, $\Delta T/T$ receives its contribution primarily from scalar fluctuations because photons are Thompson scattered from electrons moving into the wells of the gravitational potential.[8] At the larger angular scales probed by COBE, $\Delta T/T$ is sensitive to both scalar and tensor perturbations. Combining these two results would result in a determination of both the scalar and tensor fluctuations. For example, if the

South Pole experiment did not detect a signal in the near future, one may interpret that the COBE measurement contains a large contribution from gravity waves.

Using the COBE results, Souradeep and Sahni[59] have looked into the consequences of measuring gravity waves at much smaller angular scales. It unlikely that LIGO will measure the primordial background, although a space bound experiment could.

Fluctuation spectra with power-laws can also arise if the scalar field has a potential of the form[23]

$$V(\phi) = \Lambda^4 (1 + \cos(\phi/f))$$

where Λ is of the order of the GUT scale, and f is of the order of the Planck scale. In this model, one does not obtain an appreciable contribution from gravity waves. The spectral index for scalar perturbations is

$$n = 1 - \frac{m_{\mathcal{P}}^2}{8\pi f^2}.$$

However, it is difficult to produce galaxies early enough in this model and yet produce large scale power.[23]

(Note that in eq.(3.10), I have assumed that the scalar field perturbation leaves the Hubble radius with amplitude $\mathcal{P}_\phi^{1/2}(k) = H(k)/(2\pi)$, eq.(2.3). As was shown by Lyth and E. Stewart[60] using the powerful linear perturbation methods of Mukhanov, Feldman and Brandenberger,[19] the correction factor is the same for gravity waves as for the scalar field perturbations. Hence by ignoring both of these factors, one obtains the correct fluctuation spectrum and the quoted values of the biasing parameter are quite accurate. The limit on the spectral index has been confirmed by Liddle with Lyth.[54])

4. IMPLICATIONS FOR STRUCTURE FORMATION

The first 4 models discussed in Sec. 3 all produce low values of the bias parameter, $b_\rho \sim 1.0$ (compare with Efstathiou, Bond and White[61]) This is certainly a problem for the most popular biased galaxy formation scenarios where $b_\rho = 1.4$ has been favored. However, a low bias parameter does have a distinct advantage. It does produce sheets in the distribution of galaxies in qualitative agreement with the CfA survey[24] and pencil beam surveys[26] (see, for example, Weinberg and Gunn[62]). However, one of the main problems with a low bias parameter is that it gives a velocity disperson of galaxies which is higher than what is observed. However, Couchman and Carlberg[28] claim that their N-body simulations demonstrate velocity bias: the velocity is lower than one may have anticipated because of

mergers of groups. They claim to produce a satisfactory velocity dispersion even when b_ρ is as low 0.8.

However, power-law inflation does demonstrate that is quite easy to construct inflation models which give a higher value of the biasing parameter. Because of the gravity wave contribution as well as the tilt in the spectrum, this model is severely constrained by COBE and galaxy formation. A clear confirmation of velocity bias could restrict these models even more, and hence could act as a probe of the early Universe.

5. NEW PHYSICS ?

Inflation probes the highest energies that one can imagine $(10^{-5}-1)m_{\mathcal{P}}$. A natural question is whether COBE or some other microwave background experiment can discover new physics. It is clear that new physics must enter in the scenario. The inflationary model is phenomenological. It yields a Universe at critical density in agreement with IRAS measurements. It enables one to model the fluctuations for structure formation. It is unsatisfactory because it is essentially a model of a decaying cosmological constant. A comprehensive theory of the cosmological constant is still lacking, although worm-holes are the most promising mechanism of yielding zero cosmological constant at the present epoch (see, for example, Moss[63]).

I will focus my effort on finding an observable signature for new physics. Is linear perturbation theory sufficient for a treatment of fluctuations in inflation? Any refinement in the calculation should really include nonlinear quantum gravity effects. These effects are interesting because they could lead to a non-Gaussian distribution for $\Delta T/T$.

5.1 Solving for Semi-Classical Gravity

As a first step in solving the nonlinear evolution of the scalar field with gravity, I will solve the semi-classical theory. As a guide through this complex maze, I will utilize the intuitive method of stochastic inflation[64-68,39] which contains many of the essential ingredients.

In semi-classical gravity, the wavefunctional for the scalar field and the 3-metric γ_{ij} is approximated by

$$\Psi \sim e^{iS} \tag{5.1}$$

where the phase factor $\mathcal{S}$ satisfies the Hamilton-Jacobi equation for general relativity:

$$0 = \frac{16\pi}{m_P^2}\gamma^{-1/2}\frac{\delta\mathcal{S}}{\delta\gamma_{ij}(x)}\frac{\delta\mathcal{S}}{\delta\gamma_{kl}(x)}\left[\gamma_{jk}(x)\gamma_{il}(x) - \frac{1}{2}\gamma_{ij}(x)\gamma_{kl}(x)\right] - \frac{m_P^2}{16\pi}\gamma^{1/2}R + \frac{1}{2}\gamma^{-1/2}(x)\left(\frac{\delta\mathcal{S}}{\delta\phi(x)}\right)^2 + \frac{1}{2}\gamma^{1/2}\gamma^{ij}\phi_{,i}\phi_{,j} + \gamma^{1/2}(x)V(\phi). \tag{5.2}$$

This equation is extremely complicated because it is a partial differential equation for an infinite number of interacting degrees of freedom.

In order to explain how to solve this equation, it is useful to consider a trivial example. A series expansion of the form $y = \sum_{n=0}^{\infty} a_n x^n$ reduces the differential equation $d^2y/dx^2 + y = 0$ to a set of a algebraic equations satisfying the 2-term recursion relation $a_n = -a_{n-2}/(n(n-1))$. In an analogous way, $\mathcal{S}$ can be expressed in terms of a spatial gradient expansion

$$\mathcal{S} = \left[\int d^3x\gamma^{1/2}H(\phi)\right] + \left[\int d^3x\gamma^{1/2}J(\phi)R + \int d^3x\gamma^{1/2}K(\phi)\phi_{|i}\phi^{|i}\right] + \ldots . \tag{5.3}$$

as proposed by myself and J. Stewart.[69,70] Here R is the Ricci scalar for the 3-metric whereas $H(\phi), J(\phi), K(\phi)$ are arbitrary functions of the scalar field. The first terms contains no spatial gradients, whereas the second term contains two spatial gradients, and so on. The phase functional must be invariant under reparametrizations of the spatial coordinates. For example, in the first term, the combination $\int d^3x\gamma^{1/2}$ ensures reparametrization invariance if $H(\phi)$ is some arbitrary function of the scalar field. At each order of the expansion, one obtains a partial differential equation of a *finite* number of variables. For example, at zeroth order one obtains the separated Hamilton-Jacobi equation[39,69] (SHJE)

$$H^2 = \frac{m_P^2}{12\pi}\left(\frac{\partial H}{\partial\phi}\right)^2 + \frac{8\pi}{3m_P^2}V(\phi). \tag{5.4}$$

At the second order,[70] there are 2 independent equations for the two functions $J(\phi)$ and $K(\phi)$

$$-\frac{m_P^2}{4\pi}\frac{\partial H}{\partial\phi}\frac{\partial J}{\partial\phi} + JH - \frac{m_P^2}{16\pi} = 0, \tag{5.5a}$$

$$K = \frac{8\pi}{m_P^2}\left(H/\frac{\partial H}{\partial\phi}\right)\frac{\partial J}{\partial\phi}. \tag{5.5b}$$

5.2 Non-Gaussian Fluctuations from Inflation

I will now describe a simple model that produces non-Gaussian fluctuations from inflation.[29] In the long-wavelength approximation (see eq.(5.4)), one can obtain exact and general solutions for the case of two scalar fields whose logarithm of the potential is linear:[69,71] $\ln V(\phi_1, \phi_2) = a_1\phi_1 + a_2\phi_2$. Now any potential surface may be approximated by linear potentials. For illustration, one may construct a potential by continuously joining three regions with linear $\ln V$ as shown in Fig.(5). Starting with Gaussian initial conditions in the lower left hand region, non-Gaussian fluctuations arise if the scalar fields pass sufficiently close to the origin. The resulting fluctuations are essentially scale-invariant. They produce a pronounced signature in the distribution of $\Delta T/T$ as shown in Fig.(6). The exact shape of the distribution depends among other things on the potential parameters chosen. One has considerable freedom here.

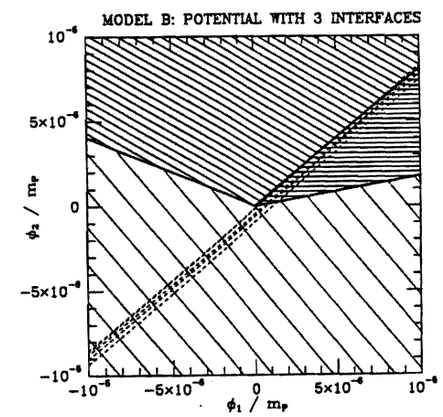

Fig.(5): Non-Gaussian fluctuations may be generated during the inflationary epoch if the scalar fields pass over 3 interfaces in their potential. The light solid curves are lines of uniform potential, whereas the heavy lines are the interfaces between regions where $\ln V(\phi_1, \phi_2)$ is linear. If the scalar field trajectories (broken lines) pass sufficiently near the origin, nonlinear effects at long-wavelengths are important. All calculations are performed in the long-wavelength approximation which is accurate to first order in spatial gradients.

Currently, COBE claims that its results are consistent with a Gaussian distribution, although the errors in their distribution are probably about 50%. Smoot has stated in this meeting that they are relatively confident that there is no long

extended tail in the distribution, but because of observational errors, they are not able to rule out deviations from a Gaussian distribution which are closer to the mean. It appears that the simple non-Gaussian described above remains viable, although one must wait for COBE to publish their data before one could proceed further.

Could the non-Gaussian signal be observed in the distribution of galaxies? I have computed the cosmological fields at the present epoch using linear perturbation theory.[72] I find that the Newtonian potential $-\Phi_H$ remains non-Gaussian. However, the peculiar velocity and the density perturbation are very close to being Gaussian. As a result, the non-Gaussian model is in apparent agreement with the observational tests of numerous authors[73,74] who claim that the peculiar velocities as well as the linear density distribution are consistent with Gaussian. (I have yet to perform the genus test.[75,76])

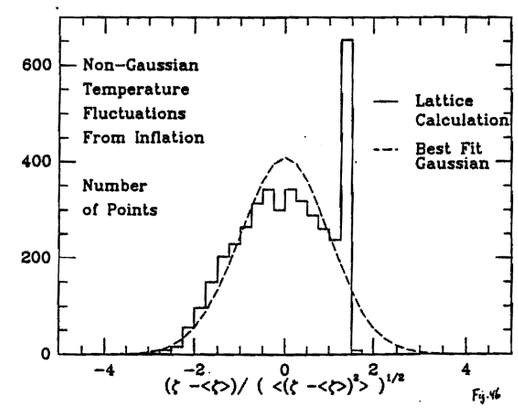

Fig.(6): For the model given in Fig.(5), the distribution in $\zeta = -15\Delta T/T$ is shown (solid curve). For comparison, a Gaussian distribution (broken curve) with the same mean and dispersion is also drawn. For this model, the most significant feature is that cold spots in the temperature anisotropy are suppressed over the usual cold-dark-matter (CDM) model with Gaussian primordial fluctuations.

Moscardini *et al*[77] and Cole and Weinberg[78] have performed N-Body simulations with non-Gaussian initial conditions. Both agree that some non-Gaussian models may produce enhanced sheet-like structures in the distribution of galaxies.

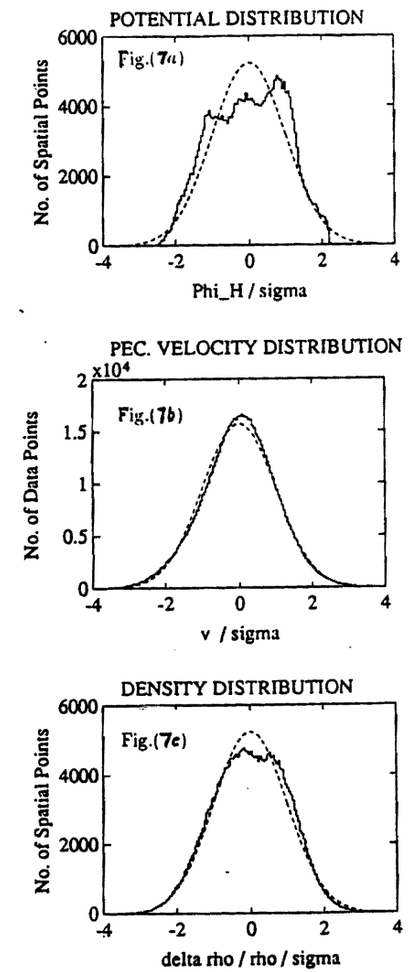

Fig.(7a,b,c): For the model considered in Fig.(5), the distributions of Φ_H, v and $\delta\rho/\rho$ are shown (solid histograms) at the present epoch for a Cold-Dark-Matter cosmological scenario. Evolution of the non-Gaussian primoridial fluctuations through the radiation and matter dominated eras was calculated using linear perturbation theory. The broken curves are the best-fit Gaussian distributions. The simulation was of a comoving cube, $64h^{-1}$Mpc to a side, consisting of 64^3 lattice points. (a) The gravitational potential, $-\Phi_H$, is strongly non-Gaussian and hence is the best signature of the model. (b) Surprisingly, the distribution of the peculiar velocity field, $v = [v_1, v_2, v_3]$, considered as a single random variable with 3×64^3 data points, is very nearly Gaussian and is in apparent agreement with observations. (c) The comoving density perturbation $\delta\rho/\rho$ is non-Gaussian, although I believe that its comparison with observations will be obscured by poorly understood astrophysical effects.

However, the two groups ultimately disagree with the viability of these models. This point will not be resolved until N-body simulations are further refined. For example, one needs a better understanding of how to identify galaxies. The importance of velocity bias is unclear.[28]

At present, the observational arguments for non-Gaussian fluctuations are not particularly persuasive. However, I encourage the observers of the microwave background anisotropy to persevere in the search for such effects because they may be signatures of new physics at energy scales $10^{-5}m_P - 1m_P$. Here I believe that the quantum gravity aspects are perhaps of the most fundamental interest. It is doubtful whether there is any other experiment in the near future that will be able to probe this question. Unfortunately, a quantum theory of the gravitational field is still in a very rudimentary stage. The best studied models are those of homogeneous minisuperspace.[18] Recently, a quantum theory of long-wavelength gravity has been proposed by myself.[30] Here it proved useful to introduce a dust field[79] into the gravitational system because one can generalize the notion of Lorentz frame to a general relativistic context.[30] It remains to be seen whether this analysis can be extended to shorter wavelengths.

In the mean time, one can indeed demonstrate that a non-Gaussian distribution in the gravitational potential can arise from inflation using a two scalar field model. It may be possible that non-Gaussian fluctuations can arise from the simplest single scalar field models, but one would have to calculate nonlinear short-wavelength quantum fluctuations within the horizon which has not yet been attempted in a serious fashion. I still believe that microwave background experiments will be in the best position to test these models since it appears likely that the strongest signature will occur in the gravitational potential. Galaxy clustering does impose some constraints on non-Gaussian models, but deviations from Gaussian can be hidden in the peculiar velocities as well as the linear density field.

6. CONCLUSIONS

Currently, the IRAS satellite provides the best support for the inflationary model. COBE's determination of the spectral index is consistent with the simplest models. It has already put stringent limits on power-law inflation because the gravity wave contribution is important. The true beauty of the COBE satellite lies in its small errors. With improved data, it offers the possibility of precision tests of inflation. Moreover, various models may be ruled out if one can determine whether velocity bias[28] occurs in N-body simulations.

In these proceedings, Brandenberger has rightfully criticized the inflationary model with a quartic potential $\lambda\phi^4/4$ because it requires a very small self-coupling

of λ. This is a difficult problem, but several solutions have been attempted. For example, the Variable Planck Mass model[20] can actually reconcile particle physics with inflation. The lesson to be learned here is that modifications of the gravitational sector are actually beneficial for chaotic inflation because the scalar field hovers around the Planck scale.

It is natural to consider extensions of the inflationary model in order to find the limits of its predictions. One of the most exciting possibilities is that large angle microwave background fluctuations may provide some insight on physics near the Planck scale. However, the calculation of non-Gaussian fluctuations from inflation is in its infancy. Even in the simplest inflation models utilizing a single scalar field, it is difficult to estimate the magnitude of the terms because of the presence of infinities in the quantum calculations. Currently, a nonlinear semi-classical analysis is employed.

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