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FREE FIELD CONSTRUCTION OF CHIRAL $SU(N)^{\hat{}}_{k=1}$ WZW MODELS

Cambridge U., DAMTP

Meifang Chu and Peter Goddard

D.A.M.T.P.
University of Cambridge
Silver Street, Cambridge, CB3 9EW
United Kingdom

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ABSTRACT

The free field realisation a la Frenkel-Kac and Segal is applied to construct the chiral vertex operators in the basic representations of the level-one $SU(N)$ WZW model. These operators are shown to be non-local and they intertwine between physical states in the chiral Hilbert space. The corresponding symmetry algebras in the chiral model are also determined.

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In this conference, we present a summarised version of our recent work[1] on the free field construction of the chiral $SU(N)$ WZW models at level one. The main motivation is to understand the symmetry structures and the formulation of chiral WZW models which contain either right-moving or left-moving degrees of freedom. The approach is to follow the one proposed in [2] and to study the chiral factorisation from the quantisation of the WZW model compactified on a circle and associated with a simple Lie group G at level k. In particular, the quantum commutation of the chiral vertex operators is derived in terms of an exchange relation[1-3] given by a braiding matrix Q. In [1], this matrix is shown to be related to the Racah matrix of $U_t(SL(N))$. In order to formulate the chiral WZW model with a well-defined Hilbert space, it is very instructive to obtain an explicit construction for the chiral vertex operators, e.g. in terms of free quantum fields. It has been known for some time that the symmetry algebras of the level-one model, i.e. level-one $SU(N)$ Kac-Moody algebra, has an unitary realisation in terms of free scalar fields a la Frenkel-Kac and Segal[4]. In this paper, we explain how to extend this construction to obtain the chiral vertex operators.

Let us recap briefly the quantisation of the level-one $SU(N)$ WZW model. Let $g(\tau,x)$, defined on a cylinder $\mathcal{M}$, take values on a Lie group $SU(N)$. The action of this model is conformal invariant at the infra-red fixed point,

$$\mathcal{A}[g] = -\frac{k}{16\pi}\left\{\int_{\mathcal{M}} \mathrm{tr}\left(g^{-1}\partial_\mu g g^{-1}\partial^\mu g\right) d^2x - \frac{2}{3}\int_{\mathcal{B}} \epsilon^{\lambda\mu\nu}\mathrm{tr}\left(\hat{g}^{-1}\partial_\lambda\hat{g}\hat{g}^{-1}\partial_\mu\hat{g}\hat{g}^{-1}\partial_\nu\hat{g}\right) d^3x\right\}. \tag{1}$$

The second term is a topological term obtained by integrating over a three-dimensional manifold $\mathcal{B}$, whose boundary is the cylinder $\mathcal{M}$ and $\hat{g}(\tau,x,y)\in SU(N)$ is defined on $\mathcal{B}$ such that it maps to $g(\tau,x)$ at $y=0$. The level k is set to be one in the following discussion. From the equations of motion, $g(\tau,x)$ can be written as a product of a right-moving group element $U(x^+)$ and a left-moving group element $V(x^-)$ where $x^\pm \equiv \tau \pm x$.

$$g(x^+,x^-) = U(x^+)e^{-iq_\nu\cdot H}V(x^-). \tag{2}$$

In this parameterisation[1], U and V have the diagonal monodromy given by the maximal torus of the group.

$$U(x^+ + 2\pi) = U(x^+)e^{i2\pi\nu\cdot H}, \qquad V(x^- - 2\pi) = e^{-i2\pi\nu\cdot H}V(x^-). \tag{3}$$

Upon quantisation in the gauge-fixed approach, the monodromy momentum ν conjugates to q_ν in (2). In the following, we will discussion mainly the quantisation of the right-moving sector. Similar results can be obtained for the left-moving sector. Under the chiral symmetry transformation, U transforms as the *chiral primary fields* of the Kac-Moody algebra,

$$[J^a_m, U(x^+)] = e^{imx^+}T^aU(x^+), \tag{4}$$

where $T^a \in \{H^i, E^\alpha\}$ is a basis of the generators in $SU(N)$. The current algebra, i.e.

$SU(N)_{k=1}$ Kac-Moody algebra, can be written in the Cartan-Weyl basis as[4,5]

$$
\begin{aligned}
&[J^i_m, J^j_n] = m\delta^{ij}\delta_{m,-n} && i,j = 1,2,..N-1. \\
&[J^i_m, J^\alpha_n] = \alpha^i J^\alpha_{m+n} \\
&[J^\alpha_m, J^\beta_n] = \begin{cases} \epsilon(\alpha,\beta) J^{\alpha+\beta}_{m+n} & \text{if } \alpha+\beta \text{ is a root} \\ \sum_{j=1}^r \alpha^j J^j_{m+n} + m\delta_{m,-n} & \text{if } \alpha+\beta = 0 \\ 0 & \text{otherwise.} \end{cases}
\end{aligned} \tag{5}
$$

The structure constants, $\epsilon(\alpha,\beta)$, are normalised to be ± 1.

Beside the monodromy and the transformation laws mentioned above, the quantum commutation among U's is specified by a braiding matrix $Q(\nu)$; e.g. in the tensor notation, $U_1 \equiv U \otimes \mathcal{I}$ and $U_2 \equiv \mathcal{I} \otimes U$,

$$
U_1(x^+)U_2(y^+) = U_2(y^+)U_1(x^+)Q\left(x^+ - y^+, \nu\right). \tag{6}
$$

The braiding matrix Q is solved explicitly in the fundamental representation in [1] for all level k. It is written in terms of a parameter $t \equiv e^{i\chi(k)}$ and $\eta(x) \equiv 2[x]+1$ where $[x]$ denotes the maximal integer less than x.

$$
\begin{aligned}
Q(x) =& t^{(\frac{N-1}{N})\eta(x)} \left(1 \otimes 1 - \sum_{\alpha\in\Phi} E_\alpha E_{-\alpha} \otimes E_{-\alpha} E_\alpha\right) \\
&+ t^{-\frac{1}{N}\eta(x)} \sum_{\alpha\in\Phi} \cos\theta(\alpha\cdot\nu) E_\alpha E_{-\alpha} \otimes E_{-\alpha} E_\alpha \\
&- t^{-\frac{1}{N}\eta(x)} \sum_{\alpha\in\Phi} t^{-\frac{\alpha\cdot\nu}{\chi}\eta(x)} \sin\theta(\alpha\cdot\nu) E_\alpha \otimes E_{-\alpha},
\end{aligned} \tag{7}
$$

$$
\text{where} \qquad \sin\theta(\alpha\cdot\nu) = \frac{1}{[\frac{\alpha\cdot\nu}{\chi}]_t}, \qquad \text{and} \qquad [A]_t \equiv \frac{t^A - t^{-A}}{t - t^{-1}}.
$$

For $t \neq \pm 1$, Q can be related to the Racah coefficients[6,7] of $U_t(SL(N))$. In the limit of $t = \pm 1$, Q is proportional to the Racah coefficients of $SU(N)$. Thus, for a given parameter t, the braiding matrix implies that there is an additional symmetry, $U_t(SL(N))$, which acts on U from the right-hand side. Whilst, the affine current acts on U from the left-hand side as in (4). Following [1,8,9], we call $U(x^+)$ in (6) the *chiral vertex operators*. According to the state/vertex operator correspondence, one can obtain the entire chiral Hilbert space from U and J. It is therefore important to understand how U transforms under this additional symmetry explicitly. The main result of this paper is to answer this problem by constructing the chiral vertex operators U explicitly in terms of free fields.

Let us briefly review the unitary realisation of (5) a la Frenkel-Kac and Segal[4] in terms of free scalar fields. Further details are given in [5]. Let us introduce $N-1$ right-moving scalar fields $\phi^j(z)$ and expand it on a complex plane with $z = e^{ix^+}$,

$$
\begin{aligned}
&\phi^j(z) = q^j - ip^j \ln z + i \sum_{n\neq 0, \in \mathbb{Z}} \frac{1}{n}\phi^j_n z^{-n}, \qquad j = 1,2,...N-1; \\
&\phi^{j\dagger}_n = \phi^j_{-n}, \qquad [\phi^i_m, \phi^j_n] = m\delta_{m,-n}\delta^{ij}, \qquad p^{j\dagger} = p^j, \qquad [q^i, p^j] = i\hbar\delta^{ij}.
\end{aligned} \tag{8}
$$

For each root β of $SU(N)$, we define a normal ordered vertex operator,

$$
\begin{aligned}
\mathcal{O}^\beta(z) &\equiv z^{\beta^2/2} \times \exp(i\beta\cdot\phi(z)) \times \\
&= z^{\beta^2/2} \exp(-\beta\cdot\sum_{n<0} \frac{1}{n}\phi_n z^{-n}) \exp(i\beta\cdot q + \beta p \ln z) \exp(-\beta\cdot\sum_{n>0} \frac{1}{n}\phi_n z^{-n}).
\end{aligned} \tag{9}
$$

Then, according to [4], the current generators can be realised as follows.

$$
\begin{aligned}
&J^\alpha(z) = \mathcal{O}^\alpha(z) e^{-i\alpha\cdot q}\hat{C}_\alpha, && \forall \ \alpha \ \text{ is a root}, \\
&J^j(z) = iz\partial_z\phi^j(z), && j = 1,2,..N-1.
\end{aligned} \tag{10}
$$

Conformal symmetry is ensured by the Virasoro generators which can be obtained from the affine currents according to the Sugawara construction,

$$
L(z) = \frac{1}{2(1+N)} \times \sum_{a=1}^{N^2-1} J^a(z) J^a(z) \times. \tag{11}
$$

In (10), we have introduced the cocycle $\hat{C}_\alpha$ defined on the root lattice, Λ_R, in order to obtain the commutators among the currents using the contour integration technique[5]. These operators obey the following cocycle condition,

$$
\begin{aligned}
&\hat{C}_\alpha\hat{C}_\beta = S(\alpha,\beta)\hat{C}_\beta\hat{C}_\alpha = \epsilon(\alpha,\beta)\hat{C}_{\alpha+\beta}, \\
\text{where} \qquad &S(\alpha,\beta) = e^{i\pi\alpha\cdot\beta}, \qquad \forall \alpha,\beta \in \Lambda_R.
\end{aligned} \tag{12}
$$

The structure constant $\epsilon(\alpha,\beta)$ is determined by the symmetry factor $S(\alpha,\beta)$ according to the following consistency requirements,

$$
\epsilon(\alpha,\beta)\epsilon(\alpha+\beta,\gamma) = \epsilon(\alpha,\beta+\gamma)\epsilon(\beta,\gamma), \tag{13a}
$$

$$
\epsilon(\alpha,\beta) = S(\alpha,\beta)\epsilon(\beta,\alpha), \tag{13b}
$$

$$
S(\alpha,\beta)S(\gamma,\beta) = S(\alpha+\gamma,\beta), \tag{13c}
$$

$$
S(\alpha,\beta) = S^{-1}(\beta,\alpha). \tag{13d}
$$

These conditions determine $\hat{C}$ up to some gauge transformation.[5,10] In general, without introducing new degrees of freedom, one can construct $\hat{C}$ explicitly as functions of the zero modes p, q of the free scalar fields.

We are now ready to extend these results to construct the chiral primary fields. In order to achieve this aim, it is necessary to introduce a finite number of additional degrees of freedom into the chiral sector. The reason is because we need to extend the cocycles to the weight lattice and we are not able to solve (13) unless extra degrees of freedom are introduced[1]. To begin with, we consider $U(z)$ as a $N \times N$ matrix in the fundamental representation. The matrix indices can be denoted by the weights in the representations.

They are normalised to be $\lambda^2 = (N-1)/N$. The conformal weight of U in this representation is given by $\Delta_N = (N^2-1)/2N(1+N)$. From the transformation of $U(z)$ in (4) as a chiral primary field of the Kac-Moody algebra, we observe that the z-dependent parts of $U_{rs}(z)$ should be $\mathcal{O}_{\lambda_r}(z)$. Then, the solution for (4) has the following form,

$$\hat{U}_{rs}(z) = \overset{\times}{\underset{\times}{}} e^{i\lambda_r\cdot\phi(z)} \overset{\times}{\underset{\times}{}} e^{-i\lambda_r\cdot q_\phi}\hat{C}^{(s)}_{\lambda_r}. \tag{14}$$

For each weight λ_s, the cocycle $\hat{C}^{(s)}_{\lambda_r}$ is defined on a weight lattice Λ_W. However, there exists no consistent solution of (13) on Λ_W. One way of resolving this problem is to introduce $(N-1)$ new pairs of harmonic oscillators, $[\overline{q}^i, \overline{p}^j] = i\hbar\delta^{ij}$, so that the cocycle is extended to $\Lambda_W \times \Lambda_W$. These new degrees of freedom commute with the right-moving Kac-Moody current and they can be thought as the replacements of the zero-modes in the left-moving vertex operator[1] $V(x^-)$.

$$\hat{C}_{(\lambda_r,\lambda_s)} \equiv e^{i(\lambda_r q + \lambda_s \overline{q})} C_{(\lambda_r,\lambda_s)}(p,\overline{p}), \qquad \forall \quad (\lambda_r,\lambda_s) \in \Lambda_W \times \Lambda_W. \tag{15}$$

The symmetry factors which satisfy the consistency conditions in (13) on the new lattice $\Lambda_W \times \Lambda_W$ are given by

$$S(\vec{\mu},\vec{\nu}) = e^{i\pi(\mu\cdot\nu - \overline{\mu}\cdot\overline{\nu})}, \qquad \forall \quad \vec{\mu},\vec{\nu} \in \Lambda_W \times \Lambda_W. \tag{16}$$

Once the symmetry factor is specified, we can determined $\epsilon(\vec{\mu},\vec{\nu})$ from $S(\vec{\mu},\vec{\nu})$ using (13b). Then, the cocycles can be constructed as functions of the momentum $\vec{p}$ according to

$$\hat{C}_{\vec{\lambda}} \equiv e^{i\vec{\lambda}\cdot\vec{q}}\epsilon(\vec{\lambda},\vec{p}). \tag{17}$$

This determines the cocycle completely up to the following gauge transformation[5,10],

$$\begin{cases} \hat{C}_{\vec{\mu}} & \longrightarrow \quad t(\vec{\mu})\hat{C}_{\vec{\mu}}\ ; \\ \epsilon(\vec{\mu},\vec{\nu}) & \longrightarrow \quad \frac{t(\vec{\mu})t(\vec{\nu})}{t(\vec{\mu}+\vec{\nu})}\epsilon(\vec{\mu},\vec{\nu}), \end{cases} \tag{18}$$

where $t(\vec{\mu}) = t(\mu)\overline{t}(\overline{\mu})$ denote some non-zero scaling constants. Subsequently, the solution for the chiral primary field in (4) is

$$\hat{U}_{rs}(z) = \overset{\times}{\underset{\times}{}} e^{i\lambda_r\cdot\phi(z)} \overset{\times}{\underset{\times}{}} e^{i\lambda_s\cdot\overline{q}} C_{(\lambda_r,\lambda_s)}(p_\phi,\overline{p}). \tag{19}$$

The advantage of having such an explicit construction is that we can determine the local properties of the chiral primary fields directly. In fact, we find that they obey the following braiding relation,

$$\begin{aligned} &\hat{U}_1(z)\hat{U}_2(w) = \hat{U}_2(w)\hat{U}_1(z)R(\arg(z/w)), \qquad \text{for} \quad |z| > |w|, \\ \text{where} \quad &R(x) = \exp\left\{i\pi H_1\cdot H_2\eta(x)\right\}. \end{aligned} \tag{20}$$

For $\eta(x) = 1$, we can identify $R(x)$ as the R-matrix[11] in the fundamental representation of $U_t(SU(N))$ with $t = e^{i\pi}$. By denoting the deformation parameter with $t = q^{\frac{1}{2}}$ rather than with q as in [11], the R-matrix is

$$R(t) = t^{-\frac{1}{N}}\left\{t\sum_{r=1}^{N} e_{rr}\otimes e_{rr} + \sum_{r\neq s=1}^{N} e_{rs}\otimes e_{rs} + (t-t^{-1})\sum_{r>s=1}^{N} e_{rs}\otimes e_{sr}\right\}, \quad t = e^{i\pi}. \tag{21}$$

Since $t = -1 \in \mathbb{R}$, we replace $U_t(SL(N))$ by $U_t(SU(N))$. The braiding parameter agrees with the one obtained in [1] for $k = 1$. However, the braiding relation given by the R-matrix in (20) is diagonal. Thus, $\hat{U}(z)$ differs from the chiral vertex operator $U(z)$ defined in (6) and (7).

Because $U(z)$ is also a chiral primary field, it can differ from $\hat{U}(z)$ in (19) only by the zero-modes. In other words, $U(z)$ can be obtained by multiplying $\hat{U}(z)$ with a N-dimensional matrix, A, which commutes with the Kac-Moody currents. In particular, let the matrix A depend on the momentum, $\overline{p}$, and denote $\hat{A} \equiv e^{iH\overline{q}}A(\overline{p})$. The chiral vertex operator obeying the braiding relation given by Q in (7) can be written as

$$U(z) \equiv e^{i\phi(z)H}e^{-iq_\phi H}\hat{C}e^{-i\overline{q}H}\hat{A}, \tag{22}$$

provided that $\hat{A}$ satisfies the following equation,

$$R(t)\hat{A}_1\hat{A}_2 = \hat{A}_2\hat{A}_1 Q(\nu,t). \tag{23}$$

This is nothing but the operator identity of the IRF-Vertex transformation[8,9,11,12] for $U_t(SL(N))$. This transformation relates the R-matrix with the braiding (Racah) matrix of $U_t(SL(N))$ via the Wigner-matrix, $\hat{A}$. This interpretation is confirmed by the results in [1,12]. One can easily verify that if (23) holds for $R(x,t)$ and $Q(x,\nu,t)$ when $\eta(x) = 1$, then it will also hold for all x.

For $U_t(SL(2))$, this operator identity had been realised explicitly for all t in [12]. It is shown that $\hat{A}$ depends on two pairs of harmonic oscillators: $(\overline{q},\overline{p})$ and (q_ν,ν), where ν is the monodromy momentum in the braiding matrix Q. They are independent degrees of freedom in the sense that they commute with the Kac-Moody current. We will explain more about the meaning of ν later on when we discuss the monodromy property of the chiral vertex operator. In the level-one model, we have to take the limit of $t = e^{i\pi}$ in the Wigner operator obtained for all t in [12]. This involves taking $[Y]_t = Yt^{(Y-1)}$ for $Y \in \mathbb{Z}$. The result is given as follows.

$$\hat{A} \equiv e^{i\frac{\sigma_3}{\sqrt{2}}\overline{q}}\begin{pmatrix} \sqrt{\frac{\nu+\overline{p}}{2\nu}} & -e^{-i\frac{\pi}{\sqrt{2}}(\nu+\overline{p})}\sqrt{\frac{\nu-\overline{p}}{2\nu}} \\ e^{i\frac{\pi}{\sqrt{2}}(\nu+\overline{p})}\sqrt{\frac{\nu-\overline{p}}{2\nu}} & \sqrt{\frac{\nu+\overline{p}}{2\nu}} \end{pmatrix} e^{i\frac{\sigma_3}{\sqrt{2}}q_\nu}. \tag{24}$$

Thus, we have completed the construction of the chiral vertex operator $U(z)$ in (22). Note that U is unitary because both $\hat{C}$ and $\hat{A}$ are unitary.

Subsequently, one expects the symmetry algebra $U_{-1}(SU(2))$ acts on the chiral vertex operator through $\hat{A}$. Because the deformation parameter is $t = -1$, this algebra is isomorphic to $SU(2)$. According to [12], its generators can be realised as

$$S^3 \equiv \frac{1}{\sqrt{2}}\overline{p}, \qquad S^{\pm} \equiv e^{\pm i\sqrt{2}\overline{q}} t^{\mp\frac{\nu}{\sqrt{2}}} \sqrt{\left(\frac{\nu}{\sqrt{2}}\right)^2 - \left(\frac{\overline{p}}{\sqrt{2}} \pm \frac{1}{2}\right)^2}. \tag{25}$$

Under this algebra, $\hat{A}$ transforms according to

$$\begin{aligned} t^{S_3}\left(A_{+b}, A_{-b}\right) &= \left(A_{+b}, A_{-b}\right) t^{S_3} t^{\frac{\sigma_3}{2}}, \\ S^{\pm}\left(A_{+b}, A_{-b}\right) &= \left(A_{+b}, A_{-b}\right)\left(S^{\pm} t^{\mp\frac{1}{2}\sigma_3} + \sigma_{\pm} t^{\pm S_3}\right). \end{aligned} \tag{26}$$

This transformation can be formulated in a covariant way in terms of the coproducts of $U_{-1}(SU(2))$ as in [12]. Furthermore, the quadratic Casimir operator for (25) is given by $(\frac{\nu}{\sqrt{2}})^2 - \frac{1}{4}$. Thus, the irreducible representations $\overline{V}_j$ of $U_{-1}(SU(2))$ are labelled by the eigenvalues of of the monodromy momentum, $j = \frac{\nu}{\sqrt{2}} - \frac{1}{2}$, for j a positive integer or a positive half-integer. Like the irreps of $SU(2)$, the dimension of $\overline{V}_j$ is $(2j+1)$. Consequently, U acts on the irreducible representations of $U_{-1}(SU(2))$ like an intertwining operator. Notice that because the quantum group structure is rather trivial in this case, there is no truncation on the spin j.

Let us now discuss the monodromy property of the chiral vertex operator. From the explicit construction of U in (22), we have

$$U_{rs}(ze^{i2\pi}) = U_{rs}(z)e^{i2\pi p_\phi \cdot \lambda_r} \tag{27}$$

On the other hand, from the definition of monodromy momentum ν in (3), it has the following monodromy:

$$U_{rs}(ze^{i2\pi}) = U_{rs}(z)e^{i2\pi\nu\cdot\lambda_s}. \tag{28}$$

Thus, these two equations impose a constraint between the monodromy momentum ν and the free-field momentum p_ϕ.

$$e^{i2\pi p_\phi\cdot\lambda_r} = e^{i2\pi\nu\cdot\lambda_s}, \qquad \forall\lambda_r, \lambda_s = \pm\frac{1}{\sqrt{2}}. \tag{29}$$

This condition is manifestly satisfied when the spin of the $U_{-1}(SU(2))$ representation differs from the spin of the Kac-Moody representation by an integer. Thus, the Hilbert space for the right-moving $SU(2)_{k=1}$ WZW model is

$$\mathcal{H}_R = \left(H_0 \otimes (\oplus_{j=0,\mathbb{Z}_+}\overline{V}_j)\right) \bigoplus \left(H_{\frac{1}{2}} \otimes (\oplus_{j+\frac{1}{2}\in\mathbb{Z}_+}\overline{V}_j)\right). \tag{30}$$

These results for the chiral vertex operators in the fundamental representation can be generalised to other basic representations of $SU(N)$. In particular, (22) and (23) apply

to these representations as well. We can obtain all the states in the chiral Hilbert space by applying the chiral vertex operators and the affine current generators to the vacuum according to the state/vertex operator correspondence. Consequently, this determines the chiral Hilbert space to be

$$\mathcal{H}_R = \bigoplus_{\lambda=0,\text{minimal}} H_\lambda \otimes \left(\oplus_{\mu\in\Lambda_\lambda}\overline{V}_\mu\right), \tag{31}$$

where H_λ's are the minimal representations of the right-moving $SU(N)_{k=1}$ Kac-Moody algebra and μ's denote all the highest weights of $U_{-1}(SU(N))$ which are in the same weight lattice as λ. The Hilbert space of the left-moving sector can be constructed in a similar way using the left-moving free scalar fields and $(N-1)$ additional pairs of harmonic oscillators.

$$\mathcal{H}_L = \bigoplus_{\lambda=0,\text{minimal}} (\oplus_{\mu\in\Lambda_\lambda} V_\mu) \otimes \overline{H}_\lambda \tag{32}$$

These new degrees of freedom correspond to $U_{-1}(SU(N))$ do not appear in the original Wess-Zumino models and they should be gauged away when we "join" the left-moving and the right-moving sectors together.

$$\mathcal{H}_{WZW} = \bigoplus_{\lambda=0,\text{minimal}} H_\lambda \otimes \overline{H}_\lambda \tag{33}$$

The formulation of this gauging process is nontrivial but it is certainly very interesting. The following message is fairly clear. In the process of factorising the chiral Hilbert space, we need to keep the same number of zero-modes as the original un-factorised theory. The additional symmetry associated with these zero-modes is hidden in the original non-chiral model and thus plays the role of an internal symmetry of the WZW model.

To conclude, the free field construction of the chiral vertex operators for the $SU(N)_{k=1}$ WZW model described in this paper gives a explicit description of how to factorise the model into two chiral sectors. A finite number of new degrees of freedom are introduced into the chiral sectors. Consequently, the chiral Hilbert space acquires an additional symmetry described by $U_{-1}(SU(N))$. This result is different from the factorisation discussed in the literatures[8,9,13], where the internal symmetry is proposed to be $U_t(SL(N))$ with $t = e^{i\frac{\pi}{1+N}}$. In this latter case, an explicit realisation of the chiral vertex operators is still lacking.

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