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The Chiral Anomaly as a Quantum Mirror

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Abstract

A model consisting of a left-handed fermion coupling to an abelian gauge field in (1+1) space-time dimensions is considered. This minimal chiral Schwinger model is quantized by applying chiral bosonization technique. An anomaly in the local gauge symmetry is shown to simulate the effect of a right-handed fermion coupling to the gauge field. Therefore the quantum solution equals exactly that of the vector Schwinger model.

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Twenty years ago Gross and Jackiw [1] showed that an asymmetric fermion content and the ensueing anomaly renders a gauge theory inconsistent and/or nonunitary. Only very recently has it been realized that their proof rests on an incorrect application of the Faddeev-Popov procedure [2]. This scrutiny of an apparently settled issue came about with ideas of Faddeev and Shatashvili [3] concerning the possibility of consistently quantizing anomalous models, and the demonstration by Jackiw and Rajaraman [4] that the chiral Schwinger model (CSM) is an appropriate example. Here a Dirac fermion couples chirally to an abelian gauge field in (1+1) space-time dimensions (2d). One should expect an anomalous breakdown of the chiral gauge symmetry upon quantization. However, the anomaly converts usually unphysical gauge degrees of freedom into physical ones, and thereby restores the symmetry. One ends up with a consistent quantum solution, which is also unitary in a limited sense [4].

The idea of an anomaly generating new physical degrees of freedom has already made a first appearance in Polyakov's approach towards 2d quantum gravity [5]. In fact this gravity is completely induced by the Weyl anomaly. But the quantization of the corresponding Weyl degrees of freedom proved difficult. Eventually progress had been made by employing the so-called chiral gauge [6], which then also prompted success with the original, conformal gauge model [7]. Nevertheless the exactly soluabe CSM has remained the favourite toy model for studies in anomalous gauge theories [8]. The most interesting recent development is probably the advent of the minimal CSM, where a chiral fermion couples to the gauge field [9,10]. The seemingly negligible difference from the original formulation [4,8], i.e. the absence of a free right-handed fermion, has a surprisingly grave effect. Standard bosonization technique is no longer applicable, and one has to apply chiral bosonization [11,12]. Strictly speaking this is also the only acceptable way of quantizing the original CSM as defined in [4]. Namely, the conventional bosonization made possible by the presence of the right-handed fermion introduces superfluous degrees of freedom in the effective

action. Unfortunately chiral bosonization is much harder to perform. Moreover there exists an ambiguity concerning the question what the correct bosonized form of the CSM is. Accordingly several solutions have been put forth, either based on chiral bosonization after Siegel [9,11] or after Floreanini and Jackiw [10,12].

In the present letter a new bosonized CSM is proposed. Heavy use is made of knowledge accumulated in the quest for defining 2d quantum gravity [5-7], in particular its chiral version [13,14]. We will see that the anomaly in the CSM has the effect of a "quantum mirror". That is to say, the quantum solution of the CSM coincides *exactly* with the solution of the Schwinger model [15], where both left- and right-handed fermions couple to the abelian gauge field.

The Lagrangian of the minimal CSM reads [16]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \psi_-^\dagger(2i\partial_+ + eA_+)\psi_- \ , \tag{1}$$

which means in our conventions that a left-handed fermion couples to A_+. Both ψ_- and A_+ depend only on x^+, i.e. they are purely left-moving. When ψ_- is integrated out,

$$\int d\psi_-^\dagger d\psi_- \exp\{i\int d^2x\mathcal{L}\} = \exp\{iW_1[A]\} \ , \tag{2}$$

we find [4]

$$W_1[A] = \int d^2x\left[-\frac{1}{4}F_{\mu\nu}^2 + \frac{e^2}{8\pi}\left(aA_+A_- - A_+\frac{\partial_-}{\partial_+}A_+\right)\right] \ , \tag{3}$$

where $-\frac{1}{4}F_{\mu\nu}^2 = \frac{1}{2}(\partial_+A_- - \partial_-A_+)^2$. The parameter a is due to a regularization ambiguity. In order to understand its origin, let us recall that the fermionic measure is not invariant under the chiral gauge transformation

$$\begin{aligned} eA_+ &\to eA'_+ = eA_+ + 2\partial_+\theta \ , \\ \psi_- &\to \psi'_- = \exp\{i\theta\}\psi_- \ . \end{aligned} \tag{4}$$

The corresponding Jacobian is calculated in the standard derivation after Fujikawa [17] with the help of $D_+ \equiv \partial_+ - i\frac{e}{2}A_+$ for a necessary regularization. However, in [17] the effect

of a so-called abelian anomaly is calculated, i.e. a breakdown of a global gauge symmetry. The local gauge symmetry is supposed to survive quantization. This is not true in the present case, and a possibility of accounting for this is to adopt a more general ansatz for the regularizing Dirac operator: $D_+ \to D'_+ \equiv \partial_+ - i\frac{e}{2}[(1-\frac{a}{2})A_+ + \frac{a}{2}A_-]$ [18]. In order to complete the bosonization of (1), the nonlocal term in (3) has to be removed. Usually, one employs the relation

$$\exp\left\{-\frac{i}{8\pi}e^2\int d^2xA_+\frac{\partial_-}{\partial_+}A_+\right\} \sim \int dh \exp\left\{-\frac{i}{8}\int d^2x\left[h\partial_+\partial_-h - 2\frac{e}{\sqrt{\pi}}h\partial_-A_+\right]\right\} \ . \tag{5}$$

The kinetic term of h introduces a right-moving contribution which renders it unsuitable for bosonizing the left-handed fermion ψ_-. Moreover, even in the case of the conventional CSM, where a free right-handed fermion is added to (1), this bosonization is unsatisfactory [10]. The only effect of such a fermion should be a constant factor to the partition function. In order to avoid the unwanted right-moving degrees of freedom, one has to take resort to a form of chiral bosonization [11,12]. With $eA_+ \equiv 2\partial_+\phi$ and $\partial_- \to \nabla_- \equiv \partial_- - ih\partial_+$, we get

$$\exp\left\{-\frac{i}{8\pi}e^2\int d^2xA_+\frac{\partial_-}{\partial_+}A_+\right\} \sim \int dh \exp\left\{\frac{i}{2\pi}\int d^2x\phi\partial_+\nabla_-\phi\right\} \ . \tag{6}$$

This describes a left-moving boson ϕ, at the price of introducing a field h [11]. In contrast to relation (5) h does not have a kinetic term, so that it is merely a Lagrange multiplier. The term $\phi\partial_+\nabla_-\phi$ is invariant under Siegel's transformation $\delta\phi = \theta\partial_+\phi$, $\delta h = \theta\partial_+h - i\nabla_-\theta$. This is vital for showing that only the left-moving field ϕ is physical, since h can be dismissed as representing gauge degrees of freedom [11].

How can this reasoning be generalized to work for the CSM, as given by the effective action (3)? Obviously, the aim cannot be to express (3) by a chiral boson, since the CSM is massive. However, one should still be able to introduce only the very minimum number

of new degrees of freedom by following Siegel's idea. Let us consider the action

$$W_2[\phi,h] = \int d^2x \left\{ \frac{1}{2e^2}\partial_+(\nabla_-\phi - \partial_+h)\Box\nabla_-^{-1}(\nabla_-\phi - \partial_+h) \right. \\ \left. -\frac{1}{2\pi}(2\phi\partial_+^2 h - \phi\partial_+\nabla_-\phi) \right\} , \tag{7}$$

which has been derived from (3) by $a = 2$, $\partial_- \to \nabla_-$, $eA_+ \equiv 2\partial_+\phi$ and $eA_- \equiv 2\partial_+ h$ [19]. Moreover, the operator $\partial_-\nabla_-^{-1}$ has been inserted in $F_{\mu\nu}^2$. The variation of (7) under

$$\delta\phi = \theta\partial_+\phi - i\partial_+\theta , \\ \delta h = \theta\partial_+ h - i\nabla_-\theta \tag{8}$$

takes on a surprisingly simple form,

$$\frac{\delta}{\delta\theta}W_2[\phi,h]\Big|_{\theta=0} = -\frac{i}{4\pi}\partial_+^3 h . \tag{9}$$

The crucial observation to be communicated in the present letter is that the partition function which defines the quantized CSM,

$$\mathcal{Z} = \int d\phi dh \exp\{iW_2[\phi,h]\} , \tag{10}$$

is indeed invariant under (8), thanks to a highly nontrivial effect. In order to show this we apply work done on 2d chiral quantum gravity [13,14]. In fact, the CSM as bosonized above resembles the chiral gauge version [14]. First ϕ is transformed into a new field F, which is defined by

$$F = \nabla_-\phi - \partial_+ h . \tag{11}$$

The partition function now reads

$$\mathcal{Z} = \int dF dh \exp\{iW_3[F,h]\} , \tag{12}$$

with

$$W_3[F,h] = \int d^2x \left\{ \frac{1}{2e^2}\partial_+ F\Box\nabla_-^{-1}F \right. \\ \left. +\frac{1}{2\pi}\left(\partial_+ F\nabla_-^{-1}F - \left[1+\frac{1}{12}\right]\partial_+^2 h\nabla_-^{-1}\partial_+ h\right)\right\} . \tag{13}$$

The term $-\frac{1}{2\pi}\partial_+^2 h\nabla_-^{-1}\partial_+ h$ originates straightforwardly from the substitution (11). It accounts for the right-hand side of the anomaly equation (9), while the F-terms are invariant. The new contribution $-\frac{1}{24\pi}\partial_+^2 h\nabla_-^{-1}\partial_+ h$ derives from the noninvariance of the measure $d\phi$ under the transformation (11) into dF [14]. For the partition function $\mathcal{Z}$ we now get

$$\frac{\delta}{\delta\theta}\mathcal{Z}\Big|_{\theta=0} = 0 , \tag{14}$$

since the noninvariance of the measure dh under (8) gives rise to yet another contribution to (13), $\frac{26}{48\pi}\partial_+^2 h\nabla_-^{-1}\partial_+ h$. This is a gravitational anomaly; the h-term in W_3 has just the form of 2d quantum gravity in the chiral gauge, and (8) is a combined coordinate reparametrization and Weyl rescaling [6,14].

There is another possibility of understanding the symmetry-restoration expressed by (14). The second line in (7) is, up to the coefficient, chiral gauge 2d chiral quantum gravity for vanishing matter central charges [14]. The analogous expression in the conformal gauge reads [13]

$$W_4[\varphi,f] = \frac{1}{4\pi}\int d^2x(\varphi\Box\varphi - f\Box f) , \tag{15}$$

where φ is the Weyl variable and f the Lorentz variable. This time it is the Weyl symmetry $\delta\varphi = \theta$, $\delta f = 0$ which is anomalously restored. We get contributions from two bosonic fields φ and f ($\frac{1}{96\pi}\varphi\Box\varphi$ each), and from the gauge fixing sector ($-\frac{26}{96\pi}\varphi\Box\varphi$).

So (8) is indeed a gauge symmetry of the quantized CSM, as described by the effective action (7). The equations of motion of the quantized fields F and h read

$$\text{for } F: \qquad \left(\Box + \frac{e^2}{\pi}\right)\partial_+\nabla_-^{-1}F = 0 , \\ \text{and} \\ \text{for } h: \qquad iF\partial_+(\nabla_-^*)^{-1}\left[\left(\Box + \frac{e^2}{\pi}\right)\partial_+\nabla_-^{-1}F\right] = 0 . \tag{16}$$

We see that the h-field represents redundant degrees of freedom and can be gauged away

by fixing the unitary gauge, $h = 0$. Then we are left with

$$\left(\Box + \frac{e^2}{\pi}\right) \partial_+ \partial_-^{-1} F = 0 \,. \tag{17}$$

Since equation (11) now reads $\partial_- \phi = F$, we see that the classically massless chiral field A_+ has gained a mass, $m^2 = \frac{e^2}{\pi}$. Therefore one ends up with exactly the same result as if one had started with the vector Schwinger model [15]

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + \psi_-^\dagger (2i\partial_+ + eA_+)\psi_- + \psi_+^\dagger (2i\partial_- + eA_-)\psi_+ \,, \tag{18}$$

and fixed the chiral gauge $A_- = 0$ in the effective action derived by integrating over $\psi_\pm$. It looks as if one could apply the gauge condition directly to the Lagrangian. The ensueing chiral anomaly acts like a quantum mirror, i.e. it restores the effect of the ψ_+-term in (18) on the quantum level. Stated differently, it provides an exact compensation for the degrees of freedom lost by the "wrongly applied gauge fixation".

It has to be stressed that this new solution of the CSM has been reached by taking (7) as its bosonized action. This action was derived by applying certain ad-hoc assumptions, a drawback which is shared by prior solutions [4,8-10]. However, our solution is singled out by being really minimal, i.e. no superfluous degrees of freedom are introduced. It is important to note that this could only be achieved for $a = 2$, which makes the equivalence to the Schwinger model complete. If the CSM is taken as a pattern for realistic (3+1)-dimensional chiral gauge theories, the present result simply means that no physical phenomenon, like e.g. mass generation [4], can ever be attributed to a chiral anomaly. Its effect is not discernable from that of a very heavy, and therefore unvisible, chiral fermion which leads to anomaly-cancellation.

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[16] Conventions: $\gamma^0 = \sigma^1$, $\gamma^1 = i\sigma^2$, $\gamma^5 = -\gamma^0\gamma^1$, $\epsilon^{01} = 1$, $x^\pm = x^0 \pm x^1$, $\partial_\pm = \frac{1}{2}(\partial_0 \pm \partial_1)$, $A_\pm = A_0 \pm A_1$, $\psi = (\psi_+, \psi_-)^T$.

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[19] Note that A_μ can be expressed by two scalars in flat spacetime, $eA_\mu = \partial_\mu\sigma + \epsilon_{\mu\nu}\partial^\nu\rho$. Then we get $eA_+ = 2\partial_+(\sigma + \rho) \equiv 2\partial_+\phi$, and $eA_- = 2\partial_-(\sigma - \rho) \equiv 2\partial_+ h$, i.e. $h = \partial_+^{-1}\partial_-(\sigma - \rho)$.