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THE HIGGS MECHANISM AS A PROBLEM OF INEQUIVALENT REPRESENTATIONS

II. Mass Generation in the Electroweak Interactions

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ABSTRACT

We present a model of electroweak interactions which shows spontaneous symmetry breaking without involving an additional scalar field. In this model, the Higgs- and Yukawa- sectors of the Glashow-Weinberg standard model are replaced by a non-local interaction term, constructed from gauge-invariant strings. This interaction term may be understood as an implementation of Gauss' law in a local quantum field theory describing charged scattering states. Then the origin of the plethora of parameters in the standard model becomes obvious. The starting point of our proposal is the assumption that elements which lie in the weak closure of the electroweak field algebra, can get involved in the dynamical description of electroweak interactions. To motivate this assumption, we recall Haag's analysis of the Bardeen-Cooper-Schrieffer model of superconductivity as well as a recent work which relates the vacuum expectation value of the Higgs field to an element of the weak closure of the field algebra.

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0. INTRODUCTION

In a companion paper, hereafter referred to as [1], we have shown that the vacuum expectation value of a bounded operator $\phi(f)$ which is charged like the Higgs field. may be understood as the expectation value of an element of the center of the field algebra. More precisely, we considered the quantity

$$\hat{\phi}(f) = \lim_{\mathcal{O}\to\infty}\frac{1}{|\mathcal{O}|}\int_{\mathcal{O}} d^4x\alpha_x(\phi(f)), \tag{0.1}$$

where α_x denotes the space-time translation automorphism, $|\mathcal{O}|$ is the volume of the bounded space-time region $\mathcal{O}$, and the set of elements $\phi(f)$ form a local field algebra $\mathcal{A}(\mathcal{O})$, defined for all functions $f \in \mathcal{S}(R^4)$ with compact support in the space-time region $\mathcal{O}$. Here $\mathcal{A}$ is the set-theoretic union $\bigcup_{\mathcal{O}}\mathcal{A}(\mathcal{O})$ of all local algebras $\mathcal{A}(\mathcal{O})$ together with its norm closure and the complete field algebra $\mathcal{A}_c$ contains the gauge-dependent matter fields $\phi(f)$ as well as all other local bounded fields of the theory under consideration. It has been shown in [1] that $\hat{\phi}(f)$ commutes with all elements of the algebra $\mathcal{A}_c$. Furthermore, $\hat{\phi}(f)$ lies in the bicommutant $\pi(\mathcal{A}_c)''$ for a class of direct integral representations which correspond to the usual path integral formulations. Indeed, for the direct integral representations $(\mathcal{H},\pi)$ of the algebra $\mathcal{A}_c$ on the Hilbert space $\mathcal{H}$ with

$$\mathcal{H} = \int_{\Lambda}^{\oplus} d\mu(\lambda)\mathcal{H}_\lambda, \tag{0.2}$$

$$\pi = \int_{\Lambda}^{\oplus} d\mu(\lambda)\pi_\lambda, \tag{0.3}$$

and the cyclic translationally invariant (vacuum) state

$$\psi = \int_{\Lambda}^{\oplus} d\mu(\lambda)\psi(\lambda), \tag{0.4}$$

it has been shown that [1]

$$\hat{\phi}(f) = \int_{\Lambda}^{\oplus} d\mu(\lambda)\langle\psi(\lambda), \pi_\lambda(\phi(f))\psi(\lambda)\rangle I_\lambda. \tag{0.5}$$

Here Λ is a compact measure space with Borel measure μ, I_λ is the identity and $\psi(\lambda)$ is a cyclic vacuum vector of the irreducible representation space $\mathcal{H}_\lambda$ for $\pi_\lambda(\mathcal{A}_c)$. (*,*)

is the scalar product on $\mathcal{H}$ and the indefinite inner product $\langle *, * \rangle = (*, \eta *)$ with metric operator η is positive definite on a quotient space of physical states obtained by the Gupta-Bleuler construction.

In this work, we discuss the possible relevance of these results for the problem of mass generation in models of the electroweak interaction. Therefore we adopt a heuristic use of terminology from operator algebra theory and we accept a concordant lack of rigour, which is not aimed at in this paper. Especially, we do not distinguish bounded and unbounded operators, in particular, we identify the Higgs field with the bounded operator $\phi(f)$. We start our discussion from the assumption that it is the vacuum expectation value of the object $\hat{\phi}(f)$ which gives masses to the fermions and gauge bosons in the Glashow-Weinberg (GW) standard model of electroweak interactions. In other words, we consider the possibility that elements of the bicommutant of the corresponding field algebra get involved in the dynamical description of electroweak interactions. There are many examples in the literature which indicate that this possibility should be taken seriously: in non-relativistic theories with long-range interactions, it is known that the time evolution automorphism α_t is not defined on the norm-closed algebra $\mathcal{A}$, but on its weak closure in a suitable representation.[4] In this context, one should mention in particular the work of Morchio and Strocchi who emphasized the importance of "variables at infinity" in spontaneous symmetry breaking. In the non-local Coulomb gauge formulation of two-dimensional electrodynamics and of the Stückelberg-Kibble model, they have demonstrated the occurrence of elements of the center of the weakly extended field algebra ("variables at infinity") in the dynamical description.[4] In both models, the mass-generation via spontaneous symmetry breaking can be shown to be a consequence of "variables at infinity" rather than of the occurrence of an additional scalar field.

In all works we are aware of, elements of the center of the field algebra play a rôle in non-local gauges only, i.e., the fact that the dynamics is not defined on the norm-closed algebra is a direct consequence of the non-locality of the interaction term. However, in relativistic field theories with Gauss law, local fields cannot create charged scattering

states from the vacuum.[4,5] Hence even in strictly local approaches to such theories, we cannot discard *a priori* the possibility that the dynamics is defined only on the weak closure of the field algebra in a suitable representation and that elements of this weak closure appear in the Lagrangian and/or the equations of motion. In the case of the electroweak interactions, these considerations point to an alternative approach to the problem of mass generation: the quasi-local object $\hat{\phi}(f)$ lies in the weak closure of the field algebra for suitable representations and it gives masses to the fermions and gauge-bosons, i.e., it gets involved in the dynamical description of the system. However, although we have used ϕ to construct $\hat{\phi}$, the phenomenological relevance of the dynamical field ϕ is unclear. The corresponding Higgs particle has not been observed. Hence we might look for other objects in the center of the weakly closed field algebra $\pi(\mathcal{A}_c)''$, which have the same properties as $\hat{\phi}$ and do not involve the dynamical Higgs field ϕ.

Here we propose a model of electroweak interactions which shows all features of such an approach. It starts from a gauge-invariant hamiltonian without involving an additional scalar field or explicit mass terms. Variables at infinity arise in its dynamical description. In arbitrary irreducible represenations the gauge symmetry is broken and all fermion and boson mass terms of the electroweak standard model are obtained. This proposal is based on some mathematical analogies with the Bardeen-Cooper-Schrieffer (BCS) model of superconductivity.[7] Hence this work is organized as follows:

In section I, we review the mathematical structure of the BCS model, following the analysis of Haag.[7] The global $U(1)$ gauge symmetry is spontaneously broken in irreducible representations of this model without giving rise to massless excitations. Following Morchio and Strocchi, we explain in I.2 why Goldstone's theorem does not apply to this case.[4] The emphasis of our discussion, however, is on the dynamical equivalence of different hamiltonians: the hamiltonian H_π is a representation-dependent object which implements the one-parameter automorphism α_t in a given representation $(\mathcal{H}, \pi)$ of the field algebra $\mathcal{A}_c$, such that

$$\pi(\alpha_t(A)) = e^{iH_\pi t}\pi(A)e^{-iH_\pi t}, \tag{0.6}$$

where $A \in \mathcal{A}_c$. The essential point is now that (0.6) determines the hamiltonian only up to an element in the commutant of the field algebra. Hence, two hamiltonians H_I and $H_I{}'$ are said to be dynamically equivalent, if

$$[H_\pi - H_\pi{}', \pi(A)] = 0 \tag{0.7}$$

for all $A \in \mathcal{A}_c$. Since the hamiltonians H_π and H'_π lie in the weak closure of the field algebra, it follows from (0.7) that $[H_\pi, H_\pi{}'] = 0$. In the BCS-model, Haag observed that a gauge-invariant quartic interaction term is replaced by a gauge-dependent quadratic one in a dynamically equivalent hamiltonian. That means that the changes which we are allowed to make to H_π without changing $\pi(\alpha_t(A))$, can be non-trivial to such an extent that the gauge symmetry might be broken.

This observation suggests the question: *Is it possible to write down a gauge-invariant interaction term, containing only the gauge fields and fermion fields of the electroweak theory, which in all irreducible (vacuum) representations leads to (gauge-dependent, quadratic) mass terms for the gauge-bosons?* In section II.1, we make the first step to answering this question: We argue that a certain type of gauge-invariant strings is the unique choice for a gauge-invariant interaction term which satisfies a certain set of additional criteria. Then the idea of dynamical equivalence of different hamiltonians is used extensively to analyze the interaction term obtained from these criteria. In section II.2, we show that in all irreducible represenations this interaction term is dynamically equivalent to a gauge-dependent term containing all mass terms of the electroweak standard model.

Section III is devoted to a discussion of several aspects of this proposal. We present a physical interpretation of the interaction term used as an implementation of Gauss' law which is difficult to embed in the framework of a local quantum field theory. From this discussion, it will become apparent that the concepts of mass and charge do not enter our proposal independent of each other. Indeed, the mass of a particle appears to be in this formulation a consequence of its charge relative to a gauged symmetry. Finally, this proposal for which we coin the notion "algebraic symmetry breaking", is compared with the mechanisms of spontaneous and dynamical symmetry breaking.

This work has been carried out in the believe that despite all difficulties, it is necessary to find models which interpolate between the phenomenological descriptions of lagrangian quantum field theory and the axiomatic concepts of algebraic quantum field theory. The model presented here supports the view that the need for a Higgs field in the GW standard model arises as a consequence of certain overidealizations made in the framework of a local lagrangian quantum field theory.

I.1. The solvability of the BCS-model

The BCS-model starts from the phenomenological hamiltonian K of an electron gas in a solid [7]

$$K = K_0 + K_I. \tag{1.1}$$

In Haag's formulation, this hamiltonian is the infinite volume limit of the hamiltonians $K_0(V)$ and $K_I(V)$, defined for the finite volume V: [7]

$$K_0(V) = \sum\nolimits_{\alpha=1,2} \int_V d^3x \psi^*_\alpha(x)[\frac{1}{2m}\hat{p}^2 - \mu]\psi_\alpha(x) + h.c., \tag{1.2}$$

$$K_I(V) = \frac{1}{V}\int \psi^*_1(x)\psi^*_2(x+z)v(z,z')\psi_2(x'+z')\psi_1(x')d^3x d^3x' d^3z d^3z'. \tag{1.3}$$

Here, ψ_α, ψ^*_α denote annihilation and creation operators for electron states with spin up ($\alpha = 1$) or down ($\alpha = 2$), satisfying time zero anticommutation relations. These symbolic quantities are the usual short-hand for the elements of an algebra of bounded operators*

$$\psi(f) = \int \psi(x)f(x)d^3x, \tag{1.4}$$

the weighted averages of ψ and their adjoints with weight functions f which are square integrable. They satisfy standard anti-commutation relations at fixed time t:

$$[\psi_\alpha(x), \psi_\beta(y)]_+ = 0, \tag{1.5}$$

* The $\psi(f)$ are bounded operators since they are Fermi fields. Hence, no additional assumption is needed in this case.

$$[\psi_\alpha(x), \psi^*_\beta(y)]_+ = \delta_{\alpha\beta}\delta^{(3)}(x-y). \tag{1.6}$$

$\hat{p}$ is the three-dimensional momentum operator in the presence of a magnetic field and the chemical potential μ is a free parameter of the theory, fixing the particle density of the ground state to its observed finite value. The function $v(z, z')$ which characterizes the attractive interaction, is assumed to decrease rapidly for large z or z' without having any $z - z'$-dependence.

Here, we do not discuss the question whether this model is a realistic model of superconductivity. Clearly, the interaction term (1.3) shows a global $U(1)$ gauge invariance while the local gauge invariance is broken explicitly. However, we shall be interested in the mathematical properties only.

The most amazing of these properties is the solvability of this model for arbitrary interactions $v(z, z')$, satisfying

$$\int | v(z, z') | d^3z d^3z' < \infty. \tag{1.7}$$

As first observed by Haag,[7] this general solvability is due to the fact that in the infinite volume limit products of an even number of Fermi fields averaged over the volume V, commute with all elements of the field algebra. The simplest example of such an object in the center of the field algebra is:

$$\varphi(z') = \lim_{V\to\infty} \frac{1}{V} \int \psi_2(x' + z')\psi_1(x')d^3x' \tag{1.8}$$

From (1.8) it follows that

$$\Delta(z) = \lim_{V\to\infty} \frac{1}{V} \int\int v(z, z')\psi_2(x' + z')\psi_1(x')d^3x'd^3z' \tag{1.9}$$

lies in the center of the field algebra and is a c-number in all irreducible representations. The interaction hamiltonian $K_I(V)$ involves two undamped integrations (over x and x') and only one factor $\frac{1}{V}$. Following Haag[7], $K_I(V)$ may be simplified by working out the commutators of K_I with $\psi(f)$ and $\psi^*(f)$:

$$\lim_{V\to\infty}[K_I(V), \psi_1(y)] = -\int \Delta(z)\psi^*_2(y + z)dz. \tag{1.10}$$

Similarly, the commutators of K_I with ψ_2, ψ^*_1 and ψ^*_2 are always linear in ψ or ψ^*. Hence, in all representations K_I can be replaced by

$$K'_I = \int [\Delta(z)\psi^*_1(x)\psi^*_2(x + z) + \Delta^*(z)\psi_2(x + z)\psi_1(x)]d^3xd^3z. \tag{1.11}$$

The Fourier transform of this expression reads

$$K'_I = \frac{1}{2\pi^3} \int [\Delta(p)\psi^*_1(-p)\psi^*_2(p) + \Delta^*(p)\psi_2(p)\psi_1(-p)]dp. \tag{1.12}$$

Solving the model is then a textbook problem. It is sufficient to diagonalize the hamiltonian $K = K_0 + K'_I$ with a Bogoliubov transformation.

For our purpose however, we are more interested in the mathematical structure of the interaction term K_I than in the well-known solution of the model. The crucial point is that the nature of symmetry breaking in the BCS-model becomes apparent in Haag's derivation. The interaction term (1.3) is obviously gauge-invariant under the global $U(1)$ gauge transformation which changes the phases of the fields ψ and ψ^*:

$$\psi \longrightarrow e^{i\alpha}\psi, \tag{1.13a}$$

$$\psi^* \longrightarrow e^{-i\alpha}\psi^*. \tag{1.13b}$$

In (1.11), this gauge symmetry is lost in all irreducible representations where $\Delta(z)$ does not transform under (1.13). The reason is that the formally gauge-dependent quantity $\Delta(z)$ is an element of the center of the field algebra. Hence, $\Delta(z)$ is a c-number in all irreducible representations, i.e. the automorphism implemented in these representations by the gauge group acts trivially on $\Delta(z)$. The gauge symmetry is broken by the specification of an irreducible representation.

I.2. Spontaneous symmetry breaking without Goldstone bosons

The BCS-model discussed in the last subsection shows a breaking of the global gauge invariance without massless excitations appearing in the spectrum. Following Morchio and Strocchi we argue here that this does not contradict Goldstone's theorem.[4] We start from a symmetry group G of the physical system which is given by a group of automorphisms $\{\alpha_g; g \in G\}$ on $\mathcal{A}_c$. We assume that there exists a conserved local current $j^\mu(x,t)$ corresponding to this symmetry, so that

$$\int_V d^3x[\partial_\mu j^\mu(x,t), A] = \frac{d}{dt}\int_V d^3x[j^0(x,t), A] + \int_{\partial V} dS_i[j^i(x,t), A] = 0 \qquad (1.14)$$

for arbitrary operators A of the physical system. Here, ∂V denotes the boundary of V and the index i goes over all spatial directions. The symmetry is broken in a representation π if there exists an operator $A' \in \pi(\mathcal{A}_c)$ such that

$$\begin{aligned} J(t) &= \lim_{V\to\infty}\langle\psi_0, [Q_V(t), A']\psi_0\rangle \\ &= \lim_{V\to\infty}\langle\psi_0, [Q_V(0), e^{-iH_\pi t}A'e^{iH_\pi t}]\psi_0\rangle \neq 0, \end{aligned} \qquad (1.15)$$

where the time-translation invariance of the vacuum vector has been used. Here, $Q_V(t)$ is the generator of the symmetry transformation given as the spatial integral over $j^0(x,t)$. The main ingredient of Goldstone's theorem is now the observation that $\frac{dJ(t)}{dt} = 0$. Let us briefly sketch a proof by heuristically inserting a complete set of orthonormal states $\{\psi_n\}_n$ in (1.15), thereby obtaining

$$\begin{aligned} J(t) = \sum_n (2\pi)^3\delta^{(3)}(P_n)[&\langle\psi_0, j^0(0)\psi_n\rangle\langle\psi_n, A'\psi_0\rangle e^{-iEt} \\ &-\langle\psi_0, A'\psi_n\rangle\langle\psi_n, j^0(0)\psi_0\rangle e^{iEt}] \neq 0. \end{aligned} \qquad (1.16)$$

Using

$$\begin{aligned} \frac{dJ(t)}{dt} = \sum_n (2\pi)^3\delta^{(3)}(P_n)E_n[&\langle\psi_0, j^0(0)\psi_n\rangle\langle\psi_n, A'\psi_0\rangle e^{-iEt} \\ &+\langle\psi_0, A'\psi_n\rangle\langle\psi_n, j^0(0)\psi_0\rangle e^{iEt}] = 0, \end{aligned} \qquad (1.17)$$

it follows from (1.16) and (1.17) that there exists a state ψ_n such that $\langle\psi_0, A'\psi_n\rangle \times\langle\psi_n, j^0(0)\psi_0\rangle \neq 0$ for which $E_n\delta^{(3)}(P_n) = 0$. ψ_n is obviously a massless state, generated by j^0 from the vacuum. This establishes Goldstone's theorem.

However it follows from (1.14) that $\frac{dJ(t)}{dt} = 0$ if and only if

$$\lim_{V\to\infty}\int_{\partial V} dS_i\langle\psi_0, [j^i(x,0), e^{-iH_\pi t}A'e^{iH_\pi t}]\psi_0\rangle = 0. \qquad (1.18)$$

If the dynamical evolution of the physical system is defined on the norm-closed algebra $\pi(\mathcal{A}_c)$, then $\pi(\alpha_t(A'))$ is a local operator for all times t and (1.18) holds true. However, if H_π contains long range interaction terms, then $e^{-iH_\pi t}A'e^{iH_\pi t}$ exists in the weak closure $\pi(\mathcal{A}_c)''$ but not necessarily in $\pi(\mathcal{A}_c)$. Hence (1.18) does not hold true necessarily, $\frac{dJ(t)}{dt} \neq 0$ and the above argument fails to apply.

We conclude that it is possible to escape the consequences of Goldstone's theorem if the dynamical description of the physical system involves elements which do not lie in the norm-closed field algebra. Especially this is the case for instantaneous long range interactions as the one specified in (1.3). Hence Goldstone's theorem does not apply.

II.1. Gauge-independent strings in the standard model

In the last section we have seen that the gauge-invariant quartic interaction term K_I leads to a gauge-dependent quadratic one in all irreducible representations. The possible relevance of this observation for the theory of electroweak interactions is obvious: the mass-terms of the intermediate vector bosons in the standard model are gauge-dependent quadratic terms too. This suggests the question: *Is it possible to write down a gauge-invariant interaction term, containing only the gauge fields and fermion fields of the electroweak theory, which in all irreducible representations leads to mass terms for the gauge bosons?* The first step in the answer to this question is the most difficult one: it is the choice of a gauge-invariant interaction term. To make this choice unique, we have to invoke further criteria. In what follows, we shall look for gauge-invariant interaction terms T_I with the additional features:

[C1] T_I contains no derivatives.

[C2] T_I may be non-local but it has to be supported in a finite region of space-time only.

[C3] T_I contains both gauge and fermion fields.

To motivate these criteria, we remark the following:

i) The criteria C1 and C2 can only be justified by some arguments of simplicity. We are looking for simple expressions and derivatives might be considered to be complications. Furthermore, C1 seems an obvious requirement if we are looking for renormalizable theories. However, there is no *a priori* reason to discard them as well as there is no a priori reason why we shouldn't investigate infinitely extended objects.

ii) Since we are interested in gauge-invariant terms which might lead to mass-terms for the vector bosons, T_I has to contain gauge fields of course. The only gauge-invariant quantities, containing only gauge fields and satisfying C1 and C2 are the closed Wilson loops $\Psi(C)$ which can be constructed out of the plaquette characters:

$$\Psi(C) = \hat{P}\exp[\oint_C A_\mu dx^\mu]. \tag{2.1}$$

Here, A_μ denotes a gauge field and $\hat{P}$ is the path-ordering operation defined below. We conclude that [C3] discards only these Wilson loop terms from our consideration.

In the abelian case, these three criteria lead to an almost unique choice. Consider the combined gauge transformation of the gauge field A_μ and matter field $\psi(x)$:

$$A_\mu(x) \longrightarrow A_\mu(x) - \partial_\mu \xi(x), \tag{2.2}$$

$$\psi(x) \longrightarrow \psi(x)\exp[ie\xi(x)], \tag{2.3}$$

where the field $\xi(x)$ satisfies

$$\Box\xi(x) = 0. \tag{2.4}$$

These gauge-transformations suggest that an exponential of a gauge field has to be combined with a fermion field. This singles out the gauge-invariant string of the type :

$$\overline{\psi}(x)\hat{P}\exp[ie\int_{\Gamma_{xy}} A_\mu(z(s))\frac{dz(s)^\mu}{ds}ds]\psi(y). \tag{2.5}$$

In what follows, we choose Γ_{xy} to be the geodesic between space points x and y. s denotes the integration variable along this path. To extend this formulation to the non-abelian $SU(2) \times U(1)$ case, let us briefly review the electroweak theory first. The standard model starts from the hamiltonian density

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I, \tag{2.6}$$

where

$$\mathcal{H}_0 = \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\ell}_L i\gamma_\mu D^\mu_L \ell_L + \overline{e}_R i\gamma_\mu D^\mu_R e_R$$
$$+\overline{q}_L i\gamma_\mu D^\mu_L q_L + \overline{u}_R i\gamma_\mu D^\mu_R u_R + \overline{d}_R i\gamma_\mu D^\mu_R d_R \tag{2.7}$$

and $\mathcal{H}_I$ denotes the Higgs and Yukawa sectors of the standard model:

$$\mathcal{H}_I = f_e\overline{\ell}_L\phi e_R + f_u\overline{q}_L\tilde{\phi}u_R + f_d\overline{q}_L\phi d_R + h.c.$$
$$+(D^\mu\phi)(D_\mu\phi) + \lambda(\phi^\dagger\phi - \phi_o^2)^2. \tag{2.8}$$

We consider one lepton family only with up and down quarks u and d, electrons e and electron neutrinos ν. The subscripts L and R denote the left- and right-handed parts. The left-handed doublets

$$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \tag{2.9}$$

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \tag{2.10}$$

transform under $SU(2)\times U(1)$, which is reflected in their covariant derivative

$$D_{L\mu} = \partial_\mu + igA^a_\mu\frac{1}{2}\sigma^a + ig'\frac{Y}{2}B_\mu. \tag{2.11}$$

Here, the Pauli matrices σ^a generate $SU(2)$ transformations, g and g' are the gauge couplings and Y is the hypercharge. The $U(1)$-field B_μ corresponds to the field tensor $F_{\mu\nu}$ while the A^a_μ correspond to the field tensors $G^a_{\mu\nu}$. The covariant derivative for the right-handed singlets, transforming under $U(1)$, is given by

$$D_{R\mu} = \partial_\mu + ig'\frac{Y}{2}B_\mu. \tag{2.12}$$

Equation (2.7) contains the Yukawa mass-terms with couplings f_e,f_u,f_d and the Higgs field ϕ with quartic self-interaction and

$$\tilde{\phi} = i\sigma^2\phi^*. \tag{2.13}$$

For one lepton family, this model already has seven independent parameters (g, g', ϕ_0, λ, f_e, f_u, f_d), the last five of which appear only in H_I.

Now we are able to formulate various electroweak gauge-independent strings. For the right-handed singlets, we again obtain (2.5):

$$\overline{e}_R(x)\exp\left[ig'Y\int_{\Gamma_{xy}} B_\mu(z(s))\frac{dz(s)^\mu}{ds}ds\right]e_R(y). \tag{2.14}$$

For the left-handed doublets, we have to look for the gauge field combinations (cf. (2.11))

$$C_\mu = A^a_\mu\frac{1}{2}\sigma^a + \frac{g'}{g}\frac{Y}{2}B_\mu. \tag{2.15}$$

This leads to an gauge-independent $SU(2)\times U(1)$-string

$$\overline{\ell}_L(x)\hat{P}\exp\left[ig\int_{\Gamma_{xy}} C_\mu(z(s))\frac{dz(s)^\mu}{ds}ds\right]\ell_L(y), \tag{2.16}$$

where the hypercharge Y takes the value of the field $e_R(y)$, $\ell_L(y)$ respectively. Hence, the most minimal expressions of the electroweak theory, satisfying the criteria C1-C3, lead to different objects (2.14) and (2.16) for the right-handed singlets and the left-handed doublets respectively. We might be interested in a more symmetric expression in which right-handed singlets and left-handed doublets are on an equal footing. Such an expression can be obtained e.g. from the following additional property:*

[C4] The electroweak string should be symmetric with respect to particles and antiparticles, i.e. for a field $\psi(x)$ at one end of the string, the field $\overline{\psi}(y)$ at the other end of the string should correspond to the antiparticle(s) of ψ.

This condition is obviously difficult to fulfil in the electroweak theory. The field which describes the antiparticle of a left-handed electron e.g., is $\overline{e}_R$. It does not appear in a doublet and transforms non-trivially under $U(1)$ only. An interaction term like

$$\overline{e}_R(x)\hat{P}\exp\left[ig\int_{\Gamma_{xy}} C_\mu(z(s))\frac{dz(s)^\mu}{ds}ds\right]e_L(y) \tag{2.17}$$

is ill-defined: the exponential is a 2×2 matrix while $\overline{e}_R$ and e_L are one-component fields. To obtain a string which satisfies the additional criterion [C4], we obviously have to alter the mathematical structure of the theory. Here we propose an alteration

* A physical motivation of this criterion is deferred to section III.

which we consider to be the most minimal one: we alter the structure by substituting e_R in (2.7) by a right-handed doublet

$$e_R \longrightarrow \ell_R = \begin{pmatrix}\nu_R\\ e_R\end{pmatrix}, \tag{2.18}$$

which transforms under $U(1)$ only. Remark that this does not affect the physical predictions of the standard model, since the right-handed neutrino decouples from all other particles. Now

$$\overline{\ell}_R(x)\hat{P}\exp\left[ig\int_{\Gamma_{xy}} C_\mu(z(s))\frac{dz(s)^\mu}{ds}ds\right]\ell_L(y) \tag{2.19}$$

is at least well defined though not gauge-invariant. To obtain a gauge-invariant string, we introduce the fermion bilinears

$$\phi_1 = \begin{pmatrix}\nu_L\nu_R\\ e_Le_R\end{pmatrix}, \tag{2.20}$$

$$\phi_2 = \begin{pmatrix}\overline{\nu}_L\nu_R\\ \overline{e}_Le_R\end{pmatrix}. \tag{2.21}$$

Both these fermion condensates lead to gauge-invariant strings

$$S_i = \overline{\phi}_i(x)\hat{P}\exp\left[ig\int_{\Gamma_{x,x+R}} C_\mu(z(s))\frac{dz(s)^\mu}{ds}ds\right]\phi_i(x+R), \tag{2.22}$$

where R is a 3-dimensional vector and $(i=1,2)$. The S_i satisfy criterion C4. We did not find other minimal alterations to the standard model satisfying all four criteria.

II.2. Mass-generation by gauge-invariant strings

Now we might introduce the gauge-invariant quantities of the last section into finite volume interaction terms H_I. Our aim is to analyze these interaction terms with methods similar to those used in section I.1. in the analysis of the BCS hamiltonian. More precisely, we aim at substituting H_I by a simpler, dynamically equivalent expression $H_I{}'$. Here, dynamically equivalent means that $H_I - H_I{}'$ lies in the commutant of the field algebra. Hence, all commutators with $H_I{}'$ are equal to those with H_I, i.e. H_I and $H_I{}'$ contain the same dynamical information.

For such an analysis, we propose the finite volume hamiltonian (for fixed time t)

$$H_I(N) = c\frac{1}{N}\sum_{k=1}^{N}\int_V [S_i(x, x+R\hat{e}_k) + S_i^\dagger(x, x+R\hat{e}_k)]d^3x. \tag{2.23}$$

Here, c is a coupling constant, the strings S_i given in (2.19), are of a fixed finite length R and the unit vectors

$$\hat{e}_k(\theta_k, \varphi_k) = \begin{pmatrix} \sin\theta_k \cos\varphi_k \\ \sin\theta_k \sin\varphi_k \\ \cos\theta_k \end{pmatrix} \tag{2.24}$$

are parametrized by pairs of spherical coordinates (θ_k, φ_k). The interaction term (2.23) averages over strings in N different directions $\hat{e}_k$. We shall be interested in the $N \to \infty$-limit of this expression where the infinitely many directions $\hat{e}_k$ are chosen so that

$$H_I = \lim_{N\to\infty} H_I(N) \tag{2.25}$$

becomes an isotropic expression. The infinite volume limit of $H_I(N)$ plays no important role in what follows. We have chosen (2.23) for our discussion since this expression provides a minimal way to obtain an isotropic hermitian hamiltonian from the gauge-invariant strings (2.22). Clearly, (2.23) has to be seen as a formal expression. For a local operator C_μ, the exponential in (2.22) does not necessarily exist. However, formally it defines the Taylor expansion of a path-ordered product in orders of g:

$$\hat{P}\exp\left[ig\int_{\Gamma_{x,x+R\hat{e}}} C_\mu(z(s))\frac{dz^\mu(s)}{ds}ds\right]$$
$$= \sum_{n=0}^{\infty}(ig)^n\int_0^1 ds_1\int_0^{s_1} ds_2 \ldots \int_0^{s_{n-1}} ds_n C_\mu(z(s_1))\frac{dz^\mu(s_1)}{ds_1}\ldots C_\nu(z(s_n))\frac{dz^\nu(s_n)}{ds_n}, \tag{2.26}$$

where the path $\Gamma_{x,x+R\hat{e}}$, $z(s) \in \Gamma_{x,x+R\hat{e}}$, has been parametrized by $s \in [0,1]$. Remark that only in the abelian case, this object can be symmetrized such that

$$\hat{P}\exp\left[ig'\int_{\Gamma_{x,x+R\hat{e}}} B_\mu(z(s))\frac{dz^\mu(s)}{ds}ds\right] = \sum_{n=0}^{\infty}\frac{(ig')^n}{n!}\left(\int_0^1 ds B_\mu(z(s))\frac{dz^\mu}{ds}\right)^n. \tag{2.27}$$

In what follows, we consider the exponential in S_i as formally given by a Taylor expansion in orders of g. Furthermore we assume that

$$\hat{P}\exp\left[ig\int_{\Gamma_{x+\epsilon\hat{e},x+(R-\epsilon)\hat{e}}} C_\mu(z(s))\frac{dz^\mu(s)}{ds}ds\right]$$
$$\xrightarrow{\epsilon\to 0} \hat{P}\exp\left[ig\int_{\Gamma_{x,x+R\hat{e}}} C_\mu(z(s))\frac{dz^\mu(s)}{ds}ds\right] \tag{2.28}$$

converges for ϵ small enough. Roughly speaking this means that the endpoints of the string S_i are associated to matter fields ϕ_i but not to gauge fields.*

We start our analysis of (2.23) by considering in some detail the case where the additional constraint $\nu_R = 0$ is used for the string S_2, i.e. we consider strings with fermion bilinears

$$\phi_2 = \bar{e}_L e_R \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{2.29}$$

We comment on the results of other cases at the end of this section. To analyse $H_I(N)$, we have to work out its commutators with all local fields, analogous to the text following (1.10). For this purpose, we specify the following equal time commutation relations:

$$[e_i(x), \bar{e}_j(y)]_+ = \gamma_0\delta_{ij}\delta^{(3)}(x-y). \tag{2.30}$$

Here, γ_0 denotes a Dirac matrix, $i,j = L,R$ and all other anti-commutators between two Fermi-fields vanish.

$$[A_\mu^a(x), \dot{A}_\nu^b(y)] = -i\delta_{ab}g_{\mu\nu}\delta^{(3)}(x-y), \tag{2.31a}$$

$$[B_\mu(x), \dot{B}_\nu(y)] = -ig_{\mu\nu}\delta^{(3)}(x-y), \tag{2.31b}$$

* This is consistent with the lattice formulation of the gauge-invariant string where matter fields are associated with lattice sites while the gauge fields are defined on the lattice links.

where $g_{\mu\nu} = (1,-1,-1,-1)$. All other commutators with at least one gauge field vanish.

Let us now investigate the commutator of $H_I(N)$ with an arbitrary fermion field $e_L(y)$ say, defined at the point $y \in V$. Using (2.30) and (2.22), we obtain

$$[H_I(N), e_L(y)] =$$

$$c\frac{1}{N}\sum_{k=1}^{N}\gamma_0 e_R(y)(0,1)\hat{P}\exp\left[ig\int_{y,y+R\hat{e}_k} C_\mu(z)dz^\mu\right]\begin{pmatrix}0\\1\end{pmatrix}\bar{e}_R(y+R\hat{e}_k)e_L(y+R\hat{e}_k)$$

$$+c\frac{1}{N}\sum_{k=1}^{N}\gamma_0 e_R(y)(0,1)\hat{P}\exp\left[-ig\int_{y-R\hat{e}_k,y} C_\mu(z)dz^\mu\right]\begin{pmatrix}0\\1\end{pmatrix}\bar{e}_R(y-R\hat{e}_k)e_L(y-R\hat{e}_k). \tag{2.32}$$

Consider now an arbitrary field $F(y')$, defined at a point $y' \in V$, $y' \neq y$. It is easy to see that

$$\lim_{N\to\infty}[[H_I(N), e_L(y)], F(y')] = 0. \tag{2.33}$$

The reason is that the two points y, y' determine a direction in V and only strings passing through both points can contribute to the double commutator (2.33). Their contribution, however, is multiplied by $\frac{1}{N}$ and vanishes in the $N \to \infty$-limit. This implies that the double commutator behaves like a commutator of $F(y')$ with $\gamma_0 e_R(y)$. (The path ordered integral in (2.32) is understood in the sense specified in (2.28), i.e. $C_\mu(y)$ does not belong to the path.) We conclude that in all irreducible representations

$$\lim_{N\to\infty}[H_I(N), e_L(y)] = c_e\gamma_0 e_R(y), \tag{2.34}$$

where c_e is a c-number. Similarly, we work out the commutators of $H_I(N)$ with $\bar{e}_L(y)$, $e_R(y)$ and $\bar{e}_R(y)$. In the $N \to \infty$-limit, the same commutators can be obtained from

$$H'_{I,e} = \int d^3y\{c_e\bar{e}_L(y)e_R(y) + c_e^*\bar{e}_R(y)e_L(y)\}. \tag{2.35}$$

This calculation can be understood in a simple pictorial way. We denote the sum of the strings $S_i(x, x+R\hat{e}_k)$ $+S_i^\dagger(x, x+R\hat{e}_k)$ by a straight line between the points x and $x+R\hat{e}_k$. In Fig. I, we depict the N different strings which contribute to the commutator $[H_I(N), e_L(y)]$ in (2.32). Only the string with the dashed line contributes

to the double commutator $[[H_I(N), \dot{B}_\nu(y)], F(y')]$ and hence, (2.33) holds. A similar analysis is possible for the commutators and double commutators of $H_I(N)$ with the gauge fields. The mathematical details of this calculation are given in appendix A. Here, we mention just that a similar pictorial representation of the calculation exists (cf. Fig. II). One concludes that in all irreducible representations, the hamiltonian

$$H_{IG}'(N) = \frac{1}{N}\sum_{k=1}^{N}\int_V d^3y\sum_{n=1}^{\infty} i^n g^n c_n(R)(0,1)[C_i(y)\dot{e}_k^i]^n\begin{pmatrix}0\\1\end{pmatrix} \tag{2.36}$$

leads to identical commutators for all gauge fields in the isotropic $N \to \infty$-limit. Here, the $c_n(R)$ are c-numbers corresponding to the elements of the center of the field algebra. These $c_n(R)$ absorbe n integrations over the string length R and the coupling constant c of (2.23). Hence, $c_m(R)$ is of order $O(M^{4-n})$ in the mass.

Furthermore, we show in appendix B that the isotropic $N \to \infty$-limit, $H'_{IG} = \lim_{N\to\infty} H'_{IG}(N)$ can be written as

$$H'_{IG} = \sum_{n=1}^{\infty}(-1)^n g^{2n} c_{2n}(R)\frac{8\pi^2}{2n+1}\int d^3y(0,1)(C_1^2(y)+C_2^2(y)+C_3^2(y))^n\begin{pmatrix}0\\1\end{pmatrix}. \tag{2.37}$$

Here, it is worth remarking that we obtain a Lorentz-invariant formulation which reduces to (2.37) in the temporal gauge $C_0(x) = 0$, by substituting

$$C_1^2(y) + C_2^2(y) + C_3^2(y) \longrightarrow C_\mu(y)C^\mu(y). \tag{2.38}$$

where the Einstein summation convention has been used on the right hand side. With this substitution, let us examine expression (2.37) to $O(g^4)$. We start from

$$H_{IG}'(R) = -g^2c_2(R)\frac{8\pi^2}{5}\int_V(0,1)C_\mu(y)C^\mu(y)\begin{pmatrix}0\\1\end{pmatrix}d^3y + O(g^4). \tag{2.39}$$

Using (2.15), the integrand of this expression can be written in the following form:

$$-g^2c_2(R)(0,1)C_\mu(y)C^\mu(y)\begin{pmatrix}0\\1\end{pmatrix}$$

$$= -g^2c_2(R)[A_\mu^1(y)A^{1\mu}(y) + A_\mu^2(y)A^{2\mu}(y) + A_\mu^3(y)A^{3\mu}(y)$$

$$-\frac{g'}{g}Y(A_\mu^3(y)B^\mu(y) + B_\mu(y)A^{3\mu}(y)) + \frac{g'^2}{g^2}Y^2B_\mu(y)B^\mu(y)]. \tag{2.40}$$

For $Y = 1$, the hypercharge of the composita (2.22), this expression denotes nothing but the gauge boson mass terms in the electroweak theory. Defining

$$W_\mu^+ = \frac{1}{\sqrt{2}}(A_\mu^1 - iA_\mu^2), \tag{2.41a}$$

$$W_\mu^- = \frac{1}{\sqrt{2}}(A_\mu^1 + iA_\mu^2), \tag{2.41b}$$

$$Z_\mu = A_\mu^3 \cos\theta_W - B_\mu \sin\theta_W, \tag{2.41c}$$

$$\tilde{A}_\mu = A_\mu^3 \sin\theta_W + B_\mu \cos\theta_W, \tag{2.41d}$$

with the Weinberg angle θ_W

$$\tan\theta_W = \frac{g'}{g}, \tag{2.42}$$

we obtain the physical fields with mass terms:

$$M_W^2 = \frac{16\pi^2}{5} g^2 c_2(R) \tag{2.43a}$$

$$M_Z^2 = \frac{16\pi^2}{5} c_2(R)\left(g^2 + g'^2\right), \tag{2.43b}$$

$$M_A^2 = 0. \tag{2.43c}$$

Similarly, (2.35) can be interpreted as a fermion mass term

$$m_e^2 = 2c_e. \tag{2.44}$$

Remark however that c_e is an arbitrary complex c-number, while it can be shown that the $c_{2n}(R)$ are real - there is a hermitian conjugate for each term contributing to $c_{2n}(R)$, while there is not necessarily one for c_e [cf. appendix A]. This seems closely related to the classical Higgs mechanism where the Higgs vacuum expectation value (which is an arbitrary c-number in all irreducible representations) enters the mass terms of the gauge bosons as a bilinear, while the fermion mass terms depend linearly on $\langle\phi\rangle$ [cf. (2.8)]. Furthermore, the value of the real number $c_2(R)$ is determined by the choice of the irreducible representation and does not depend on the length R with which we have started our calculation [cf. appendix A].

These observations lead us to the question whether we can reproduce the quark mass terms too. In analogy with (2.21), we introduce the bilinears

$$\phi_3 = \begin{pmatrix} \bar{u}_L u_R \\ \bar{d}_L d_R \end{pmatrix} \tag{2.45}$$

into the gauge-invariant string (2.22). Following step by step the analysis of (2.23), we indeed obtain mass terms for the quarks. However, the string S_3 leads to additional contributions to the gauge boson masses. These contributions stem from the equation analogous to (2.39) and have the form

$$(\Delta_u^*, \Delta_d^*) C_\mu C^\mu \begin{pmatrix} \Delta_u \\ \Delta_d \end{pmatrix}. \tag{2.46}$$

Including these contributions in the gauge boson masses, we obtain the usual mass terms in the following way: we can always find complex numbers a, b such that

$$(\Delta_u^*, \Delta_d^*) C_\mu C^\mu \begin{pmatrix} \Delta_u \\ \Delta_d \end{pmatrix} + (0,1) C_\mu C^\mu \begin{pmatrix} 0 \\ 1 \end{pmatrix} = (a^*, b^*) C_\mu C^\mu \begin{pmatrix} a \\ b \end{pmatrix}, \tag{2.47}$$

The right hand side of (2.47) is then subjected to a global $SU(2)$-transformation

$$C_\mu \longrightarrow C_\mu^U = U C_\mu U^{-1}, \tag{2.48a}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} \longrightarrow U \begin{pmatrix} a \\ b \end{pmatrix}, \tag{2.48b}$$

such that $U\begin{pmatrix} a \\ b \end{pmatrix}$ is parallel to $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. The physical fields obtained from this calculation are a reparametrization in terms of A_μ^U of those given in (2.41). This shows that an interaction term (2.23) for every fermion doublet in the electroweak theory leads to the correct masses for the fermion and gauge fields in the standard model. Hence, we have obtained an affirmative answer to the question asked at the beginning of this section: *It is possible to write down gauge-invariant interaction terms which in all irreducible vacuum representations of the infinite volume system lead to correct mass terms for the fermions and gauge bosons in the electroweak theory.* Gauge-invariant strings of the type S_2 which fulfil a certain set of criteria [C1] - [C4] provide such interaction terms.

III. The concept of algebraic mass generation

In the last section, we have investigated a special non-local gauge-invariant interaction term. We have shown that this interaction term is dynamically equivalent to a hamiltonian containing all mass terms of the electroweak interactions as well as higher order couplings in the gauge fields. In this section, we concentrate on the conceptual questions connected with this proposal. In subsection III.1, we provide a physical interpretation for the interaction term H_I, extending recent ideas of how to implement Gauss' law in local quantum field theories. III.2 contains a comparison of the model of algebraic mass-generation with other models of mass generation. In what follows, the notion "algebraic symmetry breaking" is defined by example: the symmetry of a hamiltonian is called " algebraically broken", if there exists a dynamically equivalent expression in which the symmetry is restored.

III.1. The physical interpretation of H_I

The calculation in subsection II.2 has provided the correct mass terms of the electroweak theory for strings of the type S_2. However, strings of the type S_1 satisfying the same set of criteria C1-C4 would lead to bilinear fermion terms, e.g. $e_l e_R$, which are difficult to interpret as mass terms [for a definition of S_1, cf. (2.20) and (2.22)]. Furthermore, the analysis of interaction terms constructed out of the strings (2.14) or (2.16) and satisfying the criteria C1-C3 but not C4 leads to strange linear terms in the dynamically equivalent hamiltonian. This shows that the choice of the interaction term (2.23) on the basis of symmetry arguments is not completely unique. It is obviously important to have a physical interpretation which motivates the choice of (2.23).

To find such a physical interpretation, let us recall that the physical states carrying electric charge cannot be constructed by applying a local field operator to the vacuum state. This is a direct consequence of Gauss' law which implies that the electric charge of a particle can be detected by measuring the total electric flux through an arbitrarily large surface surrounding the particle. However, as remarked by Buchholz in his investigation of the state space of quantum electrodynamics (QED),[5] it should be possible

to construct charged scattering states from the field operators

$$\psi_c(x) = \psi(x) \exp\left[ie \int d^4y c^\mu(x-y) A_\mu(y)\right], \tag{3.1}$$

which are formally related to the Fermi-field ψ and the vector-potential A_μ. Here, the functions c_μ have to satisfy the equation

$$\partial^\mu c_\mu(x) = \delta^{(4)}(x) \tag{3.2}$$

to make $\psi_c(x)$ locally gauge-invariant. The set of these functions c_μ is stable under Lorentz transformations, and for any open spacelike cone $\mathcal{S}$ containing the origin of Minkowski space there exists functions c_μ having support in $\mathcal{S}$. Buchholz concludes: "Therefore one may expect that the appropriately regularized field-operators ψ_c generate charged states..." [5]

The heuristic idea motivating $\psi_c(x)$ is obvious: while charged states cannot be constructed by applying a local field operator to the vacuum state, it should be possible to obtain them via a non-local object constructed from local covariant fields ψ and A_μ where A_μ takes the total electric flux spacelike to infinity. Equation (3.1) exhibits this important property. We might view

$$\psi_{CS}(x) = \psi(x) \exp\left[ie \int_{\Gamma_{x,\infty}} A_\mu(z) dz^\mu\right] \tag{3.3}$$

as an object closely related to the class of field operators (3.1). Here, $\Gamma_{x,\infty}$ is a path in Minkowski space, connecting the point x spacelike to infinity, i.e., $\psi(x)$ is a charged local operator connected to spacelike infinity by a flux tube of gauge fields. Gauge-invariant strings like $\psi_{CS}(x)$ have been discussed in attempts to obtain a formulation of QED in terms of gauge-invariant fields. [6]

Another way to obtain a charged scattering state describing a charged particle should be to remove the corresponding antiparticle spacelike out of the region of observation.*

* We define the "region of observation" to be the region of support of those local field operators whose commutators with the hamiltonian play a rôle in the prediction of an experimental outcome.

Roughly speaking, applying

$$\psi'_{CS}(x,y) = \psi(x)\exp\left[ie\int_{\Gamma_{x,y}} A_\mu(z)dz^\mu\right]\overline{\psi}(y) \tag{3.4}$$

to the vacuum describes a charged particle at the point x and its corresponding antiparticle at the point y both connected by a flux tube of gauge strings. If the distance $|x-y|$ is large and spacelike, then the operators $\psi'_{CS}(x,y)$ and $\psi_{CS}(x)$ should lead to the same commutation relations for a local algebra, localized in a neighbourhood of x.* The interesting feature of operators of the type ψ'_{CS} is that they map the vacuum sector onto itself while ψ_{CS} maps the superselection sector corresponding to states with n electrical charges to the one with $n+1$ electrical charges. Nevertheless, ψ'_{CS} applied to the vacuum state describes in some sense a charged state without invoking a non-trivial superselection structure. In a lagrangian setting, we might consider objects of the type ψ'_{CS} to be the most natural choice to describe charged states without violating Gauss' law. This leads us to postulate the following principle:

To describe a charged particle in a local lagrangian quantum field theory with a Gauss law, it is necessary to remove the corresponding antiparticle out of the region of observation and to connect it with the particle by a flux tube of gauge strings.

In what follows, we shall argue that the interaction term H_I in (2.25) can be understood as a heuristic implementation of the above idea in a lagrangian formulation of the electroweak theory. Our starting point is the hamiltonian density $\mathcal{H}_0$ in (2.7) which contains the massless part of the standard model without scalar field. $\mathcal{H}_0$ is the "nice" part of the standard model, specifying both the $SU(2)\times U(1)$ gauge symmetry and the different transformation properties of left-handed doublets and right-handed singlets. It contains the two gauge couplings g and g' only. These couplings fix the

* Because of the Reeh-Schlieder theorem, it is very difficult to speak of the space-time localization of states, since any local algebra is sufficient to generate the physical state space. Hence, the heuristic concept that the operators $\psi'_{CS}(x,y)$ and $\psi_{CS}(x)$ are in some sense equivalent in a neighbourhood of x, cannot naively be implemented on the level of physical states.

relative strength of the electrodynamic and weak interaction and can therefore be considered to be much more fundamental than the additional couplings appearing in the Higgs and Yukawa sector. Hence it seems consistent to assume that the dynamics of the electroweak interaction can be defined for the local fermion and gauge boson fields but that for the charged scattering states, these local fields have to appear in non-local combinations similar to (3.4). The interaction term H_I in (2.25) appears to be the most natural heuristic implementation of this idea:

choosing a negative coupling c in (2.23) turns the gauge-invariant strings S_i into an energetically favourable field configuration. Roughly speaking, this implies that the local fermion and gauge boson fields evolve dynamically in a potential which glues them together to gauge invariant strings in the $t\to\infty$ limit, i.e., as the correct operators creating charged scattering states.

Clearly, this interpretation of H_I is tentative. It is interesting to point out its conceptual advantages. The interaction term H_I is considered to be a heuristic implementation of the non-locality of charged scattering states which seem difficult to embed in a local quantum field theory. This provides a natural explanation of the origin of the plethora of parameters in the standard model. All additional free parameters of the Higgs and Yukawa sector are obtained as couplings of terms which are dynamically equivalent to H_I in all irreducible representations. Hence these parameters can be understood as part of a heuristic implementation of Gauss' law in a local quantum field theory. On the other hand, this interpretation leads to the question: does this method of algebraic mass generation explain why all electrically charged elementary particles are massive while electrically neutral elementary particles are massless (except for the Z gauge boson)? Here, we shall not pursue these speculations.

III.2. A comparison between the different mechanisms of symmetry breaking

One reason why a vector boson in an apparently massless lagrangian acquires a mass was given by Schwinger.[8] He discovered in two dimensional massless spinor electrody-

namics, that in the vacuum polarization tensor $\Pi^{\mu\nu}(q) = (g^{\mu\nu}q^2 - q^\mu q^\nu)i\Pi(q^2)$, $\Pi(q^2)$ acquires a pole. $\Pi^{\mu\nu}(q)$, which denotes the sum over all one-vector-boson irreducible graphs, is of course zero in the absence of interactions. In general, vector bosons acquire masses through suitably chosen interaction terms.

The Higgs-Kibble-mechanism provides an explicit mechanism which leads to a pole in $\Pi(q^2)$.[7] The vector boson couples to a self-interacting scalar field ϕ via the gauge-invariant term $(D^\mu\phi)(D_\mu\phi)$, [cf. (2.8)]. After spontaneous symmetry breaking,

$$\phi \longrightarrow \hat{\phi} + \phi', \tag{3.5}$$

the vacuum expectation value $\hat{\phi}$ couples to the vector bosons, giving rise to tadpole contributions to $\Pi(q^2)$ which produce a pole. Here, $\hat{\phi}$ is the minimum in the (quantum corrected) Higgs potential and ϕ' is the new dynamical field.

In principle, there are mechanisms which can give rise to a pole in $\Pi(q^2)$ without involving a scalar field. Let us briefly sketch one of these proposals: Jackiw and Johnson demonstrated in a particular field theory that a pole in $\Pi(q^2)$ can arise whenever a massless fermion acquires a mass through spontaneous symmetry breaking.[9] This observation is very interesting if one follows the standard belief that the dynamics is such that the QCD vacuum breaks the $SU(2)_{left} \times SU(2)_{right}$ symmetry of the massless QCD lagrangian to $SU(2)_{isospin}$ (throughout this paper we are working with one family of leptons and quarks). Then, the strong interaction gives rise to a non-zero vacuum expectation value

$$\langle \bar{u}u + \bar{d}d \rangle_{VEV} \neq 0 \tag{3.6}$$

and to three massless Goldstone bosons which are usually referred to as "pions". Clearly, one does not want these massless modes to appear as the longitudinal components of massive vector bosons (we do observe pions, after all!), though this is possible for suitably chosen interaction terms. The technicolour idea proposes a way out, postulating the existence of technifermions which effectively interact like quarks under a new strong force. The resulting massless technipions can appear as longitudinal components of the vector bosons without contradicting the existence of pions.

These and other ideas are well documented in literature[9] and share one common feature: unobserved particles have to be postulated to give rise to the experimentally observed masses. The mechanism of algebraic symmetry breaking proposed in section II is the only phenomenologically relevant one we are aware of which starts from a gauge invariant expression and leads to the experimentally observed mass terms without postulating the existence of new particles. More importantly: no new fundamental forces are introduced. The fermion bilinears (2.20), (2.21) and (2.45) might ressemble the vacuum expectation values (3.7) in models of dynamical symmetry breaking, conceptually they are on a very different footing: they are a consequence of criterion [C4] which in our approach lies at the heart of implementing the consequences of the presence of Gaussian charges in a local quantum field theory.

CONCLUSION

In this work, we have presented a model of electroweak interactions which shows spontaneous symmetry breaking without involving a scalar field. This proposal originates from two observations:

1) Haag's observation that one can add elements of the commutant of the field algebra to the hamiltonian without changing the dynamics. In the case of non-local interaction terms, the changes which we are allowed to make to the hamiltionian can be highly non-trivial to such an extent that they break the symmetry of the hamiltonian.
2) The observation that because of Gauss' law, charged scattering states cannot be described by the operation of local fields on the vacuum state. This motivates the assumption that elements of the weak closure of the field algebra play a rôle in the dynamics of a quantum field theory describing charged scattering states, even if the primary field operators are local. In particular, this motivates our working assumption that the vacuum expectation value $\hat{\phi}$ of the Higgs field in the standard model can be understood as the expectation value of an element of the weak closure of the electroweak field algebra and that it should be possible to find elements in this weak closure which show all the properties of $\hat{\phi}$ without involving an additional

scalar field.

Starting from these considerations, we made a formal attempt to construct Haag's mechanism of algebraic symmetry breaking in the case of a theory of electroweak interactions. This led to the interaction term (2.23) built out of gauge-invariant strings. The analysis of this interaction hamiltonian in section II.2 allowed us to identify the elements in the weak closure of the electroweak field algebra which correspond to the vacuum expectation value $\hat{\phi}$ of the standard model Higgs field. The analogy between $\hat{\phi}$ and these elements of the center of the field algebra is remarkable: there is even a correspondence for the fact that the gauge boson mass terms of the GW standard model are quadratic in $\hat{\phi}$ while the fermion mass terms depend only linearly on $\hat{\phi}$. Hence we obtained all mass-terms of the electroweak standard model from a gauge-invariant interaction term which does not involve an additional scalar field. The question whether this type of gauge symmetry breaking allows for a theory with finite radiative corrections, shall be pursued in our subsequent work.

In this work, we have restricted ourselves to giving a physical motivation for the interaction term H_I. H_I can be understood as a part of a heuristic implementation of Gauss' law in a local quantum field theory describing charged scattering states. This interpretation links the concepts of mass and charge together: in the mechanism of algebraic mass generation, the mass of a particle appears to be a consequence of its charge relative to a gauged symmetry.

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Appendix A: Commutators and double commutators of $H_I(N)$ with gauge fields

We start with an investigation of the commutator $[H_I(N), \dot{B}_\nu(y)]$. For notational convenience, we introduce $H_I(\hat{e}_k)$ and $S_i^{(n)}(x, x+R\hat{e}_k)$, defined by

$$H_I(N) = \frac{1}{N}\sum_{k=1}^{N} H_I(\hat{e}_k); \tag{A.1}$$

$$S_i(x, x+R\hat{e}_k) = \sum_n (ig)^n S_i^{(n)}(x, x+R\hat{e}_k). \tag{A.2}$$

In a first step, we compute $[H_I(V, \hat{e}_1), \dot{B}_\nu(y)]$ up to $O(g^2)$ for a special spatial direction $\hat{e}_1 = (1,0,0)$ say. Specifying $\hat{e}_1$, we are able to simplify the directed path-ordered exponential:

$$\begin{aligned}
&\int d^3x S(x, x+R\hat{e}_1) \\
&= \int d^3x \sum_{n=0}^{\infty} (ig)^n \int_0^R dR_1 \int_0^{R_1} dR_2 \ldots \int_0^{R_{n-1}} dR_n \\
&\overline{\phi}(x) C_1(x+R_1\hat{e}_1)\hat{e}_1 \ldots C_1(x+R_n\hat{e}_1)\hat{e}_1 \phi(x+R\hat{e}_1) \\
&= \int d^3x \sum_{n=0}^{\infty} (ig)^n S^{(n)}(x, x+R\hat{e}_1),
\end{aligned} \tag{A.3}$$

where the subscript i on $S_i^{(n)}$ has been omitted for notational convenience. Using

$$C_\mu = A_\mu^a \frac{1}{2}\sigma^a + \frac{g'}{g}\frac{Y}{2}B_\mu \tag{A.4}$$

and the commutation relations (2.31), this allows us to calculate

$$\int d^3x [S^{(0)}(x, x+R\hat{e}_1), \dot{B}_\nu(y)] = 0, \tag{A.5}$$

$$\begin{aligned}
&\int d^3x (ig)[S^{(1)}(x, x+R\hat{e}_1), \dot{B}_\nu(y)] \\
&= (ig)\int d^3x \int_0^R dR_1 \frac{g'}{g}\frac{Y}{2}\overline{\phi}_2(x)[B_1(x, x+R_1\hat{e}_1)\hat{e}_1, \dot{B}_\nu(y)]\phi_2(x+R\hat{e}_1) \\
&= g\int d^3x \int_0^R dR_1 \frac{g'}{g}\frac{Y}{2}\hat{e}_1\delta_{1\mu}\delta(x^1+R\hat{e}_1-y^1)\delta(x^2-y^2)\delta(x^3-y^3)\overline{\phi}_2(x)\phi_2(x+R\hat{e}_1) \\
&= g\delta_{1\mu}\frac{g'}{g}\frac{Y}{2}\hat{e}_1 \int_0^R dR_1 \overline{\phi}_2(y-R_1\hat{e}_1)\phi_2(y+(R-R_1)\hat{e}_1)
\end{aligned} \tag{A.6}$$

and

$$(ig)^2 \int d^3x [S^{(2)}(x, x + R\hat{e}_1), \dot{B}_\nu(y)]$$

$$= (ig)^2 \int d^3x \int_0^R dR_1 \int_0^{R_1} dR_2 \overline{\phi}_2(x)[C_1(x + R_1\hat{e}_1)\hat{e}_1 C_1(x + R_2\hat{e}_1)\hat{e}_1, \dot{B}_\nu(y)]\phi_2(x+R\hat{e}_1)$$

$$= g^2 \frac{g'i}{g}\frac{Y}{2}\hat{e}_1 \int_0^R dR_1 \int_0^{R_1} dR_2 \{\overline{\phi}_2(y - R_2\hat{e}_1)C_1(y + (R_1 - R_2)\hat{e}_1)\hat{e}_1\phi_2(y + (R - R_2)\hat{e}_1)$$

$$+\overline{\phi}_2(y - R_1\hat{e}_1)C_1(y + (R_2 - R_1)\hat{e}_1)\hat{e}_1\phi_2(y + (R - R_1)\hat{e}_1)\}. \qquad (A.7)$$

The commutator $[H_I(N), \dot{B}_\nu(y)]$ is given to $O(g^2)$ as the sum of the terms (A.5-7) and the contributions corresponding to the second term in (2.23), summed over the N different spatial directions $\hat{e}_k$. Similarly to (2.35), we obtain for $y' \neq y$

$$\lim_{N\to\infty}[[H_I(N), \dot{B}_\nu(y)], F(y')] = 0. \qquad (A.8)$$

The reason is that the points y and y' specify one direction $\hat{e}_{yy'}$ in space. Hence, the double commutator (A.8) has to vanish since it receives contributions from $H_I(\hat{e}_{yy'})$ only, but includes a damping factor $\frac{1}{N}$. This holds true to all orders, i.e., in all irreducible representations, $[H_I(N), \dot{B}_\nu(y)]$ can be written for any order $O(g^n)$ as a function of fields at the point $y \in V$ times a c-number. Similar results can be obtained for the commutator of $H_I(N)$ with $\dot{A}^a_\mu(y)$. We conclude that in the $N \to \infty$-limit of all irreducible representations, the hamiltonian

$$H_{IG}'(N) = \frac{1}{N}\sum_{k=1}^N \int_V d^3y \sum_{n=1}^\infty i^n g^n c_n(R)(0,1)[C_i(y)\hat{e}^i_k]^n \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad (A.9)$$

is dynamically equivalent to $H_I(N)$ with respect to the gauge fields. This is just equation (2.39), where the $c_n(R)$ are c-numbers corresponding to the elements of the center of the field algebra. From the number of line integrals we know that $c_n(R)$ is of order $O(M^{4-n})$ in the mass. Furthermore, the value of the real numbers $c_n(R)$ is determined by the choice of the irreducible representation. In particular, the actual value of $c_2(R)$ depends on the choice of the irreducible representation only, irrespective of the length R with which we have started our calculation.

Appendix B: The isotropic $N \to \infty$-limit

Here, we calculate the $N \to \infty$-limit of $H'_{IG}(N)$ where the countably many directions $\hat{e}_k$ are chosen so that

$$H'_{IG} = \lim_{N\to\infty} H'_{IG}(N) \qquad (B.1)$$

becomes an isotropic expression. Therefore we substitute in

$$H_{IG}'(N) = \frac{1}{N}\sum_{k=1}^N \int_V d^3y \sum_{n=1}^\infty i^n g^n c_n(R)(0,1)[C_i(y)\hat{e}^i_k]^n \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad (B.2)$$

the sum over the N directions by an integral over the 2-sphere:

$$\frac{1}{N}\sum_{k=1}^N \longrightarrow \int_0^\pi \int_0^{2\pi} 2\pi \sin\theta d\theta d\varphi. \qquad (B.3)$$

This substitution is justified in the sense of an explicit construction of a Riemann integral. Clearly, for very singular objects as the δ-distributions used in the analysis of the commutators of $H_I(N)$, such a replacement is not free from difficulties. In the evaluation of the isotropic $N \to \infty$-limit of (B.2), however, the operator valued distributions $C_i(y)$ just play the rôle of coefficients of the unit vectors. Hence, it is allowed to substitute (B.3) to obtain an explicitly isotropic form for (B.2). On this basis, we parametrize the unit vectors $\hat{e}_k$ by spherical coordinates. For (B.2), this leads to the substitution

$$C_i(y)\hat{e}^i_k \longrightarrow \hat{C}(y), \qquad (B.4)$$

where

$$\hat{C}(y) = C_1(y)\sin\theta\sin\varphi + C_2(y)\sin\theta\cos\varphi + C_3(y)\cos\theta, \qquad (B.5)$$

and hence

$$\frac{1}{N}\sum_{k=1}^N [C_i(y)\hat{e}^i_k]^n \longrightarrow \int_0^\pi \int_0^{2\pi} \hat{C}^n(y) 2\pi \sin\theta d\theta d\varphi. \qquad (B.6)$$

Using the formulae

$$\int_0^{\frac{\pi}{2}} \sin^{2m} x dx = \int_0^{\frac{\pi}{2}} \cos^{2m} x dx = \frac{(2m-1)!!}{(2m)!!}\frac{\pi}{2}, \qquad (B.7)$$

$$\int_0^{\frac{\pi}{2}} \sin^{2m+1} x dx = \int_0^{\frac{\pi}{2}} \cos^{2m+1} x dx = \frac{(2m)!!}{(2m+1)!!}, \qquad (B.8)$$

a lengthy but straightforward calculation shows that

$$\int_0^\pi\int_0^{2\pi}\hat{C}^{2n}(y)2\pi\sin\theta d\theta d\varphi = \sum_{(k,r,s)}\frac{(2n)!}{(2k)!(2r)!(2s)!}8\pi^2 C_1^{2k}(y)C_2^{2r}(y)C_3^{2s}(y)$$
$$\times\left(\sum_{l=0}^{s}\binom{s}{l}(-1)^l\frac{(2(k+r+l))!!}{(2(k+r+l)+1)!!}\right)\left(\sum_{l'=0}^{r}\binom{r}{l'}(-1)^{l'}\frac{(2(k+l')-1)!!}{(2(k+l'))!!}\right), \tag{B.9}$$

while the odd powers of $\hat{C}(y)$ vanish. Here, $\sum_{(k,r,s)}$ goes over triples (k,r,s) such that $n=k+r+s$. To simplify this expression, one proves by induction that

$$\sum_{l=0}^{r}\binom{r}{l}(-1)^l\frac{(2(k+l)-1)!!}{(2(k+l))!!} = \frac{(2k-1)!!}{(2(k+r))!!}(2r-1)!!, \tag{B.10a}$$

$$\sum_{l=0}^{s}\binom{s}{l}(-1)^l\frac{(2(k+l))!!}{(2(k+l)+1)!!} = \frac{(2k)!!}{(2(k+s)+1)!!}(2s-1)!!. \tag{B.10b}$$

Inserting (B.10) in (B.9), one obtains

$$\int_0^\pi\int_0^{2\pi}\hat{C}^{2n}(y)2\pi\sin\theta d\theta d\varphi = \frac{8\pi^2}{2n+1}\left(C_1^2(y)+C_2^2(y)+C_3^2(y)\right)^n. \tag{B.11}$$

Hence, the isotropic $N\to\infty$-limit H'_{IG} of $H'_{IG}(N)$ reads

$$H'_{IG} = \sum_{n=1}^{\infty}(-1)^n g^{2n}c_{2n}(R)\frac{8\pi^2}{2n+1}\int d^3y(0,1)\left(C_1^2(y)+C_2^2(y)+C_3^2(y)\right)^n\begin{pmatrix}0\\1\end{pmatrix}, \tag{B.12}$$

which is just equation (2.37).

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Figure Caption

Fig.I Depiction of the N strings contributing to the commutator $[H_I(N), e_L(y)]$. Only the k-th string denoted by a dashed line may contribute to $[[H_I(N), e_L(y)], F(y')]$.

Fig.II Some of the strings contributing to $[H_I(N), \dot{B}_\nu(y)]$. Only strings on the dashed line may contribute to $[[H_I(N), \dot{B}_\nu(y)], F(y')]$.

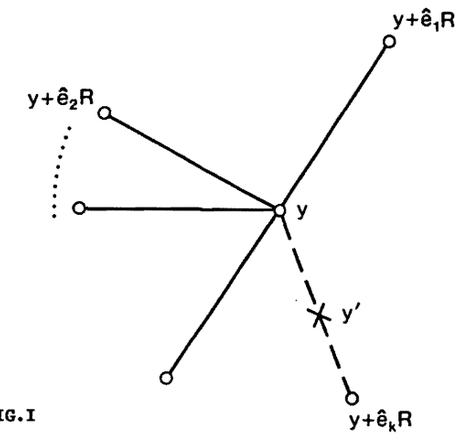

FIG.I

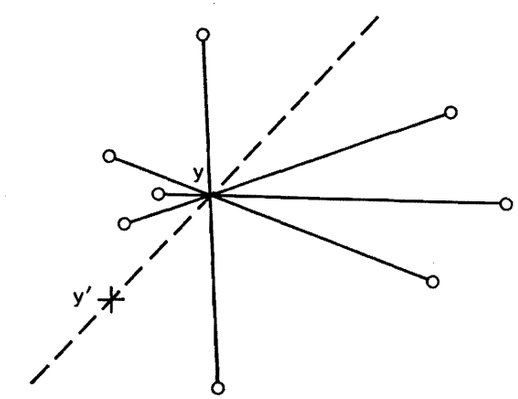

FIG.II