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Dynamics of SU(3) monopoles

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Abstract

We study geodesic flow on a totally geodesic submanifold of a moduli space of solutions to Nahm's equations, and draw conclusions about the dynamics of $SU(3)$ monopoles with minimal symmetry breaking.

1. Introduction

A powerful technique for obtaining results about soliton dynamics is due to Manton (1982). The key idea is that the dynamics of a soliton should, at least at low velocities, be closely approximated by geodesic motion on the moduli space of static solitons. To actually implement this programme involves identifying the moduli space, calculating its metric, and analysing the resulting geodesic flow. This was carried out in the case of sigma-model lumps by Ward (1985) and Leese (1990), and for $SU(2)$ monopoles of charge two by Atiyah and Hitchin (1988). It is monopole dynamics that will concern us in this paper.

The first author (Dancer 1991a) introduced an 8-dimensional moduli space M^8 of solutions to *Nahm's equations*, which are the nonlinear system of O.D.E.s

$$\frac{dT_i}{dt} + [T_0, T_i] = [T_j, T_k], \tag{1}$$

where T_i are matrix-valued functions of one variable, and (i, j, k) ranges over cyclic permutations of $(1, 2, 3)$.

M^8 carries a natural Riemannian metric, which was calculated. The metric on N^5, the quotient of M^8 by an isometric action of $SU(2)/Z_2$, was also found.

The significance of the geometry of M^8 for the study of monopoles arises as follows. Nahm (1982) has described a procedure, inspired by the ADHM construction for instantons, to produce monopoles from solutions to the equations (1). The dimension of the Nahm matrices becomes the charge of the monopole, while the behaviour of the matrices at the endpoints of the interval on which they are defined determines the gauge group. There are natural actions on the Nahm matrices which correspond under this procedure to spatial translation, spatial rotation, and gauging of monopoles. Moreover in the case of $SU(2)$ monopoles the transform from Nahm data to monopoles is known to be an isometry (Hurtubise 1990, Nakajima 1991).

It has been shown previously (Dancer 1991b) that points of M^8 give rise to $SU(3)$ monopoles with symmetry breaking to $U(2)$ (that is, with two eigenvalues of the asymptotic Higgs field being equal). If the Nahm transform is indeed an isometry, we can deduce dynamical properties of $SU(3)$ monopoles from the geodesic flow on M^8. We shall make this assumption for the remainder of the paper.

The expressions for the metric on M^8 are very complicated, and somewhat implicit. In order to study geodesic motion it is desirable to restrict ourselves to totally geodesic submanifolds where the geodesic problem is more tractable. It is, of course, the defining characteristic of such submanifolds that geodesics on them are also geodesics on the ambient space. Fortunately, a standard result of differential geometry guarantees that each component of the fixed point set of an isometry of a Riemannian manifold is totally geodesic. This was used (Dancer 1991a,1991b) to identify a totally geodesic surface of revolution Σ in M^8 and analyse its geodesic flow.

In this paper we shall study a totally geodesic surface Y in M^8. The metric is now more complicated than that of Σ, involving elliptic functions rather than elementary functions, and Y has only a discrete group of symmetries. The increased difficulty of the problem of analysing the geodesic flow leads us to adopt a numerical approach. We calculate the metric in terms of elliptic integrals and Jacobi functions, and numerically integrate the resulting geodesic equations. Finally, a physical interpretation of the

resulting geodesics is given.

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2. Moduli spaces

We first review some material (Dancer 1991a,1991b) concerning our moduli spaces.

Consider quadruples (T_0, T_1, T_2, T_3) where T_i are u(2)-valued functions on $[0,3]$ satisfying the following conditions:

(i) T_1, T_2, T_3 are analytic on $(0,3]$ with simple poles at $t=0$ of residue $-\frac{1}{2}\sigma_1, -\frac{1}{2}\sigma_2, -\frac{1}{2}\sigma_3$ respectively, where σ_i are the Pauli spin matrices. T_0 is analytic on $[0,3]$.

(ii) The T_i satisfy Nahm's equations (1).

The gauge group G of analytic $U(2)$-valued functions on $[0,3]$ which are the identity at $t=0$ acts on the space of such data by

$$\begin{aligned} T_0 &\mapsto gT_0g^{-1} - \dot{g}g^{-1} \\ T_j &\mapsto gT_jg^{-1} \quad (j=1,2,3) \end{aligned}$$

(we use $\cdot$ to denote differentiation with respect to the parameter $t \in [0,3]$).

Let $\hat{G}$ denote the normal subgroup of G consisting of gauge transformations which are the identity at $t=0,3$.

Let M^{12} denote the quotient of the data satisfying (i),(ii) by $\hat{G}$.

Theorem 2.1

M^{12} is a twelve-dimensional manifold admitting commuting actions of Spin(3) and $G/\hat{G} \cong U(2)$. The Spin(3) action is given by

$$\begin{aligned} T_0 &\mapsto gT_0g^{-1} - \dot{g}g^{-1} \\ T_i &\mapsto g(\Sigma a_{ij}T_j)g^{-1} \quad (i=1,2,3). \end{aligned}$$

Here g is a function from $[0,3]$ to $SU(2)$, $g(0) \in \mathrm{Spin}(3)$ descends to $(a_{ij}) \in SO(3)$ under the double cover $\pi : \mathrm{Spin}(3) \to SO(3)$ and $g(3) = Id$.

M^{12} also admits an action of $\mathbb{R}^3$ defined by

$$\begin{aligned} T_0 &\mapsto T_0 \\ T_j &\mapsto T_j - ix_jId \quad (j=1,2,3) \end{aligned}$$

Let M^8 be the quotient of M^{12} by $\mathbb{R}^3 \times U(1)$, where the $U(1)$ group here is the centre of $U(2)$.

M^8 is an eight-dimensional manifold with commuting actions of $SO(3)$ and $SU(2)/Z_2$, induced by the actions of Spin(3) and $U(2)$ on M^{12} respectively.

Let N^5 denote the quotient of M^8 by $SU(2)/Z_2$. N^5 is a five-dimensional manifold homeomorphic to $\mathbb{R}^5$, and admits a non-free $SO(3)$ action.

Points of M^{12} give rise, via the ADHM-Nahm construction, to $SU(3)$ monopoles of charge two with symmetry breaking to $U(2)$.

□

We can view M^8 as a moduli space of monopoles centred at the origin and with a circle phase fixed. The metric on the double cover of M^{12} is the Riemannian product of the metric on the double cover of M^8 and the flat metric on $\mathbb{R}^3 \times U(1)$. Physically, this means that the centre of mass motion and the time-evolution of the circle phase of the monopole can be split off, so to study the dynamics of centred monopoles it is sufficient to consider geodesic flow on M^8.

It will be helpful to associate *spectral curves* to our monopoles. To define these we follow the discussion due to Hitchin (1987).

Nahm's equations can be put in the Lax form

$$\frac{dT}{dt} + [T_+, T] = 0,$$

where

$$T_+ = T_0 + iT_1 + \zeta(T_3 - iT_2),$$

$$T = T_3 + iT_2 + 2i\zeta T_1 + \zeta^2(T_3 - iT_2)$$

and ζ is a *spectral parameter.*

It follows that the spectrum of T is constant in t, so the curve S with equation

$$\{(\eta, \zeta) : \det(\eta + T) = 0\}$$

is independent of t.

We call S the *spectral curve* associated to our solution of Nahm's equations.

It is clear that S is unchanged under the action of $SU(2)/Z_2$. In fact the coefficients of the spectral curve provide global coordinates on the quotient N^5 of M^8 by $SU(2)/Z_2$.

We can view S as a curve in $T\mathbb{P}^1$, the tangent bundle of $\mathbb{CP}^1$, by interpreting ζ as a coordinate on $\mathbb{CP}^1$ and η as a fibre coordinate (Hitchin 1982). Now $T\mathbb{P}^1$ is just the space of pairs (u, v) where u and v are orthogonal vectors in $\mathbb{C}^2$ with the norm of u equal to one. This space may be identified with the set of oriented straight lines in $\mathbb{R}^3$, by letting u be the direction of the line and v the point where it is closest to the origin. In the language of twistor theory, $T\mathbb{P}^1$ is the *minitwistor space* of $\mathbb{R}^3$. Reversing the orientation of lines defines a real structure on $T\mathbb{P}^1$; in our coordinates this is

$$\tau : (\eta, \zeta) \mapsto (-\bar{\eta}/\bar{\zeta}^2, -1/\bar{\zeta}).$$

Now any point (x_1, x_2, x_3) in $\mathbb{R}^3$ defines a family of straight lines passing through that point; this family is just the curve in $T\mathbb{P}^1$ with equation

$$\eta + ix_3 - x_2 - 2x_1\zeta + (ix_3 + x_2)\zeta^2 = 0. \tag{2}$$

Note that this is real with respect to our real structure on minitwistor space. We shall use this correspondence between points of $\mathbb{R}^3$ and real sections in $T\mathbb{P}^1$ later in the paper.

The spectral curve description exhibits N^5 as an open set W^0 in the irreducible five-dimensional representation W of $SO(3)$. It will also be convenient, following Hurtubise (1983), to think of W as the set of 3×3 real symmetric tracefree matrices, with $SO(3)$ acting by conjugation. This description will be useful when we come to associate *principal axes* to our monopoles in §3.

3. A totally geodesic surface

As promised, we shall now look for totally geodesic submanifolds of M^8. Let $\mathcal{D}$ denote the $Z_2 \times Z_2$ subgroup of diagonal matrices in $SO(3)$, and let

$$\Delta = \{(\alpha, \alpha) \in (SU(2)/Z_2) \times SO(3) : \alpha \in \mathcal{D}\}$$

It can be checked that the fixed point set of the Δ action consists of four isometric components, one of which is the set Y of Nahm data of the form

$$T_0 = 0, \;\; T_1 = \frac{1}{2}f_1\sigma_1, \;\; T_2 = \frac{1}{2}f_2\sigma_2, \;\; T_3 = \frac{1}{2}f_3\sigma_3.$$

where the f_i are real-valued functions.

As a component of a fixed point set of a group of isometries, Y is totally geodesic. Let us now discuss the geometry of this space.

Nahm's equations force f_1, f_2, f_3 to satisfy the Euler equations

$$\dot{f}_1 = f_2 f_3 \tag{3}$$

$$\dot{f}_2 = f_3 f_1 \tag{4}$$

$$\dot{f}_3 = f_1 f_2. \tag{5}$$

After imposing our boundary conditions on the Nahm matrices we find that the space Y of solutions is two-dimensional. The quantities

$$\begin{aligned} x' &= f_2^2 - f_1^2 \\ y' &= f_3^2 - f_1^2 \end{aligned}$$

are constant in t because of equations (3-5), and are global coordinates on Y.

In fact, it will be convenient for our depiction of the geodesic flow on Y to make a linear change of variables and use as coordinates

$$x = x' + y' \tag{6}$$

$$y = \sqrt{3}(x' - y'). \tag{7}$$

If $f_1^2 \leq f_2^2 \leq f_3^2$ we have

$$\begin{aligned} f_1(t) &= -D\frac{\mathrm{cn}_k(Dt)}{\mathrm{sn}_k(Dt)} \\ f_2(t) &= -D\frac{\mathrm{dn}_k(Dt)}{\mathrm{sn}_k(Dt)} \\ f_3(t) &= -\frac{D}{\mathrm{sn}_k(Dt)}, \end{aligned}$$

where

$$\begin{aligned} k &= +\sqrt{\frac{2y}{y-\sqrt{3}x}} \\ D &= +\sqrt{\frac{\sqrt{3}x - y}{2\sqrt{3}}} \end{aligned}$$

and $\mathrm{cn}_k, \mathrm{sn}_k, \mathrm{dn}_k$ are the Jacobi elliptic functions with modulus k.

We have required the Nahm matrices to be analytic on $(0,3]$, so we have the constraint $D < \frac{2}{3}K(k)$ where $K(k)$ denotes the complete elliptic integral with modulus k.

Similar expressions for f_1, f_2, f_3, D, k exist with other choices of ordering of f_1^2, f_2^2 and f_3^2.

If two or more of the f_i^2 are equal the Jacobi functions reduce to elementary functions. If $f_1^2 \leq f_2^2 = f_3^2$ they are given by

$$\begin{aligned} f_1(t) &= -D\cot(Dt) \\ f_2(t) &= -D\mathrm{cosec}(Dt) \\ f_3(t) &= -D\mathrm{cosec}(Dt), \end{aligned}$$

while if $f_1^2 = f_2^2 \leq f_3^2$ we have

$$\begin{aligned} f_1(t) &= -D\mathrm{cosech}(Dt) \\ f_2(t) &= -D\mathrm{cosech}(Dt) \\ f_3(t) &= -D\coth(Dt). \end{aligned}$$

These solutions correspond to taking the elliptic modulus k to be 0 or 1 respectively. In both cases the isotropy subgroup for the $(SU(2)/Z_2) \times SO(3)$ action on the Nahm data is isomorphic to $S^1 \times Z_2$, so we refer to the solutions as being *trigonometric axisymmetric* or *hyperbolic axisymmetric* as appropriate.

There is also the *spherically symmetric* solution

$$f_1(t) = f_2(t) = f_3(t) = -\frac{1}{t},$$

which, as its name suggests, has isotropy subgroup isomorphic to $SO(3)$. This solution is given in terms of the coordinates x, y by $x = y = 0$. The modulus k becomes indeterminate here.

A schematic representation of the surface Y is shown in Figure 1. We emphasise that this diagram is not intended to reflect the *metric* properties of Y.

We see that Y is divided up into six regions, which correspond to the six ways of ordering the quantities f_1^2, f_2^2, f_3^2. There is an isometric action on Y of the symmetric group on three letters, which interchanges the six regions. The lines between the regions represent axisymmetric monopoles, while the origin, at which they all meet,

represents the spherically symmetric solution. Trigonometric monopoles are given by the three half lines

(a) $y = 0, x > 0$,

(b) $y = -\sqrt{3}x, x < 0$

(c) $y = \sqrt{3}x, x < 0$.

Hyperbolic monopoles correspond to the half lines

(d) $y = 0, x < 0$,

(e) $y = -\sqrt{3}x, x > 0$

(f) $y = \sqrt{3}x, x > 0$.

In terms of our coordinates x, y, the spectral curve has equation

$$\eta^2 - \frac{y}{4\sqrt{3}}\zeta^4 + \frac{x}{2}\zeta^2 - \frac{y}{4\sqrt{3}} = 0. \tag{8}$$

We see that the hyperbolic axisymmetric monopoles of (d), (e) and (f) have spectral curves given by

$$\eta^2 + \frac{x}{2}\zeta^2 = 0 \quad (x < 0), \tag{9}$$

$$\eta^2 + \frac{x}{4}\zeta^4 + \frac{x}{2}\zeta^2 + \frac{x}{4} = 0 \quad (x > 0) \tag{10}$$

and

$$\eta^2 - \frac{x}{4}\zeta^4 + \frac{x}{2}\zeta^2 - \frac{x}{4} = 0 \quad (x > 0) \tag{11}$$

The spectral curves for trigonometric axisymmetric monopoles have the same equations but with x of the opposite sign. The spherically symmetric monopole has spectral curve $\eta^2 = 0$.

We observe that the spectral curves for hyperbolic axisymmetric monopoles are the union of two real sections. For example the curve with equation (9) is the union of the sections

$$\eta = \sqrt{-\frac{x}{2}}\zeta$$

and

$$\eta = -\sqrt{-\frac{x}{2}}\zeta$$

which are each real provided $x < 0$.

Using (2) we see that these sections correspond to the points $(\frac{1}{2}\sqrt{\frac{-x}{2}}, 0, 0)$ and $(-\frac{1}{2}\sqrt{\frac{-x}{2}}, 0, 0)$ in $\mathbb{R}^3$, which lie on the axis of symmetry of the monopole.

The spectral curves in the trigonometric case, however, cannot be expressed as a union of *real* sections. Recalling the correspondence between real sections and points of $\mathbb{R}^3$, we see that hyperbolic monopoles have an interpretation as two separated points on the axis of symmetry, but that this breaks down in the trigonometric situation. Further evidence for this comes from the calculations of the first author (Dancer 1991b), which show that the norm squared of the Higgs field of hyperbolic monopoles has two distinct minima, while in the trigonometric case it has just one minimum. Moreover the energy density peaks at two points on the axis of symmetry in the hyperbolic case, but in a ring around the axis in the trigonometric case.

We have three asymptotic regions in Y given by

(i) x large and negative

(ii) x, y large and positive

(iii) x large and positive, y large and negative.

Looking at case (i), we see that asymptotically the spectral curve is approximately $\eta^2 + \frac{x}{2}\zeta^2 = 0$, that is, the union of two sections corresponding to points of $\mathbb{R}^3$. A similar analysis holds if we consider (ii), (iii). We deduce that the asymptotic regions of Y represent well-separated monopoles, with the separation increasing as we go off to infinity in Y.

Observe that the boundary of Y represents Nahm matrices with poles at $t = 0, 3$, that is, Nahm data for $SU(2)$ monopoles. It seems reasonable to conjecture that motion towards the boundary of Y should be interpreted as convergence of $SU(3)$ monopoles to embeddings of $SU(2)$ monopoles in $SU(3)$.

We observed earlier that there is a map from M^8 onto an open set W^0 in the space of real 3×3 symmetric tracefree matrices. Under this map Y is carried bijectively

onto the set of diagonal matrices in W^0. The spherically symmetric monopole corresponds to the zero matrix, the only element of W^0 with all eigenvalues equal. The axisymmetric monopoles correspond to those matrices with two equal eigenvalues; if the two equal eigenvalues are larger than the third we have trigonometric solutions, while if they are smaller than the third we are in the hyperbolic case.

We can use this description to associate a set of three mutually orthogonal *principal axes* to our monopoles. Let a monopole be represented by a matrix $\Phi \in W^0$ with eigenvalues $\lambda_1 \leq \lambda_2 \leq \lambda_3$. We define the *main, third and Higgs axes* to be the unoriented directions in $\mathbb{R}^3$ defined by unit eigenvectors of λ_1, λ_2 and λ_3 respectively. These are known collectively as the principal axes of the monopole.

If all the eigenvalues of Φ are distinct the principal axes are well-defined as unoriented axes in space. If two eigenvalues coincide only one of the principal axes is defined; the others become indeterminate. In the spherically symmetric case, when all eigenvalues are equal, all the axes are indeterminate. In particular, we see that in the trigonometric case the third and Higgs axes are indeterminate, while for hyperbolic monopoles this occurs for the main and third axes.

Y consists of monopoles whose main, Higgs and third axes are fixed as an unordered set; each of the six regions represents a particular ordering. As we cross a line representing trigonometric monopoles the Higgs and third axes are interchanged. If we cross a line of hyperbolic solutions, on the other hand, the main and third axes are interchanged. On the lines between the regions axes lose their identity. In the well-separated domain, where our monopoles have a particle interpretation, the Higgs axis should be thought of of as the axis joining these particles. We deduce that a geodesic crossing a line of trigonometric solutions represents scattering of monopoles through ninety degrees.

4. The geodesic flow

We shall now calculate the metric on Y induced by considering it as a submanifold of M^8.

Let us work first in the region where $f_1^2 \leq f_2^2 \leq f_3^2$. From work of the first author (Dancer 1991a) we know that the tangent vectors $(\epsilon_0, \epsilon_1, \epsilon_2, \epsilon_3)$ to M^8 at $(0, \frac{1}{2}f_1\sigma_1, \frac{1}{2}f_2\sigma_2, \frac{1}{2}f_3\sigma_3)$ are given by

$$\epsilon_0 = \begin{pmatrix} \dot{f}_1 I_4 \\ \dot{f}_2 I_3 + \frac{m_3}{f_2} \\ -\dot{f}_3 I_2 - \frac{n_2}{f_3} \end{pmatrix}$$

$$\epsilon_1 = \begin{pmatrix} \dot{f}_1 I_1 \\ \dot{f}_2 I_2 + \frac{m_2}{f_2} \\ \dot{f}_3 I_3 + \frac{n_3}{f_3} \end{pmatrix}$$

$$\epsilon_2 = \begin{pmatrix} -\dot{f}_1 I_2 \\ \dot{f}_2 I_1 + \frac{m_1}{f_2} \\ -\dot{f}_3 I_4 - \frac{n_4}{f_3} \end{pmatrix}$$

$$\epsilon_3 = \begin{pmatrix} -\dot{f}_1 I_3 \\ \dot{f}_2 I_4 + \frac{m_4}{f_2} \\ \dot{f}_3 I_1 + \frac{n_1}{f_3} \end{pmatrix},$$

where

$$I_j(t) = \int_0^t \left(\frac{m_j}{f_2^2(t')} + \frac{n_j}{f_3^2(t')} \right) dt'$$

and m_j, n_j are real constants.

It follows from our description of Y that the tangent vectors to Y at $(0, \frac{1}{2}f_1\sigma_1, \frac{1}{2}f_2\sigma_2, \frac{1}{2}f_3\sigma_3)$ are obtained by taking $m_2 = m_3 = m_4 = n_2 = n_3 = n_4 = 0$ in

our expressions above for $\epsilon_0, \ldots, \epsilon_3$. The vector fields $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}$ on Y dual to the coordinates x, y are obtained by setting $m_1 = n_1 = \frac{1}{4}$ and $m_1 = -n_1 = \frac{1}{4\sqrt{3}}$ respectively. On evaluating the inner products $< \frac{\partial}{\partial x}, \frac{\partial}{\partial y} >$ etc. we find that the metric is given by

$$ds^2 = Edx^2 + 2Fdxdy + Hdy^2,$$

where

$$\begin{aligned} E &= \frac{1}{16}(h_1 + 2h_2 + h_3 + g_1 + g_2) \\ F &= \frac{\sqrt{3}}{48}(h_1 - h_3 + g_1 - g_2) \\ H &= \frac{1}{48}(h_1 - 2h_2 + h_3 + g_1 + g_2) \end{aligned}$$

and

$$\begin{aligned} h_1 &= f_1(3)f_2(3)f_3(3)(\int_0^3 \frac{1}{f_2^2})^2 \\ h_2 &= f_1(3)f_2(3)f_3(3)(\int_0^3 \frac{1}{f_2^2})(\int_0^3 \frac{1}{f_3^2}) \\ h_3 &= f_1(3)f_2(3)f_3(3)(\int_0^3 \frac{1}{f_3^2})^2 \\ g_1 &= \int_0^3 \frac{1}{f_2^2} \\ g_2 &= \int_0^3 \frac{1}{f_3^2}. \end{aligned}$$

We can obtain similar expressions in the other regions of Y.

We see that our metric is written in terms of Jacobi functions and integrals of squares of Jacobi functions, or equivalently in terms of Jacobi functions, complete elliptic integrals and incomplete elliptic integrals. Furthermore the derivatives of the metric coefficients also just involve these three kinds of special function.

The upshot is that we can write down the Christoffel symbols of the metric, that is, the coefficients of the geodesic equations, explicitly in terms of Jacobi functions and elliptic integrals. The geodesic equations can then be integrated numerically. We

evolved them using a fifth-order Runge-Kutta scheme with adaptive step size. The elliptic functions and elliptic integrals were calculated using Bulirsch's algorithms, as described by Press et al (1988). There was no need to use any great computing power; we found no problems in using a very modest PC, based on a 386SX microprocessor. As a check on the numerics we can calculate the kinetic energy $\mathcal{T} = E\dot{x}^2 + 2F\dot{x}\dot{y} + H\dot{y}^2$ which should remain constant throughout the geodesic motion (here $\cdot$ denotes differentiation with respect to the geodesic parameter). We found that $\mathcal{T}$ was typically conserved to within one part in 10^6.

Some of the resulting geodesics are shown in Figures 2-5.

Let us now discuss the interpretation of the geodesic flow in terms of monopoles.

We first of all observe that the lines $y = 0, y = \sqrt{3}x$ and $y = -\sqrt{3}x$ are geodesics, because they are the fixed point sets of involutions in the symmetric group on three letters which acts isometrically on Y. These geodesics represent monopoles axisymmetric about a fixed axis in space. We deduce that a possible process in monopole dynamics is for a monopole to remain axisymmetric about a fixed axis, either converging to an embedding of an $SU(2)$ monopole in $SU(3)$ or else coming to resemble two well-separated particles with the separation increasing as time goes on.

In figures 2-4 we have shown families of geodesics starting from the following three different initial configurations:

(i) a hyperbolic axisymmetric monopole,

(ii) a trigonometric axisymmetric monopole,

(iii) a non-axisymmetric monopole.

For each choice of initial point P we have plotted eighteen geodesics starting from there. Of course, the geodesic equations are invariant under time reversal, so our curves fit together to form nine full geodesics through P.

We observe that the geodesic behaviour is qualitatively similar in all cases. Generically the geodesics ultimately move out into the well-separated region and never return. However before they do this they may perform up to three crossings between regions in the core region of Y. In particular, we may have up to two ninety-degree scat-

terings. By keeping track of which regions of Y correspond to which orderings of the principal axes, we can see that when there are two ninety-degree scatterings these scatterings take place in two *orthogonal* planes. This process has not been observed in the dynamics of $SU(2)$ monopoles.

It appears that for each monopole there are three critical geodesics which represent motion towards the boundary of the moduli space, staying away from the well-separated region. The geodesics of axisymmetric monopoles discussed above are examples of this phenomenon.

A closed geodesic would represent a bound state of an $SU(3)$ monopole. No such geodesics were encountered in our study of geodesic motion on Y.

In Figure 5 we have plotted geodesics starting at an axisymmetric monopole in an asymptotic area of Y. Again, the geodesics generically move out into asymptotic regions and do not return. We see that the geodesics diverge from each other rapidly, suggesting that Y is negatively curved.

Finally, notice that all of our geodesics stay within Y and do not reach the boundary in finite time. Physically, we interpret this as meaning that monopoles do not get 'lost'.

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Legends

Figure 1. The two-dimensional submanifold Y. Each pair of regions (A,B) , (C,D) and (E,F) corresponds to monopoles lying along one of three mutually orthogonal axes in space.

Figure 2. Geodesics in Y passing through the point $(x, y) = (1, -\sqrt{3})$.

Figure 3. Geodesics in Y passing through the point $(x, y) = (1, 0)$.

Figure 4. Geodesics in Y passing through the point $(x, y) = (\frac{3}{2}, -\frac{\sqrt{3}}{2})$.

Figure 5. Geodesics in Y passing through the point $(x, y) = (\frac{7}{2}, -\frac{7\sqrt{3}}{2})$.

FIGURE 1

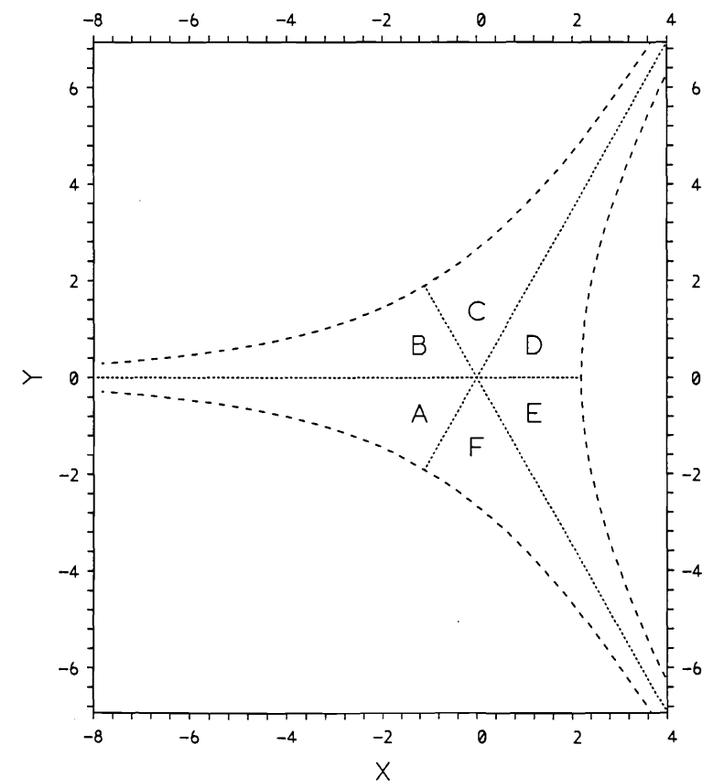

FIGURE 2

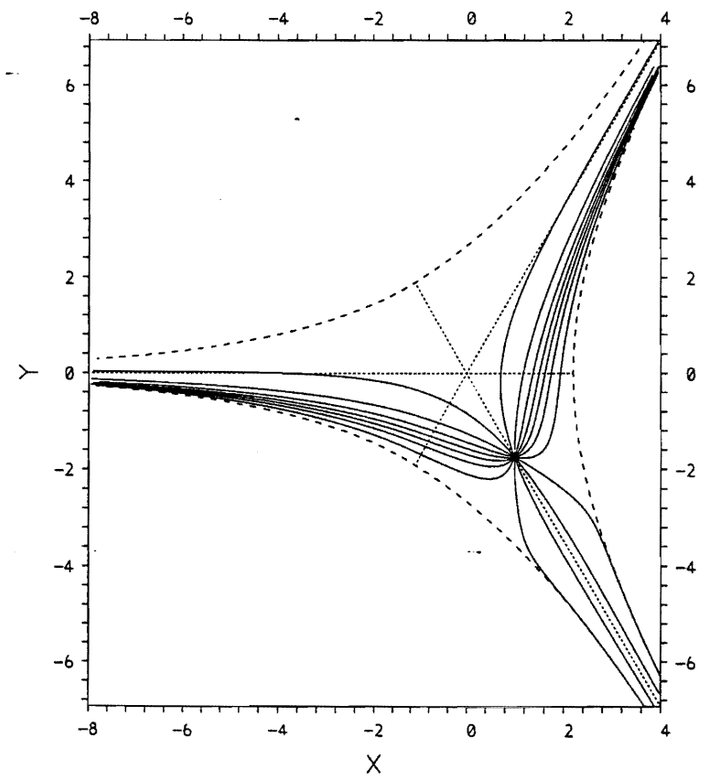

FIGURE 3

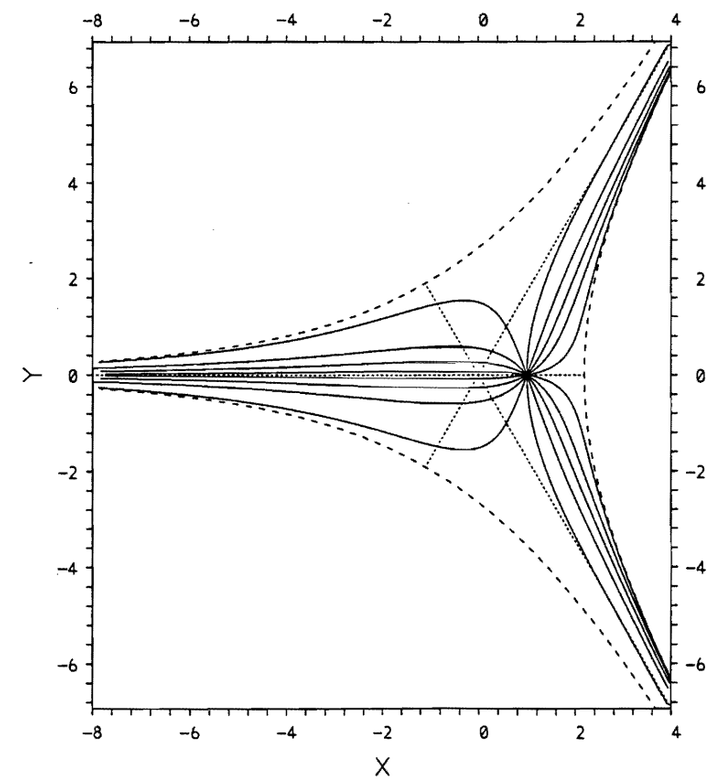

FIGURE 4

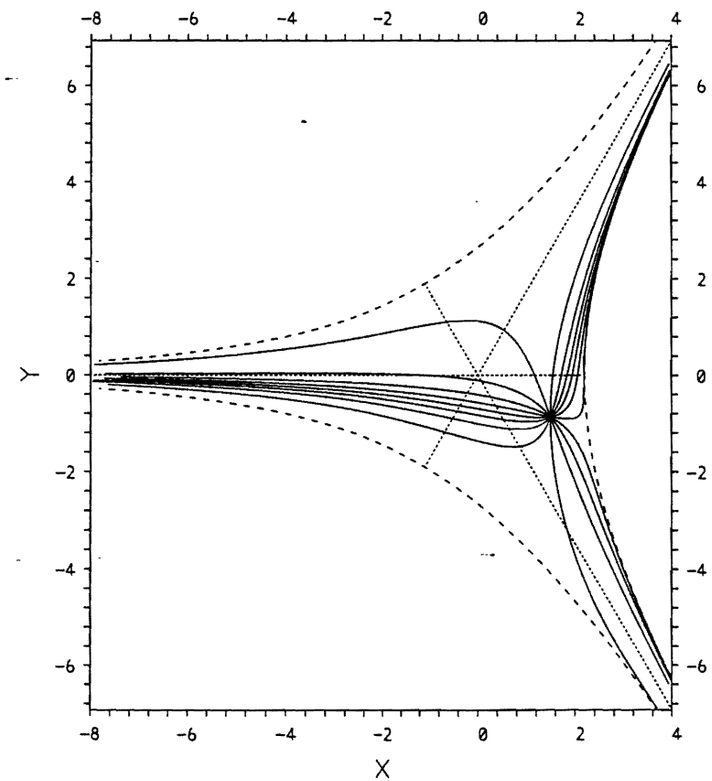
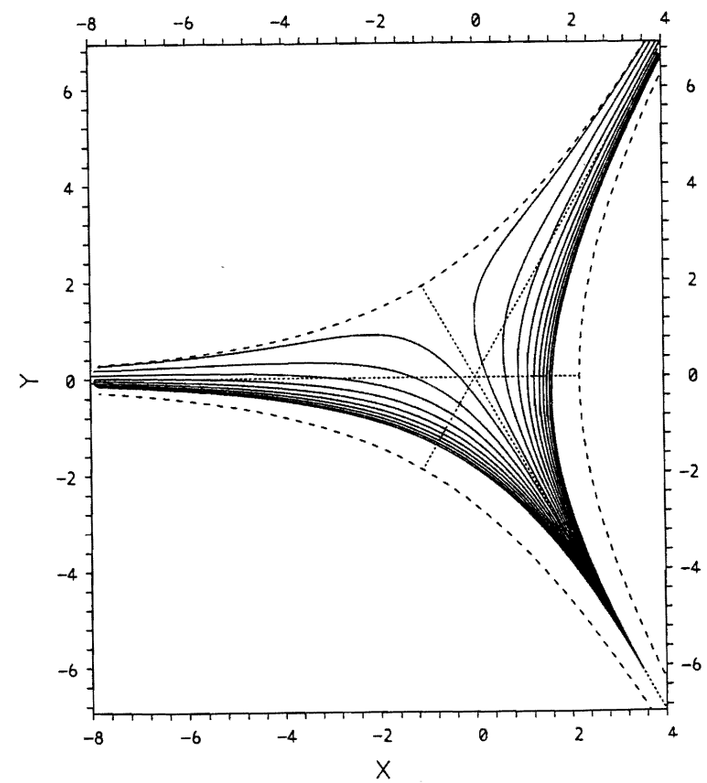
FIGURE 5