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The Gravitational "Plane Waves" of Liu & Zhou

and

the Nonexistence of Dynamic Solutions for Einstein's Equation

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Abstract

Although both the electromagnetic wave and the gravitational wave can be produced approximately from Maxwell-type equations, there are subtle differences in their respective exact equations. Since gravitational wave carries energy-momentum, the exact field equation of a gravitational wave must have a nonzero source term along its path, whereas a field equation for an electromagnetic wave does not. This explains that there is no weak wave solution of Einstein equation. Historically, neither Einstein & Rosen nor the Physical Review was aware that the nonexistence of gravitational wave solutions is due to a violation of the principle of causality. It is pointed out that the criterion of Liu & Zhou on plane-waves is valid since the principle of causality requires the existence of weak limits. However, due to the influence of the popular but unverified assumption of the existence of dynamic solutions, they made careless errors in their calculations and incorrectly concluded that their plane-waves have weak limits. It is shown that "plane-waves" of Liu & Zhou, is actually unbounded in amplitude, and have no weak limit. Therefore, Liu & Zhou provide additional evidence in supporting the nonexistence of dynamic solutions.

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Key Words: Einstein's equivalence principle, Einstein-Minkowski condition, Euclidean-like structure, dynamic solution, gravitational radiation, principle of causality, relativistic causality, plane-wave.

1. Introduction

In general relativity, a common mistake among theorists including Einstein [1] and Feynman [2], was assuming the existence of dynamic solutions for the Einstein equation [3]. This question of dynamic solutions was raised by Gullstrand [4] in his 1921 report to the Nobel Committee, and Einstein was awarded the prize mainly for his theory on the photoelectric effects [5]. Due to prevailing conceptual errors such as ambiguity of coordinates that were pointed out by Whitehead [6], Fock [7], and Zhou [8], many theorists cannot reconcile the non-existence of dynamic solutions with the three accurate predictions, which is based on static solutions. It was not until 1995 that the nonexistence of dynamic solution is proven [9,10] and related issues are further addressed in 2000 [3] and in 2002 [11, 12]. In the proof that differs from previous considerations, physical requirements such as the principle of causality [1)] are crucial [3].

Nevertheless, some still do not accept this conclusion because they have mistaken non-static solutions [2)] as valid dynamic solutions (see also Sections 4 & 5). For a linear field equation in physics, the existence of dynamic solutions is often true after static solutions are verified as valid. Formal linearization of Einstein equation also shows the existence of weak dynamic solutions. It turns out, however, such a linearization is mathematically valid only for the static cases [3]. In fact, all the existing "wave" solutions are unbounded in amplitude and thus not valid in physics. Moreover, to explain the Hulse-Taylor binary pulsar experiment, it is necessary to modify Einstein's equation with an added source term, the gravitation energy-stress tensor (with an antigravity coupling), to accommodate the waves [9, 10] and to have a valid linearized equation.

Currently, a gravitational wave is often explained in term of similarity with an electromagnetic wave because the linearized field equation for weak gravity is a set of Maxwell-type equations [13, 14]. However, there are subtle differences in physics between the electromagnetic wave whose sources are accelerated electric charges and the gravitational wave whose sources are energy-stress tensors. Since a wave carries energy-momentum, a gravitational wave should carry a source along, although an electromagnetic wave does not carry charges along.

However, if such a source term of a gravitational wave is of the second order, this term would be absent from an equation for the first order approximation although such a term must be present in the full nonlinear equation [3]. This explains why the Einstein equation is incompatible with a wave solution of the linearized equation [9]. Moreover, for the case of a massive source, this linear equation can be derived directly from [3)] Einstein's equivalence principle [3]. Thus, this field equation of weak gravity for a massive source is more appropriately called the Maxwell-Newton Approximation (MNA) [9].

To demonstrate this physical interpretation, it is useful to analyze a "plane-wave" (although the nonexistence of a gravitational plane-waves solution, which would presumably be an idealization of a wave from a distant source [15], has already been proven as due to the lack of a source term with an anti-gravity coupling [10]). In particular, Einstein's weak plane-wave solu-

tion, obtained form the Maxwell-Newton Approximation [16], can be used to illustrate that a plane-wave solution would not have a weak limit. However, this is in conflict with the claim of Liu & Zhou [17], which is based on explicit calculations.

It will be shown that their claim is actually due to mathematical errors near the end. Their incorrect derivation is understandably influenced by the prevailing but invalid assumption that dynamic solutions exist for the 1915 Einstein's equation [3] -- the same influence that led to invalid analysis of the Einstein-Rosen [18] cylindrical "wave" solution (see Appendix). This is another example illustrating that a bias belief could be decisive on the judgment of a theorist although this person may be otherwise excellent. On the other hand, this illustration of non-existence of a valid gravitational plane-wave solution is a simple but strong evidence for the conclusion that there is no dynamic solution for the 1915 Einstein equation [3, 9].

2. The Field Equations

In physics, the existence of a wave is due to that a physical cause must propagate with a finite speed [19]. This implies also that a wave carries energy-momentum. Thus, the field equation for gravity must be able to accommodate the gravitational wave, which carries away gravitational energy-momentum. However, the Einstein equation of 1915 fails this [3].

In general relativity, the Einstein equation of 1915 [20, 21] for gravity of space-time metric $g_{\mu\nu}$ is

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -K\, T(m)_{\mu\nu}\,, \tag{1}$$

where $G_{\mu\nu}$ is the Einstein tensor, $R_{\mu\nu}$ is the Ricci curvature tensor, $T(m)_{\mu\nu}$ is the energy-stress tensor for massive matter, and K (= $8\pi\kappa c^{-2}$, and κ is the Newtonian coupling constant) is the coupling constant. Thus,

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0, \qquad \text{or} \qquad R_{\mu\nu} = 0, \tag{1'}$$

would be the equation at vacuum. However, (1') also implies no gravitational wave to carry away energy-momentum [4]. An incompatibility with radiation was first discovered by Einstein & Rosen [18] in 1937. However, due to mathematical and conceptual errors, 't Hooft [22] still incorrectly believed their cylindrical wave as valid (see Section 4 and Appendix).

A major problem is a mathematical error on the relationship between (1) and its "linearization" [3]. It was incorrectly believed that the linear Maxwell-Newton Approximation [9]

$$\frac{1}{2}\partial^c\partial_c \bar{\gamma}_{\mu\nu} = -K\, T(m)_{\mu\nu}\,, \qquad \text{where} \qquad \bar{\gamma}_{\mu\nu} = \gamma_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}(\eta^{cd}\gamma_{cd}) \tag{2a}$$

and

$$\bar{\gamma}_{\mu\nu}(x^i, t) = -\frac{K}{2\pi}\int \frac{1}{r} T_{\mu\nu}[y^i, (t-r)]d^3y, \qquad \text{where} \qquad r^2 = \sum_{i=1}^{3}(x^i - y^i)^2\,. \tag{2b}$$

always provides a first-order approximation for equation (1). This belief was verified for the static case only.

For a dynamic case, however, the relation that (2) as an approximation of (1) is no longer valid because there is no bounded dynamic solution [5] for Einstein equation (1) [3, 9]. A simple but convincing evidence for the absence of a bounded dynamic solution is, as shown by Hu, Zhang & Ting [23], that gravitational radiation calculated would depend on the approach used. (A similar problem in approximation schemes such as post-Newtonian approximation [4] is that their validity is also assumed only.) Note that while the Cauchy data can be arbitrary for (2a), but is restricted for Einstein equation (1) [13].

For a dynamic situation, (2) is actually an approximation of the 1995 update of the Einstein equation [6] [9],

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -K\,[T(m)_{\mu\nu} - t(g)_{\mu\nu}], \tag{3}$$

where $t(g)_{\mu\nu}$ is the energy-stress tensors for gravity [3] and is of the first order in K. Then, since $\nabla^{\mu} G_{\mu\nu} \equiv 0$, we have

$$\nabla^{\mu} t(g)_{\mu\nu} = 0 \qquad \text{if} \qquad \nabla^{\mu} T(m)_{\mu\nu} = 0. \tag{4}$$

This equation (3) is based on the Hulse-Taylor experiment of binary pulsars and related physical requirements [9]. Thus, a gravitational radiation does carry energy-momentum, as physics requires. From (3), the equation in vacuum is

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = K\, t(g)_{\mu\nu}\,. \tag{3'}$$

The gravitational energy-stress tensor $t(g)_{\mu\nu}$ is non-zero when gravitational wave is present. Historically, a gravitational energy-stress, defined by Einstein [13, 14], is a pseudo-tensor. However, the fact that the gravitational field exchanges energy with a particle requires such an energy-stress to be a tensor but not just a pseudo-tensor [9].

The Hulse-Taylor experiment requires that, for weak gravity, $t(g)_{\mu\nu}$ be approximately equivalent to Einstein's pseudo-tensor [9]. Thus, although the exact form of $t(g)_{\mu\nu}$ with an anti-gravity coupling is not known, the radiation of weak gravity [7] can be calculated with help of the Maxwell-Newton Approximation (2) [3].

3. The Question of Weak Limits for Liu & Zhou's Plane-Waves

The analysis of Liu & Zhou [17] is based on comparison with the approximate solutions of plane waves,

$$ds^2 = dt^2 - dx^2 - (1 + 2\phi)\, dy^2 - (1 - 2\phi) dz^2 \tag{5}$$

where ϕ is a function of u (= t – x). This solution is obtained by Einstein [16] with the Maxwell-Newton Approximation (2a) without a source together with the linearized harmonic condition $\partial^{\mu} \bar{\gamma}_{\mu\nu} = 0$.[8] Liu & Zhou [17] found that the plane waves of Bondi, Pirani, & Robinson [15] contain singularities and do not satisfy the harmonic condition. Peres, Ehlers, Kundt and others

studied solutions of the "PP" waves [24-26]. These solutions are free from singularities but do not contain Einstein's solutions as approximations as expected by Liu & Zhou [17].

Apparently, they believed the unverified assumption [9] that Eq. (1) has dynamic solutions if the sources are massive matter. Nevertheless, instead of their expected results, the opposite is correct. The physical reason is that (1') cannot have weak dynamic solutions, which must be bounded (in amplitude) [27] (see also Sections 4 & 5). In other words, their solutions of (1') and any wave solution of (2) cannot be compatible, and this is independent of a gauge condition. Some theorists might disregard the relevance of investigating plane-waves on the ground that that total energy is infinite. However, it has been proven that plane-wave idealizations are useful in understanding the local properties [10, 14].

The plane wave solution of Liu & Zhou [17], which satisfies the harmonic gauge, is as follows:

$$ds^2 = dt^2 - dx^2 + 2\,F(dt - dx)^2 - \cosh 2\psi(e^{2\phi}\,dy^2 + e^{-2\phi}dz^2) - 2\sinh 2\psi\,dy\,dz. \tag{6}$$

where $\phi = \phi(u)$ and $\psi = \psi(u)$. Moreover, $F = F_P + H$, where.

$$F_P = \frac{1}{2}(\dot{\psi}^2 + \dot{\phi}^2\cosh^2 2\psi)\,[\cosh 2\psi\,(e^{2\phi}\,y^2 + e^{-2\phi}\,z^2) + 2\sinh 2\phi\,yz], \tag{7}$$

which is interpreted as the energy-flux function, and H satisfies the equation,

$$\cosh 2\psi\,(e^{-2\phi}H_{,22} + e^{2\phi}\,H_{,33}) - 2\sinh 2\psi\,H_{,23} = 0. \tag{8}$$

Obviously, $H = 0$ is a solution of (8), but F_p does not satisfy (8). Then Liu & Zhou claimed, "For the weak fields we have $1 >> |\phi|$, $1 >> |\psi|$, so that the weak field approximation of the metric (6.2) (i.e. metric (6) here) is,

$$ds^2 = dt^2 - dx^2 - (1 + 2\phi)\,dy^2 - (1 - 2\phi)dz^2 - 4\psi dydz, \tag{9}$$

This is just the Einstein weak field solution of plane waves containing two polarizations. So, we can conclude that Einstein's solution is an approximation of our exact solution, while our solution is the exact form of Einstein's approximate solution."

However, F_P is not negligible even if $1 >> |\phi|$, $|\psi| > 0$. If one examines (7) closely, then it is clear that F_P is bounded only if the first packet factor is zero since the second factor contains y^2 and z^2, which can be infinitely large. Thus, such bounded ness is possible only if $\dot{\psi} = \dot{\phi} = 0$, i.e., $\phi(u)$ and $\psi(u)$ are constants. In turn, this implies that there is no gravitational wave. As predicted, the plane-wave (6) is unbounded and invalid in physics. Mathematically, metric (6) does not have a weak wave limit since it is intrinsically unbounded.

4. Gravitational Plane-Waves and Physical Requirements

While a solution of the Maxwell-Newton Approximation is bounded, unboundedness is a problem among all "plane waves" of Eq. (1') [3] although there is no logarithmic divergent identified by Fock [28]. On the other hand, according to the principle of causality [1)], the amplitude of a wave metric component must be bounded since a wave is related to energy transfer. In addition, according to the Maxwell-Newton Approximation, the metric due to a weak or distant source must be bounded as required by Einstein's notion of weak gravity. Therefore, the rejection of unbounded plane-waves by Liu & Zhou [17] is justified. On the other hand, the violation of the principle of causality would be manifested as no bounded wave solutions.

For example, the metric of Bondi et al. [15], claimed as a wave from a distant source, is

$$ds^2 = \exp(2\phi)(d\tau^2 - d\xi^2) - u^2[\mathrm{ch}2\beta\,(d\eta^2 + d\zeta^2) + \mathrm{sh}2\beta\cos2\theta\,(d\eta^2 - d\zeta^2) - 2\mathrm{sh}2\beta\sin2\theta\,d\eta d\zeta], \tag{10}$$

where ϕ, β, θ are functions of u ($= \tau - \xi$). It satisfies the differential equation (i.e., their Eq. [2.8]),

$$2\phi' = u(\beta'^2 + \theta'^2\,\mathrm{sh}^2 2\beta). \tag{11}$$

This metric was considered as valid since physical requirement was ignored due to the misinterpretation of Pauli [11, 29] that is popular because the physical meaning of coordinates is not required.

However, in spite of being claimed as a wave from a distant source, metric (10) is not bounded because of the factor u^2 that grows anomaly large as time τ goes by. It should be noted also that metric (10) is only a plane, but not a periodic function because a smooth periodic function must be bounded. According to Eq. (2), a plane-wave with a finite source is always bounded, thus an unbounded plane-wave violates the principle of causality.

Another well known "plane wave" is the metric considered by Misner, Thorne & Wheeler [14] as follows:

$$ds^2 = dt'^2 - dx'^2 - L^2(e^{2\beta}dy^2 + e^{-2\beta}dz^2), \tag{12}$$

where $L = L(u')$, $\beta = \beta\,(u')$, $u' = t' - x'$. Then, the field equation becomes

$$\frac{d^2L}{du'^2} + L\left(\frac{d\beta}{du'}\right)^2 = 0\,. \tag{13}$$

However, in spite of their claim, it has been shown [30] that this equation has no bounded plane wave solution.

Another "plane wave", which is intrinsically unphysical, is the metric accepted by Penrose [31] as follows:

$$ds^2 = du\,dv + Hdu^2 - dx_i\,dx_i, \qquad \text{where} \qquad H = h_{ij}(u)\,x_i\,x_j \tag{14}$$

where $u = ct - z$, $v = ct + z$, $x = x_1$ and $y = x_2$, $h_{ii}(u) \geq 0$, and $h_{ij} = h_{ji}$. This metric satisfies the harmonic gauge. The cause of metric (14) can be an electromagnetic plane wave. Metric (14) satisfies

$$\eta^{\alpha\beta}\,\partial_\alpha\partial_\beta\,\gamma_{tt} = -2\{h_{xx}(u) + h_{yy}(u)\} \qquad \text{where} \qquad \gamma_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}. \tag{15}$$

However, this does not mean that causality is satisfied although metric (14) is related to a dynamic source.

The gravitational force is related to $\Gamma^{z}_{tt} = (1/2)\partial H/\partial t$. There are arbitrary non-physical parameters (the choice of origin) that are unrelated to the cause (a plane wave). Apparently, like others, Penrose [31] over-looked the physical requirements [1)] in particular the principle of causality, which may not require a clear physical meaning of coordinates. Experimentally, being unbounded, metric (14) is also incompatible with the calculation of light bending and classical electrodynamics.

These examples illustrate that there is no bounded wave solution for (1) although there is no logarithmic divergent [28]. Moreover, the principle of causality supports Einstein's notion of weak gravity [27]. Some theorists would consider the cylindrical symmetric metric of Einstein & Rosen [18] as a "wave" solution. A main reason is that they failed to understand physical principles such as the principle of causality adequately (see Appendix). Besides, one should note that this metric could only be a "standing wave" but not a solution of a propagating wave, which can be locally idealized as a plane-wave.

5. Discussions and Conclusions

The approach of Liu & Zhou [17] on the question of gravitational plane waves is different from other earlier approaches at least in four major points. They are:

1) Liu & Zhou implicitly reject a plane-wave with singularities;
2) it is recognized that a plane wave should have a weak limit, and thus bounded ness is a necessary condition;
3) the harmonic gauge is used; and
4) based on the practical calculations, Zhou has a definite notion of space-time coordinates as in special relativity [8].

Zhou's conjecture is proven essentially correct since a frame of reference of a physical space must have the Euclidean-like structure, [10)] in spite of a non-Euclidean metric [11, 12, 32].

Liu & Zhou [17] recognized that the amplitude of a plane-wave must be bounded and must have a weak limit since this is implied by the presumed validity of linearization of the exact equation [11)]. Moreover, Zhou advocated that coordinates must have physical meaning and he argued [33], "When we come to solve the field equation of moving matter, we must first define the geometrical configuration of matter, the symmetry of the configuration, its density distribution, pressure, and velocity of motion in space-time. All of them have to be expressed in terms of coordinates." On the other hand, some still unconditionally regarded any non-static solution [2)] as a dynamic solution since the physical meaning of coordinates was ambiguous. Such ambiguity was the reason that Whitehead [6] and Fock [7] did not accept general relativity as valid in physics.

Apparently, Liu & Zhou were not aware of the problem of existence related to a dynamic solution, [11] and did not know the need that the Maxwell-Newton Approximation should be derived, independent of the Einstein equation, directly from Einstein's equivalence principle [12, 27]. This Approximation is based on the theoretical framework of general relativity and, as a criterion [12], is supported by all experiments [3, 32]. Thus, the approach of Liu & Zhou [17] to use plane waves derived from the Maxwell-Newton Approximation as criterions for an exact plane wave has been recently justified.

Moreover, the harmonic gauge has useful applications in problems such as tracking the planets and spacecraft [14] although the harmonic gauge is not always valid [34]. An example is that the metric of Einstein's uniformly rotating disk does not satisfy the harmonic gauge or its linearization [12, 35]. However, this is unrelated to Zhou's conjecture of the harmonic gauge [36] since in vacuum such metrics are not asymptotically flat. Moreover, as pointed out by Eddington [37], the demand of a linearized harmonic gauge is intrinsic for the Maxwell-Newton Approximation. Thus, that the weak limit of a harmonic plane wave solution would be expected to reduce to a solution of the Maxwell-Newton Approximation.

Although the Maxwell-Newton Approximation has been established as a criterion, it is also compatible with other solutions such as the isotropic and the Yilmaz solution [38]. Thus, the problem of physical gauge cannot be solved in the first order, and it remains to consider other means to establish a physical gauge. However, it is clear that a physical gauge must be compatible with the linearized harmonic gauge, in spite of that the Einstein equation must be modified for a dynamic situation. [13] Thus, the approach of Liu & Zhou would be essentially valid for the physical gauge.

Kramer et al. [26] pointed out that many theorists had relied on the plane waves to resolve the controversies about the existence of gravitational radiation. This paper shows clearly that their analysis is inadequate. In relativity, the existence of gravitational wave is a crucial test of the field equation. Understandably, the assumption of the existence of dynamic solutions [9] is a common error among theorists, including Einstein [1], Feynman [2] and other famous physicists [3]. Unfortunately, Liu & Zhou did not manage to be an exception although they had the opportunity.

Some argued, different from Liu & Zhou, that only in a neighborhood of t–x plane rather than in the whole space, Eq. (9) serves as the weak limit of Liu & Zhou's gravitational waves. [14] Nevertheless, metric (6) still has no weak limit and is invalid in physics. A reason for the latter is that the factors y^2 and z^2 are due to choosing the two arbitrary parameters, a & b, as zeros in the factors (y - a) and (z - b) in a general form of metric (6). Since these parameters have no physical meaning, the principle of causality is violated. It follows that (6) has the same problem as metric (14) and thus is not a physical solution. This illustrates that, for an unbounded "plane-wave", a violation of the principle of causality is inevitable. Had they done their calculations carefully to the end, their work could have led to an earlier discovery of the non-existence of dynamic solutions.

At present, theorists believe Einstein's weak gravity assumption that if the source of gravity is weak, the amplitude will be small. Many also believe that, for any solution of Einstein's equation, it is always possible to change a gauge to the harmonic gauge. It thus follows that any non-static solution of Einstein's equation should be reducible to a bounded solution (in fact, of the Maxwell-Newton Approximation) by changing some physical parameters and the gauge. Nevertheless, some (for example, Penrose [31]) would accept an unbounded solution that has no physical weak limit [3], as valid in physics. Some theorists just do not understand the physics in general relativity, in particular Einstein's equivalence principle adequately [3].

Currently, a major problem was that the existence of gravitational wave solutions is based on faith instead of evidences, and related theories [14, 39] are based on hand-waving arguments [16] that cannot be rigorously substantiated [3, 40]. For some theorists, a gravitational wave solution is a "time dependent" metric without singularity, and other physical requirements such as Einstein's equivalence principle [17] and principle of causality are simply not considered. Liu & Zhou [17] are probably among the earliest, who recognize that physical requirements should be considered. For example, Bondi et al. [15] claimed that a non-static solution with *unbounded* amplitude represents a "wave" from a *distant* source. Misner et al. [14] claimed, with invalid arguments, the existence of bounded plane wave solutions. Penrose [31] ignored an obvious violation of causality.

Recently, 't Hooft [22] claimed the "cylindrical waves" of Einstein & Rosen [18] to be bounded propagating wave solutions in physics. 't Hooft did not see that this solution would violate the principle of causality (see Appendix). Historically, Einstein & Rosen [18] could be considered as the first to discover the non-existence of wave solution [41]. However, Physical Review found that the singularities they discovered are removable [42], but were unaware of that the nonexistence of gravitational wave solution is due to a violation of the principle of causality and thus also Einstein's notion of weak gravity [3, 9, 10].

A root of these problems is due to that physical space-time coordinates are ambiguous in Einstein's theory. (Such an ambiguity is a consequence of Einstein's theoretical errors on measurements [40, 43].) The correct criticisms of Whitehead [6] and Zhou [8] were not accepted since they do not provide a solid theoretical foundation for general relativity to explain impressive observational confirmations. Although the physical meaning of the space-time coordinates has been clarified recently [11, 12, 32], the existing conceptual problems still seem to grasp many theorists. Fortunately, the analysis on plane-waves initiated by Liu & Zhou [17] would give a simple illustration of the non-existence of the wave solution. It is hoped that this paper would give an added impetus for recognizing the severe problems in theories of gravitational waves.

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Appendix: The Principle of Causality and the Cylindrical Symmetric Metrics of Einstein and Rosen

> "To my mind there must be at the bottom of it all, not an equation, but an utterly simple idea. And to me that idea, when we finally discover it, will be so compelling, so inevitable, that we will say to one another, 'Oh, how beautiful. How could it have been otherwise?' " -- J. A. Wheeler [44].

It seems, the principle of causality [10] would be qualified as Wheeler's utterly simple idea. According to the principle of causality, a wave solution must be related to a dynamic source, and therefore is not just a time-dependent metric. A time-dependent solution can be obtained simply by a coordinate transformation [45]. Even in electrodynamics, satisfying the vacuum equation can be insufficient. For instance, the electromagnetic potential solution $A_0[\exp(t - z)^2]$ (A_0 is a constant), is invalid in physics. Thus, a solution free of singularities may not be valid.

A major problem in general relativity is that Einstein's equivalence principle has not been understood adequately [40, 43]. Consequently, physical principles are often neglected [12, 27]. For instance, the principle of causality was neglected such that a gravitational wave was not considered as related to a dynamic source [46]. Since the principle of causality was not understood adequately, solutions with arbitrary nonphysical parameters were accepted as valid [3].

Among the existing so-called wave solutions, not only Einstein's equivalence principle, but also others such as the principle of causality and the principle of relativistic causality [11] are not satisfied. Let us examine their cylindrical "waves" again. In cylindrical coordinates, ρ, φ, and z, the solution of Einstein & Rosen [18] is

$$ds^2 = \exp(2\gamma - 2\Psi)(dT^2 - d\rho^2) - \rho^2\exp(-2\Psi)d\varphi^2 - \exp(2\Psi)dz^2 \qquad \text{(A1)}$$

where T is the product of the velocity of light and the time coordinate, and γ and ψ are functions of ρ and T. They satisfy

$$\Psi_{\rho\rho} + (1/\rho)\Psi_{\rho\rho} - \Psi_{TT} = 0, \qquad \gamma_\rho = \rho[\Psi_\rho^2 + \Psi_T^2], \qquad \text{and} \qquad \gamma_T = 2\rho\Psi_\rho\Psi_T. \qquad \text{(A2)}$$

Rosen [27] consider the energy-stress tensor $T_{\mu\nu}$ that has cylindrical symmetry. He found that

$$T_{,45} + t_{,45} = 0, \qquad \text{and} \qquad T_4^{\,l} + t_4^{\,l} = 0 \qquad \text{(A3)}$$

where $t_{\mu\nu}$ is Einstein's gravitational pseudotensor, $t_4^{\,l}$ is momentum in the radial direction.

However, Weber & Wheeler [47] argued that these results are meaningless since $t_{\mu\nu}$ is not a tensor. They further argued that the wave is unbounded [19] and therefore the energy is undefined. Moreover, they speculated that the energy flux is non-zero. They claimed, "We concluded that many of the otherwise apparently paradoxical properties of this cylindrical wave can be understood by taking into account the analogy between gravitational waves and electromagnetic waves, and the special demands of the equivalence principle, which rules out a special role for any particular frame of reference." [20]

Thus, a basic problem is that Weber & Wheeler believed the "covariance principle", but mistook it as the equivalence principle. On the other hand "Zhou [8] correctly pointed out, "The concept that coordinates don't matter in the interpretation of Einstein's theory ... necessarily leads to mathematical results which can hardly have a physical interpretation and are therefore a mystification of the theory." Moreover, the fact is that the equivalence principle is applicable only to a physical space-time coordinate system, and this principle implies that a frame of reference must have a Euclidean-like structure [11, 12].

Instead, Weber and Wheeler [47] believed, "all correct general statements about gravitational radiation must obviously be compatible with this problem (of cylindrical waves)." Apparently, they used the word "obvious" in a way to substitute a necessary valid supporting reason for their opinions. They also did not supply any evidence for their claimed analogy between gravitational waves and electromagnetic waves. A condition for such an analogy is the Maxwell-Newton Approximation, but metric (A1) does not satisfy the linearized gauge harmonic conditions. Thus, their analysis is invalid.

Note that metric (A1) cannot satisfy coordinate relativistic causality [21]. Weber and Wheeler agreed with Fierz's analysis, based on (A2), that γ is a strictly positive where $\Psi(\rho, T) \approx 0$ for large ρ [47]. Thus, it is possible to see that there is no physical wave solution in a simpler way. Gravitational red shifts imply that $g_{tt} \leq 1$ [2]; and

$$-g_{\rho\rho} \geq g_{tt}, \qquad -g_{\varphi\varphi}/\rho^2 \geq g_{tt}, \qquad \text{and} \qquad -g_{zz} \geq g_{tt}, \tag{A4a}$$

are implied by coordinate relativistic causality [23]. Thus, according to these constraints, from metric (A1) one has

$$\exp(2\gamma) \leq 1 \qquad \text{and} \qquad \exp(2\gamma) \leq \exp(2\Psi). \tag{A4b}$$

Equation (A4) implies that $g_{tt} \leq 1$ and that $\gamma \leq 0$. However, this also means that the condition $\gamma > 0$ cannot be met [3]. This shows again that there is no physical wave solution for $G_{\mu\nu} = 0$. Thus, invalidity of the gravitational "wave" solutions, raised by Rosen [48] and Scheidigger [49], can be verified after the physical meaning of space-time coordinates is clarified [11, 12]. Moreover, space contractions would imply $\exp(2\Psi) = 1$, and thus $\Psi = 0$ and γ = constant (i.e., no wave).

The relation (A3) reflects that the cylindrical metric (A1), being function of ρ and T, cannot really have a radiation propagating in the ρ-direction, i.e., the metric being a function of (ρ - T). As seen from the following solution [47]

$$\Psi = AJ_0(\omega\rho)\cos\omega t + BN_0(\omega\rho)\sin\omega t, \tag{A5a}$$

where A and B are constants, such a "wave" represents essentially standing oscillations. (The Bessel function $N_0(\omega\rho)$ is not well defined at $\rho = 0$, but this can be fixed.) The second function γ in the special case B = A, reduces to the expression

$$\gamma = \frac{1}{2}A^2\omega\rho\{J_0(\omega\rho)J_0'(\omega\rho) + N_0(\omega\rho)N_0'(\omega\rho) + \omega\rho[(J_0(\omega\rho))^2 + (J_0'(\omega\rho))^2 + (N_0(\omega\rho))^2 + (N_0'(\omega\rho))^2]$$
$$+ [J_0(\omega\rho)J_0'(\omega\rho) - N_0(\omega\rho)N_0'(\omega\rho)]\cos 2\omega T + [J_0(\omega\rho)N_0'(\omega\rho) + N_0(\omega\rho)J_0'(\omega\rho)]\sin 2\omega T\}$$
$$- \frac{2}{\pi}A^2\omega T \tag{A5b}$$

(A5b) implies that $\exp(2\gamma)$ would become very small since γ would turn negative and large as time goes by, while $|2\Psi|$ is bounded. (Note that there are typo-errors in their paper [47], T and $\omega\rho$ should have been respectively t and $\omega\rho/c$.) This indicates a change of energy density. Nevertheless, Rosen finds that the pseudotensor that measures the density of gravitational energy and momentum in the cylindrical wave is everywhere zero. Thus, the solution of Rosen and Einstein actually supports that Einstein's pseudotensor for gravitational energy-momentum density is invalid [3].

Moreover, according (A5b) the function $\exp(2\gamma)$ gets very large as T goes to negatively large. Thus, 't Hooft's [22] claim of boundedness, against Weber & Wheeler [47], is a mistake. Although $\exp(2\gamma)$ approaches to zero as T gets positively large (thus $g_{tt} = g_{\rho\rho} \cong 0$), this means, however, that the condition for weak gravity ($|\gamma_{\mu\nu}| << 1$) would fail, independent of parameters such as the amplitude and the frequency. It follows that this metric violates the principle of causality, which requires that weak sources would give a weak gravity [3, 27]. Thus, there is another simple way to see that this metric is not valid in physics.

For the case of a propagating cylindrical plane-wave with $R_{\mu\nu} = 0$, one can consider a wave propagating in the z-direction. For instance, Au, Fang, & To [50] consider the metric as follows:

$$ds^2 = N^2(c^2dt^2 - dz^2) - L^2d\rho^2 - M^2\rho^2d\phi^2 \tag{A6}$$

where

$$N^2 = \rho^{-4}\exp(-4\int\Phi d\rho)\exp(2n_1), \qquad L^2 = \rho^{-8}(1 + \rho\Phi)^2\exp(-6\int\Phi d\rho),$$

and

$$M^2 = \exp(2\int\Phi d\rho) \qquad \text{where} \qquad n_1 = n_1(ct - z), \quad \text{and} \quad \Phi = \Phi(\rho)$$

are respectively arbitrary functions of (ct - z) and of ρ. The function $n_1(ct - z)$ makes N^2 a propagating wave.

If solution (A6) were a physical solution, M should be a bounded function of ρ, i.e.,

$$\exp(2\int\Phi d\rho) < C_1^2 \tag{A7}$$

for some constant C_1. However, this also means that N is not bounded for small ρ. Moreover, if light velocity is not larger than its vacuum velocity c, one should have N^2/L^2 and $N^2/M^2 \leq 1$. It thus follows that

$$(1 + \rho\Phi)^2 \geq \rho^4 \exp(2\int\Phi d\rho)\exp(2n_1), \quad \text{and} \quad \exp(6\int\Phi d\rho) \geq \exp(2n_1)\, \rho^{-4}. \tag{A8}$$

Hence,

$$(1/\rho + \Phi)^2 \geq \rho^{2/3} \exp(8n_1/3) \quad \text{and} \quad \Phi^2 > \sim O(\rho^{2/3}). \tag{A9}$$

Thus, condition (A9) is also inconsistent with (A7). In short, solution (A6) is unbounded in contrast to as required by the principle of causality. Clearly, Au et al. also do not understand Einstein's equivalence principle [51].

ENDNOTES

1) The time-tested assumption that phenomena can be explained in terms of identifiable causes is called the principle of causality [3, 11, 30]. This principle is the basis of relevance for all scientific investigations. This principle is commonly used in symmetry considerations. In general relativity, Einstein and subsequent theorists have used this principle implicitly on symmetry considerations [13, 14, 21] such as for a circle in a uniformly rotating disk and the metric for a spherical symmetric mass distribution [20]. Other consequences are, for instance, that parameters unrelated to any physical cause in a solution is not allowed and that a dynamic solution must be related to a dynamic source. However, this established principle is neglected in the consideration of dynamic solutions [42].

2) A solution may not be valid in physics unless all physical requirements are satisfied [3]. An example of an unphysical non-static solution is metric (14), though accepted by Penrose [31] as valid, since it violates the principle of causality. Einstein [21] proposed that for the gravity of a weak source, the deviation from the flat metric must be very small (i.e., $1 >> |\gamma_{\mu\nu}|$). Thus, the notion of unbounded (in amplitude) non-static solutions is against Einstein's proposal if $|\gamma_{\mu\nu}|$ cannot become small even the source is reduced to weak. Moreover, according to Einstein, a metric with an irreducible singularity is invalid, and thus the question of singularity is essentially unrelated to weak gravity. From Einstein's equation, it is possible to obtain mathematical "time-dependent" solutions, which are incompatible with the Maxwell-Newton Approximation. It turns out, however, such solutions are invalid in physics [3, 27, 30].

3) Some believe that Einstein's equivalence principle cannot derive anything beyond the gravitational redshifts. This is due to that they have mistaken [12, 43] Einstein's 1911 preliminary application of the equivalence between acceleration and Newtonian gravity as Einstein's equivalence principle proposed in 1921. Notably Misner et al. [9] claimed that gravitational energy-Momentum could not be located due to Einstein's equivalence principle. However, this is in disagreement with the fact

that the field energy would be exchange with the energy of a particle [9]. Thus, this is another way that Misner et al. [14] misinterpreted [16] Einstein's equivalence principle [40, 43].

4) One might argue that $G_{\mu\nu} = G^{(1)}{}_{\mu\nu} + G^{(2)}{}_{\mu\nu}$ is a sum of the first order terms and the second order terms. It has been shown that $G^{(2)}{}_{\mu\nu}$ can be considered as equivalent to the gravitational energy-stress tensor [13]. However, although $G^{(2)}{}_{\mu\nu}$ includes the effects of gravitational radiation, $G^{(2)}{}_{\mu\nu}$ is not the gravitational energy-stress tensor just as $G^{(1)}{}_{\mu\nu}$ is not the energy-stress tensor of matter although $\nabla^{\mu}T(m)_{\mu\nu} = 0$ would imply the exchange of energy between the source and the field. Moreover, it has been shown also that $G_{\mu\nu} = 0$ is not possible if a gravitational wave exists [3, 9].

5) Fock [28] is probably the earliest who found that, in harmonic coordinates, there are divergent logarithmic deviations from the radiation. The contradiction manifests that the assumption of boundedness is invalid.

6) Eq. (3) would lead to the vacuum equation (3'), which looks superficially similar to the background equation in [9],

$$G^{(B)}{}_{\mu\nu} \equiv R^{(B)}{}_{\mu\nu} - \frac{1}{2} g^{(B)}{}_{\mu\nu} R^{(B)} = K\, T^{(GW)}{}_{\mu\nu}\,. \qquad (35.16)$$

This is based on the approach devised by Isaacson [52]. However, the terms in (35.16) are defined different from those in (3'). It should be noted also that this equation is derived from the Einstein equation (1), whereas Eq. (3) is modified from (1) such that its physical deficiency would be removed. A basic problem of this approach to (35.16) remains that it is necessary to assume invalidly that bounded dynamic solutions exist for the 1915 Einstein equation.

7) For weak gravity, $Kt(g)_{\mu\nu}$ is of the fifth order in a post-Newtonian approximation, and the deceleration due to radiation is of the three and a half order in a post-Newtonian approximation [8] and the orbits of the binary pulsar can be calculated with the second-order post-Newtonian approximation by using equation (3) or "equivalently" the Einstein equation (1). It is interesting to note [3] that the analysis of Damour and Taylor [53, 54] is essentially valid except that they have mistaken the Maxwell-Newton Approximation, which approximates Eq. (3), as an approximation of the Einstein equation.

8) As Wald [55] pointed out, the linearized harmonic condition $\partial^{\mu}\bar{\gamma}_{\mu\nu} = 0$, is a problem since it lead to the linearized conservation law, $\partial^{\mu}T(m)_{\mu\nu} = 0$. The necessary condition for linearization is only that $\partial^{\mu}\bar{\gamma}_{\mu\nu}$ is of second order. However, for a plane wave, this is not a problem since a plane wave being an idealization, has no source.

9) Zhou [33] incorrectly believed, "Any problem of gravitation which has a solution in Newton's theory should also be solvable, but in a more profound way, according to Einstein."

10) The existence of a Euclidean-like structure in the frame of reference is a necessary condition for a physical space [24, 29]. For example, the Schwarzschild solution in quasi-Minkowskian coordinates [13; p.181] is the following:

$$ds^2 = (1 - 2M\kappa/\rho)c^2dt^2 - [(1 - 2M\kappa/\rho)^{-1} - 1]\, \rho^{-2}(xdx + ydy + zdz)^2 - (dx^2 + dy^2 + dz^2), \qquad (E1)$$

where

$$\rho^2 = x^2 + y^2 + z^2, \qquad x = \rho \sin\theta \cos\varphi, \qquad y = \rho \sin\theta \sin\varphi, \qquad \text{and} \qquad z = \rho \cos\theta. \qquad \text{(E2)}$$

Coordinate transformation (E2) tells that the space coordinates satisfy the Pythagorean theorem. The Euclidean-like structure represents this fact, but avoids confusion with the notion of a Euclidean subspace, determined by the metric. Clearly, metric (E1) and the Euclidean-like structure (E2) is complementary to each other in the Riemannian space. This structure is independent of the metric. These space-time coordinates forms not just a mathematical coordinate system since a light speed ($ds^2 = 0$) is defined in terms of dx/dt, dy/dt, and dz/dt [20, 21].

11) For a dynamic situation, the linearization is valid for the modified Einstein equation update (3) [3, 9]. Note that the existence of dynamic solutions for the 1915 Einstein equation had been claimed by two Princeton professors, Christodoulou and Klainerman **[56]**. However, a close examination of their proof shows that they actually made all their proofs on an empty set [57] because they failed to show that the set of their dynamic solutions is non-empty. Moreover, their set of weak dynamic solutions, if non-empty, would be in conflict with the Maxwell-Newton Approximation.

12) Although a solution of (2) gives too large a value for the perihelion of Mercury, it has found that the harmonic solution, which is compatible with the Maxwell-Newton Approximation, gives the same value [7] as the Schwarzschild solution.

13) A dynamic metric solution is related to the dynamics of its source matter, and a dynamic source would generate gravitational radiation [19]. For the perihelion of Mercury and the deflection of light, solutions of the test particles are calculated with a static metric. It was believed that the influence of a test particle to the metric could also be calculated with equation (1). However, as suspected by Gullstrand [4, 5] and conjectured by Hogarth [58], the opposite is correct.

14) However, Liu & Zhou [17] claim that plane-waves "should naturally contain Einstein's solutions as weak field approximations", and believe a plane wave should be bounded as required by physics and Einstein's notion of weak gravity. Their paper was part of Liu's thesis, but Zhou was 84 years old already. Therefore, Zhou probably provided only general guidance.

15) Formally, there is an equation whose solution would transform a given gauge to the harmonic gauge. However, this is actually conditional. It has been discovered [34] that if the source is an electromagnetic wave, such an equation may not have any solution, which is bounded in amplitude.

16) A good example of inadequate deliberations is Thorne's Theory of gravitational wave detection for LIGO (Laser Interferometer Gravitational Wave Observatory). The theory is derived on the assumption [14, 39] that LIGO can be considered as built in a free fall system. However, the radius of earth is 6.3×10^3 km, but the expected gravitational wave length is only about 15 km [14]. This means that the earth cannot be regarded as a test particle for gravitational waves. Thus, Thorne's theory, which is also incorrect, is just inapplicable to LIGO [59].

17) Pauli's [29] version of the principle of equivalence was mistakenly regarded as Einstein's principle, although Einstein strongly objected to this version as a misinterpretation [60]. For instance, Misner, Thorne, & Wheeler [14] falsely claimed that Einstein's equivalence principle is as follows:

"In any and every local Lorentz frame, anywhere and anytime in the universe, all the (Nongravitational) laws of physics must take on their familiar special-relativistic form. Equivalently, there is no way, by experiments confined to infinitestimally small regions of spacetime, to distinguish one local Lorentz frame in one region of spacetime frame any other local Lorentz frame in the same or any other region. "

However, this is only an alternative version of Pauli's because the Einstein-Minkowski condition, which requires that the local space in a free fall must have a local Lorentz frame, is missing. A misinterpretation is often not free. In their eq. (40.14) [14; p. 1107], they got a physically incorrect conclusion on the local time of the earth in the solar system because they did not understand Einstein's equivalence principle and related theorems in Riemannian space [40]. Moreover, Ohanian & Ruffini [61; p. 198] also ignored the Einstein-Minkowski condition and had the same problems as shown in their eq. (50). However, Liu [62], Straumann [63], Wald [55], and Weinberg [13] did not make the same mistake. Note that Ohanian, Ruffini, and Wheeler proclaimed themselves as non-believers of Einstein's principles [61].

18) Einstein concluded his talk on gravitational waves at Princeton University by saying [64] "If you ask me whether there are gravitational waves or not, I must answer that I do not know. But it is a highly interesting problem."

19) Some interpreted this "unbounded ness" as being of the domain of the field. This disagreement with the earlier interpretation of the unbounded ness as being of the amplitude [3], is clearly incorrect if one examines Eq. (A5b) carefully.

20) Here, Weber & Wheeler [47] have mistaken the "Covariance principle" as Einstein's equivalence principle. A consequence of Einstein's equivalence principle is the *Einstein-Minkowski condition* [21, 43] that the local space of a particle under the influence of only gravity must be locally Minkowskian [1, 13], but Misner et al. [14], Ohanian & Ruffini [61], and Will [65] failed to see this. A problem was that Einstein addressed only metrics without a crossing space-time element. This creates a false impression that the Einstein-Minkowski condition is trivial [43].

21) Validity of Coordinate Relativistic Causality is based on space contractions and the time dilation. These are clarified after the meaning of a physical space-time coordinate system has been identified [11, 12, 43].

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